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13. ABSTRACT *(Maximum 200 words)*

The objective of the bandwidth-packing problem is to decide which demands, or calls, on a list of requests should be chosen to route on a capacitated network. The objective is to maximize the revenue that is obtained by routing the chosen calls. If call splitting is allowed, then a multi-commodity flow formulation can be used. However, we are concerned with the situation where call splitting is not allowed, as with video data. Without call splitting, the problem requires that one know the paths upon which the calls are routed. There are many possible paths for each call. Oox et al [1991], Laguna and Clover [1993], Anderson, Jones, Parker and Ryan [1993], Parker and Ryan [1994], and Kang and Park [1996] have done research on this problem. A number of problems exist within the open literature as the common test bed. This problem is of interest to the US Navy as part of a larger project known as the Network Centric Warfare (NOW). NSW is concerned with applying advanced communications technology to achieve improved military effectiveness while simultaneously avoiding the expense of building large numbers of new weapon systems and platforms.Within this framework, the Navy has devoted considerable attention to nodal targeting--an offensive form of NOW. Here, the aim is to identify a select set of nodes critical to an enemy network and attack these nodes in the hopes of crippling the entire system. In short, it is the study of conducting precison engagement to reduce effectiveness of the enemy.

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Research Activity Report
Final Report

Karla L. Hoffman and Manfred W. Padberg

Subject: Grant Number F49620-00-1-0357

Subject: Research done by Manfred Padberg and Karla Hoffman

HONORS: Manfred Padberg received the prestigious Von Neumann Prize in Spring, 2000 for life's work in the field of combinatorial optimization.
Karla Hoffman was inducted as a Fellow of the Institute of Operations Research and the Management Sciences in 2002.

Bandwidth Packing Problem:

The objective of the bandwidth-packing problem is to decide which demands, or calls, on a list of requests should be chosen to route on a capacitated network. The objective is to maximize the revenue that is obtained by routing the chosen calls. If call splitting is allowed, then a multi-commodity flow formulation can be used. However, we are concerned with the situation where call splitting is not allowed, as with video data. Without call splitting, the problem requires that one know the paths upon which the calls are routed. There are many possible paths for each call. Cox et al [1991], Laguna and Glover [1993], Anderson, Jones, Parker and Ryan [1993], Parker and Ryan [1994], and Kang and Park [1996] have done research on this problem. A number of problems exist within the open literature as the common test bed.

This problem is of interest to the US Navy as part of a larger project known as the Network Centric Warfare (NCW). NSW is concerned with applying advanced communications technology to achieve improved military effectiveness while simultaneously avoiding the expense of building large numbers of new weapon systems and platforms.

Within this framework, the Navy has devoted considerable attention to nodal targeting -- an offensive form of NCW. Here, the aim is to identify a select set of nodes critical to an enemy network and attack these nodes in the hopes of crippling the entire system. In short, it is the study of conducting precision engagement to reduce the overall effectiveness of the enemy.

A second objective of the Navy is a defensive one in which one needs to determine how to route calls in wartime setting. In this situation, specific calls have significantly different value to the war effort and the network has limited capacity so that it cannot route all calls. This might occur when either part of the network is no longer functional or when one thinks that the enemy is capable of intercepting calls over certain nodes. In either case, the problem is to maximize the total value of the calls chosen to be routed on the "safe and functional" portion of the network. Finally, one might wish to solve the bandwidth-packing problem whenever one wishes to expand an existing network at least cost.

We began this research by attempting to solve all of the problems in the open literature. We have additional problems provided by IDA and will also use these problems in our testing. We designed a generate-branch-and-cut algorithm that builds on the work of both Parker and Ryan and of Kang and Park. We have strengthened the cuts known in the literature by looking at the entire set of constraints simultaneously. We have designed new branching strategies that branch on groups of variables simultaneously, and we incorporate a heuristic into the algorithm thereby quickly obtaining both good lower and upper bounds on the problem. With these techniques, we have been able to solve to *proven* optimality all problems in the open literature. We are now developing methods for handling the case where expansion of the network (with the associated fixed costs) is provided. The problems provided by IDA have this generalization.

A paper on this subject has been submitted to *SIAM Journal on Optimization.*

Fleet Assignment Problem:

The fleet assignment problem can be stated as follows: given a list of flights and an available aircraft fleet, assign a specific type of aircraft to each flight leg in the schedule such that the overall passengers carried by this assignment is as large as possible, the expenses associated with the assignment are minimized and maintenance, service and operational requirements are maintained. Operational constraints ensure (a) that federal regulations regarding the maximum number of hours a plane can fly before maintenance are satisfied, (b) that, at airports with curfews, aircraft depart and land at stations only during allowable times, (c) that noise restrictions at specific stations are maintained, (d) that ground-time standards between successive flights are maintained, that crews are available to fly the respective schedule, and (e) that certain flights connect to other flights (i.e. that the same aircraft fly two consecutive flight legs – these are known as forced "through turns"). To satisfy the maintenance constraints, most airlines require that each aircraft overnight at a maintenance station at least once in every three day period. Other operational constraints are satisfied by limiting the type of aircraft possibilities for specific flight legs and by ensuring that in the aggregate, aircraft types are assigned in proportion to crew availability.

The fleet-assignment problem has traditionally been formulated as a standard multi-commodity flow formulation with side-constraints that assure that the operational restrictions are satisfied. This time-staged network approach has been quite successful and is the standard model for most airlines. A proposed daily schedule with fleeting and associated lines-of-flying (LOFs) is provided to the rest of the company about 2 months before its effective start date. After feedback and negotiations, between operations and marketing, a final daily schedule is adopted. Other changes for days with different schedules, e.g. Saturday and Sunday must then be scheduled. Often, airlines piece the weekend to the daily schedule manually.

We have begun to study the problem from a column-generation approach similar to that used for the bandwidth-packing problem. By considering paths through the network (a possibility not allowed in the multi-commodity flow formulation) we can include the revenue that is obtained from linking flight-legs together to create "direct flights". Thus, the model will take into account the additional revenue obtained by designating two flight legs as part of a unique flight (with a single flight number assigned to both legs) as well as any penalties associated with short turn times that might be associated with connecting two flights. Similar to the bandwidth-packing problem, we will systematically generate paths as needed using the dual prices obtained from the solution when only a subset of the variables are given to the linear-programming solver. Thus, x_{ij} is a binary variable taking on the value one if that path is a path flown by equipment type j. The profit associated with such a variable includes not only the sum of the revenues and costs of each flight leg in that path, but also the additional profit associated with any direct-flights in the path and any penalties associated with short turn times. The first n rows of the constraint matrix designate each flight leg flown on a weekend and a variable x_{ij} has a value of one in that row if the path covers that flight leg and zero, otherwise. The other constraints of the problem assure that at least one-third of the paths for each fleet type overnight at a maintenance station, that crews in the aggregate have sufficient flying hours, and that only available planes are assigned.

We need to emphasize that obtaining solutions to the fleet assignment problem has important ramifications for many other scheduling problems of the airlines. Once the fleet type is determined for each flight, the crew scheduling, maintenance scheduling and rostering problems can all be decomposed into k problems, where k is the number of aircraft types. However, if the assignment is not done with proper concern for the later scheduling problems, then the other problems may either have no feasible solutions, or may have only very expensive feasible solutions. Thus, the issue is to determine if one can couple crew scheduling with that of fleet scheduling.

As one develops paths for aircraft, one can consider how the crew can "follow the plane", assure that the plane is available for routine maintenance at maintenance facilities, and include a variety of other scheduling issues not possible within the multi-commodity flow formulation.

David Johnson and I have begun the work of modeling the fleet scheduling problem using a column-generation approach. We have discovered a number of issues that have not been discussed in the literature: (1) Even using exact methods for column-generation (i.e. methods that *only* generate columns that can improve the current solution), the number of flight legs needed to be generated to prove optimality is exponential. (2) If cutting planes are used within a branch-and-cut code, the ability to use the exact column-generation methods is no longer doable! Thus, much research is needed to determine a mechanism to incorporate our current methodologies into a meta-heuristic capable of solving this very important problem

This research resulted in two talks at INFORMS meetings and a talk at Northwestern University.

3. Modeling Defense Systems Using IP (with Terry Friesz, and David Johnson): This paper describes that assists commanding officers in determining optimal maneuvers when frigates encounter over time the threat of anti-ship missiles. Currently, the Navy uses a system that determines the best set of headings for the priority one threat, and then considers other maneuvers secondarily. This paper experiments with using an optimization procedure that "sees" all of the threats simultaneously and determines the collection of moves to make over time in order to avoid all possible threats. The paper is in revision and we plan to send this to *Naval Logistics Quarterly.*

4. Real-time Planning and scheduling:
Working with a doctoral student, Martin Durbin, we have designed a scheduling and dispatching algorithm for goods shipped by truck that are perishable. Thus, the time to deliver is critical and once a good leaves the depot on a truck, that truck and its goods must complete the delivery before any additional goods can be routed on this truck.

The data available for this problem comes from a concrete company that is equipped with Global Positioning Software (GPS) and thus, we know precisely the time it takes to get to various locations at different times of the day. The procedure provides information to customer service representatives about whether to accept new orders for future days, determines the time at which each driver should arrive, provides an optimal schedule to begin the day and then provides heuristics to update the schedule whenever changes occur.

This scheduling and routing procedure has wide applicability for the military. The military has many routing and dispatching demands and quick re-scheduling when changes warrant is a difficult and important problem.

Karla Hoffman was invited to give a plenary talk at the National Science Foundation grantees meeting in January, 2001 in Tampa and has subsequently given talks at INFORMS and at Northwestern University on this topic.

5. Market clearing of goods and services: Federal and state governments often buy and sell goods and this research evaluates what auction designs are best suited to allow the government to achieve its goals of both public good and revenue maximization. This research began by looking at the design of the FCC auctions and has continued to examine the use of auctions for energy generation and distribution and for slot allocation at airports. Karla Hoffman gave a paper at the Wye River Conference sponsored by the FCC this fall and is helping to organize a conference on FAA slot allocation for Spring, 2002.

6. Cutting plane algorithms for mixed-integer programming:
The paper by Manfred Padberg "Classical cuts for Mixed-integer programming and Branch-and-Cut" appeared in *Mathematical Methods of Operations Research* **53** (2001). This paper examines the theory behind the classical Gomory Cuts and shows how these cuts require exact arithmetic to be valid. Manfred Padberg continues working along the lines outlined in my research proposal dated October 25, 1999. In this paper he reviews classical valid linear inequalities for mixed-integer programming, i.e., Gomory's fractional and mixed-integer cuts, and discuss their use in branch-and-cut. In particular, a generalization of the recent mixed-integer rounding (MIR) inequality and a sufficient condition for the global validity of classical cuts after branching has occurred are derived. Because of the current revived interest in this topic the paper sailed smoothly through the refereeing process and has been published in Mathematical Methods of Operations Research.

On the software development side, Manfred Padberg has developed a LU factorization routine that uses exact arithmetic (as described in Chapter 7.6.4 of my book Linear Optimization and Extensions, Springer Verlag, 1999). This work proceeds slowly, but he expect to have an exact linear programming code by winter, 2002.

6. Book publication:
Manfred Padberg published the textbook **Linear Optimization and Extensions: Problems and Solutions,** with D. Alevras in 2001. It is a Springer Verlag Publication,

7. Presentations of Manfred Padberg:

a) From May 7 through May 9, 2000, Manfred Padberg participated in the INFORMS meeting in Salt Lake City, Utah, where he received the 2000 John v. Neumann Theory Prize (joinly with E. Johnson of Georgia Tech).

b) From August 28 through August 31, 2000, Manfred Padberg participated in the International Colloquium on Graph Theory held on the Luminy campus of the Mediterranean University in Marseille (France).

c) From September 4 through September 7, 2000, Manfred Padberg participated in the Workshop on Combinatorial Optimization held in the Arithmeum of Bonn University in Bonn (Germany).

d) From September 17 through September 21, 2000, Manfred Padberg participated in the AIRO Annual Meeting of the Italian Operations Research Society in Milan (Italy). I gave the opening plenary speech on the "Linear Optimization over Rational Polyhedra".

e) November 26 through November 30, 2000, Manfred Padberg spent at the Konrad-Zuse-Zentrum fur Information-stechnologie Berlin. I gave a 75 minutes talk on "Cutting Plane Algorithms for Mixed-Integer programming" in the Graduierten Kolleg (for doctoral students) run by ZIB and several Berlin universities in Berlin (Germany).

8. Manfred Padberg Publications:

Padberg, M "Approximating separable nonlinear functions via mixed zero-one programs" appeared in *Operations Research Letters* **27** (2000) 1-5.

Padberg, M. "Almost perfect graphs and matrices" appeared in *Mathematics of Operations Research* (2001).

Padberg, M. "Packing small boxes into a big box" *Mathematical Methods of Operations Research* **51** (2001).

Padberg, M. "Classical cuts for mixed-integer programming and Branch-and-Cut" *Mathematical Methods of Operations Research* 53(2001)

9. Karla Hoffman Publications:

Hoffman, K. "Combinatorial Optimization: Current Successes and Directions for the Future" in the *Journal of Applied Mathematics* .

Hoffman, K. and Padberg M. "Integer and Combinatorial Programming" *Encyclopedia of Operations Research*, Second Edition (2000)

Hoffman, K. and Padberg, M. "The Traveling Salesman Problem" *Encyclopedia of Operations Research* Second Edition (2000).

Hoffman, K. and Padberg, M. "Set-covering, packing and partitioning problems" *Encyclopedia of Optimization (2000)* .

"Airline Scheduling" with Manfred Padberg, Russell Rushmeier, and Spyros Kantogeorgelis has been submitted for publication in *Operations Research.*