**Title and Subtitle:** Design Methods for Machinery Emulators

**Authors:** Dupont, Pierre, E.

**Abstract:**
The realization of electromechanical dynamic systems possessing specified input-output dynamic properties is studied. Applications of this problem include the scaled shock and vibration testing of machinery and the design of electromechanical filters. A two-step realization process was developed by which both passive and active systems can be realized. Two techniques have been developed to obtain realizable models for the design of passive mechanical systems. The first technique involves searching the parameterized space of congruent coordinate transformations relating input-output equivalent second order models for those that are realizable, i.e., those that can be directly interpreted as a network of mechanical elements. The second technique involves estimating realizable models which include both distributed and lumped mechanical elements directly from experimental machinery data. Active emulation is achieved by adding vibration sources, e.g., shakers, to the passive structure. These sources are driven under closed-loop control so as to produce the desired level of vibration at the output locations. Experimental evaluation of these techniques has been carried out through the design of a modular machinery emulator, which can be adapted to match the mass and dynamic properties of a desired machine within a frequency range of interest.

**Subject Terms:** vibration, acoustics, scale model testing, machinery emulators, equipment emulators
## Final Report

<table>
<thead>
<tr>
<th>Principal Investigator:</th>
<th>Pierre E. Dupont</th>
</tr>
</thead>
<tbody>
<tr>
<td>Organization:</td>
<td>Boston University</td>
</tr>
</tbody>
</table>
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                         | Boston, MA 02215          |
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Contents

Research Objectives ............................................................................................................. 3

Technical Approach .............................................................................................................. 3

Accomplishments .................................................................................................................. 4
  Mechanical Realization ........................................................................................................ 5
  Estimation of Canonical Models ........................................................................................ 7
  Active Emulation ................................................................................................................ 10
  Experimental Test Bed ....................................................................................................... 12

Relevance to the Navy ......................................................................................................... 15

References ............................................................................................................................ 15

List of Publications and Presentations .................................................................................. 16
  Theses ................................................................................................................................. 16
  Invited Presentations .......................................................................................................... 16
  Contributed Presentations ................................................................................................. 16
Research Objectives

The concept of a machinery emulator is illustrated in Figure 1. Rather than attempt to build an exact scale model of a machine, the proposed approach to emulator design utilizes the fact that there are an infinite number of mechanical systems that can approximate the energy flow between a machine and its foundation. The problem of emulator design is to characterize this set of approximating systems and to identify canonical members of the set that can represent a variety of machinery and are easy to fabricate. Passive elements are used to reproduce the power-off behavior of the machine while shakers are used to reproduce the active behavior of the machine.

![Diagram of Machinery and Emulator](image)

Figure 1 – Representing machinery by mechanical emulators with active elements.

The design procedure consists of two major parts. In the first, a mathematical model is developed for the machinery when it is turned off. The inputs and outputs of this model are vectors of forces and accelerations, respectively, at the foundation attachment points. While Figure 1 depicts only vertical forces and velocities, three orthogonal directions would typically be considered at each attachment location.

From this mathematical model, a corresponding mechanical system must be realized. While an infinite number of realizations exist in theory, the goal is to obtain a canonical class of mechanical systems that are easy to build and whose parameters are straightforward to compute. These systems are composed of lumped passive elements such as masses, springs and dashpots.

The final part of the design procedure is to modify this mechanical model to include one or more shakers to reproduce the effect of internal machinery vibrations at the base. In the modification process, the masses of the canonical model are considered in turn as mounting locations for shakers. Their suitability is judged analytically by comparing experimental observations of base velocity during machinery operation with those predicted by the modified emulator model. Some design discretion can be exercised here since multiple solutions for placement of the shakers may exist.

Technical Approach

The steps involved in part 1, passive emulator design, are shown in Figure 2. Data obtained from swept sine experiments is used to obtain a first-order, state space model of the form
where \( x \in \mathbb{R}^n \) is a vector of \( n \) states, \( u \in \mathbb{R}^m \) is a vector of \( m \) attachment points forces, \( m \leq n \), and \( y \in \mathbb{R}^m \) is a vector of \( m \) attachment points velocities or accelerations. Many methods are available for first-order model estimation.

Passive mechanical systems, however, are readily written, in terms of their mass, damping and stiffness matrices, \( M, C \) and \( K \). For collocated force inputs and acceleration outputs at a machine's attachment points, the second order equations are

\[
M\ddot{x} + C\dot{x} + Kx = Fu \\
y = H\ddot{x} \\
H = F^T
\]

Conversion from first- to second-order form is straightforward and is given by

\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \\
B = \begin{bmatrix} 0 \\ M^{-1}F \end{bmatrix}, \\
C = \begin{bmatrix} 0 & F^T \end{bmatrix}, \\
D = 0
\]

The inverse transformation is, however, not straightforward and when the model is put in second order form, the resulting \( M, C \) and \( K \) matrices typically cannot be interpreted as a mechanical system. The problem of mechanical realization involves applying coordinate transformations that produce mass, damping and stiffness matrices that can be readily interpreted as an interconnection of individual masses, dampers and springs. While an infinite number of transformation and thus realizations exist in theory, the goal here is to obtain a canonical class of mechanical systems that are easy to build and whose parameters are straightforward to compute.

**Accomplishments**

The two-step realization process was developed by which both passive and active systems can be realized. Two techniques were developed to obtain realizable models for the design of passive mechanical systems. The first technique involves searching the parameterized space of congruent coordinate transformations relating input-output equivalent second order models for those that are
realizable, i.e., those that can be directly interpreted as a network of mechanical elements. The second technique involves estimating realizable models which include both distributed and lumped mechanical elements directly from experimental machinery data.

Active emulation has been achieved by adding vibration sources, e.g., shakers, to the passive structure. These sources are driven under closed-loop control so as to produce the desired level of vibration at the output locations. Experimental evaluation of these techniques has been carried out through the design of a modular, single-input, single-output (SISO) machinery emulator, which can be adapted to match the mass and dynamic properties of a desired machine within a frequency range of interest. These topics are described below. Note that results relating to the realization of multi-input, multi-output (MIMO) have also been developed and appear in the thesis of Wenyuan Chen, cited in the last section of this report.

Mechanical Realization

The mechanical realization problem is to convert a general \( \{M,C,K\} \) model to a form that can be interpreted as a connected system of masses, dampers and springs. The SISO version of this problem has received coverage in the mathematical and mechanics literature. Most of this work is devoted to the realization of iso-spectral systems, i.e., systems which possess prescribed resonance and anti-resonance frequencies \([1],[2],[3],[4],[6]\) as serial or parallel connections of masses. This work does not consider preservation of the input and output as a driving point accelerance nor does it consider damping. For both these reasons, it is not applicable to emulator design. Well known within the naval community, the work of O'Hara and Cunniff demonstrates how any undamped or proportionally damped mass-spring system can be converted to a parallel model while preserving the driving-point relationship \([5]\).

This research addressed the general SISO realization problem which includes arbitrary damping and preservation of the driving point relationship. In the time domain, the mechanical realization problem is one of finding coordinate transformations that convert the system to realizable form.

The set of corresponding matrix transformations are congruence transformations, which involve pre- and post-multiplying by an invertible matrix and its transpose: \( A \rightarrow T^T AT \). Such a transformation preserves matrix symmetry as well as the input-output relationship as can be seen when it is applied to a second order system as shown below.

\[
\begin{align*}
M\ddot{x} + C\dot{x} + Kx &= f_u \\
\dot{y} &= h^T\dot{x} \\
y &= h^T x \\
h &= f
\end{align*}
\]

\[
\begin{align*}
T^T MT\ddot{y} + T^T CT\dot{y} + T^T KTq &= T^T f_u \\
y &= h^T T\ddot{y} \\
h^T T &= \left( T^T f \right)^T
\end{align*}
\]

To be in realizable form, the transformed mass, damping and stiffness matrices must satisfy certain realizability conditions, which are given in (5)-(7) below. Equation (5) indicates that the mass matrix must be diagonal with the positive masses on its diagonal. The stiffness and damping matrices must be positive semi-definite and with a single zero eigenvalue corresponding to a rigid
body mode eigenvector of \([1 \cdots 1]^T\), i.e., skyhook springs and dampers are prohibited. In addition, the diagonal elements must be positive and the offdiagonals negative or zero to ensure positive values of stiffness and damping.

\[
M = M^T > 0 \\
m_{ij} = 0, \ i \neq j
\]  
\[
K = K^T \geq 0, \ k_{ii} > 0, \ k_{ij} \leq 0, \ K = 0
\]
\[
C = C^T \geq 0, \ c_{ii} > 0, \ c_{ij} \leq 0, \ C = 0
\]  
\[
(5) \quad (6) \quad (7)
\]

Given these conditions, it can be shown that for any given system defined by \(\{M,C,K\}\), a congruence transformation \(T\) taking this system to the realizable form \(\{M^r,C^r,K^r\}\) can be written

\[
T = M^{-1/2}RM^r_{1/2}
\]  
\[
(8)
\]

The first component of this transformation, \(M^{-1/2}\), is used to mass normalize the initial model. The second component, \(R \in O(n)\), is an orthogonal matrix which has \(n(n-1)/2\) parameters, where \(n\) is the number of masses. Its role is to obtain the correct signs on the elements of \(C\) and \(K\) as given by (6) and (7). The last component is the realizable mass matrix \(M^r\), which is determined by the choice of \(R\) as shown in the equation below.

\[
(M^{-1/2}R)^T K (M^{-1/2}R) \begin{bmatrix}
\sqrt{m_1} \\
\vdots \\
\sqrt{m_n}
\end{bmatrix} = 0
\]
\[
(9)
\]

In this equation, the \(n\) masses of the realizable system are given by \(m_1,\ldots,m_n\).

To enforce the constraint that the input and output are applied and measured only at the base mass, the orthogonal matrix \(R\) can be parameterized as

\[
R = R_{\theta} R_{f0}
\]
\[
(10)
\]
in which \(R_{f0}\) aligns input/output vector \(f\) with \(e_1\) (base mass is \(m_{f1}\)) and \(R_{\theta}\) is the set of rotations preserving \(e_1\).

It can be shown that, in general, there are many realizable systems with the same drive point accelerance. Equations (8)-(10) describe the set of congruence transformations that satisfy...
necessary conditions for realizability. This set is parameterized by \( R_n \), which, for an \( n \) mass system, is of dimension \((n-1)(n-2)/2\). Thus, the set of realizing transformations is given by the subset of \( R_n \) that simultaneously satisfies the sign constraints on \( C \) and \( K \) given by (6) and (7).

**Estimation of Canonical Models**

To emulate a particular machine, we only need one solution to the realization problem and we would like to have an easy and standardized approach to identifying this solution and building it. To do this in a general way, a canonical class of emulators must be defined. This class must possess the following properties:

1. It covers (or approximates with given error) the model set.
2. Members of the class are easy to build and tune, i.e., they involve the fewest and simplest spring and damping connections between the masses.
3. Given an input-output or second order system description, an efficient numerical procedure is available to compute the associated class member.

![Figure 3. Canonical forms.](image)

Physically, the concept of canonical emulator forms is summarized in Figure 3. It is comprised of a base structure which may be as simple as a rigid plate. Alternately, it may consist of a scaled version of the frame of the actual machinery. The latter approach may be useful when the frame, as a continuous element, contributes many modes in the frequency range of interest. Attached subsystems can consist of oscillators in one or several layers and attached in parallel to the main structure. The number of connections between the oscillators would be minimal.

In general, canonical classes impose limitations on coupling of stiffness and damping matrices. In the vein of [5], we have attempted to obtain the simplest forms, i.e., minimum number of spring and damping connectors that can represent a system with general damping. Alternately, we have investigated simple forms that do not completely cover the model set, but provide good approximations of input-output behavior.

Two approaches have been considered for SISO systems. The first approach is depicted in Figure 4. The canonical form is specified through the structure of the damping and stiffness matrices. Its parameters are obtained by nonlinear numerical optimization using a cost function based on accelerance error and its inverse. This approach ensures good matching at both resonance and anti-
resonance frequencies. The dimension of the search space is determined by the number of connectors included in the canonical form.

The second approach we have investigated is an exhaustive search of the parameterized space defined by the necessary conditions on the transformations to realizable form (equations (5)-(10)). While an exhaustive search is impractical for complicated systems, this approach may provide a means to verify the minimum number of spring and damping connectors in the set of all realizations.

As an illustrative example, experimental data was collected from a laboratory model possessing modes in the 2-40 Hz range. Its accelerance is shown in Figure 5. The numerical search was employed to identify a realizable model. The canonical form embodied in this model allowed all possible connections between the masses, but the search was initialized at a parallel spring model. The resulting realizable model response is also shown in Figure 5 while the model itself is depicted in Figure 6.
This four-mass SISO example is simple enough to perform an exhaustive mapping of the three-dimensional parameter space defined by equations (5)-(10). The results of this mapping are presented in Figure 7 and Table 1. The shaded regions of parameter space in part (a) of the figure correspond to realizable models. The different regions can be shown to be contain identical models, with each region corresponding to a permutation of the four masses. Therefore, the solution set shown in part (b) of the figure comprises all distinct solutions.

The color coding illustrates the number of connectors (springs and dampers) in each realization. For a one-degree discretization of the angular parameters, the number of realizations with a particular number of connectors is given in the table. The minimum possible number for four masses is three springs while the maximum number is twelve for the case when each mass is attached to the others by both a spring and damper. The minimum number of connectors for which a realization was found is seven, which is the same number of connectors that resulted from the numerical estimation shown in Figure 6.
**Active Emulation**

Active emulation of SISO systems has been successfully demonstrated. The approach is based on a two-step design process, as shown in Figure 8, in which a passive emulator is first realized and then modified for active emulation. Two sets of experimental measurements are needed for the two steps of this process. For passive emulation, the attachment point accelerance, $M_f(\omega)$, is first measured with the machinery turned off:

$$M_f(\omega) = a_f / f_f_{\text{machine off}}$$  \hspace{1cm} (11)

For active emulation, the base acceleration $a_f$ is measured while the machine is running, but without forcing at the foundation. We can view this acceleration as the product of an unknown transfer accelerance $M_f$ and an unknown internal forcing $f_i$:

$$a_f_{\text{on}} = M_f f_i$$  \hspace{1cm} (12)

Assuming above-mount linearity of the machinery, the total base response is given by the sum

$$a_f = M_f f_f + M_f f_i$$  \hspace{1cm} (13)

The passive realization found using the measurements described by (11) is now modified to include a shaker that reproduces the effect of internal machine vibrations at the foundation as described by (12). In this approach, a known transfer accelerance based on the passive realization is substituted into (12) allowing one to solve for the appropriate internal forcing, which is applied with a shaker operating under closed-loop control.

**Table 1. Number of realizations with a given number of connectors.**

<table>
<thead>
<tr>
<th>Number of Connectors</th>
<th>Number of Realizations</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-6</td>
<td>0</td>
</tr>
<tr>
<td>7</td>
<td>224</td>
</tr>
<tr>
<td>8</td>
<td>3,579</td>
</tr>
<tr>
<td>9</td>
<td>1,382</td>
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<td>10</td>
<td>805</td>
</tr>
<tr>
<td>11</td>
<td>13,954</td>
</tr>
<tr>
<td>12</td>
<td>230,241</td>
</tr>
</tbody>
</table>

**Figure 8. Modification of a passive realization to achieve active emulation.**
Since the model of Figure 8(b), obtained when the machine is turned off, contains the entire mass of the machine, the mass to which the shaker is attached must be reduced so that total mass is preserved. In Figure 8(c), shaker mass \( m_3 \) is attached to a block of mass \( m'_2 = m_2 - m_3 \) so that the total mass remains \( m_2 \). As a result, the system of Figure 8(c) is equivalent to Figure 8(b) if a shaker force \( F_2(t) \) is added as shown in Figure 9. This shaker force is given by

\[
F_2(t) = -m_3 \ddot{\Delta}(t), \quad \ddot{\Delta}(t) = a_3(t) - \dot{a}_2(t)
\]  

(14)

The system's foundation acceleration \( a_i \) is given by the following equation.

\[
a_i(\omega) = M_{11}(j\omega)f_1(\omega) + M_{21}(j\omega)f_2(\omega)
\]

(15)

Here, the first term corresponds to base forcing of the passive realization and the second term arises by applying the shaker force through the transfer accelerance, \( M_{21} = v_i/F_2 \) \( \omega = \omega_0 \), which is obtained analytically from the passive equipment model of Figure 9.

\[
F_2, v_2 \quad \begin{array}{c}
\text{m}_2 \\
\hline
k \\
\hline
\text{c} \\

\end{array}
\quad F_1, v_1 \quad \begin{array}{c}
\text{m}_1
\end{array}
\]

Figure 9. Emulator Model with Shaker Force, \( F_2 \).

For the shaker placement of Figure 8(c) to successfully model internal machinery vibrations, the experimentally observed base velocity given by (13) must match that of the emulator model given by (15). This reduces to the following requirements on \( m_3 \) and \( \ddot{\Delta}(\omega) \):

\[
\frac{M_{21}(\omega)(-m_3 \ddot{\Delta}(\omega))}{\text{from passive model}} = \frac{M_{21}F_2(\omega)}{\text{shaker force on structure}} \quad \text{from experimental measurement}
\]

\[
0 < m_3 < m_2
\]

(16)  

The first equation imposes minimum bounds on shaker relative acceleration \( \ddot{\Delta}(\omega) \) as a function of frequency. It has a unique finite solution for all frequencies at which \( M_{21} \) is nonsingular. Equation (16) also indicates that to simulate the operating machinery, the shaker must be driven in closed loop with its acceleration given by the inverse Fourier transform \( \ddot{\Delta}(t) = \mathcal{F}^{-1}\{\ddot{\Delta}(\omega)\} \). For many types of machinery, it is anticipated that this signal will be dominated by several frequencies.
Experimental Test Bed

These concepts have been tested on the test bed shown in Figure 10. Figure 10(a) depicts the machinery model, which is composed of two rigid frames attached by flexible plates. In each frame, oscillators of different masses, frequencies and damping levels can be mounted. Machinery operation is simulated using two unbalanced electric motors, one of which is shown in Figure 11. During testing, the machinery model is suspended using flexible supports which are just visible at the top of Figure 10(a).

For emulation purposes, only planar motion is considered. Of these three degrees of freedom, dynamic matching is enforced in the horizontal direction at the labeled attachment point while rigid-body emulation is obtained in the vertical and rotational directions. The emulator test bed of Figure 10(b) achieves dynamic matching using a single structural mode in addition to the modes provided by its oscillators. Rigid-body matching is simultaneously achieved by matching the machinery total mass, moment of inertia and apparent mass at the attachment point. Details of the design procedure are provided in the Ph.D. thesis referenced at the end of this document. Figure 12 depicts the experimental results of the passive dynamic matching of horizontal accelerance at the attachment point. The two lowest modes are due to suspension of the structures and are not relevant to emulation. The higher modes are due to both the structural mode and the oscillators of the test beds.

Figure 10. (a) Machinery test bed. (b) Emulator test bed.

Figure 11. Unbalanced electric motor used to simulate internal energy generation of machinery.
As an example, two unbalanced motors were driven at frequencies within the range of the structural and oscillator modes, and the attachment point acceleration was measured. The two motors were then replaced by a single shaker as shown in Figure 10. Equation (16) calls for the transfer accelerance from the shaker to the attachment point. In this case, it was identified experimentally, however, in practice, it could be found either analytically or experimentally from the realizable emulator model.

For active emulation, the vibrations produced at the base due to the two unbalanced motors of Figure 10(a) are reproduced by the emulating shaker depicted in Figure 10(b). Given the machinery’s attachment point acceleration due to operation and the emulator’s transfer accelerance from the emulating shaker to the attachment point, (16) can be solved for the desired shaker force in the Fourier domain. Its inverse transform was used as the reference signal in the shaker control block diagram depicted in Figure 13. Note that this controller converts the desired force signal into a shaker voltage using the transfer function $V(s)/F_d(s)$, which is obtained experimentally using standard system identification techniques.
As an example of active emulation, Figure 14 compares the response of the machinery and emulator models under combined base and internal excitation. The unbalanced motors of the machinery model are driven at two different frequencies (37.69 and 43.63 Hz). As shown in the figure, an impulsive force is applied at the attachment point at about 1.75 sec. The total response due to base as well as internal excitation compares quite well between the machinery and emulator models. The mismatch that does occur after the impulse is caused by passive (rather than active) emulation error around 35 Hz. This can be seen in the frequency plot of Figure 14.

Figure 14. Example of active response of machinery and emulator to an impulsive base excitation.
Relevance to the Navy

Scaled testing of ship structures is used to assess the acoustic and shock isolation afforded by deck and mount designs. Analytical and experimental techniques for designing machinery emulators are of direct use to U.S. Navy engineers. This research has resulted in increased emulator design capabilities as summarized in Table 2 below. The results of this research have been presented to the engineering staff of the Naval Surface Warfare Center, Carderock Division, for use in future advanced structures programs.

Table 2. Machinery emulation capabilities resulting from this research.

<table>
<thead>
<tr>
<th>Capability</th>
<th>Prior</th>
<th>Current</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emulates machinery operation</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Standardized design procedures</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Design procedure includes damping</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Modular emulator component tool box</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of attachment points</td>
<td>1</td>
<td>Theory = n, Experiments = 1</td>
</tr>
<tr>
<td>Number of modes</td>
<td>4</td>
<td>Theory = n, Numerical = 20, Experiments = 5</td>
</tr>
<tr>
<td>Coordinate directions per attachment point</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

References

List of Publications and Presentations

Theses


Invited Presentations


Contributed Presentations
