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By

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for

Optimization-Based Techniques for Designing Optical Networks

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This document contains the summary of eleven research projects performed under this contract.
During the period 01-Feb-1996 through 31-Dec-2002, the PI (Kennington), Co-
PIs (Helgason and Olinick), two colleagues from the Navy Personnel Research and
Development Center (Krass and Sorensen), two colleagues from Nortel Networks
(Ortynski and Spiride) and nine graduate students (Mohammed, Mohammadi, Apte,
Jayasuriya, Whitler, Rahman, Oh, M. Lewis and K. Lewis) have been developing models
and special algorithms for a variety of operations research problems. Eleven of these
investigations are summarized in this report.

1. Linear Programming

In linear programming systems, an important component is the algorithm for obtaining a
new factorization for the inverse of the basis. The first stage in the algorithm is to
determine a permutation of the rows and columns of the basis, so that the sparsity
property of the basis will be maintained in the factorization of its inverse. This is
followed by a LU factorization (or Gaussian elimination) stage to obtain the factorization.
In the literature, the permutation of the basis is known as a pivot agenda, and there are
several algorithms for obtaining a good pivot agenda.

Most of the preassigned pivot agenda algorithms that extend the Hellerman-Rarick P3
algorithm assume that the input matrix is nonsingular. Due to the numerical instability,
this assumption may be violated and these algorithms fail. We present a modification of
the P3 algorithm which includes a procedure to recover from this type of numerical
instability. The recovery procedure is integrated into P3 in such a way that all previous
work can be maintained and it reduces the likelihood that additional recovery will be
required. This investigation has been reported in Kennington and Mohammed (1997).

1
2. Network Flows

2.1 Generalized Networks

The mathematical problem of finding a minimal cost flow through a capacitated network is a fundamental problem in both operations research and computer science. A generalization of this model which allows for either gains or losses as flows pass along an arc is known as the generalized network problem. Generalized networks have been used to model a wide array of applications in many engineering and economic areas. Applications that involve either gains or losses include electric power carried on transmission lines, cash management that involves the time value of money, and manufacturing processes of varying efficiencies. Other applications allow for flow conversion from one unit to another. We developed a specialization of the dual simplex algorithm for the generalized network problem. We use a dual two-phase method along with efficient dual partial pricing schemes and specialized routines for the dual ratio test. In comparison with CPLEX 3.0 on a set of ten benchmark problems, we found that our specialized dual code performed 20 times faster than the best CPLEX optimizer. Problems having 10 to 20 thousand arcs are routinely solved in under 10 seconds on a 60MHz DECStation 5000/260 with our rapid dual generalized network optimizer. This investigation has been reported in Kennington and Mohammed (1997).

2.2 Constrained Assignment Problem

The constrained assignment problem is to determine a least cost assignment of \( m \) people to \( n \) jobs such that an additional set of constraints is satisfied. We developed a truncated branch-and-bound algorithm to obtain a near optimal solution for this problem in which there are only a few side constraints. At each node of the branch-and-bound tree a lower
bound is obtained by solving a singly constrained assignment problem. If needed, Lagrangean relaxation theory is applied in an attempt to improve this lower bound. A specialized branching rule is developed which exploits the requirement that every man be assigned to some job. A software implementation of the algorithm has been tested on problems with five side constraints and up to 75,000 binary variables. Solutions guaranteed to be within 10% of an optimum were obtained for these 75,000 variable problems in from two to twenty minutes of CPU time on a Dec Alpha workstation. The behavior of the algorithm for various problem characteristics is also studied. This includes the tightness of the side constraints, the stopping criteria, and the effect when the problems are unbalanced having more jobs than men. This investigation has been documented in Kennington and Mohammadi (1997).

3. Navy Training

The Navy currently has approximately 400,000 sailors, each of whom is assigned to a job billet for a specific tour of duty lasting from 2 to 5 years. After the tour is completed, a sailor is assigned to a new billet for another tour of duty. Sailors assigned to sea billets are usually rotated to shore billets and vice versa. Each month, the Navy assigns thousands of new recruits and rotating sailors to vacant billets (jobs). The assignments are made by some 200 detailers within the Personnel Assignment Division of the Bureau of Navy Personnel. Enlisted personnel assignment is a complex process which often involves trade-offs among several conflicting policies.

Before assuming a new assignment, a sailor frequently is required to complete one or more Navy training classes. Experienced personnel attend advanced skills training courses while new recruits take a set of basic courses. The advanced courses are offered
by Navy C-Schools and the basic courses are offered by Navy A-Schools. Multiple offerings of over 2000 different courses must be taught each year with a course lasting from one week to over six months. The Navy’s annual budget for advances skill training exceeds $1.3 billion.

Based on the forecast demand for courses, the C-School and A-School managers develop a schedule of class offerings for use by the detailers. Until recently these schedules were prepared manually by personnel at each of the schools.

The problem of developing good schedules for Navy C-Schools has been modeled as a combinatorial optimization problem. The only complicating feature of the problem is that classes must be grouped together into sequences known as pipelines. An ideal schedule will have all classes in a pipeline scheduled in consecutive weeks. The objective is to eliminate the nonproductive time spent by sailors at C-Schools who are waiting for the next class in a pipeline. In this investigation an implicit enumeration procedure for this problem was developed. The key component of our algorithm is a specialized greedy algorithm which is used to obtain a good initial incumbent. Often this initial incumbent is either an optimal schedule or a near optimal schedule. In an empirical analysis with the only other competing software system, our greedy heuristic found equivalent or better solutions in substantially less computer time. This greedy heuristic was extended and modified for the A-School scheduling problem and was found to be superior to its only competitor. This investigation has been documented in Apte et al. (1998).
4. Cruise Missile Mission Planning

During the 1991 conflict in the Persian Gulf, the U.S. Strategy was to launch a cruise missile strike followed by manned aircraft strikes. The cruise missile strike would involve approximately 25 missiles which were launched from Navy ships and submarines stationed in relatively safe offshore positions. Communication systems and surface-to-air missile (SAM) sites were frequently targeted, which made any following manned strikes substantially safer.

A typical cruise missile strike consisting of 25 missiles requires developing a mission for each missile. A mission is defined as a path composed of line segments from a launch site to a target site. Using current technology, missions are constructed using a two-step process. In the first step, a mission planner uses a two-dimensional map to manually select a path (mission) from the launch site to the target. A software system takes this path, adds the vertical dimension, and estimates the probability of success. The probability of success is a function for the path chosen and is based on the length of the mission and the proximity of the path to known threats.

These weapons are designed to be low flying (terrain-following) which makes them extremely difficult to destroy en route to the target. The mission planners attempt to either avoid SAM sites or to fly so low that a SAM radar system cannot detect the incoming missile. However, cruise missiles are vulnerable to anti-aircraft artillery (AAA) if the ground crew is given sufficient warning. If the AAA ground crew has its guns aimed at an opportune spot, the missile can be destroyed by a round striking a key component. The best strategy to outwit AAA is to vary the missions of the cruise
missiles. If the first missile of the strike passes directly over an AAA site, then the second should fly a few hundred yards to either the left or right.

We developed a heuristic algorithm based on geometric concepts for the problem of finding a path composed of line segments from a given origin to a given destination in the presence of polygonal obstacles. The basic idea involves constructing circumscribing triangles around the obstacles to be avoided. Our heuristic algorithm considers paths composed primarily of line segments corresponding to partial edges of these circumscribing triangles, and uses a simple branch-and-bound procedure to find a relatively short path of this type. This work has been documented in Helgason et al. (2001).

5. Telecommunications Design

5.1 Link Restoration

We present a new model for the spare capacity allocation problem in a self-healing, mesh SONET telecommunications network. This model is known as the spare capacity network flow model and has not, it is believed, previously appeared in the literature. The spare capacity network flow model uses a node-arc formulation while others have used an arc-path approach. The node-arc formulation is much more compact, and its special structure can be exploited in devising solution techniques. An original algorithm, inspired by Benders’ decomposition procedure and called the network cut algorithm, is developed to solve the spare capacity network flow model. Application of decomposition principles results in an algorithm that iterates between a pure integer master problem and a set of minimum cost network flow subproblems. The algorithm incorporates a branch-and-bound procedure to solve the master problem, and a pure network dual simplex
optimizer to solve the subproblems. The network cut algorithm is much more efficient than directly solving either the continuous relaxation or the mixed integer version of the spare capacity network flow model. This work has been published in Kennington and Whitler (1999).

5.2 SONET Rings
Designing low cost SONET networks composed of self-healing rings is an important problem facing the providers of broadband services. This manuscript demonstrates how optimization technology coupled with the highly efficient CPLEX software system can be used to help solve this important problem. For the first time, provable optimal designs are presented for several small networks which appear in the literature. In an empirical analysis, we demonstrate how this same methodology can be used to find good (but not provably optimal) designs for large test cases. This work appears in Kennington et al. (1999).

5.3 MPLS Routing
This paper investigates distributed fault restoration techniques for multiprotocol label switching (MPLS) to automatically reroute label switched paths in the event of link or router failures while maintaining quality of services (QoS) requirements. Protocols for path and partial path restoration are evaluated. A backup route selection algorithm based on optimization of equivalent bandwidth is formulated and demonstrated for an example network. The complete manuscript may be found in Oh et al. (2000).
5.4 Path Restoration

This investigation presents a strategy to construct a compact mathematical model of the path-restoration version of the spare capacity allocation problem. The strategy uses a node-arc formulation and combines constraints whenever multiple working paths affected by an edge failure have identical origins or destinations. Another unique feature of this model is the inclusion of modularity restrictions corresponding to the discrete capacities of the equipment used in telecommunication networks.

The new model can be solved using a classical branch-and-bound algorithm with a linear programming relaxation. A preprocessing module is developed, which generates a set of cuts that strengthens this linear programming relaxation. The overhead associated with the cuts is offset by the improved bounds produced. A new branch-and-bound algorithm is developed that exploits the modularity restrictions. In an extensive empirical analysis, a software implementation of this algorithm was found to be substantially faster than CPLEX 6.5.3. For a test suite of 50 problems, each having 50 nodes and 200 demands from a uniform distribution with a small variance, our new software obtained solutions guaranteed to be within 4% of optimality in five minutes of CPU time on a DEC AlphaStation. This manuscript appears on Kennington and Lewis (2001).

5.5 Wavelength Assignment

All optical networks with wavelength division multiplexing (WDM) capabilities are prime candidates for future wide-area backbone networks. The simplified processing and management of these very high bandwidth networks make them very attractive. A procedure for designing low cost WDM networks is the subject of this investigation.
In the literature, this design problem has been referred to as the routing and wavelength assignment problem. Our proposed solution involves a three-step process that results in a low-cost design to satisfy a set of static point-to-point demands. Our strategy simultaneously addresses the problem of routing working traffic, determining backup paths for single node or single link failures, and assigning wavelengths to both working and restoration paths.

An integer linear program is presented that formally defines the routing and wavelength assignment problem (RWA) being solved along with a simple heuristic procedure. In an empirical analysis, the heuristic procedure successfully solved realistically sized test cases in under 30 seconds on a Compaq AlphaStation. CPLEX 6.6.0 using default settings required over 1,000 times longer to obtain only slightly better solutions than those obtained by our new heuristic procedure. This publication may be found at Kennington et al. (2003a).

5.6 Robust Optimization

The dense wavelength division multiplexing routing and provisioning problem with uncertain demands and a fixed budget is modeled as a multicriteria optimization problem. To obtain a robust design for this problem, the primary objective is to minimize a regret function that models the total amount of over and/or under provisioning in the network resulting from uncertainty in a demand forecast. Point-to-point demands are given by a set of scenarios each with a known probability, and regret is modeled as a quadratic function. The secondary objective is to minimize the equipment cost that achieves the optimal value for regret. We propose a two-phase robust optimization strategy that uses a pair of integer linear programs having a large number of continuous variables, but only
two integer variables for each link. In an empirical study, the two-phase robust optimization strategy is compared to alternative techniques using a mean value model, a worst-case model, and a two-stage stochastic integer program with recourse model. Both the worst-case model and the stochastic programming model exhibited a bias toward low-cost designs (well below the budget) at the expense of high expected over/under provisioning. For a tight budget, the mean-value model fails miserably yielding no design for comparison. The two-phase robust strategy produces the optimal design for a given budget that is the best compromise between expected regret and equipment cost. This work appears in Kennington et al. (2003b).
6. References


7. Distribution

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