We demonstrate that a microelectronic circuit, the Cooper-pair box, is a coherent, quantum two-level system whose parameters can be extracted through resonant spectroscopy. The width of the resonant features implies a worst case decoherence rate of the box which is still 150 times slower than the transition rate of the two-level system, even though it is inhomogenously broadened. Much slower than this decoherence rate is the rate of spontaneous decay of the excited state, which we measure by resolving in time the decay of the box into its ground state with a single electron transistor. We find a spontaneous decay rate which is $10^5$ times slower than the transition rate of the two-level system, even when the measurement is active. This long lifetime and the sensitivity of our measurement will permit a single-shot determination of the box's state.
Measurement of the excited-state lifetime and coherence time of a microelectronic circuit

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We demonstrate that a microelectronic circuit, the Cooper-pair box, is a coherent, quantum two-level system whose parameters can be extracted through resonant spectroscopy. The width of the resonant features implies a worst case decoherence rate of the box which is still 150 times slower than the transition rate of two-level system, even though it is inhomogeneously broadened. Much slower than this decoherence rate is the rate of spontaneous decay of the excited state, which we measure by resolving in time the decay of the box into its ground state with a single electron transistor. We find a spontaneous decay rate which is $10^3$ times slower than the transition rate of the two-level system, even when the measurement is active. This long lifetime and the sensitivity of our measurement will permit a single-shot determination of the box's state.

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Recently, microelectronic circuits have been coaxed into behaving as quantum two-level systems (TLS) \cite{1-5}. Although nature abounds with quantum two-level systems, such as the spins of nuclei in a magnetic field or the electronic states of dilute atomic gases, the TLS behavior of circuits is revolutionary because it demonstrates the quantum behavior of a macroscopic degree of freedom composed of many microscopic degrees of freedom. Quantum coherence was believed to be fragile in electrical circuits both because it required the complete suppression of the dynamics of the microscopic elements in a condensed matter system, and because the quantum oscillations of an electric or magnetic degree of freedom would efficiently radiate energy into the electromagnetic environment.

In this paper, we observe that under conditions of continuous measurement a microelectronic circuit, the Cooper-pair box, has the Hamiltonian of a two-level system. The parameters that appear in the Hamiltonian can be tuned experimentally with voltage and with magnetic field. We determine the Hamiltonian by a kind of spectroscopy, where we observe a resonant change in the box state when its transition frequency matches a multiple of the frequency of an applied oscillatory excitation. From the width of these resonances we can find a worst case estimate of the coherence time of the two-level system.

By placing the box into its excited state and watching it decay into its ground state we find the excited-state lifetime $T_1$ of the box. Based on the excited-state lifetime and the observed noise in the readout, we conclude that it is possible to perform a 'single-shot' measurement that observes the box in its excited state before it has relaxed into its ground state.

The Cooper-pair box is a microelectronic circuit composed of an isolated superconducting island, attached to a superconducting lead through a thin insulating layer across which Cooper-pairs can tunnel. An additional lead, called the gate lead, lies near the island and changes the electrostatic potential of the island with the application of a voltage $V_g$ to the gate lead through the gate capacitance $C_g$ [Fig. 1(a)]. The island's total capacitance $C_I$ is small enough that the addition of a single Cooper-pair to the island requires a large electrostatic energy, leading to suppressed fluctuations of charge on the island. Because the island is superconducting, all of the electrons form Cooper-pairs and participate in the macroscopic quantum ground state of the island. The only degree of freedom is the number of pairs $n$ on the island. Because of the large charging energy, we need only consider two states, a state $|0\rangle$ with no excess Cooper-pairs ($n = 0$), and a state $|1\rangle$ with one excess Cooper-pair ($n = 1$), as reckoned from electrical neutrality. The Hamiltonian of the Cooper-pair box circuit is

$$H = -2E_C(1 - 2n_g)\sigma_z - \frac{E_J}{2}\sigma_x$$

where $\sigma_z$ and $\sigma_x$ are the Pauli spin matrices and $n_g$ is the total polarization charge applied to the gate electrode, $n_g = C_gV_g/2e - n_{off}$, in units of a Cooper-pair's charge $\langle 8, 9 \rangle$. The offset charge $n_{off}$ accounts for the uncontrolled potential arising from charges nearby the box island. The charging energy, $E_C = e^2/2C_I$, is the electrostatic energy required to add one electron to the island and the Josephson energy, $E_J^{\text{Jass}} = h\Delta/8e^2R_J$, is the effective tunnelling matrix element for Cooper-pairs across a junction with resistance $R_J$ in a superconductor with BCS gap $\Delta$. The junction is in fact a composite of two parallel junctions connected to form a loop with 1 (\mu m)$^2$ area (Fig. 1). The effective Josephson energy $E_J$ of the pair of junctions is tuned by introducing magnetic flux $\Phi$ into this loop, as $E_J = E_J^{\text{Jass}} \cos(\Phi/\Phi_0)$, where $\Phi_0$ is the quantum of flux ($h/2e$). Equation 1 is the Hamiltonian of the Cooper-pair box circuit.
FIG. 1: (a) An SEM micrograph of the Cooper-pair box and SET electrometer. The device is made from an evaporated aluminum film (light gray regions) on an insulating SiO₂ substrate (dark gray regions) by the technique of double angle evaporation [6], which gives the double image. (b) A circuit diagram of the box and RF-SET electrometer showing: the voltage \( V_g \) and magnetic flux \( \Phi \) which control the box's Hamiltonian, the quantities \( \gamma \) and \( \omega \) which set the amplitude and frequency of the microwave excitation, the voltages \( V_{pe} \) and \( V_{ds} \) which determine the electrometer's operating point, and the capacitance \( C_C \) that couples charge between box and electrometer. \( V_{ds} \) includes both dc and \( \approx 500 \text{ MHz} \) oscillatory components [7]. The tunnel junctions (crosses in boxes) are characterized by a junction resistance \( R_j \) and capacitance \( C_j \).

In the box, states of definite numbers of Cooper pairs on the island are states of definite charge. In order to measure the charge of the Cooper-pair box, we fabricate the box next to a radio-frequency single-electron transistor (RF-SET) [6, 7], an exquisitely sensitive electrometer, so that the addition of a Cooper-pair to the box's island causes a small fraction \( \frac{C_C}{C_j} \) of the Cooper-pair's charge to appear as polarization charge on the capacitor \( C_C \) that couples charge between box and RF-SET (Fig. 1). The electrometer used here had a sensitivity of \( 4 \times 10^{-5} \text{ e/\sqrt{Hz}} \), 10 MHz of measurement bandwidth, and 3.7% of the charge on box was coupled into the electrometer. Because the RF-SET measures charge, its ac-

tion can be described as projecting the state of the box into a state of definite Cooper-pair number. In the formal terms of Eq. 1, it measures \( Q_{box} = (1 + \langle q_r \rangle) e \) where \( Q_{box} \) is further averaged over the measurement time. In the box, states of definite numbers of Cooper pairs on the island are states of definite charge.

We perform spectroscopy by applying a CW microwave stimulus to the gate of the Cooper-pair box, and sweeping \( n_g \) to tune the parameters of the TLS and find the resonance condition (Fig. 2). A measurement of \( Q_{box} \) vs. \( n_g \) shows that the box does not remain in its ground state over a range \( 0.3 < n_g < 0.7 \). This behavior is caused by backaction [10] generated by currents flowing through RF-SET [11]. We proceed by studying the box in the range of \( n_g \) where it does remain in its ground state.

When a 35 GHz microwave signal is applied to the gate, we observe clear evidence that the box is a coherent two-level system. Resonant peaks appear [Fig. 2(b)] in \( Q_{box} \) that are sharp and symmetrically spaced about \( n_g = 0.5 \).
r < 40 mK, they are in the limit caused by these measurements being made at a temperature generator which created the ramp voltage for \( V_g \).

Because the systematic deviation from linearity of the function \( 13.0 \pm 0.2 \text{ GHz} \). The uncertainties arise from the Hamiltonian, terms of the Hamiltonian, visible. Nevertheless, we are able to extract the parameters from the Hamiltonian in Eq. 1 which is \( n_q^0 \cos(\omega t)\sigma_z \), and is collinear with ground state of the quasi-spin described by Eq. 1 when \( E_J = 0 \). The microwave excitation therefore applies no torque which could excite the quasi-spin from its ground state [14].

The width of the resonant peaks we observe provides a worst-case estimate of the coherence time of the two-level system. As expected for a TLS, we find a broadening of the peak with increasing power of the microwave excitation. We express the width of a resonance \( \delta n_q \) as a width in frequency \( \delta \omega_{n_q} = (1/\hbar)(\partial E_q/\partial n_q)\delta n_q \). In the absence of inhomogeneous broadening, the half-width at half maximum inferred for zero power is the decoherence rate \( T_3 \) of a TLS [14]. From the width of a resonant peak that is just resolved at the lowest applied microwave power, we estimate an inhomogeneous ensemble coherence time \( T_2^* \) of about 325 ps [15]. We observe both, that the resonant peaks have a Gaussian shape, and that \( n_{qff} \) drifts an amount comparable to \( \delta n_q \) during the two minutes required to complete a measurement, due to the well-known 1/f noise of single-electron devices [16]. These observations imply that the width of the peaks expresses not the intrinsic loss of phase coherence due to coupling the TLS to the environment, but rather the degree to which an ensemble of measurements are not identical. We emphasize that this coherence time is a worst-case estimate because it is extracted while the system is measured continuously by the RF-SET and because it represents an ensemble average of many single measurements that require about two minutes to complete. Nevertheless, \( T_2^* \) is about 150 times longer than \( 1/\delta \omega_{n_q} \) [Fig. 2(c)].

In order to measure the excited-state lifetime \( T_1 \), we excite the box and then measure the time required to relax back to the ground state. A 38 GHz signal is continuously applied to the gate and the box gate is tuned to \( n_g = 0.248 \) and \( E_J = E_J^{\text{mas}} \) so that the microwaves resonantly couple the ground and excited state through a two-photon transition. Abruptly, \( n_g \) is then shifted to \( n_g = 0.171 \) in 30 ns, slowly enough to be adiabatic but much faster than \( T_1 \). The microwave excitation no longer resonantly couples the ground and excited state, and the probability of being in the excited state decays in a time \( T_1 \). By averaging many of the transient responses to this stimulus, we find \( T_1 = 1.3 \) \( \mu s \) (Fig. 4). The lifetime is a quantity which is insensitive to slow drifts in \( n_{qff} \) and demonstrates that the intrinsic quality factor [17] of the TLS, \( Q_1 = T_1/\omega_{n_q} = 6 \times 10^9 \).

We can compare this long lifetime, with the spontaneous emission rate induced by the quantum fluctuations of a generic electromagnetic environment. Calculating

\[
\text{Width of the resonant peaks: } \delta \omega_{n_q} = (1/\hbar)(\partial E_q/\partial n_q)\delta n_q.
\]

\[
\text{Inhomogeneous ensemble coherence time: } T_2^* \approx 325 \text{ ps}.
\]

\[
\text{Excited-state lifetime: } T_1 = 1.3 \mu s.
\]

\[
\text{Intrinsic quality factor: } Q_1 = T_1/\omega_{n_q} = 6 \times 10^9.
\]
FIG. 4: $Q_{\text{box}}$ vs. time $t$, (triangles), relative to $t = 0$, when $n_g$ is shifted from $0.248$ to $0.171$ in $30$ ns, with $38$ GHz microwaves applied. The shift in $n_g$ brings the box out of resonance with the microwave excitation. An exponential fit to the data implies $T_1 = 1.3 \mu$s (line).

The spontaneous emission rate using Fermi's golden rule gives

$$\frac{1}{T_1} = \left( \frac{C_{\text{tot}}}{C_0} \right)^2 \left( \frac{\epsilon}{\hbar} \right)^2 \sin^2(\theta) S_{\text{v}}(\omega_0)$$  \hfill (2)$$

where $S_{\text{v}}(\omega) = 2\hbar \omega (\text{Re}(Z_0))$ is the voltage spectral density (per Hz) of the quantum fluctuations of an environment with an impedance $Z_0$ at frequency $\omega$ and $\sin \theta = E_J/\hbar \omega_0$ [9]. The quantity $C_{\text{tot}}^2$ is the total capacitance of the box to nearby metal traces, including intentional coupling to the gate lead and other unintended capacitive coupling (Fig. 1). We calculate $T_1$ for a $50$ $\Omega$ environment to be between $0.25$ and $1$ $\mu$s, extracting $C_{\text{tot}}^2$ with a factor of two uncertainty from an electrostatic simulation of the chip layout [9, 11]. We do not claim to have demonstrated that the lifetime is limited by spontaneous emission; however, if there are additional relaxation processes, either due to the nearby electrometer or fluctuations of some microscopic degree of freedom in the box, their influence is at most comparable to that of spontaneous emission into a typical ($Z_0 \approx 50$ $\Omega$) electromagnetic environment.

In these experiments, we demonstrate that a Cooper-pair box is a coherent two-level system with a long excited-state lifetime. With spectroscopy, we determine the box's Hamiltonian and estimate the rate of spontaneous emission of the box into a typical environment. We measure an excited-state lifetime of box that is remarkable for two reasons. First, it shows that a quantum-coherent microelectronic circuit can have a $T_1$ that approaches the limit set by spontaneous emission of a photon into the electromagnetic environment. Second, it is achieved while the two-level system is continuously measured. This means that the coherence time in the Cooper-pair box can be long lived, if the sources of inhomogeneous decoherence can be reduced [5, 17]. Furthermore, given the observed electrometer sensitivity of $4 \times 10^{-5}$ $e/\sqrt{Hz}$, the excited-state lifetime is long enough that a single measurement can discriminate between the box in its excited state and the box in its ground state. Both of these are vital to implementing a quantum computer.

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[10] We have observed that the shape and size of the step near $n_g = 0.5$ depends sensitively on the operation point of the SET electrometer.  
[12] We determine $E_c$ by measuring the thermal broadening of the transitions between charge states when the box is driven into its normal state. This method is used in reference [8].  
[15] We refer to the inverse linewidth as $T_2^*$ because our measurements share some features of liquid-state NMR in a
spatially inhomogenous magnetic field. As in liquid state NMR, we believe $T_2$ is a worst case estimate of the intrinsic coherence of box.


[17] For quantum computation, the relevant quantity is the number of state manipulations which can be performed; expressing the quality factor relative to the maximum operation rate ($E_J/h$) gives a theoretical maximum $Q = T_1 E_J/h = 5 \times 10^4$. 
