EEG Analysis Based on Chaotic Evaluation of Variability

Maurice E. Cohen¹,², Donna L. Hudson²
¹California State University, Fresno, CA 93740, USA
²University of California, San Francisco, Fresno, CA 93703, USA
cohen@ucsfresno.edu

Abstract – Electroencephalogram (EEG) analysis remains problematic due to both lack of understanding of the origins of the signal and inadequate evaluation methods. In spite of these shortcomings, the EEG is a valuable tool in the evaluation of some neurological disorders as well as in the evaluation of overall cerebral activity. It becomes more useful when combined with other clinical parameters. The focus of the work described here is twofold. New chaotic methods are introduced for EEG evaluation coupled with a hybrid system approach that permits the combination of the EEG results with clinical parameters to form a comprehensive decision model. The system is illustrated in an application for diagnosis of dementia. Extensions can easily be made to applications such as evaluation of brain activity during surgery.

Keywords – Chaotic analysis, biomedical time series, EEG analysis, brain activity levels

I. INTRODUCTION

The exact mechanism of the generation of electroencephalogram (EEG) signals is not understood, due in part to the lack of appropriate theoretical models and appropriate measurements to adequately describe and dissect the EEG signals. Basic approaches to signal analysis have relied on Fourier analysis, cross-correlation, auto correlation, and other techniques to determine if the signal is stationary [1]. While these approaches have proved useful in many areas, analysis of many medical time series such as ECGs and EEGs are still problematic. Conventional EEG evaluation methodologies are useful but limited and potentially problematic form both theoretical and practical standpoints. EEG signals are considered as the results of the combined dynamic activity of neuronal populations. Models including excitatory and inhibitory circuits with feedback loops have been adopted to explain the oscillation property of EEG activity [2]. Clinical correlations of the dominant signal frequencies and visual detection of paroxysmal events such as spikes or sharp waves have been the mainstay of clinical neurophysiological interpretation of EEG recording. The traditional approach to EEG analysis, Fourier analysis provides a quantitative tool to examine signal frequencies and relative loads. It is almost certain that conventional Fourier analysis cannot represent the entire spectrum of biological activities. In addition, some of the assumptions such as the stationarity of the signal are not valid. Signal averaging and analysis based on short intervals ranging from one to four sections are inadequate. These problems may bias the analysis [3].

Clinical utility of the EEG is also limited by the frequent lack of specificity of the EEG abnormality. Generalized slowing during an EEG tracing unrelated to drowsiness can be an indication of generalized cerebral dysfunction due to metabolic derangement, neurodegenerative disorders, or infectious or inflammatory diseases. Conventional EEGs include 18 channels with only limited resolution for localization, imposing yet another limitation. More comprehensive linear and nonlinear analyses of the EEG signals described here not only have practical utility [4] but can also open new windows for studying the significance of the EEG signal in the understanding of the basic neurophysiological functioning of the human cerebral cortex. A nonlinear approach using continuous chaotic modeling that provides measurements of the level of variability of the EEG is described below. The method is illustrated in an application for diagnosis of dementia and can be extended to analysis of brain activity during surgery.

II. METHODOLOGY

A. Theoretical Basis for Chaotic Analysis of Time Series

The basic common thread in chaos theory is the recursive evaluation of seemingly simple functions that produce unexpectedly complex results. An iterative function does not suddenly become chaotic, but rather goes from the stage of convergence to a single value to a bifurcation, or convergence to two values. Additional bifurcations occur, and finally chaos results. As an example, consider logistic equation

\[ a_n = A a_{n-1} (1 - a_{n-1}) \quad 2 \leq A \leq 4 \quad (1) \]

where A is a constant whose value changes the behavior of the function. The recursion is dependent on the selection of \( a_0 \), which must be chosen between 0 and 1. For increasing values of A, the equation progresses from single value convergence to chaos. Within the chaotic area, regions of stability unexpectedly appear. For integer values of n, this function exhibits chaotic properties for A > 3.57. These properties include apparent lack of periodicity and sensitivity to initial conditions. The picture changes, however, if continuous values instead of integers are considered. The exact solution of (1) at A = 4 is:

\[ a_n = \frac{1}{2} \left[ 1 - T_{2^r} (1 - 2 a_0) \right] \quad (2) \]

where \( T_{2^r} (x) \) is the Chebyshev function [5]. The authors have derived a soft solution for (1) for all values of A [6]:

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**Performing Organization Name(s) and Address(es)**
California State University Fresno, CA 93740

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**Abstract**

**Subject Terms**

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Assume a solution of the type

$$a_n = \sum_{k=0}^{l} a_k T_k(2^n x)$$  \(3\)

where \(T_k(x)\) is the Chebyshev function of the first kind and \(n\) is a real number. We assume \(l\) to be the number of points in the interval \(0 \leq n \leq 1\). Thus

$$a_n^2 = \sum_{k=0}^{l} a_k^2 T_k^2(2^n x) + \sum_{j=0}^{l-1} a_i a_j T_i(2^n x) T_j(2^n x)$$  \(4\)

The conjecture adopted is that going from one point to another implies adding a Chebyshev polynomial. Hence

$$2l$$

$$a_{n+1} = \sum_{k=0}^{l} b_k T_k(2^n x)$$  \(5\)

where \(n\) is assumed to be a real number. By imposing appropriate boundary conditions one obtains a unique solution to these nonlinear equations involving 300 variables. Values for \(n > 1\) are obtained by applying the logistic equation to the points obtained for \(0 \leq n \leq 1\).

Plots of the continuous solution show no dramatic change in behavior at \(A = 3.57\), but rather a well-defined increase in variability, as illustrated by examining the second-order difference plots generated by the soft solution. Second order difference plots are generated by plotting \(a_{n+2} - a_{n+1}\) versus \(a_{n+1} - a_n\), where \(a_n\) is the value of the time series at time \(n\). Theoretical plots at \(A = 3.57\) and \(A = 4.0\) are shown in Fig. 1. Equivalent plots can be used for the evaluation of time series, as shown in Fig. 2 for electrocardiogram (ECG) evaluation of a normal patient and a patient with congestive heart failure. In this approach, rather than defining a time series as chaotic or not chaotic, it is evaluated in terms of degree of variability or chaos. To quantify the level of variability, the central tendency measure (CTM) is used, which is computed by selecting a circular region around the origin of radius \(r\), counting the number of points within the radius, and dividing by the total number of points \(t\). Then

$$n = \frac{\sum_{i=1}^{t-2} \delta(d_i)/t(2)}{t-2}$$  \(6\)

where \(\delta(d_i) = 1\) if \([(a_{n+2} - a_{n+1})^2 + (a_{n+1} - a_n)^2] < r\) and 0 otherwise.

The CTM measure has been shown to be effective in analysis of ECG data in several ways: as an independent measure, combined with other ECG measures in a neural network model, and combined with clinical parameters in a neural network model [7]. Preliminary studies have shown its feasibility for use in EEG analysis [8].

**B. Implementation for Rapid Evaluation**

Another problem encountered with EEG analysis is the large number of data points. For a 10-minute run for each channel, approximately 75,000 points are recorded, with a minimum of eighteen channels. In an application such as surgery, any useful evaluation must be capable of running in real time. While plotting a second-order difference plot is time-consuming, the CTM measure can be done directly and rapidly enough to produce real-time results.

**C. Preprocessing for EEG Signal Analysis**

EEG signals require preprocessing for removal of noise and identification of peaks. Two methods that are currently being tested are the use of a peak identification algorithm developed by the authors [9] and wavelet processing that allows the identification of peaks of varying amplitudes [10].

**D. Hybrid System for Data Analysis**

The hybrid system Hypermerge is used to combine multiple EEG results with clinical parameters [11]. Hypermerge has three components:

- Rule-based component (EMERGE)
- Data-based component (Hypernet)
- Chaotic Analysis of Time Series (CATS).
In this application, the rule-based component is used to include expert opinion, the neural network model is used to combine EEG summary results with clinical parameters and neuropsychological testing results, and the chaotic analysis is used for evaluation of the EEG. The knowledge-based component can include a wide range of information, ranging from impressions of mental status to human interpretation of imaging and EEG results.

III. RESULTS

A. Collection of CTM Variability Data

Preliminary EEG data has been collected to evaluate the feasibility of this approach. Data were collected at a rate of 250 samples/second with a periodic 2-second delay for storage requirements. Digital EEG runs lasted approximately 10 minutes and consisted of approximately 75,000 points. Each data point consists of a consecutive number and two channels of output. The output value for each channel is a positive or negative integer indicating the current amplitude. Two channels are selected from the 21 available for this preliminary analysis. The channels selected for recording were T3-T5 and T4-T6. T3 to T6 are based on standard EEG electrode placement. T3-T5 locates over the left temporal area with T4-T6 over the right.

B. Comparison of Lead Activity

Symmetry is always an important medical indicator of abnormal states. This is especially true in symmetric organs such as the brain and is often used in the evaluation of medical images. It can also be used in the evaluation of EEGs. Electrodes are placed to cover each of the different lobes of the brain, frontal, parietal, occipital, and temporal, and are symmetrically placed on the right and left sides, as described in the data collection above. Thus the summary measures for corresponding areas can be compared to each other to see if a similar level of activity is occurring.

Table I shows CTM measures based on two methods of analysis for leads placed on the left and right temporal lobes for 6 Alzheimer’s patients and 2 normal controls. Two methods of analysis were used:

- The second-order difference plot generated based on each point in the time series: This analysis is based on amplitude values indicating the level of electrical activity.
- Time between peak occurrence: Time between peaks was used as \( a_n \), the \( n \)th point in the series, to generate the second-order difference plot. This analysis is based on frequency values of the occurrence of the peaks as determined by the peak identification algorithm.

C. Hybrid Evaluation

Hypermerge, the hybrid system developed by the authors, is used to implement the decision strategy. The knowledge-based component and neural network are described briefly.

<table>
<thead>
<tr>
<th>Rule #</th>
<th>Premise</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>IF</td>
<td>EEG shows low variability in temporal lobe</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Score &lt; 11 on MMSE</td>
<td>0.4</td>
</tr>
<tr>
<td></td>
<td>Family hx of Alzheimer’s</td>
<td>0.2</td>
</tr>
<tr>
<td>THEN</td>
<td>Prescribe aricept</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Threshold 0.6</td>
<td></td>
</tr>
</tbody>
</table>

Knowledge-Based Component (Emerge)

The knowledge base is in the form of rules that are derived from expert input. An example of a rule used to supplement EEG analysis for the diagnosis of dementia is given below:

\[
\text{Am.: Amplitude; Fr.: Frequency}
\]

Table I

<table>
<thead>
<tr>
<th>ID #</th>
<th>Left Temporal (Am. (r=0.1) Fr. (r=0.05))</th>
<th>Right Temporal (Am. (r=0.1) Fr. (r=0.05))</th>
</tr>
</thead>
<tbody>
<tr>
<td>N_1</td>
<td>0.54 0.29</td>
<td>0.50 0.31</td>
</tr>
<tr>
<td>N_2</td>
<td>0.59 0.57</td>
<td>0.59 0.55</td>
</tr>
<tr>
<td>A_1</td>
<td>0.81 0.52</td>
<td>0.80 0.53</td>
</tr>
<tr>
<td>A_2</td>
<td>0.40 0.62</td>
<td>0.41 0.62</td>
</tr>
<tr>
<td>A_3</td>
<td>0.67 0.28</td>
<td>0.66 0.26</td>
</tr>
<tr>
<td>A_4</td>
<td>0.60 0.44</td>
<td>0.60 0.43</td>
</tr>
<tr>
<td>A_5</td>
<td>0.68 0.40</td>
<td>0.68 0.40</td>
</tr>
<tr>
<td>A_6</td>
<td>0.58 0.18</td>
<td>0.58 0.22</td>
</tr>
</tbody>
</table>

N_: Normal controls; A_: Alzheimer’s Patients

Neural Network Model (Hypernet)

The neural network model is used in two ways: to combine different measures of variability obtained from the chaotic analysis and to combine the chaotic measures with other clinical and neuropsychological parameters. Variables for input nodes along with their sources for dementia evaluation are given in Table II. If the application is adjusted for surgery, the first 12 parameters are still relevant. Nodes 13-20 may also be relevant. The clinical inputs can be replaced with any pertinent clinical components. A new decision model is then easily obtained using the Hypernet learning algorithm. Once the decision model is established, computation of patient-specific input values can easily be accomplished in real time.

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IV. DISCUSSION

The approach to continuous chaotic modeling described here has previously been shown to be useful in differentiation of categories of cardiac disorders using ECG analysis. In these applications, the combination of the chaotic analysis with clinical parameters through the use of a neural network decision model increased sensitivity, specificity, and accuracy of results. In preliminary results in diagnosis of dementia, the same approaches look promising for the analysis of EEGs. The same paradigm is followed in which the chaotic analysis is used as part of a more general neural network model. As a third step, these two modalities are included in a comprehensive hybrid system that also permits monitoring of patients.

V. CONCLUSION

Analysis using chaotic parameters presents a novel approach for the extraction of information from the complex mix of signals that make up the EEG. Early work using both frequency and amplitude analysis looks promising. Use of the chaotic parameters in a comprehensive decision model expands the potential for using the EEG as an important clinical parameter in the diagnosis of disease and in the monitoring of patients.

TABLE II

<table>
<thead>
<tr>
<th>Node #s</th>
<th>Contents</th>
<th>Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>N₁₁-N₁₂</td>
<td>CTM</td>
<td>Chaotic Analysis</td>
</tr>
<tr>
<td>N₁₃-N₁₄</td>
<td>CTM Difference</td>
<td>Chaotic Analysis</td>
</tr>
<tr>
<td>N₁₅-N₁₆</td>
<td>Activity levels</td>
<td>Functional Imaging</td>
</tr>
<tr>
<td>N₁₇</td>
<td>MMSE</td>
<td>Neuropsychological Test</td>
</tr>
<tr>
<td>N₁₈</td>
<td>Genetic Factor (y/n)</td>
<td>Genetic Testing</td>
</tr>
<tr>
<td>N₁₉</td>
<td>Family Hx (y/n)</td>
<td>Interview</td>
</tr>
<tr>
<td>N₂₀</td>
<td>Visible impairment (y/n)</td>
<td>Exam</td>
</tr>
</tbody>
</table>

Location Codes: T (temporal), P (parietal), O (occipital), F (frontal)
L (left), R (right)

Node Definitions:
N₁₁-N₁₃   TL, TR, PL, PR, OL, OR, FL, FR
N₁₄-N₁₆   T, P, O, F
N₁₇-N₁₉   TL, TR, PL, PR, OL, OR, FL, FR

All variables are continuous unless otherwise indicated.

ACKNOWLEDGMENT

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REFERENCES