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RETRIEVE TETHER SURVIVAL PROBABILITY

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Abstract

[1]The survival probability with time of the Rendezvous, Examine, and Tethered Return for Immolative EVaporation Experiment (RETRIEVE) tether due to cuts of the tether by meteorites and orbital debris, is calculated to be 99.934% for the planned experiment duration of six months or less. This is equivalent to odds of 1 out of 1500 of the tether being severed during the six-month duration of the experiment. Nearly all of this relatively low risk is due to the unlikely event of a strike by a large piece of orbital debris greater than 1 meter in size cutting all the lines of the tether at once. The probability of the tether surviving multiple cuts by meteoroid and orbital debris impactors smaller than 5 cm in diameter is 99.9993% at six months, so severing of the tether by that mode has odds of less than 1 in 150,000. The tether survival probability with time will remain above 99% until after 5 years of exposure to cuts. After three decades of cuts, it will have fallen to 90%.

RETRIEVE Experiment

The Rendezvous, Examine, and Tethered Return for Immolative EVaporation Experiment (RETRIEVE) was a candidate for a secondary payload experiment on the Air Force XSS-11 Mission, but was not selected. The experiment was designed to be initially dormant, and set into operation only after the XSS-11 microspacecraft had completed its primary mission of demonstrating near-autonomous rendezvous and examination of one or more resident space objects. The purpose of the 2.4 kg RETRIEVE apparatus was to demonstrate that an electrodynamic tether operated in the drag mode could autonomously deorbit the 120 kg XSS-11 vehicle from its nominal 51.6 degree inclination, 500 km altitude circular orbit into an Earth atmosphere burn-up orbit. To minimize the technological risks involved with the RETRIEVE experiment, we wanted the tether to have a high probability of surviving the duration of the orbit-lowering phase, which may last as long as six months.

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Tether Survival Probability With Time

In the following, we calculate the survival probability of the tether with time. Calculating the survival probability of an interconnected multiline tether is not simple since it involves calculating the survivability probability of the individual tether line segments between the interconnection points, and combining those properly to calculate the survivability of the entire tether [1]. This survival probability depends upon the structure of the tether, the stress loads on the tether, the diameter of the tether lines capable of carrying the stress loads, and the flux of meteoroid and orbital debris impactors at the operational altitude(s) of the tether.

Tether Structure

The initial design for the RETRIEVE tether consists of a 2-km long, two-primary-line, two-secondary-line Hoytape™, which has the two secondary lines soldered alternately at connection points or "solder joints" to the two primary lines (black) in the pattern shown in the structural schematic in Fig. 1. The structural schematic specifically shows a finite length to the solder joint portion, as a cut in that short segment, although unlikely, results in the cutting of two lines at once, and thus can affect the overall survival probability of the tether. In the design in Fig. 1, the secondary lines are not connected to each other where they cross. If desired, they could be soldered to each other where they cross, which will increase the survival probability significantly at the cost of a more time consuming fabrication process and additional solder mass at the joints. As we shall see, the survival probability is already good enough for the expected mission.

The four lines in the tether will be made of Dupont ARACON® conductive yarn. For this initial design, we have chosen an ARACON® yarn type that is readily available in production quantities despite it being slightly heavier than we might ultimately want. The yarn chosen is ARACON® type XN0200E, consisting of 89 fine strands of KEVLAR®, each about 0.015 mm (0.6 mils) diameter and massing 0.25 grams per kilometer, or 22 grams per kilometer for the 89 strands.

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Fig. 1 - Structural schematic of two-primary two-secondary soldered Bi-Line Hoytape™.

Each KEVLAR® strand is separately coated with electroless nickel using a chemical deposition process, which forms a tight bond between the surface of the KEVLAR® and the nickel. The nickel layer is then electroplated with many layers of copper, and finally finished with a flash of nickel, silver, gold or other metal wanted by the customer. We will probably use the standard nickel outer coating, unless some other metal is found to be better for best resistance to atomic oxygen. The finished yarn is very flexible despite its high metal content. It has a total lineal mass of 70 g/km, of which 22 g/km (31.4% by weight) is KEVLAR® with a specific density of 1.44, and 48 g/km (68.6% by weight) is copper with a specific density of 8.93. The resistance is 9180 ohms/km at 20 degrees C.

The yarn is normally delivered with a twist, but we have special-ordered untwisted lengths of yarn. The untwisted yarns "puff" out significantly under zero or low load because of the slight bending strength or "stiffness" of the metal-covered fibers. The measured "effective" diameters are 0.5 mm for a "taut" primary line under a very slight load and 1.3 mm for the normally slack secondary line. The primary and secondary lines in the Hoytether™ are then soldered together where they interconnect to form a soldered joint. Measurements on a number of solder joints produced an estimate of the two diameters of the elliptically-shaped solder joint of 0.048 mm by 0.023 mm, which is equivalent to a circular diameter of 0.038 mm and an average solder joint length of 17.5 mm between the points where the primary lines and secondary lines start to separate away from each other. The average excess solder mass over the mass of the two yarns in the solder joint is 50 mg of solder per joint.

The number of intervals between repeats of the interconnection pattern will be determined by a combination of: 1) desired minimum survival lifetime, 2) ease of manufacture, and 3) desired minimum mass of the tether, since each repeat of the interconnection pattern involves two more solder joints, which adds the solder mass of two joints to the fiber mass of the tether. If we assume that the 2 km long tether has m=1000 interconnection intervals, each interval being two meters in length, then there will be 2000 solder joints with a total solder mass of 100 grams. The total tether mass of this 2 km long tether using this available Dupont product is 660 g, of which 560 g is contributed by the four lines, each 2.0 km long with a lineal mass of 70 g/km, and 100 g is contributed by the solder. Decreasing the number of interconnection levels to 500 will decrease the total mass by 50 g to 620 g or 7.5%, but the length of the intervals between solder joints now becomes 4 meters or 12 feet, which makes it difficult to arrange the tether in the holding and tensioning jig before soldering the joints. Increasing the number of interconnection levels to 2000 will increase the total mass by 100 g to 770 g, or 15%. The length of the intervals between solder joints is now a reasonable one meter, but there are now 4000 joints to solder. As we shall see, the survival probability of the tether is very high in all cases, so the choice of the number of interconnection levels is determined more by ease of manufacture and tether mass considerations than tether survival concerns. As of now, we have decided to use m=1000.

We will, however, carry out the tether survival probability calculations using the number of interconnection intervals m as a variable. In prior studies, it was found that when the length of the joints was long, there were an "optimum" number of interconnection levels. Too few levels and it didn't take many cuts by space impactors before the tether failed due to low redundancy. Too many levels and the length of the joints became a significant fraction of the total length of the tether, and cuts at a joint cut two lines at once, increasing the cutting effect of each impactor. For the short junction length of the soldered joints in the RETRIEVE tether, we did not find such an optimum. The more levels in the tether, the higher the survival probability with time. More levels, however, adds to the number of soldered joints and hence to the total tether mass.

The "width" of the Bi-Line Hoytape™ tether can be selected pretty much independently of the other parameters. In fact, the "width" can be adjusted by merely using larger "spreaders" at the ends of the tether. Since, however, we are trying to make the tether deployer small, we will want the tether width to be small. For this analysis, we will assume that the two primary lines in the Bi-Line Hoytape™ will be 5 cm (2 inches) apart. If the interconnection interval is 200 cm (m=1000), then the angle of the secondary line to the primary line is arctan(5/200)=1.4 degrees, not the ~30 degrees shown in the schematic of Fig. 1. This means the secondary lines are almost in line with the primary lines - ready to take up the load if a primary

line is cut. The length of the secondary lines between connection points on the primary lines is deliberately made slightly longer than the 200.06 cm distance between the connection points, so the secondary line will remain slack under the normal load on the tether. By designing the secondary lines to be initially slack, the Hoytape™ does not "neck in" under normal load, increasing the survival probability. The secondary line segments will go taut and pick up the load when a primary line is cut.

Stress Load On Tether Lines

The maximum expected nominal gravity-gradient tension on a 2 km long tether with an 120 kg XSS-11 microsatellite at one end and a 2 kg tether deployer endmass at the other end would be roughly $2GMmL/(R+h)^3=10$ mN, where $G=6.67\times10^{-11}$ $m^3/kg\text{-}s^2$ is the Newtonian gravitational constant, $M=6\times10^{24}$ kg and R=6371 km are the mass and radius of the Earth, h=500 km is the altitude, m=2 kg is the deployer mass, and L=2 km is the length of the tether. This 10 mN of stress is so low that both the primary and secondary lines will stay "puffed" when in space. If significant tether dynamic oscillations occur and no attempt is made to control them, transient loads up to 5 N could occur. A single line of XN0200E ARACON® yarn has a measured breaking strength in excess of 30 N. Thus, even in a worst case scenario, where the methods for controlling tether dynamics fail, a single line of tether provides a stress safety factor of 6. Based on this, we will assume that as long as a single line of tether survives at each and every two-meter-long "interconnection interval" along the tether, the tether will not part.

Most Probable Tether Cut Scenarios

When a tether has a multiple number of well-separated interconnected lines, there are two scenarios that can cause the tether to part and fail. First, the tether can be cut by a very large piece of space debris, which is larger in size than the 5-cm width of the tether, so that all the tether lines are cut at the same time. The probability of this happening is low, since there are a small number of objects larger than 5 cm. The risk is finite, however, and will be calculated in a later section and included in the tether survival probability calculations. Since large objects are tracked, this risk could be lowered by controlling the rate of descent of the tether so that the orbit of any large tracked object that might approach the tether is out of phase with the orbit of the tether. We will find that this operational complexity is not needed for the short six-month duration of the RETRIEVE experiment.

The other probable tether cut scenario involves the cutting of a large number of the individual segments of the various lines in the tether by small meteoroid or orbital debris impactors. Since a single line in the RETRIEVE tether can support the expected load, for the RETRIEVE tether to be severed, all of the four tether lines that bridge a given two-meter-long "interconnection interval" must be cut. The probability of that happening is very low. As a result, the survival probability of the tether due to that failure mode remains above 99% for many years.

Meteoroid And Orbital Debris (M/OD) Flux

The cut rate for the individual tether line segments is determined by the flux of meteoroids and orbital debris impactors, the broadside area of the tether segment, and the susceptibility of the tether to being cut by a high speed impactor. NASA maintains an office that is constantly trying to improve our knowledge of the flux of the larger impactors. They can be visited on the web site: http://orbitaldebris.jsc.nasa.gov/. They have generated Technical Manuals [2-4] that discuss how to calculate both the meteoroid flux and the orbital debris flux at different altitudes and inclinations. The meteoroid flux comes in from outer space at very high speeds and is essentially constant with altitude and inclination near the Earth. The orbital debris flux varies considerably in near-Earth space, so NASA has produced a computer model ORDEM96 [3] to compute the orbital debris flux for different altitudes and inclinations. Unfortunately, this computer model does not include the meteoroid model since NASA was only interested in damage due to impactors greater than a few millimeters in diameter, where the orbital debris flux is orders of magnitude greater than the meteoroid flux. For space tether lines, however, the diameter of impactors capable of cutting the tether line segments would be much smaller. In this size range, the meteoroid flux is roughly equal to the orbital debris flux, so the meteoroid flux must be calculated separately and added to the orbital debris flux obtained from ORDEM96 [3]. This is done by reading the meteoroid flux graph in Fig. 7-2 of reference [2] for the flux of meteoroids larger than the diameter of the meteoroid that is capable of cutting the tether line. An alternate source (less readable) for the meteoroid flux graph can be found on the internet in reference [4].

Lethality Factor

The typical impactor velocity is so high (15-55 km/s for meteoroids and 5-12 km/s for orbital debris) that the impactor does not "cut" the tether. Upon first contact, the large amount of kinetic energy in the impactor is instantly converted into heat energy, which turns the impactor, and the portion of the tether the impactor has touched, into an exploding ball of plasma. Inspection

of aluminum plates returned from the Long Duration Experiment Facility (LDEF) found that all the pits were near-perfect hemispheres that had been "melted" into the aluminum plate as if by a ball of plasma exploding equally in all directions. There were no ellipsoidal holes, as would be expected if the interaction were modeled as a directed shock front from the impactor "plowing into" the solid metal surface at an angle. It was estimated, using a number of different clues, that the meteoroids or orbital debris paint flakes that made the pits were typically 1/3 the diameter of the hemispherical pit. This has led to the use of a "lethality factor" for strikes on a tether of 1/3. In other words, if a tether is 1 mm in diameter, an impactor with a diameter of 1/3 mm will create an exploding ball of plasma three times the diameter of the impactor that will sever or damage the tether line segment severely enough that the line segment will fail to carry its nominal stress. Other lethality factors from 1/2 to 1/5th have been used. This uncertain choice of lethality factor introduces a very large uncertainty into the "lethal" cut rate. The flux for impactors 1/5th the size of the tether can be 10-20 times larger than the flux for impactors 1/2 the size of the tether.

We do not know how an impactor will interact with a tether at these very high speeds. We cannot shoot particles fast enough to find out. We thus have to make a guess, with a lot of uncertainty in it, as to how big an impactor has to be in order to cut the tether. There is some indication that multi-fiber tethers react differently than the solid aluminum plates of LDEF and it may take a larger impactor to cut a tether line segment than make a pit in an LDEF plate. The Tether Physics & Survivability (TiPS) experiment [5] consists of a non-conducting tether 4 km long and 2.0 mm in diameter, massing 5.5 kg, connecting a 37.7 kg spacecraft to a 10.3 kg spacecraft. The experiment was launched into a 1022 km, 63.4 degree orbit on 20 June 1996 and it is still uncut after more than 5 years in space. The tether was a hollow braid of eight SPECTRA® lines, each line of which was made up of a number of smaller filaments. Inside the hollow braid was ordinary household yarn to keep the hollow braid "puffed" out. The load on the tether is very small, only 0.08 N, so the tether has probably stayed at its 2.0 mm design diameter. If we assumed a "lethality factor" for this tether of 1/3, then an impactor of 2.0/3=0.67 mm diameter should cut it. The flux of 0.067 cm orbital debris particles at 1022 m is estimated by ORDEM96 at 0.143 particles/yr-m^2, which is higher than normal because there is a band of heavy orbital debris flux predicted at 1400 km altitude. The flux of meteoroids is significantly less, 0.030 meteoroids/yr-m^2. The broadside area of the tether is A=DL=2 mm x 4 km=8 m^2. The predicted cut rate would thus be 1.4 cuts per year. The probability of the TiPS tether surviving 5 years is only 0.001. Either the ORDEM96 flux prediction for that altitude is wrong or the lethality factor is wrong. It could be that for "puffed" out tethers which cannot propagate (and fact may "damp") a plasma explosion, a lethality factor of 1/2 or even higher may be appropriate.

For the purposes of this analysis we will make the assumption that since the primary and secondary lines in the Hoytether™ naturally "puff" out under low or zero load and are not twisted into a compact mass that can easily propagate a plasma shock wave, that it will require an impactor with a diameter larger than 1/2 the diameter of the tether to either cut the tether line segment or damage it so badly that it can no longer carry a nominal load. For the soldered joint, with a density that is close to that of solid metal, we will assume the more conservative lethality factor of 1/3 found suitable for the LDEF metal plates.

Cutting Impactor Flux

If we assume a lethality factor of 1/2 or 1/3, then for a given tether line diameter D, we can estimate the cutting impactor diameter as d=D/2 or D/3. We then look up the flux of meteoroids with diameters greater than d on the meteoroid graph [2] and use ORDEM96 [3] to get an estimate of the flux of orbital debris objects with diameters greater than d.

Flux Adjustment For Fixed Radial Orientation: The flux obtained from the meteoroid graph [2] and ORDEM96 [3] is given as the total flux from all directions passing through a randomly tumbling aperture. If the aperture (for a tether, this is the broadside area A=DL) is not randomly tumbling, then the flux has to be adjusted to take into account the velocity of the aperture through the flux. The RETRIEVE tether will be gravity gradient stabilized and will always be aligned with the radial direction to Earth. Thus, the broadside area presented by the tether will always be moving into the impactor flux at right angles to its 7.5 km/s orbital velocity vector. This tether orbital velocity is equal to the velocity of the orbital debris flux in magnitude (although not in direction), and is comparable to, but smaller than, the average meteoroid flux velocity. Mike Matney of the JSC Orbital Debris group has recommended that for a radially oriented tether we should double the meteoroid flux to take into account this velocity "aberration" effect, but not the orbital debris flux, since ORDEM96 [3] already takes the aberration effect into account. The adjusted total impactor flux is the sum of these two components.

Tether Segment Cut Rate

To get the cut rate for a given segment of the tether, we multiply the adjusted total impactor flux by the broadside area of the tether segment, which is the length of the tether segment times the effective width w of the tether. The effective width consists of the physical diameter of the tether D plus the diameter of the impactor d, if the impactor is larger than the tether. For a lethality factor of 2, the minimum cutting impactor size is d=D/2. All of the minimum-sized impactor must hit the tether to produce a large enough plasma ball to completely cut the weakly loaded tether. The tether width in this case is w=D. The impactor flux, however, is the total flux of all objects D/2 or larger. Many of the impactors are larger in diameter than D/2 and can partially miss the tether and still produce a large enough plasma ball to cause the tether to be cut. The larger the object, however, the lower its contribution to the total flux. Mike Matney of the JSC Orbital Debris group has recommended the use of an "effective" width of the tether of w=2D. For this particular Hoytether™ design, the primary and secondary line segments have essentially the same length. The length of a line segment is the length L of the tether divided by the number of interconnection intervals m, which for the RETRIEVE soldered tether is m=1000, so a line segment length is L/m=2 km/1,000=2 m.

Cut Rate for Primary Line Segment: A slightly taut primary line has an effective diameter of 0.5 mm. If we assume a lethality factor of 1/2, the diameter of the cutting impactor is 0.25 mm or 0.025 cm. The meteoroid flux [2] is found to be 1 cut/yr-m^2. The orbital debris flux from ORDEM96 [3] for 51.6 degrees inclination and 500 km altitude in the year 2004 is found to be 0.7 cuts/yr-m^2. If we double the meteoroid flux to account for velocity aberration, we obtain a total impactor flux of F=2.7 cuts/yr-m^2.

The broadside area of a combined segment is:

$$A=wL/m=2DL/m=2 \times 0.5 \text{ mm} \times 2 \text{ km}/1000=2/1000 \text{ m}^2$$

so the cut rate for a primary line segment at F=2.7 cuts/yr-m^2 is:

$$C_P=FA=5.4/1000 \text{ cuts/yr}=5.4\text{x}10^{-3} \text{ cuts/yr}=1 \text{ cut in 185 years.}$$

Note that we are explicitly keeping in our equations the number of interconnection intervals in the tether, which is m=1000 interconnection intervals for a 2 km long tether where the primary lines are interconnected with the secondary lines every 2 m. This is because the survivability of the tether can be increased simply by increasing the number of interconnection levels, so we want the final equation for the survival probability of the tether to explicitly include the number m=1000 of interconnection intervals so we can later vary the parameter m if desired.

Cut Rate for Secondary Line Segment: A "puffed out" slack secondary line has an estimated effective diameter of 1.3 mm. If we assume a lethality factor of 1/2, the diameter of the cutting impactor is 0.65 mm or 6.5x10^{-2} cm. The meteoroid flux [2] is found to be 0.03 cuts/yr-m^2. The orbital debris flux from ORDEM96 [3] for 51.6 degrees inclination and 500 km altitude in 2004 is found to be 0.025 cuts/yr-m^2. If we double the meteoroid flux to account for velocity aberration, we obtain a total impactor flux of F=0.085 cuts/yr–m^2.

The broadside area of a secondary line segment is

$$A=wL/m=2DL/m=2 \times 1.3 \text{ mm} \times 2 \text{ km}/1000=5.2/1000 \text{ m}^2$$

so the cut rate for a segment of the secondary line at F=0.085 cuts/yr–m^2 is:

$$C_S=FA=0.44/1000 \text{ cuts/yr} = 4.4\text{x}10^{-4} \text{ cuts/yr} =1 \text{ cut in 2250 years.}$$

Cut Rate for a Soldered Joint Segment: A soldered joint segment has an estimated effective diameter of 0.38 mm and a length of 17.5 mm. If we assume a lethality factor of 1/3, the diameter of the cutting impactor is 0.38 mm/3 = 0.13 mm or 1.3x10^{-2} cm. The meteoroid flux [2] is found to be 3 cuts/yr-m^2. The orbital debris flux from ORDEM96 [3] for 51.6 degrees inclination and 500 km altitude in 2004 is 6 cuts/yr-m^2. If we double the meteoroid flux to account for velocity aberration, we obtain a total impactor flux of 12 cuts/yr-m^2. The length L and mean diameter D of a soldered joint are constants fixed by the soldering method and the length of the solder joint, and (to first order) does not depend the number of interconnection intervals m in the tether. With a fixed diameter D and length L, the broadside area of a soldered joint segment is:

$$A=wL=2DL=2 \times 0.38 \text{ mm} \times 17.5 \text{ mm}=1.33\text{x}10^{-5} \text{ m}^2$$

so the cut rate for a soldered joint segment at F=12 cuts/yr-m^2 is:

$$C_J =FA=1.6\text{x}10^{-4} \text{ cuts/yr}=1 \text{ cut in 6,300 years.}$$

Tether Segment Survival Probability With Time

Now that we have the average yearly cut rates C for the individual tether segments, we can use those to

calculate the survival probability with time of the segments. The average number of cuts N(t) in a tether segment with increasing time t in years, at a cut rate of C cuts per year, is simply N(t)=Ct. A tether segment survives if it has zero cuts, and fails if it has one or more cuts. The number of cuts with time of a given segment is obtained using Poisson statistics, in which the probability of exactly n cuts in a given time t, when the average number of cuts is N, is given by:

$$P_N(n,t) = \frac{N^n}{n!} e^{-N} = \frac{(Ct)^n}{n!} e^{-Ct}$$

The probability of a given tether line segment surviving, by experiencing n=0 cuts when the average number of cuts is N, is then given by:

$$S(t) = P(0,t) = \frac{N^0}{0!} e^{-N} = e^{-N} = e^{-Ct}$$

Alternatively, the probability of the given tether line segment failing is just unit probability minus the survival probability, or:

$$F(t) = (1 - S(t)) = (1 - e^{-Ct})$$

Typically, the survival probability for a given tether segment will be very high for short time intervals, while the failure probability will be very low. As time goes on, the average number of cuts N begins to rise, and this reverses.

Tether Cut Rate By Multiple Small Impactors

For the RETRIEVE tether to be severed, there must be two, three or four line segments cut at the SAME two-meter-long interconnection interval along the tether. The eight cut scenarios that will produce a severing of the tether are shown in Fig. 2.

Scenario 1 in Fig. 2 requires the cut of two joint segments at the same tether interval, which each have a cut rate of C_J. The probability of both the blue joint AND the red joint being cut and causing a failure of the tether is given by the PRODUCT of the failures of the individual joint segments:

$$F_1 = F_J F_J = (1 - e^{-C_J t})^2$$

Scenario 2 in Fig. 2 requires the failure of three tether segments at the same tether interval, one of a joint segment with a cut rate of C_J, one of a primary line segment with a cut rate of C_P, AND one of a secondary line segment with a cut rate of C_s. The probability of these three segments ALL being cut and causing a failure of the tether is given by the PRODUCT of the failures of the individual segments:

$$F_2 = F_J F_P F_S = (1 - e^{-C_J t})(1 - e^{-C_P t})(1 - e^{-C_S t})$$

Scenarios 3, 4 and 5 in Fig. 2 also require the same set of three segment failures as Scenario 2, so the failure rates for these scenarios are the same as that for Scenario 2.

$$F_5 = F_4 = F_3 = F_2$$

Scenarios 6, 7 and 8 in Fig. 2 each require the failure of four line segments at the same tether interval, two cuts of primary line segments each with a cut rate of C_P, AND two cuts of secondary line segments, each with a cut rate of C_S. The probability of ALL these four failures occurring and causing a failure of the tether is given by the PRODUCT of the failures of the individual line segments:

$$F_8 = F_7 = F_6 = F_P F_P F_S F_S = F_P^2 F_S^2 = (1 - e^{-C_P t})^2 (1 - e^{-C_S t})^2$$

The probability of failure F_I of the given tether interval I by its being cut by one OR the other of ANY of these eight tether cut scenarios is then the SUM of the eight failure modes:

$$\begin{aligned} F_I &= F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 = F_1 + 4F_2 + 3F_6 \\ &= (1 - e^{-C_J t})^2 + 4(1 - e^{-C_J t})(1 - e^{-C_P t})(1 - e^{-C_S t}) + 3(1 - e^{-C_P t})^2 (1 - e^{-C_S t})^2 \end{aligned}$$

Alternatively, the probability of the given tether interval surviving (and thereby the tether surviving) is given by unity probability minus all the failure modes:

$$S_I = 1 - F_I = 1 - [(1 - e^{-C_J t})^2 + 4(1 - e^{-C_J t})(1 - e^{-C_P t})(1 - e^{-C_S t}) + 3(1 - e^{-C_P t})^2 (1 - e^{-C_S t})^2]$$

This survival probability for a given tether interval is typically very close to unity, even for a tether that is about to fail due to many cuts. But, for the tether to survive, ALL of the m=1000 tether intervals must survive. So the probability that the tether will survive due to multiple cuts by small impactors is given by the PRODUCT of the survival probabilities of all the m=1000 tether intervals:

$$S_S = S_I^m = \{1 - [(1 - e^{-C_J t})^2 + 4(1 - e^{-C_J t})(1 - e^{-C_P t})(1 - e^{-C_S t}) + 3(1 - e^{-C_P t})^2(1 - e^{-C_S t})^2]\}^m$$

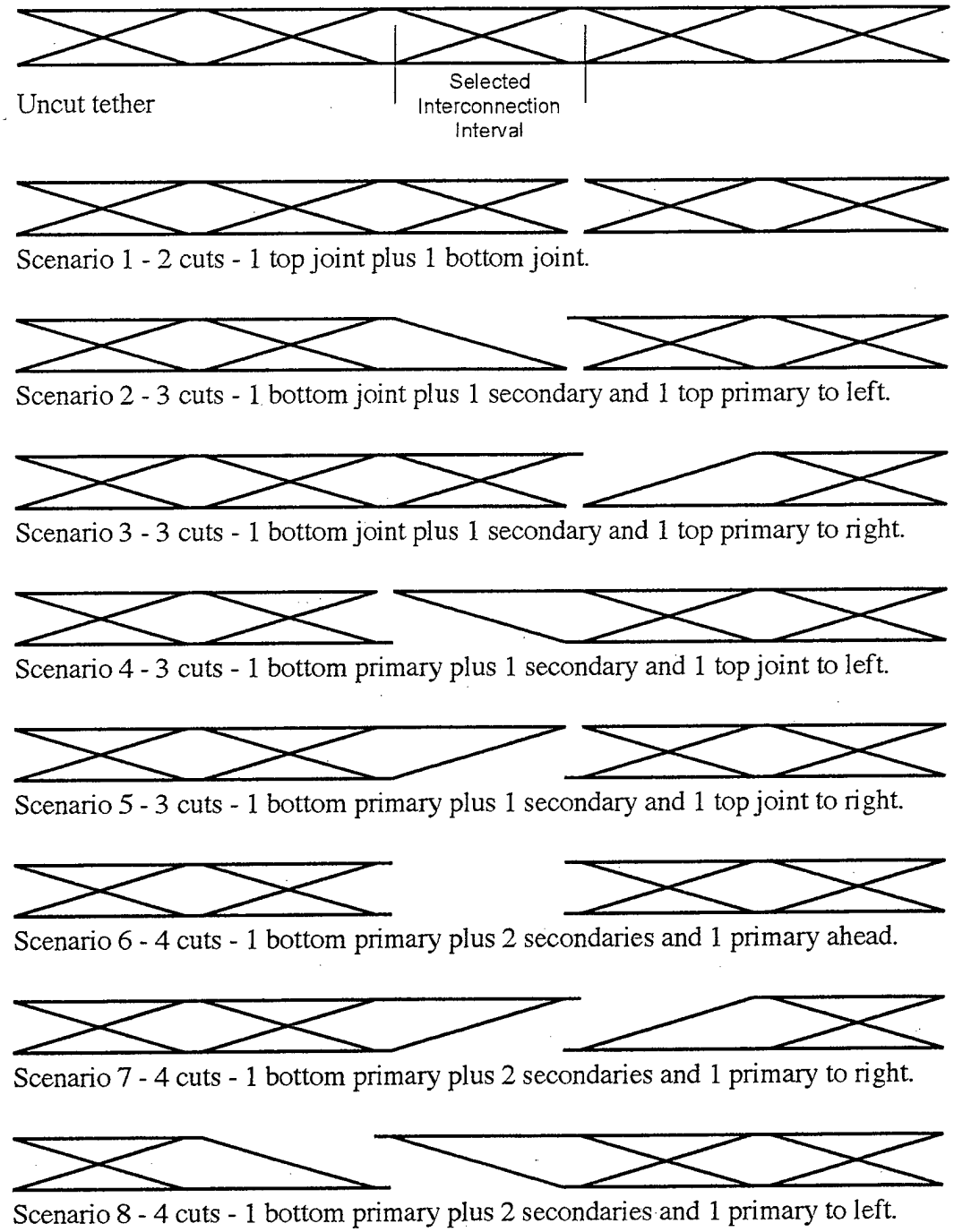

Fig. 2 - Tether segment cut scenarios that result in severing of the tether.

Tether Cut Rate By Large Impactors

The other way the tether can be cut is for a large piece of orbital debris to strike the Bi-Line Hoytape™ and cut all the lines at the same time. (The large meteoroid flux is so small compared to the large orbital debris flux that the meteoroid flux can now be neglected.) This cut rate is more difficult to calculate, since the effective broadside area A of the tether is NOT the area of the tether, but instead is the length L of the tether times the width W of the orbital debris minus the width w of the tether, A=L(W-w). We carried out such an analysis in a previous study [6] and we will go through an abbreviated version of the analysis here - which is summarized in Table I.

If we assume that the width of the tether is w=5 cm, then objects larger than 5 cm can cut all the lines in the tether. The total flux [3] of large orbital debris objects greater than 5 cm in size at 500 km altitude and 51.6 degrees inclination in 2004 is 9.84×10^{-7} objects/yr-m^2.

In Table I we have divided the total flux up into bands and assigned the appropriate portion of the total flux to each size band according to the flux [3] for that size of impactor. The flux in a given size band is then multiplied by the effective area of the tether, which in turn depends upon the size band. The cut rates of each size band are then added to get the total cut rate for large impactors C_L.

The total cut rate due to all debris over 5 cm in diameter is estimated to be about $C_L=1.3 \times 10^{-3}$ cuts/year or one cut in 770 years. The largest portion of this cut rate comes from a small number of large spacecraft with widths greater than 1 meter. The tether survival probability with time for cuts by large impactors $S_L(t)$ is then given using Poisson statistics as:

$$S_L(t) = e^{-C_L t}$$

Table I - Large debris cut rate of a 2 km long by 5 cm wide Hoytape™

Debris Size (cm)	Mean Size (cm)	Incremental Flux (#/yr-m^2)	Cut Width (W-w) (m)	Area=(W-w)L (m^2)	Cut Rate=FA (cuts/yr)
600-inf.	650	3×10^{-9}	6.45	12,900	4×10^{-5}
500-600	550	5×10^{-9}	5.45	10,900	6×10^{-5}
450-500	475	5×10^{-9}	4.70	9,400	5×10^{-5}
400-450	425	9×10^{-9}	4.20	8,400	8×10^{-5}
350-400	375	15×10^{-9}	3.70	7,400	11×10^{-5}
300-350	325	28×10^{-9}	3.20	6,400	18×10^{-5}
250-300	275	41×10^{-9}	2.70	5,400	22×10^{-5}
200-250	225	44×10^{-9}	2.20	4.400	19×10^{-5}
150-200	175	33×10^{-9}	1.70	3,400	11×10^{-5}
100-150	120	26×10^{-9}	1.15	2,300	6×10^{-5}
50-100	75	44×10^{-9}	0.70	1,400	6×10^{-5}
30- 50	40	51×10^{-9}	0.35	700	4×10^{-5}
15- 30	22.5	126×10^{-9}	0.175	350	4×10^{-5}
5- 15	10	554×10^{-9}	0.05	100	6×10^{-5}
	Total Flux	984×10^{-9}		Total Cut Rate	130×10^{-5}

Overall Tether Survival Probability With Time

The joint probability that the tether will survive as a function of time is then given by the product of the probabilities that the tether survives both large impactors AND small impactors, which is given by the PRODUCT of the two survival probabilities:

$$S_T = S_L S_S = e^{-C_L t}\{1-[(1-e^{-C_J t})^2 + 4(1-e^{-C_J t})(1-e^{-C_P t})(1-e^{-C_S t}) + 3(1-e^{-C_P t})^2(1-e^{-C_S t})^2]\}^m$$

If we put in the parameters for the RETRIEVE tether, reformat the equation so it can be used in a graphing calculator program, and multiply the time by 10 to get the survival probability with time graphed in terms of decades rather than years, we obtain the following, which is plotted in Fig. 3.

```
S_T(t)=exp(((-1)*10*t/770))*(1-(((1-exp(((-0.00016)*10*t)))^2
    + 4*(1-exp(((-0.00016)*10*t)))*(1-exp(((-5.4)*10*t/1000)))*(1-exp(((-0.44)*10*t/1000))))
    + 3*(1-exp(((-5.4)*10*t/1000)))^2*(1-exp(((-0.44)*10*t/1000)))^2))^1000
```

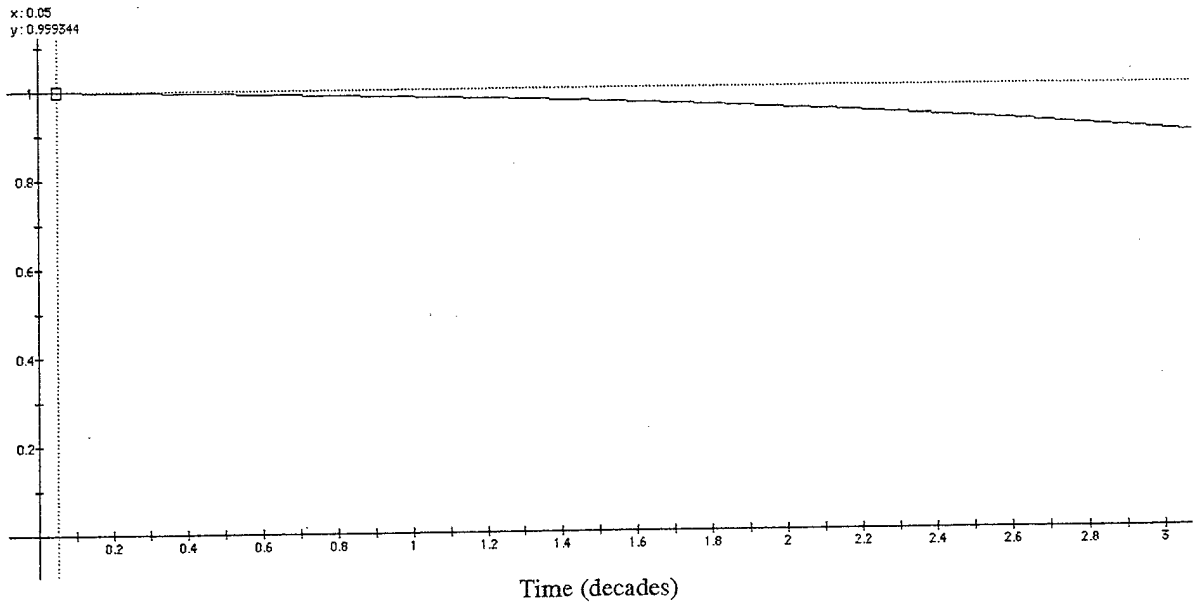

Fig. 3 - Survival probability of RETRIEVE tether under meteorite and orbital debris cuts.

From Fig. 3 we see that the probability that the tether will survive uncut for the estimated deorbit time of six months (0.05 decade) is 99.934%, which is equivalent to odds of 1 out of 1500 of the tether being severed during the six month duration of the experiment. Nearly all of the risk is due to strikes by large pieces of orbital debris greater than one meter in size cutting all the lines of the tether at once. The probability of the tether surviving multiple cuts by M/OD impactors smaller than 5 cm in diameter is 99.9993% at six months, so severing of the tether by that mode has odds of less than 1 in 150,000. As seen in Fig. 3, the tether survival probability will remain above 99% until about 6 years (0.6 decades) of exposure. After nearly three decades of cuts, it will have fallen to 90% and will begin to drop rapidly with each decade following.

Total Line Segment Cuts In Six Month Interval

Using the cut rates estimated previously, we can expect that during the six month deorbit interval there will be 5.4 cuts of the 2000 primary line segments, 0.44 cuts of the 2,000 secondary line segments, and 0.16 (~0) cuts of the 2000 soldered joint segments. This sums to a total of about 6 cuts of tether line segments out of a total of 4,000 tether line segments, which means only 0.15% of the tether line segments are being cut during a six month time interval (which is why the tether will survive for many years).

The Rendezvous, Examine, and Tethered Return for Immolative EVaporation Experiment (RETRIEVE) was a candidate for a secondary payload experiment on the Air Force XSS-11 Mission, but was not selected. The experiment was designed to be initially dormant, and set into operation only after the XSS-11 microspacecraft had completed its primary mission of demonstrating near-autonomous rendezvous and examination of one or more resident space objects. We calculated the survival probability with time of the RETRIEVE tether due cuts by meteorites and orbital debris of all sizes to be 99.934% for the planned experiment duration of six months or less. This is equivalent to odds of only 1 out of 1500 of the tether being severed during the six-month duration of the experiment. Nearly all of this relatively low risk is due to the unlikely event of a strike by a large piece of orbital debris or a spacecraft greater than 1 meter in size cutting all the lines of the tether at once. This risk could be mitigated by using the Space Command's catalog of large orbiting objects to control the deorbit rate to avoid those large objects. Even without avoidance of large orbiting objects, the tether survival probability will remain above 99% until it has experienced almost 5 years of exposure to cuts. After three decades of cuts, the survival probability will have fallen to 90% and will begin to drop rapidly with each decade.

Acknowledgments

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