1 OVERVIEW OF PROGRAM

This program addresses the Navy need for extended firm track range for low altitude cruise missiles through the integration of multiple sensors. Track-Before-Declare (TBD) techniques that utilize signal features are proposed for the synergistic integration of an Electronically Scanned Array (ESA) radar with other sensors for the detection of weak targets. A comparison of the performances of a notional multisensor system with that of a federated system of sensors is planned. More specifically, the integration of an ESA radar and Infrared Search and Track (IRST) sensor for a shipboard combat system was proposed. The computer simulation models of the radars and IRST sensor will include the effects of many issues such as finite sensor resolution, limitations on the sensor resources, atmospheric refraction, sensor pointing errors, sea-surface induced multipath, nonhomogeneous clutter, sea clutter, etc. that are omitted in most of the legacy simulations. The computer simulation models will be utilized to develop tracking benchmark problems for broad distribution. These benchmark problems will serve to educate the research community on many of the “real-world” problems that are faced in actual tracking systems and the integration of multiple sensors.

The technical objectives of the project include the following:

- Development and demonstration through computer simulation of algorithms that provide enhanced detection of weak targets through the integration of multiple sensors. More specifically, Track-Before-Declare (TBD) techniques that utilize signal features are proposed for the synergistic integration of an electronically scanned array (ESA) radar with other sensors for the detection of weak targets. The integration of the ESA radar with the Infrared (IR) sensor is to be considered. The detection performance of a notional multisensor system will be compared with that of the federated system of sensors.
Development and demonstration through computer simulation of efficient radar resource allocation techniques that maximizes the information procured by the multisensor suite, while accommodating remote cues and/or warnings and adapting to changes in the characteristics of the targets of interest, weather conditions, etc. The waveforms and revisit times of the ESA will be selected to maximize the information procured by the multisensor system. The detection performance of a notional multisensor system with the new resource allocation techniques will be compared to that of the multisensor system with conventional resource allocation and that of the federated system of sensors.

This report summaries the progress and accomplishments made during October 1, 1999 and September 30, 2001. Since only a small amount of funding was provided for this program, the progress toward the goals of this program are very limited and include a few publications and other related activities. Section 2 summaries the accomplishments of this program, while Section 2 lists the papers that have been published or submitted for publication. Section 3 summarizes related activities that were supported as part of this project.

2 PROJECT ACCOMPLISHMENTS

The two primary accomplishments for the first year of this program were the development of a phased array radar model with search and track management functions for multiple targets as well as the development of a sea-clutter model with moving target indicator (MTI) waveform designs.

The development of the radar model for this project was to involve the addition of a horizon search function to an existing computer simulation of a phased array radar that was developed by the Naval Surface Warfare Center, Dahlgren Division (NSWCDD). After further consideration, the structure of the NSWCDD computer simulation was determined to be inappropriate for the inclusion of a horizon search function. The following list gives some of the reasons that the NSWCDD simulation was determined to be inappropriate.

- The existing simulation allows for the simultaneous tracking of at most two targets, while the horizon search function will need to allow for the tracking of many targets.
- The existing simulation requires that the number of targets at the beginning of a Monte Carlo experiment (or run) be the same as the number at the end of an experiment.
- The existing simulation does not include the infrastructure for starting false tracks, while false tracks are key to performance assessment of a horizon search function.
- The existing simulation includes only one array, while most shipboard phased array radars have multiple arrays to search the horizon.
- The existing simulation includes no method for the introduction of new targets after the scenario starts or the reacquisition of lost targets, while the purpose of the horizon search is to detect new targets and put them into track.
- The existing simulation does not include a model for sea clutter, while sea clutter is a major issue for horizon search.
These issues and the associated structure of the simulation program precluded its use without a major rewrite of the simulation. Thus, a new computer simulation with an alternative structure was developed for this project. The new radar model is an event-driven simulation with search, track, and dual track test modes as discussed in Appendix A. The search mode is a pseudo-search that is included to allow for automatic acquisition of new targets or reacquisition of targets that have been lost. In the pseudo-search mode, search dwells and the resulting detections are generated near targets and persistent clutter. This pseudo-search is readily extendable to accomplish a full search for the horizon as discussed in Appendix A. In the track mode, a track dwell and the resulting detections are generated for the specified target and used for tracking that target. The dual track mode is included to eliminate redundant or dual tracks. Appendix A gives an overview of the phased array radar model.

As the problem of realistically modeling sea backscatter is itself a difficult research problem, the modeling effort is proceeding in stages with the goal of balancing realism with implementability. The model will be a purely statistical model that captures the phenomenology that is most relevant to the detection and tracking process. Thus, the problem initially is to generate a stream of random numbers that have a given non-Rayleigh amplitude fluctuation and that have the proper correlation properties. During this first year, a model was developed and coded for K-distributed sea clutter without any spatial or temporal correlation. Appendix B gives an overview of the calculation of the average RCS and spectral distribution of the sea clutter, while Appendix C describes the generation of the random variables for the K-distributed sea clutter.

3 PUBLICATIONS

The following eight papers have been accomplished through funding from this grant.


4 RELATED ACTIVITIES

The following related activities have been supported at the request of the Office of Naval Research.


2. Hosted the *4th ONR/GTRI Workshop on Target Tracking and Sensor Fusion*, May 15-16, 2001 at the Naval Postgraduate School in Monterey, California. The workshop included approximately 60 attendees and 20 technical presentations. Proceedings of the presentations were produced as a bound copy and on CD-ROM.

3. Participated in the *Workshop on Estimation, Tracking, and Fusion: A Tribute to Yaakov Bar-Shalom*, May 17, 2001 at the Naval Postgraduate School in Monterey, California. The workshop included approximately 70 attendees and 20 technical presentations, and panel discussions. Proceedings of the presentations were produced as a bound copy and on CD-ROM.

4. Hosted the *3rd ONR/GTRI Workshop on Target Tracking and Sensor Fusion*, May 17-18, 2000 at the Cobb Research Facility of the Georgia Tech Research Institute. The workshop included approximately 50 attendees and 20 technical presentations. Proceedings of the presentations were produced as a bound copy and on CD-ROM.


6. Participated in several meetings with personnel from the United Kingdom on a Cooperative R&D Program on Data Fusion.
   - February 21-22, 2000 in NAVAIR in Lexington Park, MD: Participated in discussions on clutter modeling, data fusion, and naval weapons systems in UK and USA that utilize data fusion. A presentation on the JCTN Benchmark was given.
   - September 14-15, 2000, DERA in Portsmouth, UK: Participated in discussions with UK’s DERA personnel on the UK’s R&D programs that involve data fusion. A brief entitled “Multiplatform/Multisensor Tracking” was given (see Enclosure 7).

8. Assisted NSWCDD personnel with corrections to their multipath model for the phased array radar simulation for ONR/NSWC Tracking Benchmark IV.

9. Assisted GTRI personnel with the modeling of sea-surface induced multipath into the PESM simulation for ONR/NSWC Tracking Benchmark IV.

10. Supported meetings with Northrop-Grumman on multisensor integration for the E2-C.

11. Supported meetings at NSWCDD and ONR for planning of projects for airborne and shipboard multisensor integration.

Respectfully,

William Dale Blair, Principal Research Engineer
Project Director
Appendix A
Overview of Phased Array Radar Simulation Model

The radar model is an event-driven simulation with search, track, and dual track test modes as shown in Figure 1. The search mode is a pseudo-search that is included to allow for automatic acquisition of new targets or reacquisition of targets that have been lost. In the pseudo-search mode, search dwells and the resulting detections are generated near targets and persistent clutter. This pseudo-search is readily extendable to accomplish a full search for the horizon as discussed below. In the track mode, a track dwell and the resulting detections are generated for the specified target and used for tracking that target. The dual track mode is included to eliminate redundant or dual tracks.

The phased array radar model is currently configured as a notional Shipboard S-Band Phased Array Radar that operates at 3.4 to 4.0 GHz with four arrays that provide 360 degrees of azimuthal coverage and 70 degrees of elevation coverage. The four arrays are tilted to 15 degrees in elevation and located at 0, 90, 180, and 270 degrees relative to the ship's heading. Linear motion of the platform is included as well as roll, pitch, and yaw. Each array has uniform illumination across the array and each element has cosine illumination. As shown by Figure 2, the 3-dB beamwidth of the radar varies from 1.7 degrees at a broadside angle of 0 to 3.5 degrees at of broadside angle of 60 degrees. The radar model includes single pulse and 3-pulse moving target indicator (MTI) waveforms. The single pulse waveforms include four subpulses at distinct frequencies, while the MTI waveforms include two subpulses at distinct frequencies. Five, single-pulse waveforms that differ by 3 dB in energy for a total difference of 15 dB are included with waveform #1 denoting the highest energy. Each subpulse is bi-phase coded with a bandwidth of 15 MHz to give a range resolution of 10 m for each range cell. Two MTI waveforms have been designed for inclusion in the simulation.

The first action of the pseudo-search mode is to identify the targets that are in the search space that is specified by the function call. Each call to the search mode includes a search of the horizon and a fraction of the above horizon search region for each array. The search space is discretized into beam pointing angles and range intervals. The beam positions overlap resulting in a cluster of beams that may contain the target. The detection signals are generated for all of the range cells in beam angles and range intervals that include targets or persistent clutter tracks. For a full search of the horizon, the range intervals are defined to extend from the minimum detection range to the maximum detection range and detection signals are generated for all azimuth beaming pointing angles. Sensitivity time control (STC) is included in the search dwells to prevent the detection of very small objects at short ranges. The energy from clutter, targets, multipath, etc., and the receiver noise are included in each range cell. The subpulse energies are integrated noncoherently and an SNR threshold is applied to each range cell for detection. Monopulse measurements and an interference test for the detection of unresolved objects are computed for each detection. The measurements are then transformed from sine space at the array to a shipboard reference coordinate frame. The current model allows for biases in the location and orientation of each array.
Figure 1. Function flow of shipboard s-band phased array radar model
The second action of the search mode is cross gating that is required to prevent the initiation of new tracks on targets that are already under track. The cross gating is performed by first applying a course range and angle gates of 5 km and 2 degrees and then applying a chi-square test. The detections obtained in this search call are cross gated with each other to eliminate adjacent beam detections on a common target. The detections that remain are then cross gated with the tracks. Detections that do not correlate to any existing track are then assigned a track number and a confirmation dwell is scheduled for the track mode. If an error occurs in the cross gating so that a detection that should have been correlated with an existing track, a dual or redundant track is more than likely to be formed. Confirmation dwells are scheduled 0.1 s after the search mode. The final action of the search mode involves calling the event manager to schedule the time and sector for the next search and the confirmation dwells for the detections that did not correlate with existing tracks.

The first action of the track mode is to exit if the specified track has been dropped by the dual track mode. The first action for an active track is to predict the position of the target at the current time and generate the detection signals for all of the range cells in beam and range window. The energy from clutter, targets, multipath, etc., and the receiver noise are included in each range cell. The subpulse energies are integrated noncoherently and an SNR threshold that is specified by the tracking function is applied to each range cell for detection. Monopulse measurements and an interference test for the detection of unresolved objects are computed for each detection. The measurements are then transformed from sine space at the array to a shipboard reference coordinate frame. The current model allows for biases in the location and orientation of each array. The second action for an active track is the track filtering and data association. A nearly constant velocity Kalman filter is used for the first five measurements and then an IMM estimator with two nearly constant velocity models is initialized. The measurement-to-track data association is accomplished by picking the measurement with the range that is closest to that of the predicted range of the specified target and standard gating to verify the validity of the measurement. The fact that a measurement might actually belong to a target near the target specified in the function call is not considered at this time. If a misassociation occurs between to targets, a track swap or track loss is likely. A five sample median estimator is used to select the waveform so that a desired SNR for tracking is
maintained. The sample period and waveform are scheduled for transition-to-track and missed measurements. If a valid detection occurs in the confirmation dwell, transition-to-track is initiated. Otherwise, the track initiated with the search detection is dropped. The transition to track is accomplished by a set of scheduled revisit periods [e.g., 0.1, 0.1, 0.1, 0.1, 0.25, 0.25, 0.5, 0.5, 1.0, 1.0, 1.5, 1.5, 2.0 s], where the revisit periods are set at 2 s until a missed detection occurs. After a missed detection occurs, the revisit period is set to 0.1 s. Adaptive revisit time and waveform and revisit time scheduling for measurements with interference will be addressed later. The time update portion of the track filters includes platform motion. Tracks are dropped after five consecutive missed detections. The request dwell times are then perturbed by a positive uniform random number so that measurements are not received at the exact time requested by the track filter. The final action for an active track is a call to the event manager to schedule the time for the next track dwell on the target specified in the function call.

In the dual track test, the tracks are cross-gated with each other. The dual track test is performed every 5 s and if two tracks correlate on two consecutive test, the newer track is dropped.

Two PRFs are needed for the radar model. The two primary issues in PRF selection are maximum unambiguous range and the position of the Doppler nulls. Since this model is to be representative of operational systems, the PRF is limited to a maximum of 1 kHz, implying a maximum unambiguous range of about 150 km. With this constraint, the PRF pair was selected to minimize the impact of Doppler nulls on the detection performance.

Assuming an unstaggered PRF, an MTI filter has Doppler nulls at integer multiples of the PRF. For a radar operating at wavelength $\lambda$, the Doppler frequency $f_d$ of a target with radial velocity $v$ is given by

$$ f_d = \frac{2v}{\lambda} $$

The resulting MTI response for 3.5 GHz radar using a 1 kHz PRF is shown in Figure 3, where deep nulls appear periodically at multiples of 42.8 m/s. The selection of the two different PRFs was made to achieve a response that minimizes the number of deep nulls.

Several performance indices were considered. For example, minimizing the ratio of the region below $-10$ dB to the regions above $-3$ dB was considered. Ultimately, the integral of the area under the composite MTI function was maximized (note that all optimization performance indices considered produced different PRF pairs, but their composite responses were substantially similar). Let $PRF1$ and $PRF2$ be such that

$$ \{PRF1, PRF2\} = \arg \max \left\{ \int_0^{331} \max[MTI(PRF1, v), MTI(PRF2, v)] dv \right\} $$

where $MTI()$ is the MTI filter response defined in Appendix B, and the limits of integration were (somewhat arbitrarily) taken over the 0 to 1 Mach interval. Using this criteria, a two dimensional optimization procedure was developed to find $PRF1$ and $PRF2$ for PRF values near, but below, 1 kHz. The optimal pair was found to be {977, 891}, and their composite response is shown in Figure 4. As can be seen the number of deep nulls (<20 dB) is reduced from seven to two, and these two nulls are actually only $-34$ dB and $-22$ dB instead of $-\infty$. 
Figure 3. MTI response for 3-pulse canceler

Figure 4. Two-PRF composite MTI response
Appendix B
Average Sea Clutter RCS Modeling for Shipboard MSI

Prepared by
Philip D. West

1 INTRODUCTION
This appendix documents the equations, tables and concepts used in the development of the sea clutter model. We consider five primary facets of the problem:

- Scenario Geometry
- Average clutter RCS
- Statistical Sample generation
- Clutter Doppler Signature
- MTI Improvement Factor

Scenario geometry deals with the angles and distances between the sea surface and the radar antenna. The primary quantities of interest in the geometry problem are the depression angle, grazing angle, and clutter patch size. After these parameters have been determined, a table of reference RCS values, indexed by radar frequency, is used to generate a reference clutter radar cross section (RCS) for a particular reference condition. Next, analytical interpolation and correction/adjustment factors are applied to compensate for differing sea states, polarizations, grazing angles, and frequency. After all of these computations have been made, a statistical model can be used to generate random samples to represent clutter returns for each range cell of interest. Of course, our primary interest is not on clutter phenomenology, but rather on its impact on radar detection and tracking performance. It is realized that all modern radar systems will use some type of moving target indicator (MTI) or Doppler processing circuitry to remove or separate low velocity targets from the targets of interest. In the final two sections, we first present the Doppler (frequency domain) signature of sea clutter, and then present a closed-form methodology for computing the expected clutter output power from an MTI filter using this clutter model.

2 GEOMETRY
We employ the standard clutter geometry scenario used in many texts [see, e.g. 1] as shown in Figure 1. In this figure, the radar antenna is located at some height $H_R$ meters above the sea surface illuminating a patch of clutter at range $R$. The Depression Angle of the clutter with respect to the radar is $\theta_d$ and the Grazing Angle is $\theta_g$. The grazing angle and depression angle may be computed as:
\[ \theta_s = \sin^{-1}\left( \frac{2a_e H_R + H_R^2 - R^2}{2a_e R} \right) \]

\[ \theta_d = \sin^{-1}\left( \frac{2a_e H_R + H_R^2 + R^2}{2R(a_e + H_R)} \right) \]

where \( a_e \) is the effective radius of the earth.

Figure 1. Clutter Geometry

Next, we seek to compute the physical area on the sea surface that will be illuminated by the radar beam. For range-gated radars with range resolution \( \Delta R \) and beamwidth \( BW \), the area will be defined by a strip with width equal to the radar’s range resolution and width proportional to the radar azimuthal beamwidth. Specifically:

\[ A = R \cdot \Delta R \sin(BW) / \cos(\theta_s) = R \cdot \Delta R \cdot BW \]
3 MEAN CLUTTER RCS

In this section, we follow the development advocated in the APL report "Clutter Models for Shipboard Radar Applications: 0.5 to 70 GHz" [2]. Computing the mean sea clutter RCS, $\sigma_0$, is a multistep procedure involving first computing the nominal mean clutter RCS as a function of frequency and area and then computing and applying corrections for:

- Wind Speed (effectively sea state) and Direction ($S$)
- Polarization ($P$)
- Grazing Angle ($\Psi$)
- Look Direction ($D$)

3.1 Fundamental Equation

The fundamental sea clutter RCS equation is:

$$\sigma_0 = \sigma_{ref} + K_\Psi + K_S + K_P + K_D$$

(3)

where $\sigma_{ref}$ is the reference RCS corresponding to $S = 5, \Psi = 0.1^\circ, P = V$, and $D = 0^\circ$, and $K_\Psi, K_S, K_P$ and $K_D$ are the corrections for grazing angle, windspeed, polarization and wind direction, respectively. We now consider each term individually. To obtain the RCS in a given resolution cell, the mean clutter RCS (equation(3)) is simply multiplied by the area given by equation (2):

$$\sigma = \sigma_0 A$$

(4)

3.1.1 Reference RCS

This value, $\sigma_{ref}$, is obtained from a table assuming sea state 5, vertical polarization, look direction = 0 (directly upwind) and grazing angle = $0.1^\circ$. The values as a function of frequency are:

<table>
<thead>
<tr>
<th>F (GHz)</th>
<th>0.5</th>
<th>1.25</th>
<th>3.0</th>
<th>5.6</th>
<th>9.3</th>
<th>17.0</th>
<th>35.0</th>
<th>70.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>dBsm</td>
<td>-72.0</td>
<td>-63.0</td>
<td>-54.0</td>
<td>-47.0</td>
<td>-41.0</td>
<td>-38.0</td>
<td>-37.0</td>
<td>-36.0</td>
</tr>
</tbody>
</table>

Interpolation between the tabulated values is done linearly with respect to log(frequency).

3.1.2 Grazing Angle Adjustment

We first define the transitional grazing angle as

$$\Psi_t = \sin^{-1}(0.066\lambda / \sigma_S)$$

(5)

where $\sigma_S$ is the RMS wave height (see Section 3.1.5). With this, the grazing angle adjustment can be partitioned into six regions with different functional adjustments:
for $\Psi_t > \Psi_r = 0.1^\circ$

$$K_\Psi = \begin{cases} 0 & \Psi < \Psi_r \\ 20 \log(\Psi / \Psi_r) & \Psi_r < \Psi < \Psi_t \\ 20 \log(\Psi_r / \Psi_f) + 10 \log(\Psi / \Psi_r) & \Psi_r < \Psi < 30^\circ \end{cases}$$

(6)

for $\Psi_t < \Psi_r$

$$K_\Psi = \begin{cases} 0 & \Psi \leq \Psi_r \\ 10 \log(\Psi / \Psi_r) & \Psi > \Psi_r \end{cases}$$

3.1.3 Polarization Adjustment

For vertical polarization, $K_p$ is 0. Horizontal polarization adjustment is a function of wave height and frequency. Specifically if the polarization is Horizontal:

$$K_p = \begin{cases} 1.7 \ln(h_a + 0.015) - 3.8 \ln(\lambda) - 2.5 \ln\left(\frac{\Psi}{573} + 0.0001\right) - 22.2 & f < 3 \text{GHz} \\ 1.1 \ln(h_a + 0.015) - 1.1 \ln(\lambda) - 1.3 \ln\left(\frac{\Psi}{573} + 0.0001\right) - 9.7 & 3 \text{GHz} \leq f < 10 \text{GHz} \\ 1.4 \ln(h_a) - 3.4 \ln(\lambda) - 1.3 \ln\left(\frac{\Psi}{573}\right) - 18.6 & f \geq 10 \text{GHz} \end{cases}$$

(7)

where $h_a$ is the average wave height in meters (see section 3.1.5).

3.1.4 Wind Direction Adjustment

The wind direction adjustment, $K_d$, is a function of the radar look angle with respect to the wind, $\Theta$.

$$K_d = \left(2 + 1.7 \log \frac{0.1}{\lambda}\right) \left(\cos(\Theta) - 1\right)$$

(8)

Figure 2 shows this cosinusoidal variation of sea clutter RCS with look angle for measured data.
3.1.5 Sea State Adjustment

The sea state adjustment is:

\[ K_s = 5(S - 5) \]  

(9)

where \( S \) is the sea state. For blue-water scenarios, wind speed and sea state are related as shown in Table 2.

<table>
<thead>
<tr>
<th>Sea State</th>
<th>Wind Speed (kn)</th>
<th>Wave Height, ft</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (smooth)</td>
<td>&lt;7</td>
<td>1</td>
</tr>
<tr>
<td>2 (slight)</td>
<td>7-12</td>
<td>1-3</td>
</tr>
<tr>
<td>3 (moderate)</td>
<td>12-16</td>
<td>3-5</td>
</tr>
<tr>
<td>4 (rough)</td>
<td>16-19</td>
<td>5-8</td>
</tr>
<tr>
<td>5 (very rough)</td>
<td>19-23</td>
<td>8-12</td>
</tr>
<tr>
<td>6 (high)</td>
<td>23-30</td>
<td>12-20</td>
</tr>
<tr>
<td>7 (very high)</td>
<td>30-45</td>
<td>20-40</td>
</tr>
</tbody>
</table>

Functional relationships between sea state \( S \), windspeed, \( V_w \), RMS wave height \( \sigma_z \), and average wave height \( h_a \) can be defined through the following equations.

\[
V_w = 3.2S^{0.8} \\
\sigma_z = 0.031S^2 \\
h_a = 0.08S^2
\]  

(10)
which implies that \( \sigma_z = 0.3875 h_0 \).

4 RANDOM CLUTTER SAMPLES

The APL report [2] advocates generating random clutter samples with a Weibull density function. Instead, we prefer to use Gamma distributed variates as described in Appendix A and advocated in [3]. When these Gamma distributed variates are used as clutter RCS samples and combined using standard I&Q signal processing, they lead to the K-Distribution for the received signal power. One downside of using the Gamma model is that generation of these variates is computationally intensive. As a baseline, we used the Gamma distribution 'm-file' available on the Mathworks web-site software archive. To generate random variates, this algorithm generated one sample at a time using nonlinear transformation of uniform variates. Because of the required looping and conditional processing this method is inefficient for generating large vectors of samples. Thus, the code was 'vectorized', using the same numeric algorithm to process entire vectors of samples rather than single samples. The resulting code produces samples with equivalent distributions, but in about 15% of the required CPU time.

5 CLUTTER DOPPLER SPECTRUM

Sea surface wave motion imposes an aspect dependent Doppler shift on the clutter waveform. The clutter spectrum is modeled as Gaussian \( \mathcal{N}(V_D, \sigma_v^2) \) where:

\[
V_D = \begin{cases} 
0.85 \sqrt{V_w} \cos \Theta & \text{Polarization = H} \\
0.15 V_w \cos \Theta & \text{Polarization = V} 
\end{cases} \\
\sigma_v = 0.23 V_w \text{ (m/s)}
\]

(11)

Clutter attenuation by MTI filtering is considered under a separate memo.

6 MTI IMPROVEMENT

Moving target improvement (MTI) circuits are used to cancel radar returns from stationary or low speed objects such as land and trees or even sea surface-induced returns. Of particular interest is a standard 3-pulse tapped delay line MTI filter as shown in Figure 3. Here, the [complex] output pulse sample is a weighted sum of the current pulse and the prior two pulses with the indicated weightings. The frequency domain response for a simple feedforward MTI filter is [4]:

\[
|H(f_d)| = 2^N \sin^N \left( \frac{\pi f_d}{PRF} \right)
\]

(12)

where \( N \) is the number of delays, \( f_d \) is the Doppler frequency and \( PRF \) is the pulse repetition frequency of the radar. As can be seen, this MTI response has the desired null at zero, but the null repeats periodically at integer multiples of \( f_d = PRF / \pi \). Recalling that the Doppler offset for a target flying with velocity \( v \) and being observed by a radar with wavelength \( \lambda \) is \( f_d = \frac{2v}{\lambda} \), we see that the MTI filter will have nulls...
every 42.8 m/s for a 3.5 GHz radar using a 1 KHz PRF (see Appendix A—Radar Model for radar parameters and PRF design). A composite graph showing the Doppler signature for vertically polarized sea clutter with 15, 30 and 45 Knot winds along with the 3-pulse MTI response is shown in Figure 4. As can be seen, as the windspeed increases, the width and mean of the sea clutter spectrum increases. As this happens, less and less of the distribution falls within the clutter notch suggesting that as windspeed (sea state) increases, the clutter rejection ratio for the MTI filter will decrease. Our task, then is to compute the MTI output power as a function of windspeed.

Figure 3. 3-Pulse MTI Canceler

Figure 4. MTI Response and Sea Clutter Doppler Spectrum
The output clutter power will be proportional to the area under the product of the clutter spectrum and the MTI response. This conceptually simple formulation leads to an integral that did not yield to our closed-form integration attempts, so we follow another approach to compute the output clutter power. Following [5], we realize that we wish to find the output power of the MTI filter with weight vector \( W \) and with random input signal \( X \). Thus:

\[
\sigma_c^2 = E\{W'XX^*W^*\} = W'M_cW^*
\]

where \( M_c \) is the covariance matrix of the clutter, \( * \) denotes complex conjugation and \( ' \) denotes transposition. Given the Gaussian clutter spectrum specified in Section 5 with variance \( \sigma_v^2 \) and mean \( V_p \):

\[
P_c(f) = \frac{1}{\sqrt{2\pi\sigma_v}} e^{-\frac{(f-V_p)^2}{2\sigma_v^2}}
\]

the covariance matrix is defined as:

\[
M_c(k,l) = \exp\{-2\pi^2 \sigma_v^2 (k-l)^2 T^2 - j2\pi V_p(k-l)T\}
\]

where \( T \) is the pulse recurrence interval \( (1/PRF) \). Evaluating equation (13) with \( W=\{1, -2, 1\} \) we get

\[
\sigma_c^2 = 6 + 2e^{-8\pi^2\sigma_v^2 T^2} \cos(4\pi V_p T) - 8e^{-8\pi^2\sigma_v^2 T^2} \cos(2\pi V_p T)
\]

By defining the MTI improvement factor to be \( I = \sigma_c^2 / \max\{|H(f)|^2\} = \sigma_c^2 / 16 \) we can evaluate the performance of the MTI filter as function of windspeed (using equation (11)), radar frequency and PRF. Using the 1 KHz PRF and 3.5 GHz center frequency of interest yields Figure 5.

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1 Care must be taken when evaluating this equation to ensure that all units are scaled to the proper Doppler domain.
As can be seen in the figure, cancellation is quite good for lower windspeeds, achieving 30 dB or better for winds below 10 Knots. As the windspeed increases, clutter rejection becomes poor, approaching -10 dB as the wind increases to 45 Knots.

7 Summary
A sea clutter model has been developed and implemented in the Matlab® programming environment. The model includes the following features:

- Sea Clutter RCS as a function of:
  - Grazing Angle
  - Sea State (windspeed)
  - Polarization
  - Frequency
  - Radar look direction w.r.t wind direction
- Clutter Doppler Signature
- Statistical (random) RCS Sample Generation

Additionally, the MTI improvement factor was computed for this sea clutter model, and performance curves were plotted for a low PRF waveform using a 3-pulse canceler.
8 REFERENCES

   ("Performance of Cascaded MTI and Coherent Integration Filters in a Clutter Environment" by G.A. Andrews)
Appendix C
Simulation of Non-Gaussian, Complex Sea Scatter

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1.0 Gaussian Clutter

Simulation of complex, non-Gaussian sea scatter builds on the simulation of complex Gaussian sea scatter. We assume the returns are independent and identically distributed (i.i.d.) in both time and space. Hence it suffices to describe how to generate a single sample of sea scatter. Let

\[ u_a = \text{uniform random number} \]
\[ u_b = \text{uniform random number} \]
\[ c_i = \text{phase clutter return} \]
\[ c_q = \text{clutter return} \]
\[ c = c_i + c_q = \text{complex clutter return} \]

Then

\[ c_i = \sqrt{-2\sigma_0 \ln(u_a)} \cos(2\pi u_b) \]
\[ c_q = \sqrt{-2\sigma_0 \ln(u_a)} \sin(2\pi u_b) \]

generates a single, complex Gaussian return whose amplitude PDF is

\[ f(|c|) = \frac{|c|}{\sigma_0} \exp\left(-\frac{|c|^2}{2\sigma_0}\right) \]
\[ = \text{Rayleigh distributed} \]

where \( \sigma \) represents the clutter power level.

2.0 K-Distributed Clutter

To build a simulation for non-Gaussian clutter we will let the amplitude statistics be \( K \) distributed of which the Rayleigh distribution is a special case. The extension of the Gaussian simulation
of the non-Gaussian simulation with $K$ distributed amplitude statistics is straight forward. We simply replace $\sigma_0$, which in the Gaussian case is a fixed number, with a gamma-distributed random variable $\sigma$ where

$$f(\sigma) = \frac{\nu^\nu}{\sigma_0 \Gamma(\nu)} \sigma^{\nu-1} \exp\left(-\frac{\nu \sigma}{\sigma_0}\right) = \text{Gamma PDF}$$

In this distribution

$$E\{\sigma\} = \sigma_0 = \text{average power level}$$
$$\nu = \text{shape parameter}$$

As $\nu \to \infty$, the distribution reverts to Gaussian. Thus, in this case we generate

- $u_a = \text{uniform random number}$
- $u_b = \text{uniform random number}$
- $\sigma = \text{gamma distributed random variable}$
- $c_i = \sqrt{-2\sigma \ln(u_a)} \cos(2\pi u_b)$
- $c_q = \sqrt{-2\sigma \ln(u_a)} \sin(2\pi u_b)$

In this case the PDF of the amplitude is given by

$$f(|c|) = \int_0^\infty \frac{|c|}{\sigma} \exp\left(-\frac{|c|^2}{2\sigma}\right) \frac{\nu^\nu}{\sigma_0 \Gamma(\nu)} \sigma^{\nu-1} \exp\left(-\frac{\nu \sigma}{\sigma_0}\right) d\sigma$$

$$= \left(\frac{\sqrt{2\nu}}{\sigma_0}\right)^{\nu+1} \frac{|c|^\nu}{2^{\nu-1} \Gamma(\nu)} K_{\nu-1} \left(\frac{2\nu}{\sigma_0} |c|\right)$$

where $K_{\nu-1}(\cdot)$ is the modified Bessel function of the second kind.

The question now arises as to how to choose the parameter $\nu$ as a function of the range resolution length. Over a number of years the general observation has been made that as resolution increases (i.e., as the resolution length decreases), the clutter is observed to be more non-Gaussian (i.e., spikier). Since spikier clutter implies a smaller $\nu$, we are led to hypothesize a relationship of the form

$$\nu = A f^q$$
where

\[ \nu = \text{is the shape parameter in the } K \text{ distribution} \]
\[ l = \text{is the range resolution length (meters)} \]
\[ A = \text{is a constant} \]
\[ q = \text{is a constant} \]

A study by Watts and Ward (IEE Proceedings vol. 134 Pb F, No. 6, October 1987, pp 526-532) suggests that a good choice for \( q \) is

\[ q = \frac{5}{8} \]

However, this same study shows that \( A \) can vary greatly as a function of sea state, location, etc. To start, I would recommend setting \( A = 0.1 \), for a 1 \( \mu \)s pulse width (i.e., \( l = 300 \) m), this gives \( \nu = 3.5 \), which is moderately spiky clutter. For a 5 \( \mu \)s pulse width it gives \( \nu = 9.6 \), which is probably indistinguishable from Gaussian clutter. Initially, we recommend choosing

\[ \nu = 0.1l^{\frac{5}{8}} \]
This program addresses the Navy need for extended firm track range for low altitude cruise missiles through the integration of multiple sensors. Track-Before-Declare (TBD) techniques that utilize signal features are proposed for the synergistic integration of an Electronically Scanned Array (ESA) radar with other sensors for the detection of weak targets. The computer simulation models of the sensors will include the effects of many issues such as finite sensor resolution, limitations on the sensor resources, atmospheric refraction, sensor pointing errors, sea-surface induced multipath, nonhomogeneous clutter, sea clutter, etc. that are omitted in most of the legacy simulations. The two primary accomplishments for the first year of this program were the development of a phased array radar model with search and track management functions for multiple targets as well as the development of a sea-clutter model with moving target indicator (MTI) waveform designs.