WAVES AND VIBRATIONS IN ELECTROPLATED PIEZOELECTRIC PLATES AND SUBSTRATE SYSTEMS

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13. ABSTRACT *(Maximum 200 words)*

In the present research, a new, infinite system of plate equations is obtained. They are applicable to piezoelectric plates of any crystal symmetry and requiring no correction factors. Firstly, a system of first-order plate equations is extracted from the infinite system, and they are employed to study the thickness shear and flexure vibrations of rectangular plates of AT- and SC-cuts of quartz and are shown to give accurate predictions in dispersion relations, frequency spectra, and capacitances without any corrections. Secondly, a system of second-order plate equations is extracted from the infinite system by a new truncation procedure, and these equations are employed to study the thickness-stretch, symmetric thickness-shear, and extensional vibrations of finite strips and circular disks of ceramics. The accuracy of the prediction is validated by the close comparison with experimental data, without any correction.

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WAVES AND VIBRATIONS IN ELECTROPLATED PIEZOELECTRIC PLATES AND SUBSTRATE SYSTEMS

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I. STATEMENT OF THE PROBLEM STUDIED

Accurate prediction of resonance frequencies and coupled motions is, perhaps, one of the most important objectives and demands in modeling the vibrations of piezoelectric crystal resonators for applications in frequency control and sensors. However, the accuracy of the prediction depends very much on the accuracy of the governing equations employed for the solution and computation. In current and well-known governing equations for plates, such as those by Timoshenko, Mindlin, and Lee, correction factors were introduced in order to correct the prediction. These correction factors may not always be readily available, since they usually depend on the material properties, cut orientation and the mode of vibration of the plate resonators.

In the present research, a new, infinite system of plate equations is obtained. They are applicable to piezoelectric plates of any crystal symmetry and requiring no correction factors. Firstly, a system of first-order plate equations is extracted from the infinite system, and they are employed to study the thickness shear and flexure vibrations of rectangular plates of AT- and SC-cuts of quartz and are shown to give accurate predictions in dispersion relations, frequency spectra, and capacitances without any corrections. Secondly a system of second-order plate equations is extracted from the infinite system by a new truncation procedure, and these equations are employed to study the thickness-stretch, symmetric thickness-shear, and extensional vibrations of finite strips and circular disks of ceramics. The accuracy of the prediction is validated by the close comparison with experimental data, without any corrections.

II. SUMMARY OF THE MOST IMPORTANT RESULTS

1. A new and infinite system of two-dimensional governing equations for piezoelectric plates with general crystal symmetry and electroded faces are deduced from the three-dimensional equations of linear piezoelectricity by a trigonometric series expansion with a linear term newly added in mechanical displacement [See III(a)1 and III(b)1]. These equations will be shown to give accurate predictions without any correction factors for plates undergoing predominantly thickness-shear and flexural vibrations as well as for plates undergoing predominantly thickness-stretch and extensional vibrations.

2. A system of two-dimensional first-order equations is extracted from the infinite system. These equations are employed to study the vibrations of rectangular AT-cut of quartz. The accuracy of the predictions is validated by the close comparison of the dispersion curves with those from the three-dimensional equations, and by the close agreement of predicted frequency spectra with the experimental data by Koga and Fukuyo and that of Nakazawa, Horiuchi, and Ito [See III(a)1 and III(b)1].

3. The two-dimensional first-order equations are employed to study the piezoelectrically forced vibrations of rectangular SC-cut quartz plates, which have the most general constitutive relations for piezoelectric crystals. It is shown that computational results from these equations are accurate without using any corrections [See III(a)2 and III(b)2].

4. By using a new truncation procedure, a system of two-dimensional second-order plate equations is extracted from the infinite system. These equations are shown to predict accurate dispersion curves for frequencies up to and including the cut-off frequencies of the first symmetric thickness-stretch and the second symmetric thickness-shear modes. Furthermore, a system of one-dimensional equations is deduced by averaging the two-dimensional equations. Predicted frequency spectra from the one-dimensional equations are compared with the experimental data by Onoe and Pao and calculated results by Medick and Pao for strips of barium titanate with good agreement, again, without using any correction factors [See III(b)3].

5. Extensional, thickness-stretch and symmetric thickness-shear vibrations of circular disks of barium titanate are studied by employing a system of two-dimensional second-order plate equations similarly obtained. Computed dispersion curves are compared with those from the three-dimensional equations, and predicted resonance frequencies are compared with experimental data by Shaw. Both comparisons show good agreement without any corrections [See III(b)4].

III. LIST OF PUBLICATIONS AND TECHNICAL REPORTS

(a) Papers in peer-reviewed journals

1. P.C.Y. Lee, J.D. Yu, and W.S. Lin, "A New 2-D Theory of Vibrations of Piezoelectric Crystal Plates," *J. Appl. Phys.* **83** (3), pp. 1213-1223, 1998.

2. P.C.Y. Lee and W.S. Lin, "Piezoelectrically Forced Vibrations of Rectangular SC-cut Quartz Plates," *J. Appl. Phys.* **83** (12), pp. 7822-7833, 1998.

(b) Papers in conference proceedings

1. P.C.Y. Lee, J.D. Yu, and W.S. Lin, "A New 2-D Theory for Vibrations of Piezoelectric Crystal Plates with Electroded Faces," *Proc. 1996 IEEE Ultrasonics Symposium*, pp. 1591-1994, 1996.

2. P.C.Y. Lee and W.S. Lin, "Piezoelectrically Forced Vibrations of Rectangular SC-cut Quartz Plates," *Proc. IEEE International Frequency Control Symposium*, pp. 659-664, 1997.

3. P.C.Y. Lee and N.P. Edwards, "Extensional Vibrations of Piezoelectric Crystal Plates and Strips," *Proc. of Joint Meeting of the 13th European Freq. and Time Forum and 1999 IEEE Int. Freq. Control Symposium*, pp. 754-757, 1999.

4. P.C.Y. Lee, R. Huang, W.S. Lin, and J.D. Yu, "Extensional, Thickness-stretch and Symmetric Thickness-shear Vibrations of Piezoelectric Ceramic Disks," *Proc. 1999 IEEE Ultrasonics Symposium*, pp. 873-876, 1999.

IV. LIST OF PARTICIPATING SCIENTIFIC PERSONNEL

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