Naval Research Laboratory
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NRL/MR/6790--01-8505

Apparent Superluminal Propagation of a Laser Pulse in a Gain Medium

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January 30, 2001

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (*Leave Blank*)

2. REPORT DATE
January 30, 2001

3. REPORT TYPE AND DATES COVERED
Interim

4. TITLE AND SUBTITLE
Apparent Superluminal Propagation of a Laser Pulse in a Gain Medium

5. FUNDING NUMBERS

6. AUTHOR(S)
Phillip Sprangle, Joseph Peñano,* and Bahman Hafizi**

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Washington, DC 20375-5320

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/MR/6790--01-8505

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research
Arlington, VA 22217
Department of Energy
Germantown, MD 20864

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
*Leading Edge Technologies (LET), 4431 MacArthur Blvd., Washington. DC 20007
**Icarus Research, Inc., P.O. Box 30780, Bethesda, MD 20824-0780

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (*Maximum 200 words*)

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14. SUBJECT TERMS
Superluminal
Anomalous dispersion
Pulse distortion
Laser pulse propagation
Gain medium
Causality

15. NUMBER OF PAGES
16

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std 239-18
298-102

Apparent Superluminal Propagation of a Laser Pulse in a Gain Medium

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The distortion of a laser pulse propagating in a dispersive gain/absorptive medium is analyzed. The relationship between the distortion of the pulse and superluminal propagation is discussed. We present an analytical approach based on the laser envelope equation that is readily applicable to arbitrary input pulse shapes. This analysis is used to interpret recent experiments that claim to have observed distortionless superluminal laser pulse propagation.

I. Introduction

In a recent article in *Nature* [1], titled ***Gain-assisted superluminal light propagation***, researchers reported observing superluminal propagation of a laser pulse through an amplifying medium by a new mechanism that does not distort the pulse. In this experiment, a long laser pulse was passed through an amplifying medium consisting of a specially prepared caesium gas cell of length $L = 6$ cm, as depicted in Fig. 1. The laser pulse of duration $T = 3.7$ μsec was much longer (1.1 km) than the gas cell, so that at any given instant only a small portion of the pulse was inside the cell. By measuring the pulse amplitude at the exit, it is claimed that both the front and the back edges of the pulse were shifted forward in time by the same amount relative to a pulse that propagated

Manuscript approved November 14, 2000.

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through vacuum. In contrast to earlier works that have interpreted apparent superluminal propagation as a pulse reshaping effect [2,3], it is claimed in Ref. 1 that superluminal propagation is observed "while the shape of the pulse is preserved" and "the argument that the probe pulse is advanced by amplification of its front edge does not apply". This article generated a great deal of press attention around the world [4]. The objective of this paper is to provide a general analysis of laser pulse propagation in an anomalously dispersive gain/absorptive medium. In addition, this analysis is used to comment on the conclusions reached in Ref 1.

It is well known that in regions of anomalous dispersion the group velocity of an electromagnetic pulse can be abnormal, i.e., greater than c (the speed of light in vacuum) or negative [5,6]. While it has been claimed that group velocity "is just not a useful concept" in regions of strong anomalous dispersion [6], others have shown that for a Gaussian pulse, the group velocity represents the velocity of the peak of the pulse even when it is abnormal [2,3,7]. This apparent superluminal propagation results from a pulse reshaping effect by which a gain medium preferentially amplifies the front or absorbs the back of the pulse. This effect has been described theoretically using a Fourier transform method. For analytical tractability a Gaussian pulse was used and the refractive index expanded to keep only the lowest order group velocity dispersion (GVD) term [2,3,7].

Our analysis is based on an envelope equation that describes the propagation of arbitrary pulse shapes and can include higher order dispersion effects analytically. For a pulse with a well-defined leading edge, we show that the lowest order effect in a gain medium is that the pulse propagates with velocity c and undergoes a distortion in which the front of the pulse is amplified more than the back, i.e., *differential gain*. This leads to apparent superluminal pulse propagation; that is, the peak of the pulse travels faster than

c. However, the velocity of the leading edge of the pulse does not exceed c. A related effect can also take place in an absorptive medium. Our analysis indicates that differential gain occurred in the experiment of Ref. 1 and can account for the observed pulse advancement. Hence, the interpretation in Ref. 1 that superluminal propagation occurs without amplification of the leading edge of the pulse is incorrect.

II. Analytical Model

The following analysis considers a laser pulse propagating in a general dispersive medium characterized by a frequency dependent complex refractive index $n(\omega)$. We assume that the deviation of the refractive index from unity, i.e., $\Delta n(\omega) = n(\omega) - 1$, is small and neglect reflections of the laser pulse from the medium boundaries at $z = 0$ and $z = L$. To determine the evolution of the pulse envelope we represent the laser electric field as $E(z,t) = (1/2)\ A(z,t)\exp[i(k_o z - \omega_o t)] + c.c.$, where $A(z,t)$ is the slowly varying, complex pulse envelope, $k_o = \omega_o n(\omega_o)/c$ is the complex wavenumber, ω_o is the carrier frequency and c.c denotes the complex conjugate. The field is polarized in the transverse direction and propagates in the z-direction. Since k_o is complex, the factor $\exp(-\mathrm{Im}(k_o)\, z)$ represents an overall amplification/absorption of the pulse at frequency ω_o and does not cause pulse distortion. The actual laser pulse amplitude is $|A(z,t)|\exp[-\mathrm{Im}(k_0)z]$.

The equation describing the evolution of the laser pulse envelope, including all higher order dispersive effects, is given by [8]

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)A(z,t) = -\left[\frac{\partial(\beta - \omega/c)}{\partial \omega}\right]_0 \frac{\partial}{\partial t}A(z,t)$$
$$+ \frac{i}{2k_0}\left\{\frac{\partial^2}{\partial z^2} + \sum_{m=1}^{\infty}\frac{i^m}{m!}\left[\frac{\partial^m \beta^2}{\partial \omega^m}\right]_0 \frac{\partial^m}{\partial t^m}\right\}A(z,t), \tag{1}$$

where $\beta = \omega n(\omega)/c$ is the frequency dependent wavenumber, $[\ ]_0$ implies that the quantity in brackets is to be evaluated at $\omega = \omega_0$, and the laser pulse envelope at the input to the amplifying medium $A(z=0,t)$ is assumed given. Equation (1) was derived by substituting the representation for the laser electric field into the wave equation and performing a spectral analysis [8,9] that involves expanding the refractive index about the carrier frequency ω_o. If the spectral width of the pulse is sufficiently narrow, it is valid to limit the analysis to terms of order $\partial^2/\partial t^2$, i.e., lowest order GVD effects. With this approximation, together with neglecting the small term proportional to $\partial^2/\partial z^2$, Eq. (1) reduces to

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)A(z,t) = -\left(\kappa_1 \frac{\partial}{\partial t} + \frac{i}{2}\kappa_2 \frac{\partial^2}{\partial t^2} + \ldots\right)A(z,t), \qquad (2)$$

where $\kappa_\ell = [\partial^\ell \kappa(\omega)/\partial \omega^\ell]_0$, $\ell = 1,2,\ldots$, and $\kappa(\omega) = \omega \Delta n(\omega)/c$. This approximation, which requires both a sufficiently short interaction length and long pulse duration, is sufficient for the present purpose. For pulse propagation in vacuum, $\Delta n(\omega) = 0$ so that the right hand side of Eq. (2) vanishes and the laser envelope is given by $A(z,t) = A(0, t - z/c)$, indicating that the pulse propagates with velocity c without distortion.

Equation (2) can be solved iteratively assuming that terms on the right hand side get progressively smaller. However, to indicate where inconsistencies in the ordering of approximations can arise we proceed with a spectral analysis and show how to recover a consistent ordering. Equation (2) is Fourier transformed in time and the resulting differential equation in z is solved for the transformed envelope. Inverting the transformed envelope yields the solution

$$A(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\nu\, \hat{A}(0,\nu) \exp(-i\nu(t - z/c)) \exp\left(i\kappa_1 \nu\, z + i\kappa_2 \nu^2 z/2\right), \quad (3)$$

where $\hat{A}(0,\nu)$ is the Fourier transform of the envelope at z = 0, and ν is the transform variable.

It is assumed that the following inequalities hold: $1 \gg |\kappa_1 \nu z| \gg |\kappa_2 \nu^2 z/2|$, where $\nu \approx 1/T$ and $z \approx L$. To correctly evaluate the integral in Eq. (3), the exponentials in the small quantities should be expanded to an order of approximation consistent with Eq. (2), otherwise unphysical solutions may result. For example, if the lowest order GVD term, $\kappa_2 \nu^2 z/2$, is neglected in Eq. (3), the laser envelope is given by

$$A(z,t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\nu\, \hat{A}(0,\nu) \exp(-i\nu(t - z/c)) \exp(i\kappa_1 \nu\, z). \quad (4)$$

Equation (4) can be integrated exactly to give

$$A(z,t) = A(0, t - z/v_p), \quad (5)$$

where the quantity $v_p = [\partial(\omega n/c)/\partial\omega]^{-1} = c/(1 + c\kappa_1)$ defines the group velocity of the pulse. The exact solution, given by Eq. (5), to the approximate envelope equation can clearly lead to unphysical results since it implies that, to the lowest order of approximation, the entire pulse propagates undistorted with velocity v_p. This interpretation is due to retaining terms beyond the order of the approximation. For example, neglecting κ_2 terms in the exponent of Eq. (4) is equivalent to keeping terms proportional to $(\kappa_1 \nu z)^2$ while neglecting terms proportional to $\kappa_2 \nu^2 z$ which are of the same order.

In a dispersive gain/absorptive medium, v_p can be abnormal. For example, if $-1 < c\kappa_1 < 0$, the pulse velocity exceeds c. For the parameters in Ref. 1, however, $c\kappa_1$ is essentially real and < -1, giving a negative pulse velocity, $v_p = -c/310$. A negative pulse velocity implies superluminal propagation if one considers the pulse delay time [1,10]. The delay time, $\Delta T = L/v_p - L/c$, is defined as the difference in the transit times of an arbitrary point on the pulse in the dispersive gain/absorptive medium and in vacuum. Negative delay times, which is the case for the parameters of Ref. 1, imply superluminal propagation. While it is physically possible for points on the pulse to have negative delay times, e.g., the peak of the pulse, this should not be interpreted as superluminal propagation of the *entire* pulse since the pulse distorts.

To properly describe higher order effects it is necessary to solve Eq. (3) by keeping the order of approximation consistent. Expanding the exponential terms in Eq. (3) to second order yields

$$A(z,t) = (1/\sqrt{2\pi}) \int_{-\infty}^{\infty} d\nu\, \hat{A}(0,\nu)\left(1 + i\kappa_1 z\nu + (1/2)(i\kappa_2 z - \kappa_1^2 z^2)\nu^2\right) \times \exp\left(-i\nu(t - z/c)\right). \tag{6}$$

Equation (6) can be integrated to give

$$A(z,t) = \left(1 - \kappa_1 z\frac{\partial}{\partial t} - \frac{1}{2}\left(i\kappa_2 z - \kappa_1^2 z^2\right)\frac{\partial^2}{\partial t^2} + \ldots\right)A(0, t - z/c). \tag{7}$$

In Eq. (7) the first term on the right hand side denotes the vacuum solution, the second term represents lowest order differential gain, while the third and higher order terms are small and denote higher order effects. Equation (7) shows that the pulse propagates at the speed of light while undergoing differential gain (distortion). The quantity κ_1 can be negative in the presence of gain or absorption. In the case of gain, when $\kappa_1 < 0$, the front portion of the pulse is amplified more than the back. Note that the differential gain effect, can be recovered from Eq. (5) through a Taylor expansion. However, this is simply equivalent to expanding Eq. (5) so that the proper order of approximation is recovered, as was done in deriving Eq. (7).

It is interesting to note that for a Gaussian pulse, the integral in Eq. (3) can be evaluated exactly if the expansion of the refractive index is carried up to κ_2, i.e., lowest order GVD. Taking the input laser pulse to have the form $A(0,t) = a_0 \exp(-t^2/2T^2)$, where a_0 is the peak amplitude, the Fourier transform is $\hat{A}(0,\nu) = a_0 T \exp(-\nu^2 T^2/2)$. For this pulse form, the integral in Eq. (3) can be evaluated to give [3]

$$A(z,t) = \frac{a_0}{\sqrt{1 - i\kappa_2 z/T^2}} \exp\left\{ \frac{-[t - (1 + c\kappa_1) z/c]^2}{2T^2(1 - i\kappa_2 z/T^2)} \right\}, \tag{8}$$

where $\mathrm{Re}(1 - i\kappa_2 z/T^2) > 0$, i.e., $-\mathrm{Im}(\kappa_2) z/T^2 < 1$ is required for convergence of the integral. This analysis shows that the pulse propagates with velocity v_p (even if v_p is negative or greater than c) and remains Gaussian but with a different amplitude and width. This result is specific to Gaussian pulse which does not have a well-defined beginning or end [2,3].

III. Interpretation of Experimental Observations [1]

The results of our analysis may be used to interpret the experiment of Ref. 1. As in Ref. 1, the frequency-dependent susceptibility of the medium is taken to have the following form near the resonance frequencies

$$\chi(f) \cong \frac{\Delta n(\omega)}{2\pi} = \frac{M_1}{f - f_1 + i\gamma} + \frac{M_2}{f - f_2 + i\gamma}, \tag{9}$$

where $M_{1,2} > 0$ are related to the gain coefficients. The susceptibility in Eq. (9) represents a medium with two gain lines of spectral width γ at resonance frequencies f_1 and f_2. The gain spectrum for $M_{1,2} = M = 0.18$ Hz, $f_1 = 3.5 \times 10^{14}$ Hz, $f_2 = f_1 + 2.7$MHz and $\gamma = 0.46$ MHz, is shown in Fig. 2(a) (solid curve). For these parameters the deviation of the refractive index from unity $\Delta n(\omega)$ shown in Fig. 2(b) closely approximates that in Fig. 3 of Ref. 1. The input laser pulse envelope, is taken to have the form

$$A(z=0,t) = \begin{cases} a_o \sin^2(\pi t/2T), & 0 < t < 2T \\ 0, & \text{otherwise} \end{cases}, \tag{10}$$

where a_o is the pulse amplitude and $\omega_0 / 2\pi = (f_1 + f_2)/2$ is the carrier frequency. The spectrum associated with the input pulse is shown by the dashed curve in Fig. 2 and has no significant spectral components at the gain lines.

For the parameters of the experiment we find that the first order correction in Eq. (7), i.e. the term proportional to $\partial / \partial t$, is of order $\kappa_1 L/T \approx -1.6 \times 10^{-2}$ while the second order correction is $\kappa_2 L/T^2 \approx -10^{-3} i$. Hence, the expansion performed to obtain Eq. (7) is valid.

The differential gain effect requires that $\kappa_1 < 0$. Using Eq. (9) we find that κ_1 is approximately given by

$$c\kappa_1 \cong -8\pi \frac{(f_1 + f_2)}{(f_2 - f_1)^2} M ,$$

when $|f - f_{1,2}| >> \gamma$. In this case it is clear that a gain medium $(M > 0)$ is required for κ_1 to be negative. For this case, the gain coefficient is given by $-\mathrm{Im}(k_0) = 8M\gamma/(f_2 - f_1)^2$. Note that in an absorptive medium $(M < 0)$, κ_1 can also be negative provided $\left|f - f_{1,2}\right| << \gamma$. In this case differential absorption occurs in which the back of the pulse is absorbed more than the front [2,3].

The validity of Eq. (7) was verified by numerically solving the envelope equation to all orders in κ_ℓ. Figure 3 compares the solution given by Eq. (7) at the exit of the gain medium (dotted curves) with the vacuum solution $|A(0,t-L/c)|$ (solid curves). Panel (a) shows the entire pulse profile. Consistent with the experimental measurements, the leading edge is shifted forward in time relative to the vacuum solution by 62 nsec. Panel (b) shows three curves: the solid curve denotes the vacuum solution, the dotted curve shows the result obtained from Eq. (7), and the dashed curve shows the result obtained from Eq. (5). The dotted curve shows that the front of the pulse propagates with velocity c; the propagation is not superluminal. The unphysical solution, given by the dashed curve, shows the front of the pulse propagating at superluminal velocities. Panel (c) is an expanded view near the peak of the pulse showing that the front is amplified more than the back.

IV. Conclusions

We have analyzed the propagation of a laser pulse in a dispersive gain/absorptive medium using an approach based on the pulse envelope equation. Using this approach allows analysis of higher order dispersive effects and arbitrary input pulse shapes. We find that to properly describe pulse propagation, a consistent ordering of the approximations is necessary. We show that in a gain medium, the lowest order effect is that the pulse propagates with velocity c and undergoes differential gain, i.e., a distortion in which the front of the pulse is amplified more than the back. Our analysis indicates that this differential gain effect is misrepresented as a newly observed mechanism for superluminal propagation in Ref. 1. This effect should not be viewed as superluminal propagation, but is the result of pulse distortion due to the addition of photons to the front of the pulse.

Acknowledgements

This work was supported by the Office of Naval Research and the Department of Energy.

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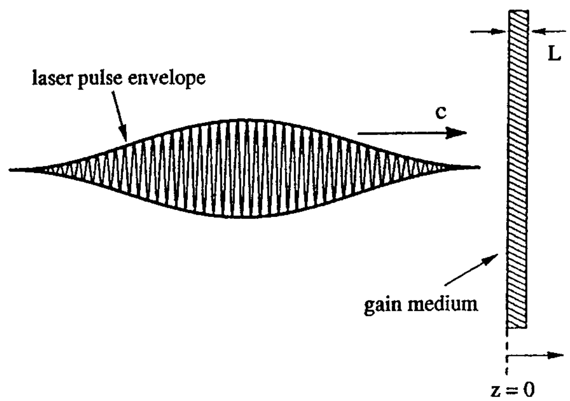

Figure 1. Schematic showing a long laser pulse entering a gain medium.

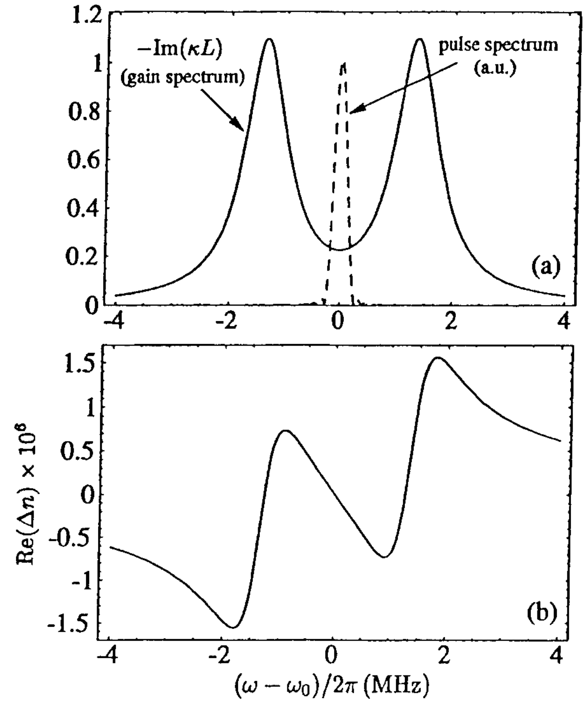

Figure 2. Gain spectrum (solid curve) obtained using the susceptibility in Eq. (9) for the parameters $M = 0.18$ Hz, $f_1 = 3.5\times10^{14}$ Hz, $f_2 = f_1 + 2.7$ MHz, and $\gamma = 0.46$ MHz. The dashed curve shows the spectrum associated with the pulse envelope of Eq. (10) with $T = 3.7$ μsec.

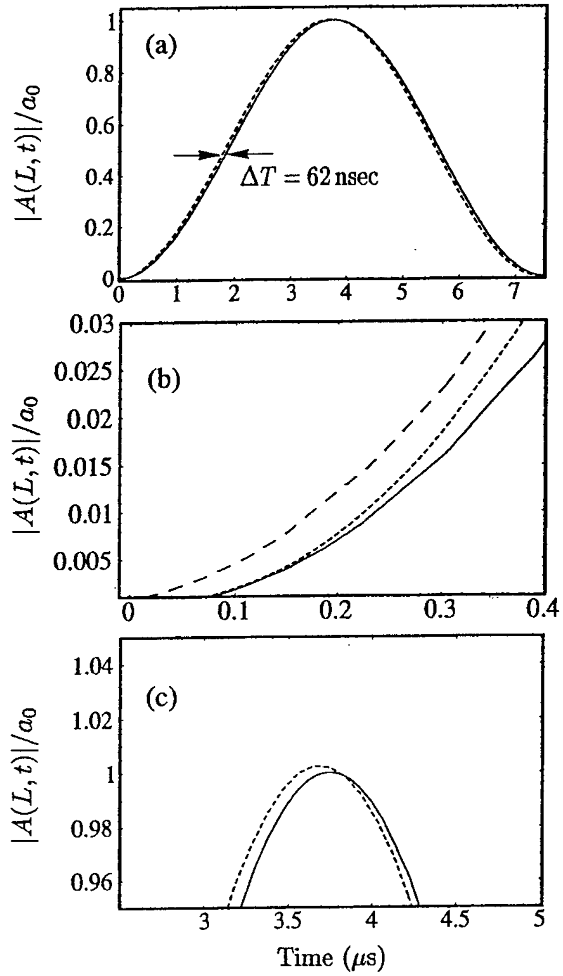

Figure 3: Dotted curves show the pulse envelope amplitude $|A(L,t)|$ at $z = L$ obtained from Eq. (7). Solid curves denote a pulse that has traveled a distance L through vacuum. The dashed curve in panel (b) is the unphysical solution obtained from Eq. (5) showing superluminal propagation. Panels (b) and (c) are expanded views of the front and peak of the pulse, respectively. The parameters for this figure are the same as in Fig. 2.