Automated Finite Element Modeling Procedures for Metal Forming Simulations

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1. Project Information

Principal Investigator: Mark S. Shephard
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2. Objectives

This project is concerned with the development of finite element procedures for effectively performing metal forming simulations in an automated analysis environment. Due to their ability to efficiently provide results to the required levels of accuracy, the desired formulation should support hp-adaptive analysis techniques. These requirements present a technical challenge due to the inability of standard finite element formulations to satisfy the Babuska-Brezzi conditions for general combinations of displacement (velocity) and pressure interpolations needed for hp-adaptivity in the presence of incompressibility constraints. To address this problem we have investigated stabilized finite element formulations for this class of problem. The second area considered was the development of a geometry-based simulation framework capable of supporting automated hp-adaptive technologies using such element formulations on problems.

3. Stabilized Finite Element Formulation

Typical metal forming processes involve large, nearly incompressible, deformations. In order to handle the incompressibility, mixed formulations, where the displacements and pressures are interpolated separately, have proved to be effective. However, it has been shown that these mixed formulations are not, in general, stable unless they satisfy a certain stability criterion, the so-called Babuska-Brezzi condition. This criterion puts restrictions on the relationship between the displacement and pressure finite element interpolation functions. This restriction limits computational efficiency since the order of interpolation is defined by this stability condition rather than by numerical accuracy, and thus makes p-adaptivity all but impossible. Because metal forming processes are generally three-dimensional in nature and involve very large deformations, adaptivity is critical to obtaining accurate results efficiently. A combination of h- and p- adaptivity is most desirable for gaining optimal efficiency. In this work, a stabilized finite element formulation for large deformation processes which eliminates the need to satisfy the Babuska-Brezzi condition is presented. The formulation is based on the stabilized formulations developed for linear problems by Hughes et al. Examples involving large deformation, three-dimensional, hyperelasticity with linear interpolations for both the displacement and pressure are presented to demonstrate the algorithm. The results show that the method can work for large deformation, nonlinear, three-dimensional problems, and is therefore a promising technique for effectively modeling metal forming processes.

Given a continuum body $\mathcal{P}$ we introduce the deformation $\varphi: P \to \Re^3$ and use the notation $B_o = \varphi_o(P) \subset \Re^3$ for the reference configuration and $B = \varphi(P) \subset \Re^3$ for the current configuration. A point in the reference configuration X is mapped into a point $x = \varphi(X) = X + u(X)$ in the current configuration where u denotes the displacement field. We define the deformation gradient $F = \nabla\varphi(X) = 1 + \nabla u$ and use the notation $C = F^T F$ for the right Cauchy-Green tensor. The symbols $\nabla[\bullet]$ and $\mathrm{Div}[\bullet]$ denote the gradient and divergence, respectively of a quantity $[\bullet]$ with respect to the coordinates $X \in B_o$. Assuming the volumetric deformation is purely elastic, we can write $p = \kappa U'(J)$, where p is the mean stress, κ is the bulk modulus, and U is the volumetric part of the stored energy depending only on the Jacobian J of the deformation Gradient F. The second Piola-Kirchhoff stress S can be decomposed into $S = JpC^{-1} + \tilde{S}$ where the first term is the volumetric part and $\tilde{S}$ is the remainder. In the following we present a stabilized mixed displacement-pressure formulation in material quantities defined in the reference configuration. The boundary value problem for equilibrium in the absence of body forces is given as: Find a displacement field u such that

$$\begin{aligned} -\mathrm{Div}[FS] &= 0 \quad \text{in } B_o \\ [FS]n_o &= g_o \text{ on } \Gamma_{oN} \\ u &= \bar{u}_o \quad \text{on } \Gamma_{oD} \end{aligned} \tag{1}$$

n_o is the exterior unit normal on Γ_o, the boundary of B_o, and g_o is a prescribed traction load on part of the boundary Γ_{oN}. We consider prescribed dirichlet boundary conditions $\bar{u}_o$ on Γ_{oD}. Γ_{oD} and Γ_{oN} describe the complete boundary of B_o. The stabilized mixed finite element formulation can now be derived from the strong form Eqn. (1) by multiplying with the perturbed weighting function $\overset{*}{u} + (\alpha h^2)/(2\mu)F^{(-T)}\nabla\overset{*}{p}$, and integrating over the domain

$$\int_{B_o} \mathrm{Div}[FS] \cdot \overset{*}{u} dV + \sum_{e=1}^{n_{el}} \frac{\alpha h^2}{2\mu} \int_{B_o^e} \mathrm{Div}[FS] \cdot F^{-T} \nabla\overset{*}{p} dV = 0 \tag{2}$$

where the perturbation term $(\alpha h^2)/(2\mu)F^{-T}\nabla\overset{*}{p}$ is applied element wise. α is a stabilization parameter, h is a mesh size parameter, and μ is shear stiffness. The first term in Eqn. (2) is integrated by parts as usual. We will focus our attention on the derivation of the stabilization term. Using the additive decomposition of the second Piola-Kirchhoff stress we obtain

$$\begin{aligned} \mathrm{Div}[FS] &= \mathrm{Div}[pJFC^{-1}] + \mathrm{Div}[F\tilde{S}] \\ &= JF^{-T}\nabla p + \mathrm{Div}[F\tilde{S}] \end{aligned} \tag{3}$$

The second step in the formulation was achieved by applying the Piola Identity $\mathrm{Div}[JF^{-T}] = 0$. Introducing Eqn. (3) into Eqn. (2) and introducing the pressure

$p = \kappa U'(J(\boldsymbol{u}))$ as an independent variable, the stabilized mixed weak formulation of Eqn. (1) is then: Find $(\boldsymbol{u}, p) \in V \times Q$ such that for all $(\overset{*}{\boldsymbol{u}}, \overset{*}{p}) \in V \times Q$

$$\int_{B_o} \tilde{S}:[F^T \nabla \overset{*}{\boldsymbol{u}}]\mathrm{d}V + \int_{B_o} JpC^{-1}:[F^T \nabla \overset{*}{\boldsymbol{u}}]\mathrm{d}V = L_{ext}(\overset{*}{\boldsymbol{u}})$$

$$\int_{B_o} \left(U'(J(\boldsymbol{u})) - \frac{1}{\kappa}p\right)\overset{*}{p}\mathrm{d}V - \sum_{e=1}^{n_{el}} \frac{\alpha h^2}{2\mu} \int_{B_o^e} JC^{-1}:[\nabla p \otimes \nabla \overset{*}{p}]\mathrm{d}V - \tag{4}$$

$$- \sum_{e=1}^{n_{el}} \frac{\alpha h^2}{2\mu} \int_{B_o^e} \mathrm{Div}[F\tilde{S}] \cdot (F^{-T} \nabla \overset{*}{p})\mathrm{d}V = 0$$

In the case when only linear shape functions are used, the divergence term in Eqn. (4) vanishes. We consider a hyperelastic material model with an additive stored energy function $W = \kappa U(J) + \tilde{W}(C)$ so that

$$S = 2\frac{\partial W}{\partial C} = 2\kappa U'(J) \cdot \frac{\partial J}{\partial C} + 2\frac{\partial \tilde{W}(C)}{\partial C} = JpC^{-1} + \tilde{S}, \tag{5}$$

We choose a Neo-Hookean material model where $U(J) = 1/2 \cdot (J-1)^2$ and $\tilde{W}(C) = 1/2 \cdot \mu[J^{-2/3}\mathrm{tr}C - 3]$

More complete details on the formulation and numerical results showing the superiority of the formulation are presented in references [18,19].

4. Geometry-Based Object-Oriented Simulation Framework

To date consideration of oriented programming in simulation software has focused on flexible structures with code reuse, application of symbolic computing, operating in parallel, linking with design processes and supporting interacting multiphysics simulations. Building on these efforts and the needs of adaptive simulation technologies we have constructed a geometry-based simulation frameworks that supports parallel adaptive simulation capabilities. This system, referred to as Trellis is based on [2,4]:

- A set of geometry-based structures which can support; (i) the direct linkage with company CAD information, (ii) all forms of adaptivity without introducing geometric approximation errors [8], and (iii) the high level integration of multiscale, multi-physics analysis methodologies.
- A careful decomposition of the geometry, physics, mathematical model, discretization and numerical methods into interacting classes. These structures support a variety of equation discretization methods. Both finite element [18,19] and partition of unity methods have been implemented [20].
- Adaptive control of each step of the simulation process from the selection of the mathematical model, through the model and domain discretization, to the selection of application of the numerical methods to solving the discrete system.

Conceptually Trellis is built on the view of an analysis as a transformation between three levels of description. The highest level description is that of the physical problem which is posed in terms of physical objects interacting with their environment. Since the goal of the analysis is to obtain reliable estimates of the response of the system the second level is a mathematical problem description that introduces some level of idealization, which also needs to be controlled to yield the desired accuracy. The third level is the numerical discretization constructed from a mathematical problem that involves another set of idealizations which also need to be controlled.

The structures used to support the problem definition, the discretizations of the model and their interactions are central to Trellis. The two structures of the geometric model and attributes are used to house the problem definition. The analysis discretizations are housed in the mesh structure. The final structure is the field structure which houses numerical solution results.

The geometric model representation is a non-manifold boundary representation based. The representation used for a mesh is similar to that used for a geometric model [3]: a hierarchy of regions, faces, edges and vertices. In addition, each mesh entity maintains a relation, called classification, to the model entity that it was created to partially represent. Understanding how the mesh relates to the geometric model is critical for both mesh adaptivity and understanding how the solution relates back to the original problem description. The topological representation can also be used to great advantage in performing adaptive p-version analyses as polynomial orders can be directly assigned to the various entities [8,22].

A problem with many "classic" numerical analysis codes is that the solution of an analysis is given in terms of the values at a set of discrete points. Trellis eliminates this problem by introducing a construct known as a field which describes the variation of a tensor over one or more entities in a geometric model. The spatial variation of the field is defined in terms of interpolations defined over a discrete representation of the geometric model entities, which can be a mesh.

The Trellis analysis process is a series of transformations of the problem from the original mathematical problem description through to sets of algebraic equations approximately representing the problem. The mathematical problem description level is described by a ContinuousSystem class, which contains the geometric model and the attributes which apply to that model, specified by a particular case node in the attribute graph. An instance of a ContinuousSystem is then transformed to an instance of the class DiscreteSystem which represents the discretized version of the model and attributes and the weak form of the partial differential equation (PDE). The particular analysis class that is used depends on the selected weak form of the PDE to be solved.

The DiscreteSystem class represents the problem in terms of contributions from a set of objects that live on the discrete representation of the model. These objects are called SystemContributors. There are three types of SystemContributors: StiffnessContributors contribute coupling terms between degrees of freedom of the system, ForceContributors contribute terms to the right hand side vector, and Constraints set specific values or constraints to given degrees of freedom. These objects are created by the Analysis object and correspond to an interpretation of attributes consistent with the weak form that the Analysis implements.

The Analysis class creates all of the SystemContributors and adds them to an instance of a DiscreteSystem. The DiscreteSystem is transformed into an AlgebraicSystem, an Assembler object. Multiple linear solvers can be used to solve the AlgebraicSystem. The most extensive capability included is the Portable, Extensible Toolkit for Scientific Computation (PETSc) from Argonne National Laboratory. These procedures have the dual advantage of working effectively in an object-oriented analysis framework and providing an efficient set of linear algebra routines.

5. Personnel

- Mark Shephard, faculty, Johnson Professor of Engineering, Director SCOREC.
- M. Beall, Assistant Director of SCOREC, part time student, Ph.D. in Mechanical Engng., Aeronautical Engng. and Mechanics, expected completion 5/99. (not supported by the project)
- O. Klaas, Postdoctoral Research Associate SCOREC.

6. Publications

[1] Adjerid, S., Flaherty, J.E., Jansen, K. and Shephard, M.S, "Parallel finite element simulations of Czochralski Melt Flows", *Modeling and Simulation-Based Engineering*, Tech Science Press, Palmdale, CA, pp. 821-8264, 1998.

[2] Beall, M.W. and Shephard, M.S., "A Geometry-Based Analysis Framework", *Advances in Computational Engineering Science*, Tech. Science Press, 557-562, 1997.

[3] Beall, M.W., and Shephard, M.S., "A General Topology-Based Mesh Data Structure", *Int. J. Num. Meth. Engng.*, 40(9):1573-1596, 1997.

[4] Beall, M.W., Klaas, O. and Shephard, M.S., "Adaptive Analysis within a Geometry-Based Framework", *Proc. Fourth US Nat. Cong. Computational Mechanics*, USACM, pp. 16, 1997.

[5] Beall, M.W., "A Geometry-Based Framework for Reliable Numerical Simulations", *Proc. Fourth US Nat. Cong. Computational Mechanics*, USACM, pp. 136, 1997.

[6] Beall, M.W. and Shephard, M.S., "An Object-Oriented Framework for Reliable Numerical Simulations", to appear *Engineering with Computers*, 1999.

[7] Bottasso, C.L., Klaas, O. and Shephard, M.S., "Data structures and mesh modifications for unstructured multigrid adaptive techniques", *Engineering with Computers,* 14(3):235-247, 1998.

[8] Dey, S., Shephard, M.S. and Flaherty, J.E., "Geometry-based issues associated with p-version finite element computations", *Comp. Meth. Appl. Mech. Engng.*, 150:39-55, 1997.

[9] Dindar, M., Lemnios, A.Z., Shephard, M.S., Flaherty, J.E., Jansen, K. and Kenwright, D., "Effect of tip vortex resolution on UH-60 rotor-blade hover performance", *American Helicopter Society Proceedings of the 1998 54th Annual Forum*, AHS, Part 1, pp. 45-57, 1998.

[10]Fish, J., Shek, K.-L., Pandheeradi, M., and Shephard, M.S., "Computational plasticity for composite structures based on mathematical homogenization: Theory and practice", *Comp. Meth. Appl. Mech. Engng.*, 148-53-73, 1997.

[11]Flaherty, J.E., Loy, R.M., Ozturan, C., Shephard, M.S., Szymanski, B.K., Teresco, J.D. and Ziantz, L.H., "Parallel structures and dynamic load balancing for adaptive finite element computations", *Applied Numerical Mathematics,* 26(1-2):241-263, 1998.

[12]Flaherty, J.E., Loy, R.M., Scully, P.C., Shephard, M.S., Szymanski, B.K., Teresco, J.D. and Ziantz, L.H., "Load balancing and communication optimization for parallel adaptive finite element methods", *SCCC '97,* IEEE, 246-255, 1997.

[13]Flaherty, J.E., Loy, R.M., Shephard, M.S., Szymanski, B.K., Teresco, J.D. and Ziantz, L.H., "Adaptive local refinement with octree load balancing for the parallel solution of three-dimensional conservation laws", *J. of Parallel and Distributed Computing*, 47:139-152, 1997.

[14]Flaherty, J.E., Loy, R.M., Shephard, M.S., Szymanski, B.K., Teresco, J.D. and Ziantz, L.H., "Predictive load balancing for parallel adaptive finite element computations", Arabnia, H.R., ed., *Int. Conf. on Parallel Distributed Processing Techniques and Applications,* PDPTA '97, 1:460-469, 1997.

[15]Flaherty, J.E., Loy, R.M., Shephard, M.S., Szymanski, B.K., Teresco, J.D. and Ziantz, L.H., "Adaptive local refinement with octree load-balancing for the parallel solution of three-dimensional conservation laws", IMA, Inst. for Mathematics and its Applications, U. Minn. IMA Report 1483, to appear.

[16]Garimella, R. and Shephard, M. S., "Boundary layer meshing for viscous flows in complex domains", *Proc. 7th International Meshing Roundtable '98*, SAND 98-2250, Sandia Nat. Labs., Albuquerque, NM, pp. 107-118, 1998.

[17]Klaas, O. and Shephard, M.S., "Automatic generation of octee-based three-dimensional discretizations for partition of unity methods", *Comp. Meth. Appl. Mech. Engng.,* to appear 1999.

[18]Klaas, O., Maniatty, A. and Shephard, M.S., "A stabilized finite element method for finite elasticity", *Comp. Meth. Appl. Mech. Engng.,* to appear 1999.

[19]Klaas, O., Maniatty, A. and Shephard, M.S., "Three-dimensional stabilized finite element model for finite deformation processes", *Proc. Plasticity '99*, pp. 365-368, 1999.

[20]Klaas, O. and Shephard, M.S., "Automatic generation of partition of unity discretizations for three-dimensional domains", *Modeling and Simulation-Based Engineering*, Tech Science Press, Palmdale, CA, pp. 89-94, 1998.

[21]O'Bara, R.M., Beall, M.W. and Shephard, M.S., "Specifying Analysis Information within a Geometry-Based Framework", *Proc. Fourth US Nat. Cong. Computational Meth.*, USACM, pp. 137, 1997.

[22]Shephard, M.S., Dey, S., and Flaherty, J.E., "A Straight Forward Structure to Construct Shape Function for Variable p-Order Meshes", *Computer Meth. Appl. Mechanics and Engng.*, 147:209-233, 1997

[23]Shephard, M.S., Beall, M.W. and O'Bara, R.M., "Revisiting the elimination of the adverse effects of small model features in automatically generated meshes", *Proc. 7th International Meshing Roundtable '98*, SAND 98-2250, Sandia Nat. Labs., Albuquerque, NM, pp. 119-131, 1998.

[24]Shephard, M.S., Flaherty, J.E., de Cougny, H.L., Bottasso, C.L. and Ozturan, C., "Parallel automatic mesh generation and adaptive mesh control", M. Papadrakakis, ed., *Solving Large Scale Problems in Mechanics: Parallel and Distributed Computer Applications,* John Wiley & Sons, Ltd., Chichester, 459-493, 1997.

7. Honors/Awards

- Fellow - Int. Association for Computational Mechanics
- Computational and Applied Sciences Award of the U.S. Association for Computational Mechanics - 8/97
- Fellow - U.S. Association for Computational Mechanics - 6/95
- Assoc. Fellow AIAA - 5/94

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