Optical Measurement of the Speed of Sound in Air Over the Temperature Range 300-650 K

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OPTICAL MEASUREMENT OF THE SPEED OF SOUND IN AIR OVER THE TEMPERATURE RANGE 300–650 K

ROGER C. HART*, R. JEFFREY BALLA†, AND G. C. HERRING‡

Abstract. Using laser-induced thermal acoustics (LITA), the speed of sound in room air (1 atm) is measured over the temperature range 300–650 K. Since the LITA apparatus maintains a fixed sound wavelength as temperature is varied, this temperature range simultaneously corresponds to a sound frequency range 10–15 MHz. The data are compared to a published model and typically agree within 0.1–0.4% at each of 21 temperatures.

Key words. laser thermometry, speed of sound, optical diagnostics

Subject classification. Fluid Mechanics

1. Introduction. The speed of sound in air near room temperature has been intensely studied over a range of various parameters such as frequency, humidity, and pressure. This body of work, stretching back three centuries, has been reviewed by Wong [1]. Most of this work was conducted in the audio (20–20,000 Hz) or hundreds-of-kHz frequency range. Less frequently, work was carried out in the MHz range. At room temperature, sound speed models for air [1] and pure gases [2] agree with measurement to 0.03% or better. However, previous work on the speed of sound in air is limited to temperatures of ~300 K and below. In this letter we present measurements of the speed of sound (10–15 MHz) in air at 1 atm over the 300–650 K temperature range and compare these data with theoretical values. This letter is an outgrowth of our earlier work [3] in developing remote diagnostics for flow temperature in wind tunnels.

We compare our experimental data to a model synthesized by Zuckerwar [4]. The speed of sound is written in the standard Laplacian form with corrections for various gas-phase phenomena. Thus,

\[ V_{\text{sound}} = \left( \frac{\gamma_0 R T}{M} (1 + C_1)(1 + C_2)(1 + C_3) \right)^0.5 \]

where \( \gamma_0 \) is the ratio of specific heats, \( R \) is the universal gas constant, \( T \) is the temperature, and \( M \) is the molecular mass. \( C_1 \) is a correction for the temperature variation of \( \gamma_0 \), \( C_2 \) is a correction for virial (non-ideal gas) effects, and \( C_3 \) is a combined correction for the vibrational, rotational, and translational (classical thermal and viscous) relaxations. For our experimental conditions, \( C_1 \), \( C_2 \), and \( C_3 \) are each \( \leq 0.05 \). Additional details of these corrections are described in Refs. [3] and [4].

2. Experimental Method. The experimental method is laser-induced thermal acoustics (LITA) [5]. Detailed descriptions of the LITA apparatus, procedure, and data analysis are described in Ref. [3]. Sound waves are produced by crossing two high-peak-power pulsed laser beams (532 nm) at \( \sim 1 \) deg, forming interference fringes at the crossing inside an oven. Two counter-propagating ultrasonic sound wave packets are launched by electrostriction. The propagation of these two acoustic waves is monitored by probing them with a long-duration laser pulse (750 nm) that is Bragg-diffracted into a fast optical detector. This probe signal decays as the wave packets propagate out of the sample volume (defined by the probe beam) and

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‡AMDB, NASA Langley Research Center, Hampton, VA 23681-2199.
Table 3.1
Summary of Model and Measurement for the Speed of Sound in Air from 300 to 650 K Using LITA

<table>
<thead>
<tr>
<th>$T$ (K)</th>
<th>$N$</th>
<th>$N_r$</th>
<th>$H$</th>
<th>$f^{LITA}$ (MHz)</th>
<th>$V^{LITA}$ (m/s)</th>
<th>$V^{mod}$ (m/s)</th>
<th>$\Delta V$ (m/s)</th>
<th>$\sigma_m$</th>
<th>$\sigma_{\Delta V}$</th>
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</thead>
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<tr>
<td>296.9</td>
<td>1432</td>
<td>15</td>
<td>34</td>
<td>10.18</td>
<td>347.5</td>
<td>346.2</td>
<td>1.1</td>
<td>0.018</td>
<td>0.51</td>
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<td>303.8</td>
<td>445</td>
<td>6</td>
<td>33</td>
<td>10.21</td>
<td>350.7</td>
<td>350.2</td>
<td>0.73</td>
<td>0.032</td>
<td>0.75</td>
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<td>313.0</td>
<td>282</td>
<td>3</td>
<td>46</td>
<td>10.55</td>
<td>354.2</td>
<td>355.7</td>
<td>-1.5</td>
<td>0.043</td>
<td>0.081</td>
</tr>
<tr>
<td>321.1</td>
<td>294</td>
<td>3</td>
<td>29</td>
<td>10.51</td>
<td>360.3</td>
<td>360.0</td>
<td>0.31</td>
<td>0.034</td>
<td>0.035</td>
</tr>
<tr>
<td>346.0</td>
<td>292</td>
<td>3</td>
<td>30</td>
<td>10.92</td>
<td>374.6</td>
<td>373.7</td>
<td>0.92</td>
<td>0.036</td>
<td>0.095</td>
</tr>
<tr>
<td>374.7</td>
<td>288</td>
<td>3</td>
<td>25</td>
<td>11.29</td>
<td>388.7</td>
<td>388.9</td>
<td>-0.20</td>
<td>0.054</td>
<td>0.066</td>
</tr>
<tr>
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<td>282</td>
<td>3</td>
<td>27</td>
<td>11.48</td>
<td>392.8</td>
<td>392.3</td>
<td>0.46</td>
<td>0.042</td>
<td>0.15</td>
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<tr>
<td>408.4</td>
<td>277</td>
<td>3</td>
<td>43</td>
<td>12.28</td>
<td>407.6</td>
<td>406.3</td>
<td>1.2</td>
<td>0.060</td>
<td>0.25</td>
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<tr>
<td>415.2</td>
<td>501</td>
<td>6</td>
<td>31</td>
<td>11.95</td>
<td>409.6</td>
<td>409.5</td>
<td>0.30</td>
<td>0.033</td>
<td>0.14</td>
</tr>
<tr>
<td>431.7</td>
<td>299</td>
<td>3</td>
<td>45</td>
<td>12.39</td>
<td>415.8</td>
<td>417.8</td>
<td>-2.0</td>
<td>0.053</td>
<td>0.23</td>
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<tr>
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<td>26</td>
<td>12.31</td>
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<td>423.0</td>
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<tr>
<td>467.8</td>
<td>660</td>
<td>7</td>
<td>34</td>
<td>12.75</td>
<td>434.7</td>
<td>434.8</td>
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<tr>
<td>482.6</td>
<td>497</td>
<td>6</td>
<td>32</td>
<td>12.84</td>
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<td>441.6</td>
<td>-1.1</td>
<td>0.05</td>
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<tr>
<td>499.3</td>
<td>903</td>
<td>10</td>
<td>32</td>
<td>13.17</td>
<td>449.2</td>
<td>449.3</td>
<td>-0.03</td>
<td>0.038</td>
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<tr>
<td>520.6</td>
<td>776</td>
<td>9</td>
<td>40</td>
<td>13.57</td>
<td>458.0</td>
<td>458.9</td>
<td>-1.0</td>
<td>0.043</td>
<td>1.34</td>
</tr>
<tr>
<td>528.4</td>
<td>496</td>
<td>6</td>
<td>34</td>
<td>13.54</td>
<td>461.5</td>
<td>462.2</td>
<td>-0.58</td>
<td>0.053</td>
<td>0.38</td>
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<tr>
<td>551.4</td>
<td>257</td>
<td>3</td>
<td>36</td>
<td>13.87</td>
<td>472.7</td>
<td>472.3</td>
<td>0.45</td>
<td>0.071</td>
<td>0.13</td>
</tr>
<tr>
<td>570.2</td>
<td>188</td>
<td>3</td>
<td>41</td>
<td>14.25</td>
<td>481.0</td>
<td>480.5</td>
<td>0.5</td>
<td>0.12</td>
<td>0.18</td>
</tr>
<tr>
<td>603.1</td>
<td>274</td>
<td>3</td>
<td>43</td>
<td>14.67</td>
<td>492.7</td>
<td>494.3</td>
<td>-1.6</td>
<td>0.10</td>
<td>0.31</td>
</tr>
<tr>
<td>631.3</td>
<td>257</td>
<td>3</td>
<td>41</td>
<td>14.99</td>
<td>504.5</td>
<td>505.7</td>
<td>-1.3</td>
<td>0.10</td>
<td>0.41</td>
</tr>
<tr>
<td>653.2</td>
<td>498</td>
<td>6</td>
<td>41</td>
<td>15.29</td>
<td>514.7</td>
<td>514.6</td>
<td>0.24</td>
<td>0.089</td>
<td>1.13</td>
</tr>
</tbody>
</table>

as the waves are attenuated through relaxation. In principle, the speed of sound can be determined from
the wavelength (derived from beam crossing angle) and frequency of the temporal modulation of the signal.
This modulation arises from the beating of two individual diffracted light beams (one from each sound wave
packet) that are Doppler shifted relative to each other.

Since we cannot measure our 1-deg crossing angle to 0.1% accuracy geometrically, we effectively measure
it with another LITA apparatus. We simultaneously generate a second LITA signal with the same crossing
angle in a reference cell at known temperature of ~ 295 K. The sound speed in the oven is then determined
from the ratio of the LITA frequencies observed in the oven, $f^{LITA}$, and reference cell, $f^{ref}$, and the known
sound speed $V^{ref}$ in the reference cell. Oven temperatures were read with a precision of 0.2 K with a
thermocouple, which had been calibrated by comparison with a precision NIST-traceable Pt thermometer.
Allowing for small temperature gradients in the oven, we estimate an absolute uncertainty in the measurement
of the oven temperature of 0.33% (±1 K at 300 K, growing to ±2 K at 600 K). The small day-to-day variations
in atmospheric pressure produce negligible changes in the speed of sound, relative to the accuracy that we
achieve here.

3. Experimental Results. The results of our experiment are summarized in Table 3.1. In what
follows, quantities associated with a single laser shot use subscript $i$, quantities averaged over a single run
(100 shots in ~ 2 min) use subscript $j$, and all quantities appearing in the table (without a subscript $i$ or $j$)
are averaged over a temperature bin of 3–15 runs. A total of 107 one-hundred-shot runs were made, of which more than 9500 shots yielded usable data. The data shown have been binned into 21 temperatures, the width of the bins varying from 0 K (all runs at the same temperature) to 5.0 K. The reason for consolidating the 107 runs into 21 temperature bins is to reduce the length of Table 3.1. For each good shot of each run, the oven sound speed \( V_i^{LITA} \) was determined, and for each run, an unweighted average \( V_j^{LITA} \) was found. Using the observed oven temperature, humidity, and the unweighted average of measured sound frequency \( f_j^{LITA} \) for each run, we calculated \( V_j^{mod} \) from the model (Eq. (1.1)).

\[
\begin{align*}
\text{Velocity Difference (AV from Table 3.1) between the measurements and the model as a function of temperature.} \\
\text{Uncertainties due to temperature uncertainties and sound velocity uncertainties (\( \sigma_m \) from Table 3.1) were added in quadrature to obtain the total uncertainties, indicated by the error bars.}
\end{align*}
\]

In Table 3.1, \( N_r \) is the number of runs and \( N \) is the total number of shots used in each bin. The average temperature \( T \), relative humidity \( H \), and model velocity \( V^{mod} \) for each bin were calculated as the average of per run values weighted by the number of good shots in each run, e.g. \( T = \Sigma(N_jT_j)/\Sigma N_j \). Average values of \( f_j^{LITA} \) and \( V_j^{LITA} \) were weighted with the per run standard deviations of the mean \( \sigma_j \). The quantity \( \sigma_m \) was calculated from the per run standard deviations of the mean \( \sigma_j \) weighted by \( N_j \) according to \( \sigma_m = [\Sigma(N_j(N_j - 1)\sigma_j^2)/\Sigma(N_j(N_j - 1))]^{1/2} \); thus, \( \sigma_m \) is the standard deviation of the mean of the distribution if all the runs in a bin are normalized to have the same mean value. The average of the differences \( \Delta V_j = V_j^{LITA} - V_j^{mod} \), weighted by per run standard deviations, is denoted by \( \Delta V \), while the standard deviation of the \( \Delta V_j \), again weighted by per run standard deviations, is denoted by \( \sigma_{\Delta V} \).

Comparison of the \( \sigma_m \) and \( \sigma_{\Delta V} \) allows evaluation of the short (\( \leq 10 \) min) and long-term (\( \geq 1 \) day) repeatability of the method. All bins containing three runs comprise data taken over a ten minute period; here \( \sigma_{\Delta V} \) is only a few times \( \sigma_m \) (average value = 2.7 for \( N_r = 3 \)). Bins containing more than three runs contain data from more than one day and show substantially higher values of \( \sigma_{\Delta V}/\sigma_m \) (\( \sim 30 \) for the \( N_r = 15 \) bin). Clearly our implementation of the reference cell technique does not provide long-term stability comparable to the short-term precision. The uncertainty due to temperature and the statistical uncertainty \( \sigma_m \) are comparable, and adding them in quadrature results in the total uncertainties (1 \( \sigma \)) shown as error bars in Fig. 3.1. Our measurements are in complete agreement with the model, and significantly smaller
uncertainties will be necessary to find deviations from the model.

4. Conclusion. Our results lead us to believe that this noninvasive method for generating and detecting free-field ultrasonic waves is promising for future work in gas-phase acoustics. The statistics of our sound speed measurements can be improved by averaging more laser shots. In addition, improvements in future LITA apparatus and ovens are possible. Thus, we anticipate that LITA can be used for characterization of the speed of sound for a variety of gases, frequencies, and temperatures with improved uncertainties compared to those we have achieved here.

We gratefully thank A. Zuckerwar for helpful discussions.

REFERENCES