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- COMMUNIST CHINA -

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COUPLING BETWEEN A RECTANGULAR WAVEGUIDE AND A CIRCULAR WAVEGUIDE OR A CYLINDRICAL CAVITY RESONATOR THROUGH AN APERTURE*

- COMMUNIST CHINA -

/Following is a translation of an article by Lin Wei-kan (2651 3634 1626), Ch'eng-tu Institute of Radio Engineering, in Wu-li Hsüeh-pao, Volume 15, Number 7, July 1959, pp. 368-376./

Abstract

Three distinct systems of coupling through an aperture are treated in this paper by means of approximations. They are, namely: co-axial coupling between a rectangular and a circular wave guide; coupling between a $TE_{1,0}$ mode in a rectangular wave guide and a TM_{120} mode in a circular cavity; and, finally, coupling between a $TE_{1,0}$ mode in a rectangular wave guide and a TM_{mn} mode in a circular wave guide. Through certain approaches to these problems, it is hoped that the reader will understand the necessity of treating complicated, fundamental microwave problems from the standpoint of physical concepts. It is believed that the formulae suggested herein are brand new, and will have certain applications which have not been adequately examined previously. Also, the techniques introduced here can be applied to other constantly arising problems which must be dealt with in microwave systems.

1. Foreward

Within certain microwave systems, we should use wave

* Received 6 February 1959.

guides or cavity resonators of varying geometries. In order to calculate the external characteristics of these systems, we have to consider the effects of discontinuity as they bear upon the problem of simultaneous employment of wave guide couplings of varying geometries. A calculation of this type is not a simple matter. In fact, it is almost impossible to achieve any accuracy when the change of geometry is appreciable. Therefore, in the study of microwave systems, many different methods of approximation must be used in the calculation of couplings of different elements, all in the hope of obtaining a high degree of accuracy as well as simplicity.

The purpose of this paper is to employ the calculations for three different types of coupling system in an effort to introduce some methods of approximation which might be useful in problem solving. The reader will readily see the necessity for a high degree of skill in solving these problems. Because of this factor, the methods used in this paper are not believed to be the best. Consequently, a more probing investigation will reveal even better techniques.

Even though the accuracy of the results presented in this paper have not been practically (i.e., experimentally) verified, the author has employed the first and the second coupling systems in actual practice and the results have been satisfactory.

2. End-on Coupling Between Rectangular and Cylindrical Wave Guides

Fig. 1 illustrates a coupling system consisting of a rectangular wave guide and a cylindrical wave guide. The rectangular wave guide is at $z<0$, the cylindrical wave guide is at $z>0$. At $z=0$, the two wave guides are coupled together through a small circular hole. The rectangular wave guide has sides $x=\pm a/2$, $y=\pm b/2$; the cylindrical wave guide has radius R. Before solving this problem we will normalize the normal waves propagating in the wave guides, in the following manner: taking z as the direction of

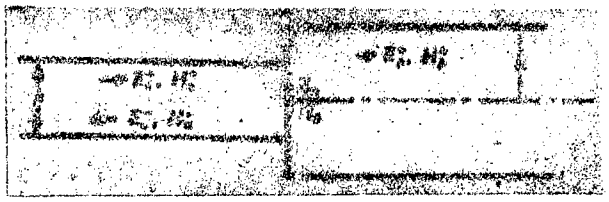

Figure 1

propagation, normalize the transverse components E_{tn} and H_{tn} on the xy plane; first normalize H_{tn}

$$\int_s |H_{tn}|^2 ds = 1 \tag{1}$$

then, according to wave guide theory, E_{tn} is also normalized:

$$\int_s |E_{tn}|^2 ds = Z_{on}^2, \tag{2}$$

where subscript t represents the value of the transverse component, n is the order of the normal waves; e.g., E_{tn} is the transverse component of the nth normal wave, Z_{on} is the wave impedance of the nth normal wave

$$Z_{on} = \sqrt{\frac{\mu}{\epsilon}} \frac{\lambda_{gn}}{\lambda},$$

where μ and ϵ are the permeability and dielectric constants, respectively, of the dielectric in the wave guide; λ is the operating wavelength, and λ_{gn} is the wavelength of the wave guide to the nth normal wave; and, s is the cross-section of the wave guide.

Assuming that the rectangular wave guide and the cylindrical wave guide can only propagate the lowest TE mode, then the transverse magnetic field components which satisfy the normalization conditions of equations (1) and (2) are (for a cylindrical wave guide):

$$\left.\begin{aligned} H_r &= \frac{1}{0.3455\sqrt{\pi}R^2} J_1\left(1.841\frac{r}{a}\right)\cos\phi\, e^{-j\frac{2\pi}{\lambda_g(b)}z}, \\ H_\phi &= \frac{1}{0.3455\sqrt{\pi}R^2} \frac{J_1\left(1.841\frac{r}{a}\right)}{\frac{1.841r}{a}} \sin\phi\, e^{-j\frac{2\pi}{\lambda_g(b)}z}; \end{aligned}\right\} \tag{3}$$

while, in a rectangular wave guide, they are:

$$H_x = \frac{1.414}{\sqrt{ab}} \sin\frac{\pi x}{a} e^{-j2\pi z/\lambda_g(a)}. \tag{4}$$

Now, let us assume that an incoming wave of unit amplitude, and a reflected wave of amplitude Γ are present in the rectangular wave guide; in the cylindrical wave guide there is only one outgoing wave of amplitude T. In the coupling aperture, the electric and magnetic fields, E_o and H_o, respectively, are yet to be determined.

We already know that if two electric and magnetic fields E_1, H_1, and E_2, H_2, of the same frequency satisfy

Maxwell's homogeneous equation, then in any defined region enclosed by a curved surface S, the following is true

$$\int_S (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \mathbf{n}\, ds = 0, \tag{5}$$

where n is the outward curve of surface S. In the problem at hand, we take a surface S in a cylindrical wave guide and apply eq. (5): a plane z=o, on which the coupling aperture lies, inside surface of the wave guide and a plane z=z, distant from plane z=o. We take the outgoing wave in the cylindrical guide as E_1, H_1, and take a normal wave with an am plitude of one, which can, and does propagate toward plane z=o, as E_2, H_2. Therefore, the EM wave on plane $z=z_1$ is composed of two oppositely propagating waves. Since we have assumed that the waves are already normalized, and that the amplitude of the outgoing wave in the cylindrical guide is T, we now have

$$\int_{z=z_1} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot d\mathbf{s} = 2TZ_E^{(b)} \tag{6}$$

where subscript b denotes the value of the cylindrical guide, and subscript a denotes the value of the rectangular guide, as shown in Fig. 1. On the inner surface of the guide, Eq. (6) takes a zero value; on the plane z=o, it takes the following form:

$$\int_{\substack{\pi \\ (\text{Nul.E})}} (\mathbf{E}_0 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_0) \cdot \mathbf{n}\, ds \tag{7}$$

Therefore, we arrive at the following result:

$$2TZ_E^{(b)} = \int_{\pi} (\mathbf{E}_0 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_0) \cdot \mathbf{n}\, ds = \mathbf{H}_2 \cdot \int_{\substack{\pi \\ (\text{Nul.E})}} \mathbf{n} \times \mathbf{E}_0\, ds + \int_{\substack{\pi \\ (\text{Nul.E})}} \mathbf{n} \cdot \mathbf{H}_0 \times \mathbf{E}_2\, ds. \tag{8}$$

In the above integral we assume that E_2 and H_2 are constant over the aperture. The integral value of n x E_0 is proportional to the magnetic moment of a magnetic dipole and it, in turn, is proportional to the magnitude of the incoming wave. Thus, applying the results of Eq. (5), we obtain

$$\left.\begin{aligned} T\frac{\lambda g^+}{\lambda} &= \frac{2\pi j}{\lambda} M \mathbf{H}_{2t} \cdot \mathbf{H}_{0t}, \\ M &= \frac{4}{3}\left(\frac{d}{2}\right)^3, \end{aligned}\right\} \tag{9}$$

where d is the diameter of the aperture at origin, and H_{2t} and H_{0t} are differentiated into the tangential components

(transverse components) of H_2 and H_0. Since what we are treating is of the $TE_{1,1}$ mode, there is no longitudinal component and, hence, no moment of electric dipole effect. We take the sum of the EM field on both sides of the aperture as the EM field, E_0 and H_0, over the aperture. Fig. 1 shows the EM fields on both sides of the aperture, so that, on plane z=o, we arrive at

$$\mathbf{H}_{at}=\mathbf{H}_{at}(1+\Gamma)+\mathbf{H}_{bt}T. \tag{10}$$

Substituting in Eq. (9)

$$T=\frac{2}{1-\frac{H_{at}^2}{\mathbf{H}_{at}\cdot\mathbf{H}_{bt}}+j\left(-\frac{\lambda_g^{(b)}}{2\pi M\mathbf{H}_{at}\cdot\mathbf{H}_{bt}}\right)} \tag{11}$$

Let us take two transmission lines of characteristic admittance Y_a and Y_b, and use reactive conductivity B of the shunt to couple them together; then, the coefficient of reflection Γ, and the coefficient of transmission T are, respectively:

$$\left.\begin{aligned}\Gamma&=-\frac{1-\left(\frac{Y_b}{Y_a}+j\frac{B}{Y_a}\right)}{1+\left(\frac{Y_b}{Y_a}+j\frac{B}{Y_a}\right)},\\ T&=1-\Gamma=\frac{2}{1+\frac{Y_b}{Y_a}+j\frac{B}{Y_a}}.\end{aligned}\right\} \tag{12}$$

When B/Y_a has a high value, Eq. (12) can be expressed as follows:

$$T=2/j\frac{B}{Y_a}. \tag{13}$$

Comparing Eq. (13) with Eq. (11), if the value of $(1-H_{at}^2/H_{at}\cdot H_{bt})$ is not high, then the normalized reactive conductivity b can be stated as

$$\left.\begin{aligned}\frac{1}{b}&=\frac{Y_a}{B}=\frac{2\pi M}{\lambda_g^{(b)}}\mathbf{H}_{at}\cdot\mathbf{H}_{bt},\\ \text{or}\quad b&=\frac{B}{Y_a}=-\frac{\lambda_g^{(b)}}{2\pi M}\frac{1}{\mathbf{H}_{at}\cdot\mathbf{H}_{bt}}.\end{aligned}\right\} \tag{14}$$

These two equations, when applied to the aperture coupling between two completely similar wave guide systems, give a degree of accuracy. If there is a conductive diaphram with a centered aperture, in the cylindrical guide; then, substituting the value of Eq. (3), and knowing that $\frac{J_1(x)}{x}\Big|_{x=0}\to\frac{1}{2}$, we have

$$b=\frac{0.236\,R^2\lambda_g}{M},\quad M=\frac{4}{3}\left(\frac{d}{2}\right)^3 \tag{15}$$

Eq. (14) can be arbitrarily applied to any coupling problem in a wave guide with (two) different cross-section areas, or cross-section shapes, provided we take the correct values of the normalized H_{at} and H_{bt} on both sides of the coupling aperture. In cases where the aperture is not circular, Eq. (14) still may be applied if we choose the proper M. For instance, when two wave guides of different cross-sections are coupled together through an aperture, this equation gives us the value of the reactive conductivity due to the factor of discontinuity. As to the problem of centered aperture coupling between rectangular guide and cylindrical wave guide, we have, from Eq. (14)

$$b = 0.487\lambda_g^{(b)}(\pi R^2 ab)^{1/3}/d^3 \quad (16)$$

In Fig. 2, a numerical example with plot is shown. In this example, we have had to include a conversion factor (due to the finite thickness of the aperture. The conversion factor is introduced by considering the membrane as a short transmission line, separating the two sides of the discontinuous plane, as shown in Fig. 3. However, thus far we have discussed only the lowest EM wave in the aperture, so in Fig. 3 t is the thickness of the conductive membrane in which the aperture is located. z_0' and λ_g' are, respectively, the wave impedance of the wave guide having the same cross-section as the aperture (correctly, the value after normalization of one wave guide impedance), and wave guide wavelength, $Y_0' \; 1/z_0'$. When the thickness t is not large, the series arm in Fig. 3 can be disregarded.

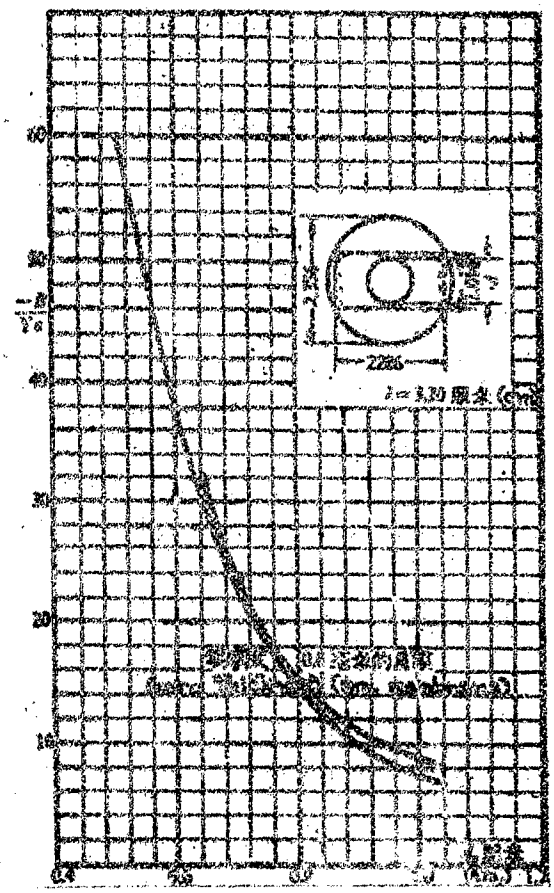

Figure 2. Inductive Coupling Reactive Conductivity Between Rectangular and Cylindrical Wave Guides.

In Fig. 2, the effect of the series arm was not con-

sidered when calculating the conversion factor of t 0.8mm.

Fig. 3 represents an equivalent circuit of the coupling system between two guides having wave impedances $Z_0^{(a)}$ and $Z_0^{(b)}$. Normalized reactive conductivity, representing discontinuity, can be calculated from Eq. (16).

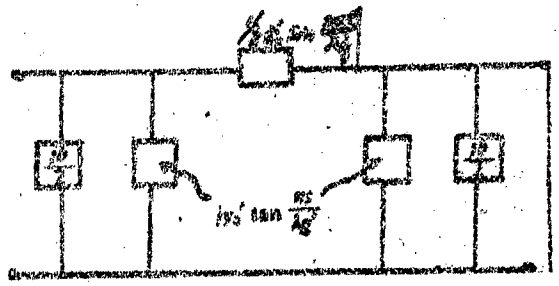

Figure 3. An Equivalent Circuit of Thick Aperture

3. Coupling Between $TE_{1,0}$ Mode in a Rectangular Wave Guide and TM_{120} Mode in a Cylindrical Cavity Resonator

As shown in Fig. 4, the short side of the rectangular guide is parallel to the axis of the cylindrical cavity resonator. The center of the rectangular wave guide cross-section is located on the perpendicular which bisects the surface of the cylindrical guide. The cross-section area of the rectangular guide is a x b, a>b, and the cylindrical cavity resonator has a radius and height of one (1). At the center of the rectangular guide there is a circular aperture of diameter d to couple the rectangular guide and the cylindrical cavity resonator together.

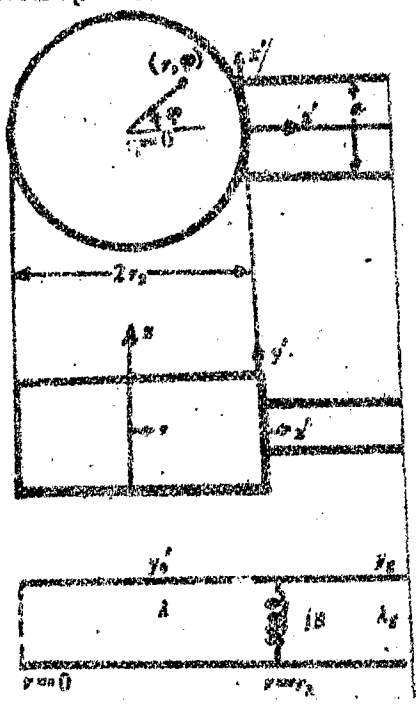

Fig. 4. Coupling Between Radial Transmission Line and Uniform Transmission Line.

Assume that the propagating mode in the rectangular wave guide is $TE_{1,0}$, that there are only H_x', E_y', and H_z' components. These three components will excite H_y, E_z, and H_r in the cylindrical wave guide. We shall consider this resonator as a radial transmission line. Also, we shall assume there is only a TM_{120} mode existing in this radial transmission line. This transmission line is coupled through an inductive admittance to a uniform line which can only support a $TE_{1,0}$ mode as shown in Fig. 4. The coordinate system used in the coupling together of these two wave guides is also shown in Fig. 4. Finally, we can assume that the

energy is propagated from the radial transmission line to the uniform line through the coupling element.

If the linear dimensions of the coupling aperture are much less than that of the wavelength, then the radius of curvature of the side surface of the cylindrical cavity resonator is comparable with the operating wavelength. Also, it is very large when compared with the linear dimensions of the aperture; and, therefore, the field distribution in the proximity of the aperture in the cavity is approximately similar to that of a $TE_{1,0}$ mode in a rectangular guide of width $a'=\pi r_2$, height $b'=1$. Hence, as a first approximation we can consider this aperture as the coupling between two wave guides of different cross-sections, one of cross-section $(\pi r_2) \times 1$, and the other $a \times b$. The reason that we take $a'=\pi r_2$, instead of $a'=2\pi r_2$ is because, in treating a TM_{120} mode, there are two half-sine wave variations along the circumference, and one half-sine variation occupies one-half of the circumference. Hence, the results of Eq. (14) can be applied to this problem -- i.e., we can take the value of b, in the following equation, as the coupling reactive conductivity of this problem:

$$b_1 = \frac{8\lambda_g}{a^2}\sqrt{\frac{abr_2 l}{4\pi}} \tag{17}$$

In the following, we incidentally calculate the transformation of the natural wavelength of the TM_{120} mode oscillation due to the loading of b_1 to the cylindrical cavity resonator. We already know that the condition for resonance is: the total reactive conductivity at $r=r_2+0$ should be zero -- i.e.,

$$y + jb_1 = 0, \tag{18}$$

where y represents the relative admittance at $r=r_2$, with respect to that of an open circuit at $r=0$:

$$\left.\begin{aligned} y &= -j\frac{J_1'(Kr_2)}{J_1(Kr_2)} \\ \text{or} \quad & \\ \frac{J_1\left(\frac{2\pi}{\lambda}r_2\right)}{J_1'\left(\frac{2\pi}{\lambda}r_2\right)} &= +\frac{1}{b_1}. \end{aligned}\right\} \tag{19}$$

where J_1' is the derivative, with respect to its variable, of the Bessel function, first kind first order. λ is the operating wavelength; also, it is the resonant wavelength whenever Eq. (19) is applicable.

In order to solve for the value of r/λ, we assume that the presence of b does not perceptibly change λ; hence, we can now introduce an infinitesimal quantity ϵ. Therefore,

let

$$r_2 = \frac{7.02}{2\pi}\lambda\left(1+\frac{\epsilon}{7.02}\right) \tag{20}$$

where 7.02 is the second root of $J_1(x)$ -- we do not count the first root zero -- take the next root, 3.83, as the first one. When $b_1 \to \infty$, $\epsilon \to 0$; therefore, $\lambda_0 = 2\pi/7.02 \cdot r_2$. Substituting Eq. (20) into Eq. (19), we have

$$\frac{J_1(7.02+\epsilon)}{J_1'(7.02+\epsilon)} = -\frac{1}{b_1} \tag{21}$$

To solve for ϵ, apply the Taylor series expansion to the second root (7.02) of the Bessel function; for the infinitesimal ϵ, omitting terms of a higher order than ϵ^2, we get

$$\epsilon = \frac{1}{b_1 - 1/7.02} \tag{22}$$

Eq. (22) gives the conversion for radius r_2 with a fixed resonant wavelength λ and a fixed coupling reactive conductivity b_1. Conversely, if we take r_2 as fixed at $7.02/2\pi \cdot \lambda_0$, then the presence of coupling reactive conductivity b_1 will cause a rate of change $\Delta\lambda$ of natural wavelength.

$$\Delta\lambda = \frac{2\pi}{7.02} r_2 \frac{\epsilon}{7.02} \tag{23}$$

Variations in height l do not affect the value of the natural wavelength, its applicable value can be determined from other circuit factors.

4. Coupling Between Rectangular Wave Guide and Cylindrical Wave Guide Through an Aperture on the Side Surface of the Cylindrical Guide

Assume that the short side of the rectangular guide is parallel to the axis of the cylindrical wave guide, and that there is only a $TE_{1,0}$ mode propagating in the rectangular wave guide. The $TE_{1,0}$ mode field components are H_z', E_y', and H_x', these components will excite H_r, E_z, and H, in the cylindrical guide.

As we did previously, assume that there is an incoming wave of unit amplitude and a reflected wave of amplitude Γ. As in Fig. 5, there will be two outgoing waves in the cylindrical guide propagating along a $+z$ and $-z$ orientation with amplitudes T^+ and T^-, respectively. Also, assume that there is only one mode existing in the rectangular wave guide and it excites only one mode in the cylindrical guide. Therefore, in the cylindrical guide, the transverse compo-

nent of the EM field can be expressed as

$$\left.\begin{aligned} E_t &= T^+ E_a e^{-\gamma z}, \\ H_t &= T^+ H_a e^{-\gamma z}, \quad (z>0) \\ E_t &= T^- E_a e^{\gamma z}, \\ H_t &= -T^- H_a e^{\gamma z}, \quad z<0 \end{aligned}\right\} \qquad (24)$$

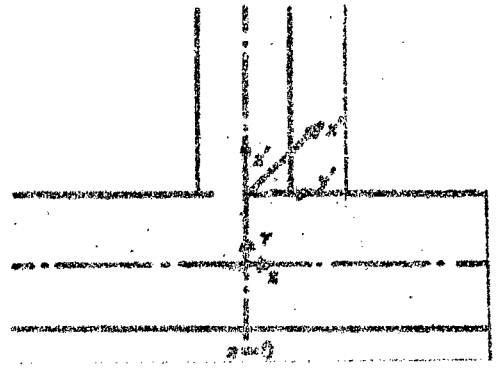

Figure 5. Coordinate System Used in Coupling System Between Rectangular and Cylindrical Wave Guides.

where the positive subscripts represent a physical quantity propagating along the positive z direction, and those with minus signs denote a physical quantity propagating along the negative z direction. Subscript a indicates the physical quantity of the normal wave propagating in the cylindrical guide, and subscript t denotes the transverse component of the EM field. γ is the propagation constant, $\gamma = 2\pi/\lambda_{ga}$.

We use the following formula once again

$$\int_S n' \cdot (E_1 \times H_2 - E_2 \times H_1) ds = 0 \qquad (5)$$

where n' is an outward normal to the surface S. We then have the well-known problem of slot antennae. We shall use Eq. (5) twice to arrive at the T^+ and T^- in Eq. (24).

First, assume that E_1 and H_1 are the excited EM waves (Eq. 24) in the cylindrical guide that pass through the aperture; as E_2, H_2, we take a normal wave propagating from left to right; as S, take the two planes, $z=\pm z_1$, and the inner surface of the guide containing the aperture, with the aperture centered at $z=0$. Now we integrate Eq. (5): on plane $z=z_1$, both EM waves, E_1, H_1, and E_2, H_2, are propagating in the same direction; from orthogonic theorems (3) and (8), and normalization conditions (1) and (2), we know that the integral has a zero value; on plane $z=-z_1$, where E_1, H_1, and E_2, H_2, are propagating in opposite directions, the integral takes a value of $2Z_{on}T^-$; on the inner surface of the guide, the integral value of the second term of Eq. (5) is zero, since $E_2 \times H_1 \cdot n = H_1 \cdot (n' \times E_2)$, and $n' \times E_2$ is always zero, when E_2 is the electric field of a normal wave. However, the value of the first term is not cancelled out, so that

$$2Z_{on}T^- = +\int_{n} E_1 \times H_2 \cdot n' ds$$

Secondly, assume E_1 and H_1 are still the EM waves excited in the cylindrical wave guide when passed through the small hole. This time, though, take a normal wave propagating from right to left as E_2, H_2; then, take the same surface as in the previous, as S. Now, along plane $z=-z_1$, Eq. (5) gives a value of zero; along plane $z=+z_1$ it gives $2Z_{on}T^+$, so that

$$2Z_{on}T^+ = \int_{\Pi} \mathbf{E}_1 \times \mathbf{H}_2 \cdot \mathbf{n}\, ds$$

Substituting Eq. (24) into the above two integrals, expressing Z_{on} as Z_0 for simplicity, we have

$$\left.\begin{aligned} 2T^+Z_0 &= \int_{\Pi} (-jE_{1s}H_{as} - E_{1s}H_{as})\, ds, \\ 2T^-Z_0 &= \int_{\Pi} (-jE_{1s}H_{as} + E_{1s}H_{as})\, ds. \end{aligned}\right\} \qquad (25)$$

where quantities with subscript "1" are components of the EM field on the aperture; whereas, those with subscript "a" are normalized components of the normal EM wave.

In our problem, we discuss the EM waves of $TM_{m,n}$ mode in the cylindrical guide, so $H_{az}=0$, and we have

$$T^+ = -T^- = -\frac{1}{2Z_0}\int_{\Pi} H_{as}E_{1s}\, ds \qquad (26)$$

When the aperture is very small, we then have

$$T^+ = -\frac{H_{as}}{2Z_0}\int_{\Pi} E_{1s}\, ds = -H_{as}\frac{4\pi jM}{2Z_0\lambda}\sqrt{\frac{\mu_0}{\epsilon_0}}H_{0t} \qquad (27)$$

where H_{0t} is the tangential component of the magnetic field on the aperture. As it was pointed out in the second paragraph, we can take

$$H_{0t} = 2T^+H_{at} + (1+\Gamma)H_{bt}$$

where H_{bt} is the tangential component of the normalized magnetic field of the normal wave in the rectangular wave guide at the center of the aperture. Solving for T^+, we have

$$T^+ = \frac{2}{2\left(1-\frac{H_t^2}{\mathbf{H}_a\cdot\mathbf{H}_t}\right) - j\frac{\lambda^2}{2\pi\lambda_g M\mathbf{H}_a\cdot\mathbf{H}_t}} \approx \frac{2}{j\left(-\frac{\lambda^2}{2\pi\lambda_g M\mathbf{H}_a\cdot\mathbf{H}_t}\right)} \qquad (28)$$

From Eq. (26) we may observe that, when speaking of a "b"-type wave, or a cylindrical wave guide, H_t is continuous at the aperture, while E_t is not continuous; therefore, this particular aperture can be represented, as far as the effect is concerned, by a series element, connected in series with an "a"-type wave transmission line. An equiva-

lent circuit diagram of this coupling system is shown in Fig. 6, where $\lambda_g^{\prime}$ is the wave guide wavelength of an $H_{1,0}$ (or $TE_{1,0}$) mode in the rectangular guide, and $\lambda_g^{\prime}$ is the wave guide wavelength of an $E_{m,n}$ (or $TM_{m,n}$) mode in the cylindrical wave guide. Therefore, the value of b is

$$b = -\frac{2\pi\lambda_g^{\prime} M M_1 \cdot M_2}{\lambda^2} = -2\pi \frac{\lambda_g^{\prime}}{\lambda} \frac{1}{\lambda} \frac{d^2}{3} \frac{1}{\sqrt{ab\pi R^2}} \qquad (29)$$

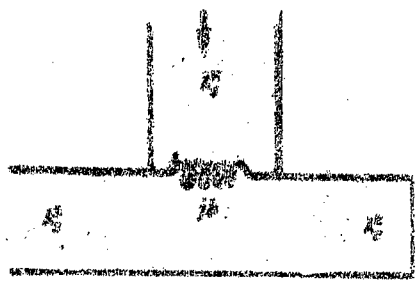

Figure 6. An Equivalent Circuit of the Coupling System Between Rectangular and Cylindrical Wave Guides.

5. Conclusion

In the second paragraph, we found a formula for coupling between the lowest $TE_{1,0}$ (dominant mode) mode in a rectangular wave guide and the lowest $TE_{1,1}$ (dominant mode) mode in a cylindrical wave guide. We also introduced a method for converting the thickness of the membrane. In the third paragraph, we applied the concept of radial transmission lines to the coupling system between a TM_{120} mode oscillation operating on the cutoff wavelength and a rectangular wave guide, which enables us to use the results of the second paragraph. In the fourth paragraph, we applied the accumulated knowledge of slot antennae to the coupling system between a rectangular wave guide and a $TM_{m,n}$ mode oscillation of a cylindrical wave guide therein obtaining useful results. In the third and fourth paragraphs, of course, we could apply all of the methods introduced in the first paragraph which were concerned with necessary conversion of membrane thickness.

These results are very useful in microwave filters, as well as other microwave circuits -- such as ferrite amplifiers, or frequency modulation circuits. In practical application, we frequently must convert the parallel discontinuous reactive conductivity into an ideal transformer, such as that illustrated in Fig. 7, where we have omitted the series arms. These series arms are of the numerical order $1/b$. When the coupling aperture is small, then the value of b is large, proportionately so -- and, it is permissible to omit them.

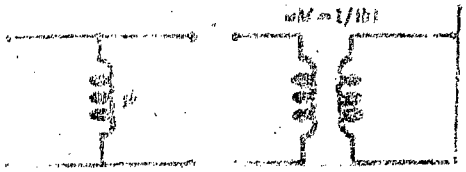

Figure 7. An Equivalent Ideal Transformer for Parallel Coil.

6. Author's Note

This paper was first completed early in 1957 for presentation before the opening meeting of the National Electronics Institute. Since the meeting was postponed several times, it was consequently never presented.

Recently, however, the author has had several opportunities to apply the conclusions stated in this paper. Therefore, I have rewritten the original paper for the purpose of publication -- in the hope that it might act as a stimulus during the "Great Leap Forward".

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