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Silicon Nitride Hybrid Bearing Fatigue Life Comparisons

15 January 1999

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Research to improve high-speed ball bearings for spacecraft applications has led to development of ceramic materials for bearing components, and the need to acquire sufficient fatigue life data to show the merits of various ceramic materials and fabrication processes, in comparison with the vast amount of steel bearing fatigue data acquired over many decades.

In order to eliminate bias from such evaluations it is best to conduct comparative fatigue tests with steel bearings that are geometrically similar and in the same type of test rig.

This report addresses some recent fatigue tests of hybrid bearings with silicon nitride (Si_3N_4) balls and Crucible Steel Company M62 steel raceways and a comparison set of all 52100 steel bearings.

Results indicate that the bearings with ceramic (silicon nitride) balls are superior to the steel bearings.

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1. Introduction

High-speed ball bearings for spacecraft applications have been subject to intensive research and development in efforts to achieve longer reliable fatigue life under higher operating load levels (1,2). This research has led to development of ceramic materials for bearing components, and the need to acquire sufficient fatigue life data to show the merits of various ceramic materials and fabrication processes, in comparison with the vast amount of steel bearing fatigue data acquired over many decades. Such comparison testing may involve a limited number of developmental bearings, and some limits in time available to conduct standard fatigue testing.

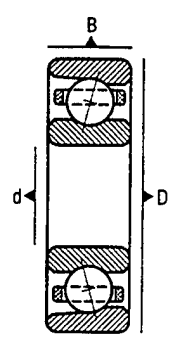
In order to eliminate bias from fatigue life evaluations of new materials and configurations, it is best to also conduct fatigue tests with all steel bearings that are geometrically similar and in the same type of test rig. Tests commonly used are individual bearing tests or sets of bearings tested to first failure.

This report addresses some recent fatigue tests (3) of hybrid bearings, using Norton NBD200 silicon nitride balls and Crucible Steel Company M62 steel raceways, and a comparison set of all 52100 steel bearings. The bearings were manufactured by Miniature Precision Bearings Company (Keene, NH) and tested by Timken Research (Canton, OH). The bearings were all of 207H size as shown in Figure 1.

Tests of the steel bearings were conducted in sets of four bearings each. Upon the first failure in the set, all were replaced and testing was resumed. Six such bearing sets of four were tested, giving six values of fatigue life corresponding to the least values in each set of four. These bearings were tested at 3000 lb load each, at a shaft speed of 5400 rpm. Fatigue life of the steel bearings varied from 17 h (5.6 million shaft cycles) to 910 h (294 million shaft cycles).

The hybrid bearings with silicon nitride balls were tested in the same rig. There were 24 bearings tested at 1500 lb load each, at 5400 rpm. Suspension times varied from about 2600 h to 2800 h. The analysis here uses a value of 2500 h, to be inherently conservative.

The 3000 lb load on the 52100 steel bearings, and the 1500 lb load on the hybrid bearings, produce an equivalent 272 ksi Hertz contact stress; however, bearing design is dictated by absolute load. In order to evaluate the merits of the silicon nitride hybrid bearings with respect to the 52100 steel bearings at equivalent load, it is necessary to correct for the different loading and also to establish a consistent comparison between the six observed least-of-four steel bearing fatigue lives, and the observed non-failure of 24 silicon nitride bearings for the reference 2500 h suspension.



Size 207H

D = 2.8346 in.

d = 1.3780 in.

B = 0.6693 in.

35 mm (1.3780 in.) shaft

Ball Diam. = 0.46875 in.

Figure 1. Bearing tested in fatigue.

This report describes the approach to the data analyses, and the methods of normalizing these data for an "apples-to-apples" comparison. The report covers a brief background on the Weibull Statistical Distribution and its use for fatigue life scatter, size effect, and life prediction, and provides rationale for the ground rules established for this comparison between the silicon nitride ball bearing and the 52100 steel counterparts.

The data from these tests are shown in the Tables 1 and 2 in Section 3.

The ground rules for comparative analysis were:

Compare life at a bearing load of 1500 lb

Fatigue Life Weibull Slope use $e = 1.5$

Load-Life Exponent, from $(LP)^p = \text{constant}$, use $p = 3$

Anchored at the *derived* 52100 single steel bearing median life for a load of 1500 lb:

L50 = 1,521 million shaft cycles

2. Summary

By the ground rules defined in this report, the Group B, silicon nitride hybrid ball bearings have greater fatigue life than corresponding Group A, all 52100 steel bearings.

At 50% confidence Group B (silicon nitride) has 4X greater life than Group A (52100 steel).

At 65% confidence Group B (silicon nitride) has 3X greater life than Group A (52100 steel).

At 90% confidence Group B (silicon nitride) has 2X greater life than Group A (52100 steel).

3. Reference Test Data and Problem Statement

Test data were generated on size 207H bearings, with alternative materials, one having steel 52100 balls and the other silicon nitride balls. The data are shown in Table 1 and Table 2:

Table 1. Group A All-52100 Steel Bearing Fatigue Data
(4 bearings per test to first failure)

At 3000 lb	Group A Data (52100)	Group A Data (52100)
rank j	time to fail h	shaft cycles to fail @ 5400 rpm
1	17.4	5,637,600
2	46.7	15,130,800
3	127	41,148,000
4	339	109,836,000
5	364	117,936,000
6	910	294,840,000
Median =	**233**	**75,492,000**
Average =	**301**	**97,421,400**

Table 2. Group B Silicon Nitride Hybrid Bearing Fatigue Data

At 1500 lb	Group B Data (silicon nitride)	Group B Data (silicon nitride)
Number Tested	Reference Test Time h	Total Cycles @ 5400 rpm
24	2500	810,000,000
Fatigue Life = Suspended with no failures		

In order to determine the relative merits of Group B vs Group A bearings, the comparison must account for two particular distinctions between Group A and Group B:

1. Group A data represent in each case the first failure among a group of four bearings, tested simultaneously, with a new group of four bearings submitted for test upon the first failure. The data for Group B are for 24 individual bearings tested for 2500 hours, with none failing.

2. Group A bearings were tested at 3000 lb load, whereas the Group B bearings were tested at 1500 lb. By the defined ground rule below, Group A fatigue life is multiplied by 8 to normalize the load effect to Group B load.

The defined ground rules were to use the Weibull Distribution for fatigue life, and the median life of Group A individual bearings as a reference anchor point, with:

Weibull Life Modulus, $e = 1.5$

Load vs Life Exponent, $p = 3$

4. Weibull Distribution - Brief Overview

4.1 Background

Weibull published his two seminal papers on statistical theory of brittle fracture (6,7) in the Proceedings of the Royal Swedish Academy of Engineering Sciences, the same publication used by Lundberg and Palmgren (4,5) to present their analytical models for rolling element bearing fatigue life. Consequently, their use of the Weibull model to correlate fatigue life was influenced by their direct and early knowledge of the work. It was not until the late 50's that Weibull's work gained a significant following in the United Sates, following his publications in American technical journals. Weibull's impact on the technology of brittle material was more or less simultaneous with the spreading use of his statistical functions to represent fatigue data (9). The Weibull functional form is a generalized exponential distribution that yields several standard distribution forms by simply varying a single parameter. It has the very desirable character that the cumulative form may be written in closed form, and is easy to apply to data, always giving a rational fit to at least portions of many types of observations. One of Weibull's most successful publications in promoting use of his theory is a paper entitled "A Distribution Function of Wide Applicability," which dwelt on its general utility, rather than the underlying rationale of weakest link phenomena and asymptotic extreme value distributions.

The original Weibull Theory of Statistical Brittle Strength (1939) is given by:

$$S = \mathrm{Exp} - \left\{ \int_{V_T} \left(\frac{\sigma}{\sigma_o} \right)^m dV \right\}$$

S is the cumulative survival fraction for the variable σ , in this case the tensile stress distributed throughout a volume V, which is scaled by σ_o and raised to an exponent m, called the Weibull Modulus (m), Weibull Slope (e), or shape factor (α).

The Weibull theory accounts in very natural way for several important characteristics applicable to fatigue testing as well as to strength of brittle materials. One such characteristic is the size effect, according to which the strength of a large brittle structure of volume V is inherently weaker than a smaller structure of the same material. The usual analogy cited is a chain failing at its weakest link. This applies to the case of a group of bearings simultaneously under test, with tests terminated upon the first failure in the group.

The size effect is expressed in the following equations for survival probability, for volume V, and for unit volume:

$$S_V = \mathrm{Exp} - V \left(\frac{\sigma_V}{\sigma_o} \right)^m \qquad S = \mathrm{Exp} - \left(\frac{\sigma}{\sigma_o} \right)^m$$

At equal probability, the small article is stronger by the factor $V^{1/m}$. In a similar manner, the Weibull Theory allows rational inference of the individual bearing lifetime distribution from observations of the least among a set of N test bearings tested simultaneously.

The cumulative survival probability of the individual bearings, S, is related to the survival probability, S_N, of the least among N by:

$$S_N = \text{Exp} - N\left(\frac{L_N}{L_o}\right)^m \qquad S = \text{Exp} - \left(\frac{L}{L_o}\right)^m$$

At equal probability, the individual bearing lifetimes are greater than the least life among N bearings by the factor $N^{1/m}$.

4.2 Normalized (Standardized) Forms of the Weibull Distribution

The scale factor, Lo, can be eliminated by normalizing to an arbitrarily selected reference value of life, Lr, and its corresponding survival probability, Sr:

$$S = \text{Exp} - \left[\text{Ln}\left(\frac{1}{S_r}\right)\left(\frac{L}{L_r}\right)^m\right] \qquad S = S_r^{\left(\frac{L}{L_r}\right)^m}$$

The most convenient normalization reference for bearing fatigue is the median life, $L50$, or sometimes the 10% failure probability life, $L10$, giving the forms below:

$$S = \text{Exp} - \left[\text{Ln}(2)\left(\frac{L}{L_{50}}\right)^m\right] \qquad S = (0.5)^{\left(\frac{L}{L_{50}}\right)^m}$$

$$S = \text{Exp} - \left[\text{Ln}\left(\frac{1}{0.9}\right)\left(\frac{L}{L_{10}}\right)^m\right] \qquad S = (0.9)^{\left(\frac{L}{L_{10}}\right)^m}$$

The Weibull Modulus or Weibull slope, m or e, is usually estimated by a least-square linear fit to a plot of the double logarithm of the above equations. The survival probability, at observed life Lj, is assigned by $(j - 0.3)/(N+0.4)$, where j is the rank number from shortest life, and N the total number of tests, or sample size:

$$\text{LogLn}\left(\frac{j - 0.3}{N + 0.4}\right) = m\text{Log}\left(\frac{L_j}{L_r}\right) + \text{LogLn}\left(\frac{1}{S_r}\right)$$

For example, for median normalized data, at $L50$:

$$\text{LogLn}\left(\frac{j - 0.3}{N + 0.4}\right) = m\text{Log}\left(\frac{L_j}{L_{50}}\right) + \text{LogLn}\left(\frac{1}{0.5}\right)$$

Several other useful forms of the Weibull Distribution are presented below. These are forms that may be used to estimate the Weibull slope based on observed *L10* and *L50* from experimental tests.

Estimate Weibull Modulus from two L values, e.g. ratio of *L50* to *L10*:

$$\mathbf{m} = \frac{\mathbf{Ln}\left(\dfrac{\mathbf{Ln}\left(\frac{1}{S_x}\right)}{\mathbf{Ln}\left(\frac{1}{S_y}\right)}\right)}{\mathbf{Ln}\left(\frac{L_x}{L_y}\right)} \quad \textbf{from } \mathbf{L}_{50}\ (\mathbf{S} = 0.5) \textbf{ and } \mathbf{L}_{10}\ (\mathbf{S} = 0.9) \quad \mathbf{m} = \frac{\mathbf{Ln}\left(\dfrac{\mathbf{Ln}(2)}{\mathbf{Ln}\left(\frac{1}{0.9}\right)}\right)}{\mathbf{Ln}\left(\frac{L_{50}}{L_{10}}\right)}$$

$$\mathbf{m} = \frac{1.8839}{\mathbf{Ln}\left(\frac{L_{50}}{L_{10}}\right)}$$

Compute ratio of *L50* to *L10*:

$$\frac{L_{50}}{L_{10}} = \left(\frac{\mathbf{Ln}(2)}{\mathbf{Ln}\left(\frac{1}{0.9}\right)}\right)^{1/m} = (6.579)^{1/m}$$

Compute *Lx* from *L50*, *L50* from *Lx*, *Lx* from *Ly:*

a. $$L_x = L_{50}\left(\frac{\mathbf{Ln}\left(\frac{1}{S_x}\right)}{\mathbf{Ln}\,(2)}\right)^{1/m}$$

b. $$L_{50} = L_x\left(\frac{\mathbf{Ln}\,(2)}{\mathbf{Ln}\left(\frac{1}{S_x}\right)}\right)^{1/m}$$

c. $$L_x = L_y\left(\frac{\mathbf{Ln}\left(\frac{1}{S_x}\right)}{\mathbf{Ln}\left(\frac{1}{S_y}\right)}\right)^{1/m}$$

The above analytical treatments use the term *m* for the Weibull Modulus. In the remainder of this report the terminology *e, Weibull Slope,* will be used, as is more customary in publications dealing with the Weibull Distribution applied to dynamic bearing fatigue life.

5. Bearings Historical Database and Analysis Approach

5.1 Defined Analysis Ground Rules

The seminal work of Lundberg and Palmgren (4,5) is based on Weibull's Theory of statistical brittle fracture (6,7), in a bivariate form, applied to fatigue life instead of the strength of brittle materials. This method has gained wide acceptance through much of the bearing industry for correlating rolling element bearing fatigue life, and establishing design levels. Lundberg and Palmgren utilize power law assumptions for the variates of load (stress) and for life, following the recommendations of Weibull. Related application of the bivariate Weibull Distribution to static fatigue is given by Robinson (11).

With S the cumulative probability of Survival (also called the Reliability), c the Weibull Modulus for the load variate, and e the Weibull Modulus (or Weibull Slope) for the fatigue life variate, Lundberg and Palmgren deduce the following relations (ref 1, Eqn 37 and Eqn 53):

$$\mathrm{Ln}(1/S) \text{ is proportional to } (\text{Life})^{e}\ (\text{Stress or load})^{c}$$

Life L vs Load F $\qquad LF^{p} = constant,$ note that $p = c/e$

A considerable database has been amassed over many years of testing, for example, Lieblein and Zellen (8), and Tallian (9). As part of the present report a statistical analysis of the Weibull Slopes tabulated by Lieblein and Zellen was conducted to determine the overall tendencies of the fatigue life scatter. Data from three bearing companies were examined.

This analysis, shown in Figure 2, reveals that e, the Weibull Slope, tends to a value of $e = 1.5$ with a variation from 0.5 to 4.5 .

The Lieblein and Zelen data, as well as the data published by Tallian (9) confirm the a-priori prescribed ground rule use of:

$$e = 1.5 \qquad \text{(Weibull Slope)}$$

$$p = 3$$

for "global" fatigue data correlation of Group A and Group B data (Leveille, 1998).

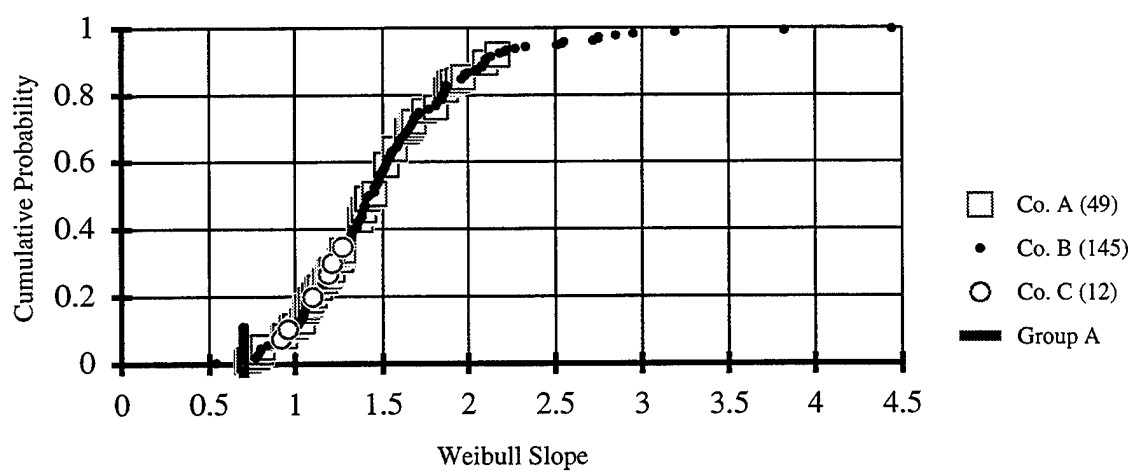


Figure 2. Weibull Modulus for bearing fatigue.

6. Analysis of Data

6.1 Group A Data Analysis

Group A Data represent a series of tests to the first failure among four bearings. In order to compare with the Group B data of single bearing tests, the Group A data were transformed to equivalent single bearing tests, as follows:

If the probability of survival of a single bearing is S, then the probability that all four survive is S^4. Probability is assigned by the median rank plotting position: $(j - 0.3)/(N + 0.4)$

The complementary probability F_4, of at least one failure observed in a group of 4, is

$$F_4 = 1 - S^4 \qquad \text{[This is referred to as a size-4 test article]}$$

The failure probability, $F = 1 - S$, of a single bearing is therefore given by:

$$F = 1 - (1 - F_4)^{1/4} \qquad \text{[This is referred to as a size-1 test article]}$$

These computations were carried out for the Group A data as shown in Table 3.

Table 3. Group A All-52100 Steel Bearing Fatigue Data
(4 bearings per test to first failure)

At 3,000 lbs N = 6	Group A Data (52100) Observed	Group A Data (52100) Observed	Size-4 test article $F_4 = (j-0.3)/(N+0.4)$	**Size-1 test article $1 - (1 - F_4)^{0.25}$**
rank (j)	time to fail (h)	shaft cycles to fail @ 5400 rpm	Failure Probability	**Failure Probability**
1	17.4	5,637,600	0.11	**0.029**
2	46.7	15,130,800	0.27	**0.074**
3	127	41,148,000	0.42	**0.128**
4	339	109,836,000	0.58	**0.194**
5	364	117,936,000	0.73	**0.282**
6	910	294,840,000	0.89	**0.425**
Median =	**233**	**75,492,000**		**infer via m = 1.5**
Average =	**301**	**97,421,400**		

The data sample (size-4 observations) median is determined in this case by the average of the rank 3 and rank 4 observations, i.e., midway between the two center observations in a complete sample with even number of specimens.

When transformed to equivalent single bearing observations, in the right column, it is seen that the single bearing median (50%) life has not yet been observed.

The observed fatigue life data may be used to fit a Weibull Distribution and to infer the statistics of size-4 and single bearing lifetime statistics. However, by the ground rules here, the defined reference Weibull Slope $e = 1.5$, and the corresponding Weibull size effect is used to infer the median life of single steel bearings. This is given by:

$$\text{Single Bearing Median} = (\text{size-4 Median})\,(4)^{1/1.5}$$
$$= (75{,}492{,}000)(2.52)$$
$$\textbf{Single Bearing Median} = \textbf{190.23 mllion cycles}$$

This median estimate is conservative for the observed data, as may be seen in the cumulative probability plot of Figure 3. If the projection were made on the basis of the best-fit Weibull Distribution, with $e = 0.7$, the size effect is then much more pronounced, and the single bearing median life will be predicted much higher than with $e = 1.5$, as done here.

Since the load on Group B bearings was 1500 lb, the life of Group A bearings at a load of 3000 lb is adjusted by exponent $p = 3$, as defined in the ground rules for this analysis, giving a factor of 8 increased predicted life at the lower load. Therefore, the reference Group A single bearing median life at 1500 lb load is given by:

Group A Single Bearing Median Life Prediction at 1500 lb load = (8)(1.90227 E8)

Group A Single Bearing L_{50} = 1.52 E9 cycles (1520 million shaft cycles)

Using the median normalized Weibull Distribution, with $e = 1.5$:

$$L = L_{50}\{Ln(1/S)/Ln(2)\}^{1/1.5}$$

S = 0.9	L = L10 = 0.285 x 1,521,823,358 =	433,719,657	cycles
S = 0.99	L = L1 = 0.059 x 1,521,823,358 =	89,787,578	cycles
S = 0.999	L = L0.1 = 0.013 x 1,521,823,358 =	19,783,704	cycles

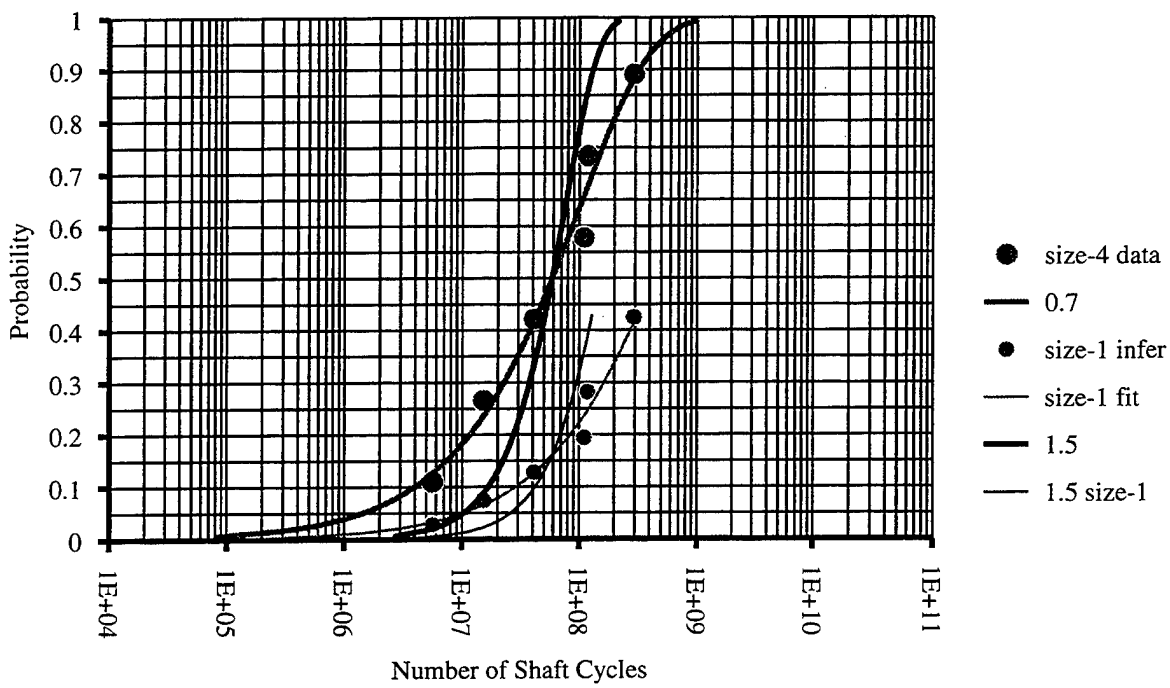


Figure 3. Group A bearing life data. Six size-4 test articles, Size-1 inferred.

6.2 Group B Data Analysis

Group B comprises the hybrid bearing, with silicon nitride balls and M62 steel races. A total of 24 such bearings were tested under 1500 lb load in excess of 2500 h, equal to 810 million shaft cycles, with none failing. Comparison of Group A and Group B bearing statistics is based on the likelihood that all 24 Group B bearings will survive the test conditions.

If the failure probability of a bearing under the test conditions were high, say 10%, then it would be very unlikely that all 24 bearings would survive the test. In other words, with 24 no-fail data, the confidence is high that the bearing probability is not a large value. On the other hand, if the probability were very low, it would be a cinch that all 24 bearings would survive.

If F is the probability that a single Group B bearing will fail under the test condition, then the probability that 24 bearings will survive is given by $(1 - F)^{24}$. The complement of this probability, $1 - (1 - F)^{24}$, is the confidence that bearing failure probability is no greater than F.

If the 24 no-failures event were taken as an expected value, with even odds of 50%, then we might observe 24 no-failures half the time if the single bearing probability were $F = 0.0285$, since $(1 - 0.0285)^{24} = 0.5$.

Table 4 shows the confidence in single bearing probability of failure, F, given N no-fail observations. Figure 4 displays the Table 4 data.

Table 4
Confidence in Single Bearing Probability
Given 24 no-fail Observations

Single Bearing Fail Prob	Likelihood of 24 no-fail	Prob. of at least 1 fail
F	$(1 - F)^{24}$	Confidence
1.00E-04	0.998	0.002
0.001	0.976	0.024
0.005	0.887	0.113
0.010	0.786	0.214
0.018	0.647	0.353
0.023	0.572	0.428
0.028	**0.500**	**0.500**
0.030	0.481	0.519
0.040	0.375	0.625
0.043	**0.350**	**0.650**
0.050	0.292	0.708
0.075	0.154	0.846
0.091	**0.100**	**0.900**
0.117	0.050	0.950
0.150	0.020	0.980
0.200	0.005	0.995
0.250	0.001	0.999
0.3	0.000	1.000

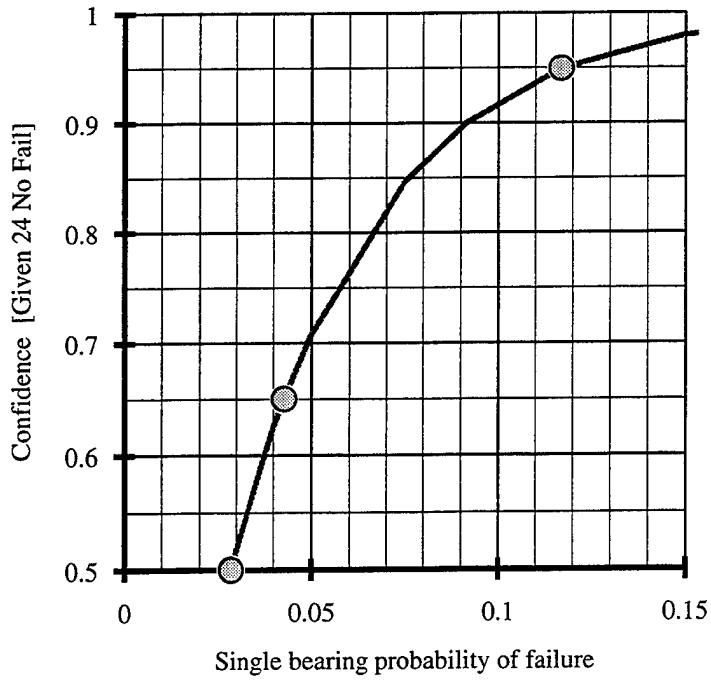


Figure 4. Confidence in single bearing probability of failure - Group B.

For comparison of these Group B data with Group A, the previously stated ground rule is employed: a Weibull bearing life distribution with Weibull Modulus of 1.5 applicable to all bearings. This ground rule gives *L50, L10*, and *L1* for any selected confidence level, denoted as C in the equations below, based on the observed no-fail fatigue life.

The reference no-fail bearing life for Group B, $\boldsymbol{L} = 810$ million shaft cycles

and

$$L_F = \boldsymbol{L}\ \{\mathrm{Ln}(1/S_F)/\mathrm{Ln}(1/C)\}^{1/e}$$

$$L_F = \boldsymbol{L}\ \{\mathrm{Ln}(1/S_F)/\mathrm{Ln}(1/C)\}^{1/1.5}$$

A summary of the findings for Group B is shown in Table 5.

Table 5. Group B L50, L10, and L1 at Confidence Values of 50%, 65%, and 90%

Bearing F-Fail	Bearing S = 1 - F	Confidence %	L_{50} (S_F= 0.5)	L_{10} (S_F= 0.9)	L_1 (S_F= 0.99)
0.027	0.973	50	6,985,102,791	1,989,486,909	415,345,533
0.043	0.957	65	5,093,898,719	1,450,836,891	302,891,474
0.091	0.9087	90	3,031,369,575	863,390,313	180,250,148

7. Conclusion - Comparison of Group A vs Group B Bearing Fatigue Life

The following statements may now be made about the relative merits of Group A vs Group B fatigue life:

With confidence 50% the Group B life exceeds Group A life by a factor of 4.5

With confidence 65% the Group B life exceeds Group A life by a factor of 3.3

With confidence 90% the Group B life exceeds Group A life by a factor of 2.0

In other words, we are very confident that Group B life exceeds Group A by a factor of 2, and moderately sure (50% odds) that Group B life exceeds Group A life by a factor of 4.5 .

Since the Group B testing was arbitrarily terminated at 2500 h, the chances are in fact that Group B exceeds Group A fatigue life by even greater factors.

The combined comparison results of this study are collected in Table 6.

Table 6. Final Comparison of Group A and Group B Bearing Fatigue Data.
Tabulations are number of shaft cycles

Group A Single Bearing Fatigue Life at 1500 lb Load, with *e = 1.5* anchored at median life				
L50	**L10**	**L1**	**L0.1**	
1,521,823,358	432,924,209	90,381,664	19,413,542	
Group B Single Bearing Inferred Fatigue Life at 1500 lb Load, with *e = 1.5*				
L50	**L10**	**L1**	**L0.1**	**at confidence**
6,985,102,791	1,989,486,909	415,345,533	87,047,501	50%
5,093,898,719	1,450,836,891	302,891,474	65,059,615	65%
3,031,369,575	863,390,313	180,250,148	39,107,128	90%
Ratio of B vs A				
L50	**L10**	**L1**	**L0.1**	
4.60	4.60	4.60	4.60	50%
3.30	3.30	3.30	3.30	65%
2.00	2.00	2.00	2.00	90%

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Appendix A - Monte Carlo Evaluation of Sample Size Effect on Weibull Modulus Estimate

The Group A data are well approximated by a Weibull Distribution with Weibull Modulus $m = 0.7$, but such a Weibull slope differs from the large database of fatigue life data, accumulated over many years of bearing fatigue tests, where the value tends to $m = 1.5$.

Consequently, a brief Monte Carlo study was conducted to evaluate the scatter of Weibull Modulus for small sample sizes drawn from a parent population with $m = 1.5$.

The computation is based on the fact that cumulative probability is a uniformly distributed random number from 0 to 1. Using the Weibull Median Normalized Form:

$$S = \mathrm{Exp}\ -\{\mathrm{Ln}(2)\ [L/L_{50}]^{m}\}$$

is inverted, solving for the variable in terms of the cumulative probability:

$$L/L_{50} = \{\mathrm{Ln}(1/S)/\mathrm{Ln}(2)\}^{(1/m)}$$

and, with Rand = random number from 0 to 1, replacing S in the equation, pseudo random Weibull Observations are given by:

$$L/L_{50} = \{\mathrm{Ln}(1/\mathrm{Rand})/\mathrm{Ln}(2)\}^{(1/m)}$$

This equation was used to generate ten sets of pseudo-random Weibull observations with sample sizes of 6 and of 24. The least-square Weibull Slope was fitted to each sample and the results arranged in rank order, as shown in Figure A-1.

Among the ten random samples of six observations each, there are two cases where the choice Weibull Slope is $m = 0.7$. Previous extensive Monte Carlo studies (12) have shown that the coefficient of variation of the Weibull Modulus tends to $CV = 1/\sqrt{N}$, also shown in Table A-1. The Group A data fit with $m = 0.7$ is therefore a plausible statistical variation drawn from a parent distribution with $m = 1.5$.

These examples justify the imposed use of $m = 1.5$, and, as noted in the report, the forecasts are more conservative for longer life than those with a lower Weibull Slope.

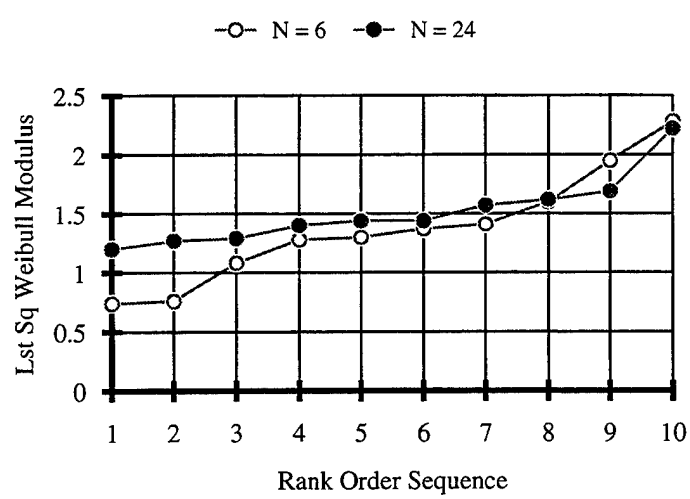

Figure A-1. Weibull Modulus ranked variations for fixed parent population with m = 1.5.

Table A-1. Results of Monte Carlo Weibull Distribution Simulation and m - Estimation by Least Square Method

Trial # (m = 1.5)	Lst Sq m N = 6	Lst Sq m N = 24	Lst Sq m N = 6 sorted	Lst Sq m N = 24 sorted
1	1.28	1.57	0.74	1.2
2	1.3	1.4	0.76	1.27
3	1.08	1.44	1.08	1.29
4	0.74	1.69	1.28	1.4
5	1.37	2.22	1.3	1.44
6	1.41	1.27	1.37	1.44
7	1.95	1.44	1.41	1.57
8	1.6	1.2	1.6	1.62
9	0.76	1.62	1.95	1.69
10	2.28	1.29	2.28	2.22
avg =	1.377	1.514		
CV =	0.370	0.205		
$1/\sqrt{N}$	0.408	0.204		

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