**Abstract**

A uniform algebraic procedure is presented for deriving both epipolar geometry and three-dimensional object structure from general stereo imagery. The procedure assumes central-projection cameras of unknown interior and exterior orientations. The ability to determine corresponding points in the stereo images is assumed, but no prior knowledge of the scene is required. Epipolar geometry and the fundamental matrix are derived by algebraic elimination of the object-variables from the imaging equations. This provides a transfer identified in the new perspective. Next, invariant coordinates for the scene-points are derived by algebraic elimination of the camera parameters from the imaging equations. Identical coordinates are obtained from any stereo images of non-occluding scene points as long as the same set of 5 corresponding points can be identified in both stereo pairs. The procedure extends methods utilizing the cross-rations of determinants and cyclopean vectors, presented in earlier work. A technique for reconstructing the 3-dimensional object from the invariant coordinates is also given.
FINAL REPORT, IMAGE INVARIANTS RESEARCH

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(2) OBJECTIVES:
Invariant methods determine quantitative relationships among image points that are independent of the perspectives from which the images were acquired and thus are invariant from view to view. The advantage of the invariant methods is that model matching and object recognition is based on measurements made on images, without requirement for direct quantitative measurements of object or camera parameters. In this study we have developed robust invariant methods for transferring features and three-dimensional models from photographic images to maps, to SAR images and other image sensors including high range resolution (HRR) radar. We have adapted these invariant methods to rapid updating of maps and geospatial databases from uncontrolled imagery, an application of current importance for military operations. The invariant methods will facilitate image fusion, perspective-independent model-based object recognition, object reconstruction and change detection for these different sensor types.

(3) STATUS OF EFFORT (December 1997):
Invariant methods for object representation and model matching exploit relationships among object and image features that are independent of camera parameters or object orientation, hence the term invariant. The types of applications investigated in this research effort included: three-dimensional (3-D) object reconstruction, model transfer, and view synthesis. In reconstruction, two or more reference images function as the object model. It is assumed that image points can be put into correspondence, and that the 3-D position are known for at least 5 non-coplanar fiducial object points. Reconstruction is achieved by representing object points as functions of reference image points and object fiducial points. In model transfer, conjugate points on two or more reference images are transferred to points in a new view or map. Transfer is achieved by representing the points in the new view or map as functions of corresponding points in the reference images. Model transfer to a fictitious but realizable camera view is called synthesis.

The geometric methods we have introduced for deriving these invariant algorithms are extensible from the linear fractional central projection camera model to weak perspective and certain non-central projection camera models including high range resolution (HRR) radar. We demonstrate that stability to adverse geometries and measurement error can be enhanced by using redundant fiducial points and images to determine the transfer and reconstruction functions. Our work has validated the successful application of this theory to updating maps of military/industrial facilities with features extracted from uncontrolled aerial imagery (from real images to real maps) without requiring 3-D object models or digital elevation models (D.E.M.s), and has been extended to include reconstruction of 3D distributions of radar scattering centers from multiple 1D HRR “views”.

(4) ACCOMPLISHMENTS AND NEW FINDINGS (1994-1997):
In 1994-1995:
• Extension of Model Cities simulation environment for analysis of sensitivity of invariant model registration algorithms to adverse viewing geometries, random measurement errors, and systematic errors,
• Development of robust invariant transfer algorithms stabilized with respect to adverse viewing geometry and random measurement errors, validated in Model Cities experimental simulations,
- Development of algorithms for image synthesis, i.e. creation of new images of object points as they would appear when viewed from a user-selected vantage point, without requiring a 3-D object model or digital elevation model (D.E.M.).
- Initial implementation of the PI Toolkit; invariant transfer algorithms and Model Cities simulation environment on Macintosh Power PC and laptops for portability to ARPA and AFOSR research communities. Initial Users Guide for the PI Toolkit.

In 1995-1996:
- Successful application of theory to updating of real maps of military and industrial facilities with structural features extracted from uncontrolled aerial imagery. Invariant methods did not require prior knowledge of 3D object models, terrain elevation models, nor position/orientation of the platforms acquiring the imagery.
- Continuing development of robust invariant transfer and image synthesis algorithms stabilized with respect to adverse viewing geometry and random measurement errors. Algorithms validated in experimental computer simulations with computer-generated 3D scenes, with real aerial imagery, and with maps.
- Continuing implementation of the PI Toolkit; invariant transfer algorithms and simulation environment on Macintosh Power PC and laptops for portability to DARPA and AFOSR research communities. Extensions to Users Guide for the PI Toolkit.

In 1996-1997:
- Continuing and successful application of theory to updating of real maps of military and industrial facilities with structural features extracted from uncontrolled aerial imagery. The invariant methods we developed did not require prior knowledge of 3D object models, terrain elevation models, nor position/orientation of the platforms acquiring the imagery.
- We have developed algorithms for invariant transfer of conjugate lines which exploit the synergy of line and point transfer. Computation of the object model...the digital elevation model (DEM)...is not required in this approach.
- We have developed for central-projection imaging systems a natural progression of cross-ratio invariant theorems extending from one through three dimensions. In each dimension there is an invariant quantitative relationship between combinations of geometric entities in image space, and combinations of corresponding geometric entities in object space. We have demonstrated that these cross-ratios of points in the images, which we can observe directly, are equal to the corresponding cross-ratios of points in the objects, which generally are not directly accessible.
- We have developed a simplified geometric model of high-resolution radar (HRR), applied to 3D distributions of isotropic scattering centers contained in a three-dimensional region of limited extent. A new six-point/three-view invariant algorithm for reconstructing the 3D distributions non-coplanar reflecting centers, was developed and validated using the XPATCH radar simulation software.
- Continuing implementation of the PI Toolkit; invariant transfer algorithms and simulation environment on Macintosh Power PC and laptops for portability to DARPA and AFOSR research communities.
(5) PERSONNEL SUPPORTED (1994-1997):

- Dr. Eamon Barrett, Lockheed Martin Space Systems Division,
- Mr. Paul Payton, Lockheed Martin Research and Development Division,
- Dr. Gregory Gheen, Lockheed Martin Research and Development Division

(6) PUBLICATIONS (1994-1997):

(The numbers #17-#30 in the following list of publications and abstracts refer to the sequence in which these publications appear in the unpublished Lockheed-Martin report “Selected Research Publications and Presentations on Image Invariance in Computer Vision” dated October 1997, by Eamon Barrett, Paul M. Payton, and Peter Marra. This report is available upon request).

(17) “Representation of Three-Dimensional Object Structure as Cross-Ratios of Determinants of Stereo Image Points”
E.B. Barrett, G. Gheen, P.M. Payton,
Applications of Invariance in Computer Vision, Second Joint European-US Workshop, Ponta Delgada, Azores, Portugal, October 1993, Proceedings
Springer-Verlag Lecture Notes in Computer Science No. 825
J.L. Mundy, A. Zisserman, D. Forsyth (Eds.), pp. 47-68.

ABSTRACT
A uniform algebraic procedure is presented for deriving both epipolar geometry and three-dimensional object structure from general stereo imagery. The procedure assumes central-projection cameras of unknown interior and exterior orientations. The ability to determine corresponding points in the stereo images is assumed, but no prior knowledge of the scene is required. Epipolar geometry and the fundamental matrix are derived by algebraic elimination of the object-variables from the imaging equations. This provides a transfer procedure to any other perspective as long as 8 or more corresponding points can be identified in the new perspective. Next, invariant coordinates of the scene-points are derived by algebraic elimination of the camera parameters from the imaging equations. Identical coordinates are obtained from any stereo images of non-occluding scene points as long as the same set of 5 corresponding points can be identified in both stereo pairs. The procedure extends methods utilizing the cross-ratios of determinants and cyclopean vectors, presented in earlier work. A technique for reconstructing the 3-dimensional object from the invariant coordinates is also given.

(18) “Image Invariance Research and Applications”
E.B. Barrett, G. Gheen, P.M. Payton
Imagery Technology R&D Vision to 2014; EXRAND Technology Conference,
Warrenton, Virginia, April 26-28, 1994

OVERVIEW
The objective of these investigations is to develop automated techniques for recognizing the same objects in images that differ in scale, tilt, and rotation, i.e. to develop a computational model for perspective-independent object recognition. The eye can do this (fairly well); can computer vision technology emulate this cognitive function?
ABSTRACT
Invariant methods for object representation and model matching exploit relationships among object and image features that are independent of camera acquisition geometry or object orientation... hence the term "invariant". The objectives of invariant methods are to simplify the task of model preparation, and to reduce the prior knowledge about object, scene, and image acquisition parameters required for model matching and object identification applications.
In this report, we describe algorithms for deriving invariants from the imaging equations by algebraic elimination procedures; we demonstrate the application of these invariants to model transfer and object reconstruction; and we illustrate the types of errors affecting model transfer accuracy under various imaging scenarios.

ABSTRACT
Projective geometry is concerned with properties of figures that remain invariant under projection. The formation of images by cameras can be described mathematically as projections, through the optical centers of the camera lenses, from three-dimensional object space to two-dimensional image planes. The appearance of a three-dimensional object in its two-dimensional images is highly variable, depending on its orientation and distance from the cameras. This variability greatly complicates the task of automated object recognition by computer image processing. Computer scientists are currently investigating properties of objects that are projectively invariant, as cues for orientation-independent object recognition. In this context, the classical theorems of Pappus, Desargues, and the projective invariance of the cross-ratio take on new meaning and significance.

ABSTRACT
Invariant methods for object representation and model matching develop relationships among object and image features that are independent of the quantitative values of the camera parameters or object orientation, hence the term invariant. Three-dimensional models of objects or scenes can be reconstructed and transferred to new images, given a minimum of two reference images and a sufficient number of corresponding points in the images. By using redundant reference images, redundancy can be exploited to increase robustness of the procedure to pixel measurement errors and systematic errors (i.e. discrepancies in the camera model). We present a general method for deriving invariant relationships based on two or more images. Simulations of model transfer and reconstruction demonstrate the positive effect of additional reference images on the robustness of invariant procedures. Pixel measurement error is simulated by adding random noise to coordinate values of the features in the reference images.
Invariant methods develop relationships between object and image features that are independent of the quantitative values of the camera parameters ... hence the term "invariant". Two or more images are generally required to capture 3-D object geometry. We demonstrate two invariant techniques; in model transfer a collection of conjugate points are determined on a set of reference images, and "transferred" to the matching conjugate points on a new view of the object without computation of camera geometry or scene reconstruction. In the scene reconstruction technique, general object points are represented as functions of non-coplanar fiducial points and corresponding conjugate points across multiple images. In this technique the object points are "reconstructed" once quantitative values are specified for the fiducial points. Stability to measurement error is enhanced by using redundant fiducial points and images to determine the transfer and reconstruction functions.

We introduce non-standard methods of deriving algebraic invariants and demonstrate two types of applications of these invariants. In model transfer a collection of conjugate points are determined on a set of reference images, and "transferred" to the matching conjugate points on a new view of the 3-D object, without prior computation of camera geometry or scene reconstruction. In object reconstruction, general 3-D object points are represented as functions of non-coplanar fiducial points and corresponding conjugate points across multiple images. In this application the object points are "reconstructed" once quantitative values are specified for the fiducial points. The methods we introduce for deriving these invariant algorithms are extensible from the linear fractional central projection camera model to weak perspective and certain non-central projection camera models. Stability to adverse geometries and measurement error can be enhanced by using redundant fiducial points and images to determine the transfer and reconstruction functions. Extensibility and stability are indications of the robustness of these methods.
Invariant methods for object representation and model matching exploit relationships among object and image features that are independent of camera parameters or object orientation, hence the term *invariant*. Three types of applications are considered in this report: three-dimensional (3-D) **object reconstruction**, **model transfer**, and **view synthesis**. In reconstruction, two or more reference images function as the object model. It is assumed that image points can be put into correspondence, and that the 3-D position are known for at least 5 non-coplanar fiducial object points. Reconstruction is achieved by representing object points as functions of reference image points and object fiducial points. In **model transfer**, conjugate points on two or more reference images are transferred to points in a new view. Transfer is achieved by representing the points in the new view as a function of corresponding points in the reference images. Model transfer to a fictitious but realizable camera view is called **synthesis**.

The methods we introduce for deriving these invariant algorithms are extensible from the linear fractional central projection camera model to weak perspective and certain non-central projection camera models. We demonstrate that stability to adverse geometries and measurement error can be enhanced by using redundant fiducial points and images to determine the transfer and reconstruction functions. Simulations results under adverse conditions illustrate the improved robustness provided by additional reference images.

A Users Guide for an invariant algorithm tool kit is provided. This tool kit is written in C and is available upon request (paul.payton@lmco.com), subject to approval by the sponsoring agency.

The reference data consists of two or more central-projection images of a three-dimensional distribution of object points, i.e., a 3D scene. The positions and orientations of the cameras which generated the reference images are unknown, as are the coordinates of all the object points. We derive and demonstrate invariant methods for synthesizing nadir views of the object points, i.e., 2D maps of the 3D scene. The techniques we will demonstrate depart from standard methods of resection and intersection to recover the camera geometry and reconstruct object points from the reference images, followed by back-projection to create the nadir view.

Our approach will be to perform the image measurements and computations required to estimate the image invariant relationships linking the reference images to one another and to the nadir view. The empirically estimated invariant relationships can thereafter be used to transfer conjugate points from the reference images to their synthesized conjugates in the nadir view. Computation of the object model...the digital elevation model (DEM)...is not required in this approach. The method also differs from interpolation in that the 3D structure of the scene is preserved, including the effects of partial occlusion. Algorithms are validated, initially with synthetic CAD models and subsequently with real data consisting of uncontrolled aerial imagery and maps with occasional missing or inaccurately delineated features.
ABSTRACT
We present some recent results in the use of uncontrolled aerial imagery and interferometric SAR (IFSAR) DEMs to update 2D maps and 3D models of military and industrial facilities. We demonstrate the application of image invariant methods to the practical problem of updating and augmenting the 2D and 3D structural content of such geospatial databases. The theory of image invariants gives rise to useful, non-standard geometric and algebraic algorithms for extracting quantitative information about 3D objects from images with unknown acquisition parameters.

ABSTRACT
The reference data consists of two or more central-projection images of a three-dimensional distribution of object points, i.e., a 3D scene. The positions and orientations of the cameras which generated the reference images are unknown, as are the coordinates of all the object points. We derive and demonstrate invariant methods for synthesizing nadir and perspective views of the object points, e.g., 2D maps of the 3D scene. The techniques we will demonstrate depart from standard methods of resection and intersection which first recover the camera geometry, then reconstruct object points from the reference images, and finally back-project to create the nadir or perspective views.

The first steps in our “invariant methods” approach are to perform the image measurements and computations required to estimate the image invariant relationships linking the reference images to one another and to the new nadir and perspective views. The empirically estimated invariant relationships can thereafter be used to transfer conjugate points and lines from the reference images to their synthesized conjugates in the nadir and perspective views. Computation of the object model...the digital elevation model (DEM)...is not required in this approach. In this paper we develop algorithms for invariant transfer of conjugate lines which exploit the synergy of line and point transfer. We validate our algorithms with synthetic CAD models. A subsequent paper will validate the line transfer algorithms with uncontrolled aerial imagery and maps with occasional missing or inaccurately delineated features.

ABSTRACT
We will demonstrate for central-projection imaging systems a natural progression of cross-ratio invariant theorems extending from one through three dimensions. In each dimension there is an invariant quantitative relationship between combinations of geometric entities in image space, and combinations of corresponding geometric entities in object space. In one dimension, when the object points and image points are colinear, these entities are line segments formed by corresponding pairs of object and image points. The “mother of all invariants” is the invariant relationship between cross-ratios of products of the lengths of these corresponding line segments in object and image. In two dimensions these geometric entities are triangles formed by corresponding triplets of points in the object and in the
image. There exists an invariant relationship between cross-ratios of products of areas of these corresponding triangles in object and image. The one- and two-dimensional results are well known. Not so well-known is the fact that for the case of multiple images of three-dimensional scenes and objects the geometric entities are triangles (in the images) and tetrahedra (in the objects), and that there exist invariant linear relationships between cross-ratios of products of the areas of image-triangles and volumes of object-tetrahedra\(^1\)\(^2\). One objective of our paper is to demonstrate that these linear relationships are established by a uniform pattern of algebraic arguments that extends the cross-ratio invariants in a natural progression from lower to higher dimensions. A second objective is to demonstrate that the resulting cross-ratio invariants can be interpreted as metric properties of geometric entities (e.g. areas of triangles formed by triplets of image points, and volumes of tetrahedra formed by four-tuples of object points). A third objective is to demonstrate that these cross-ratios of points in the images, which we can observe directly, are equal to the corresponding cross-ratios of points in the objects, which may not be directly accessible. We will use computer simulations to validate the algebraic results we derive in this paper, and 3D graphics to visualize them.

(29) “HRR Invariants in Space”
Paul M. Payton, Eamon B. Barrett, W. Kober, J. Thomas, T. Nichols
Proc. Sixth ATR System and Technology Symposium, Redstone Arsenal, Alabama, 28-30 October, 1997
ABSTRACT
In this technical paper we describe a simplified mathematical model of high-resolution radar (HRR), where objects being imaged by the sensor are assumed to consist of a collection of isotropic scattering centers contained in a three-dimensional region of limited extent. A new six-point/three-view invariant algorithm for non-coplanar reflecting centers is developed. We validate both the presented mathematical model of HRR and the new three-view HRR invariant using the XPATCH SAR/HRR simulation system. The 3D invariant algorithm requires recognition of a specified number of conjugate (i.e. corresponding) features in each of the three HRR views. For radar the relevant features will be estimated using analytic scattering models for dihedrals and trihedrals. In the paper we also discuss model-indexing for efficiently searching the model database using those extracted features and their relative geometric arrangements.

(30) “View-Based Methods for Relative Reconstruction of 3D Scenes from Several 2D Images”
Eamon Barrett, Paul Payton, Peter Marra
(manuscript in preparation, draft submitted to SPIE Symposium on Aerospace/Defense Sensing and Controls, 13-17 April 1998, Orlando FLA )
ABSTRACT
Suppose we have two or more images of a 3D scene. From these views alone, we would like to infer the \((x,y,z)\) coordinates of the object-points in the scene...i.e. to “reconstruct” the scene. The most general standard methods require either prior knowledge of the camera models (intersection methods) or prior knowledge of the \((x,y,z)\) coordinates of some of the object points, from which the camera models can be inferred (resection, followed by intersection). When neither alternative is available, a special technique called relative orientation enables a scale model of a scene to be reconstructed from two images, but only when the internal parameters of both cameras are identical. In this paper, we will present an alternative to relative orientation that does not require knowledge of the internal parameters of the imaging systems. The technique will be called view-based relative reconstruction.
(7) INTERACTIONS/ TECHNOLOGY TRANSITION:

In 1994-1995:
(a) Presentations:
- “Classical Projective Geometry in Twenty-First Century Imaging”
  E.B. Barrett
  Presentation at: 78th Annual Meeting of the Mathematical Association of America
  (MAA), January 5th, 1995
- “Stable Invariant Methods for Model Transfer and Scene Reconstruction”
  E.B. Barrett
  Presentations at: David Sarnoff Research Center, Princeton N.J., June 22, 1995, and at
- (Presentations at SPIE Symposia listed in (6) above)

(b) Consultive and Advisory Functions:
- Member, Image Understanding Advisory Group. In support of ARPA and ORD, this group
  meets periodically to review proposals submitted by academia and industry under the RADIUS
  (Research and Development in Image Understanding Systems) Program. The Advisory Group Chair
  is Dr. Joe Mundy of General Electric Corporate Research and Development Laboratories

(c) Transitions:
- To industry: Continuing implementation of SAR-EO invariants applied to fusion of FLIR and SAR
  data in Phase II SBIR contract, “Sensor Fusion for Air-to-Ground Precision Targeting”,
  (Data Fusion Corporation, Aurora, Colorado, to US Air Force, AF94-122, Dr. Woody Kober, Principal Investigator)
- To academia: photogrammetric analysis of invariants is the subject of a continuing
  university research project (Photogrammetric Analysis of Invariance, School of Civil
  Engineering, Purdue University, West Lafayette, Indiana; Prof. Edward M. Mikhail, Principal Investigator)
- To academia: Projectively adapted harmonic analysis is the subject of a continuing
  university research project (Dept. of Computer and Mathematical Sciences, University of Houston, Prof. Jacek Turski, Principal Investigator)

In 1995-1996:
(a) Presentations:
- (Presentation at Europto Symposium and Solutioneering Symposium listed in (6) above)
- “Image Invariants: Theory and Applications”
  (Presentation by Eamon Barrett at: Bay Image Seminar, Stanford University, March
  21st, 1996. Seminar Chairman: Prof. Tom Binford, Stanford University)

(b) Consultive and Advisory Functions:
- To government/industry: Member, Image Understanding Advisory Group. In support of ARPA and ORD, this group
  meets periodically to review proposals submitted by academia and industry under the RADIUS (Research and Development in Image
Understanding Systems) Program. Advisory Group Chairman: Dr. Joe Mundy of General Electric Corporate Research and Development Laboratories

- To academia: Transfer of publications and software to support research. (University of Texas-Houston, Prof. Jacek Turski; University of Texas-Arlington, Sharon Barber; University of Washington-Seattle, Prof. Robert Haralick, Sinan Karasu)

(c) Transitions:
- To industry: Continuing implementation of SAR-EO invariants applied to fusion of FLIR and SAR data in Phase II SBIR contract, “Sensor Fusion for Air-to-Ground Precision Targeting”, (Data Fusion Corporation, Aurora, Colorado, to US Air Force, AF94-122; Dr. Woody Kober, Principal Investigator)

In 1996-1997:

(a) Presentations:
- (Presentation at SPIE and ATR Symposia as listed in (6) above)

(b) Consultive and Advisory Functions:

- To academia: Transfer of publications and software to support research. (Purdue University, School of Engineering, Prof. Ed Mikhail, Hazem Barakat).

(c) Transitions:

- To industry: Continuing implementation of SAR-EO invariants applied to fusion of FLIR and SAR data in Phase II SBIR contract, “Sensor Fusion for Air-to-Ground Precision Targeting”, (Data Fusion Corporation, Aurora, Colorado, to US Air Force, AF94-122; Dr. Woody Kober, Principal Investigator)

(8) DISCOVERIES, INVENTIONS, PATENTS:

(a) Discoveries in FY '95:
New algorithms for invariant transfer, reconstruction, synthesis, as described in the open literature publications listed above

(b) Patent Disclosures in FY '95:
None

In 1995-1996:

a) Discoveries:

- New algorithms for invariant transfer, reconstruction, synthesis, as described in the publications listed above,
- New image invariant algorithms for updating facility maps with uncontrolled aerial imagery, experimentally validated with real data, as described in the publications listed above, Section 6.
(b) Patent Disclosures:
• None

In 1996-1997:

(a) Discoveries:
• New algorithms for invariant transfer, reconstruction, synthesis, as described in the publications listed above, Section 6.
• New image invariant algorithms for High Range Resolution (HRR) Radar, as described in the publications listed above, Section 6.

(b) Patent Disclosures:
• None

(9) HONORS/ AWARDS:
• None

DATE PREPARED:
• December 23rd, 1997