**Nonlinear Control of Mechanical Systems in the Presence of Magnitude & Rate Saturation**

**Richard M. Murray**

California Institute of Technology  
1200 E. California Blvd.  
Pasadena, CA  91125

Office of Navel Research

Approved for Public Release

Annual technical report for grant, covering period June 1, 1996 to May 31, 1997. This report discrives recent analytical and experimental results on nonlinear control of systems with simultaneous magnitude and rate saturations.
Nonlinear Control of Mechanical Systems in the Presence of
Magnitude and Rate Saturations

Richard M. Murray
Mechanical Engineering
California Institute of Technology

Annual Report, Grant N00014-96-1-0804
1 June 1996 to 31 May 1997

This project is aimed at developing systematic techniques for control of mechanical systems in
the presence of magnitude and rate saturations, with particular emphasis on problems arising in
the context of high performance aircraft. Magnitude and rate saturations are a major source of
nonlinearity in all flight control systems and are a fundamental mechanism of instability in both
automated and piloted flight. Recent theoretical developments in nonlinear control theory as well
as increasing computational power in offline and online computation are enabling the use of more
powerful techniques for control of these systems. This project builds on an established base of work
in nonlinear control of mechanical systems and stabilization of strongly nonlinear systems to explore
new approaches to this problem. In addition to developing theoretical tools for analysis of flight
control systems with saturations, experimental validation of the techniques is being performed using
a flight control experiment at Caltech that exhibits many of the essential features of aircraft systems
while remaining simple enough to allow meaningful testing of fundamental feedback mechanisms.

Our results to date have focused on two different approaches to modifying high performance
control laws to maintain stability and performance in the presence of rate saturations. The first
approach is the use of a nonlinear gain scheduling technique that degrades the performance of the
system when actuators are saturated in such a way as to insure stability. Experimental results on
the Caltech ducted fan show this approach to be very promising and these results will be presented
at the AIAA Guidance, Navigation, and Control conference in August, 1997 [1]. Analytical results
have been derived giving proof of convergence of the algorithm and indicating how and why the
controller works [2].

The second approach that we have been pursuing is the use of homogeneous control techniques
to convert a linear controller into a nonlinear controller that obeys magnitude and rate limits.
This follows an idea based on the work of Praly (in France) and also builds on previous work at
Caltech in the construction of homogeneous feedback laws. To date, we have made some progress
on applying these ideas to linear systems that are marginally stable (no unstable eigenvalues) and
are continuing to develop these techniques and understand how to apply them. The goal of this
approach is to design controllers that can “wrap around” existing controllers to allow operation in
the presence of magnitude and rate limits and we expect further progress in the coming year.

In addition, we have collaborated with Professor Andrew Teel at the University of Minnesota
to test some of his controllers on our flight control experiment. Teel has developed techniques for
combining local, high performance controllers with global, stabilizing controllers. These stabilizing
controllers have poor performance, but can stabilize the system in the presence of actuator limits.
As a consequence, it should be possible to achieve very good local performance using aggressive control designs while maintaining stability of the system at the operating envelope. Experimental testing of these controllers has demonstrated the efficacy of the approach as was recently presented at the 1997 American Control Conference [3].

Finally, we are beginning to pursue the development of real-time trajectory generation techniques in the presence of magnitude and rate saturations. At the present we are limiting ourselves to so-called differentially flat systems, which are nonlinear systems for which trajectory generation is conceptually simple and computationally tractable. This work is just beginning and we have not achieved any significant new results to date, but we are expecting this to be a very fruitful avenue of research with many applications to aerospace systems.

References

