The Enhanced Backscattering of Light from Random Surfaces and Related Phenomena

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The scattering of electromagnetic waves from, and their transmission through, randomly rough surfaces has been studied theoretically and experimentally. The first rigorous, computer simulation, results for the in-plane and out-of-plane, co- and cross-polarized scattering of electromagnetic waves from two-dimensional random metal surfaces have been obtained. It has been shown theoretically that the angular distribution of the intensity of the diffuse component of the electromagnetic waves scattered from, or transmitted through, a thin dielectric or metallic film that supports two or more guided or surface waves displays satellite peaks in addition to the enhanced backscattering and enhanced transmission peaks. New features (dips) have been predicted to occur at special scattering angles in the scattering electromagnetic waves from one-dimensional random metal surfaces that are periodic on average rather than planar. Experimental studies have confirmed the existence of enhanced backscattering of electromagnetic waves from one-dimensional, deterministic, metal and dielectric surfaces defined by $x_3 = \zeta \cos(ax_1 + bx_2^3)$. The limits of validity of various perturbation theories for the coherent scattering of p- and s-polarized electromagnetic waves from dielectric and metallic surfaces have been determined through simulation and experimental studies.
A. Statement of the Problem Studied

Theoretical and experimental studies of multiple-scattering phenomena occurring in the scattering of electromagnetic waves from, and their transmission through, one- and two-dimensional randomly rough surfaces have been carried out, and new physical phenomena predicted and observed.

B. Summary of the Most Important Results

Nineteen papers acknowledging support from Army Research Office Grant DAAL03-92-G-0239 were published during the grant period. In what follows we present some of the highlights of the results obtained.

Several papers (4,5,12,13) were devoted to studies of the scattering of electromagnetic waves from two-dimensional, randomly rough surfaces. In (5) the first rigorous, computer simulation, calculations of the in-plane and out-of-plane, co-polarized and cross-polarized scattering of electromagnetic waves from a two-dimensional randomly rough surface were carried out. The scattered field was expressed through the Franz formulas in terms of two vectors \( \vec{J}_H = \hat{n} \times \vec{H} > \) and \( \vec{J}_E = \hat{n} \times \vec{E} > \), where \( \hat{n} \) is the unit vector normal to the surface at each point directed into the vacuum region, and \( \vec{H} > \) and \( \vec{E} > \) are the magnetic and electric fields in the vacuum region evaluated on the surface. (Note that \( \vec{J}_H \) and \( \vec{J}_E \) have only two independent components since \( \hat{n} \cdot \vec{J}_H = \hat{n} \cdot \vec{J}_E = 0 \).) The independent components of \( \vec{J}_H \) and \( \vec{J}_E \) were obtained from a numerical solution of the Stratton-Chu equations within a square region of the \( x_1x_2 \)-plane of edge \( L = 12.8 \lambda \), where \( \lambda \) is the wavelength of the incident electromagnetic field. The latter had the form of a two-dimensional Gaussian beam of half-width \( w = 3 \lambda \). A set of six coupled, inhomogeneous integral equations had to be solved for the four independent components of \( \vec{J}_H \) and \( \vec{J}_E \), since the Stratton-Chu equations included \( \rho_H = \hat{n} \cdot \vec{H} > \) and \( \rho_E = \hat{n} \cdot \vec{E} > \) as well. (In subsequent, as yet unpublished, work \( \rho_H \) and \( \rho_E \) were expressed in terms of surface divergences of \( \vec{J}_H \) and \( \vec{J}_E \), resulting in a system of 4 coupled inhomogeneous integral equations.) The edge \( L \) of the square region of rough surface was divided into 128 intervals of length \( 0.1 \lambda \), so that a grid of 16,384 points was used in converting the integral equations into matrix equations. The resulting system of 98,304 equations in that number of unknowns was solved by Neumann-Liouville
iteration. Typically six iterations for each of 100 realizations of the random surface sufficed to yield converged results for the degrees of surface roughness assumed, viz. rms height $\delta = \lambda$ and transverse correlation length $a = 2\lambda$. The surface was a silver surface, whose dielectric constant at the wavelength of the incident light, $\lambda = 0.4579\mu m$, is $\epsilon(\omega) = -7.5 + i0.24$. The incident beam was p-polarized, and was incident normally on the surface. In all cases studied a well-defined enhanced backscattering peak was present in the angular dependence of the intensity of the diffuse component of the scattered light. A comparison of the results for silver with the corresponding results for a perfectly conducting surface showed that the latter is a poor approximation to silver under the conditions assumed.

In (4) a manifestly reciprocal form of phase perturbation theory is presented for the scattering of a scalar plane wave from a two-dimensional random surface on which the Dirichlet boundary condition is satisfied. The original version of phase perturbation theory for this scattering problem (J. Shen and A. A. Maradudin, Phys. Rev. B22, 4234 (1980)) was not reciprocal, and Ref. 4 corrects this deficiency of that earlier work.

A many-body theory of the scattering of a scalar plane wave from a two-dimensional random surface on which the Neumann boundary condition is satisfied is presented in (12). The need to use many-body theory to solve this problem is caused by the fact that low-order small-amplitude perturbation predicts a singular contribution to the mean differential reflection coefficient from the diffuse component of the scattered field. The singularity is cured by reformulating the scattering problem in terms of the averaged single-particle Green's function, whose roughness-induced complex self-energy eliminates the divergence obtained when the unperturbed Green's function (i.e. for a planar surface) is used. The calculation carried out in (12) also predicts the occurrence of enhanced backscattering in this scattering problem.

Finally, a low-order perturbation theory of the scattering of electromagnetic waves from a two-dimensional randomly rough metal surface is carried out in (13). The scattering amplitudes are calculated to third order in the surface profile function, which is sufficient to yield the contribution to the mean differential reflection coefficient from the diffuse component of the scattered electromagnetic field exactly to fourth order in the surface profile function. The point of this calculation was to show that if weakly rough two-dimensional random metal surfaces are fabricated on the basis of two-
dimensional generalizations of the power spectrum recently used by C. S. West and K. A. O'Donnell (J. Opt. Soc. Am. A12, 390 (1995)) in experimental studies of the enhanced backscattering of light from randomly rough one-dimensional metal surfaces, which allow the incident light to couple into the surface plasmon polaritons supported by the vacuum-metal interface more strongly than is possible if, say, a Gaussian power spectrum is used, enhanced backscattering through the surface plasmon polariton mechanism (A. R. McGurn, A. A. Maradudin, and V. Celli, Phys. Rev. B31, 4866 (1985)) should be readily observable in scattering from two-dimensional random metal surfaces. Until now, this effect has not been observed experimentally, although it has often been seen in scattering from large rms height, large rms slope two-dimensional metal surfaces, where the surface plasmon polariton mechanism is not significant due to the strong roughness-induced attenuation of the surface plasmon polaritons.

A second set of papers (7,8,14,18,19) was devoted to theoretical studies of the satellite peaks that accompany the enhanced backscattering peak, in scattering of electromagnetic waves from bounded systems that support two or more guided or surface waves (7,8,14,18) or the enhanced transmission peak, in the transmission of electromagnetic waves through such systems (8). Such systems include a free-standing metal film, which always supports two p-polarized surface plasmon polaritons, and a dielectric film on a perfectly conducting substrate, which can support two or more guided waves of p- and s-polarization for a given dielectric constant, provided it is thick enough relative to the vacuum wavelength of the incident light. It can be shown that if such a structure supports $N$ surface or guided waves, where $N$ is greater than or equal to two, at the frequency $\omega$ of the incident light, and the wave numbers of these waves are $q_1(\omega), q_2(\omega), \ldots, q_N(\omega)$, then satellite peaks occur in the angular dependence of the intensity of the diffuse component of the electromagnetic field scattered from a film whose illuminated surface is a one-dimensional random surface at scattering angles $\theta_s$ given by

$$\sin \theta_s = -\sin \theta_0 \pm \frac{c}{\omega} (q_m(\omega) - q_n(\omega)) \quad m \neq n,$$

where $\theta_0$ is the angle of incidence. Similarly, satellite peaks occur in the angular dependence of the intensity of the diffuse component of the electromagnetic field transmitted through a film whose illuminated surface is a
one-dimensional randomly rough surface at angles of transmission $\theta_t$ given by

$$\sin \theta_t = -\sin \theta_0 \pm \frac{c}{\omega} (q_m(\omega) - q_n(\omega)) \quad m \neq n.$$ 

These satellite peaks occur in addition to the enhanced backscattering and enhanced transmission peaks, that occur at $\theta_s = -\theta_0$ and $\theta_t = -\theta_0$, respectively, and, like the latter, are multiple-scattering effects that arise due to the coherent interference of a given multiple-scattering sequence with its time-reversed partner.

A variety of theoretical techniques have been employed in these papers in calculating the contributions to the mean differential reflection and transmission coefficients from the diffuse components of the scattered and transmitted electromagnetic fields. These include small-amplitude perturbation theory\(^{(7,8,14)}\), many-body perturbation theory\(^{(7,18)}\), numerical simulations\(^{(7,8,18)}\), and numerical solutions of the reduced Rayleigh equation for the scattering amplitude\(^{(19)}\). In the last of these approaches (T. R. Michel, J. Opt. Soc. Am. A11, 1874 (1994)) a segment of a one-dimensional random surface of length $L$ is generated numerically, and is then replicated periodically to cover the entire $x_1$-axis. The matrix equation for the amplitudes of the Bragg beams obtained by the method of reduced Rayleigh equations (F. Toigo, A. Marvin, V. Celli, and N. R. Hill, Phys. Rev. B15, 5618 (1977)) is then solved numerically for several hundred different realizations of the random surface, and the results used to calculate the mean differential reflection coefficient.

The remaining theoretical papers (1,2,6,10,11,15) are devoted to a variety of topics in the general area of optical interactions at random surfaces. In (1) it is shown that if a one-dimensional random metal surface is periodic on average rather than planar, the angular dependence of the intensity of the diffuse component of $p$-polarized light scattered from it displays features (dips) at scattering angles $\theta_s$ that are related to the angle of incidence by $\sin \theta_s = -\sin \theta_0 + n(\lambda/d)$, where $n$ is a nonzero integer, $\lambda$ is the wavelength of the incident light, and $d$ is the period of the average surface. These features are multiple scattering effects caused by the coherent interference of each multiply-scattered surface plasmon polariton path with its time-reversed partner. The results also display the phenomenon of diffuse bands, which are peaks in the angular dependence of the intensity of the diffuse component of the scattered light that are already present in the single-scattering ap-
proximation. They occur at scattering angles that correspond to the wave numbers of surface plasmon polaritons on a periodically corrugated vacuum metal interface at the frequency of the incident light, at which the Green's functions for these surface electromagnetic waves possess (damped) poles. The frequency of the incident light must lie on the second or higher branch of the surface plasmon polariton dispersion curve in the reduced zone scheme, i.e. in the radiative region, for these bands to be seen. The corresponding surface waves, therefore, possess a finite, albeit quite long, lifetime, due to radiation damping, which governs the widths of the diffuse bands.

In the numerical simulation calculations of the scattering of p- or s-polarized light from one-dimensional randomly rough metal surfaces, the scattered field is expressed as an integral whose integrand contains the values of the magnetic (electric) field and its normal derivative in the vacuum region, evaluated on the rough surface, in p- (s-) polarization. These two source functions satisfy a pair of coupled inhomogeneous integral equations. In (2) this pair of equations is transformed into an alternative pair of coupled inhomogeneous integral equations in which the inhomogeneous terms are the Kirchhoff (i.e. single-scattering) approximations to the corresponding source functions. A solution of this system of equations by Neumann-Liouville iteration generates a multiple-scattering expansion, in which the first iterate yields the double-scattering contribution, the second iterate yields the triple scattering contribution, ... , to the source functions. The result is used to show that enhanced backscattering of p- and s-polarized light from large rms height, large rms slope, one-dimensional random metal surface is a multiple-scattering phenomenon that is already present in the double-scattering approximation. The transformed equations, and their iterative solution, should be useful (i.e. the iterative solution should converge rapidly) in application to the scattering of light from random metal surfaces for which the Kirchhoff approximation is a good one.

In (6) a low-order perturbation theory of the scattering of electromagnetic waves from a perfectly conducting sphere with a randomly rough surface is presented.

In (10) the coherent scattering (reflectivity) of p- and s-polarized light from a one-dimensional randomly rough dielectric surface is calculated by small-amplitude perturbation theory, self-energy perturbation theory, phase perturbation theory, and the Kirchhoff approximation. The results are com-
pared with the results of a computer simulation calculation of the reflectivity, to determine the limits of validity of these different approximations. It is found that, for a given value of \( a/\lambda \), where \( a \) is the transverse correlation length of the surface roughness and \( \lambda \) is the wavelength of the incident light, as the rms height of the surface \( \delta \) is increased, phase perturbation theory predicts the reflectivity of p- and s-polarized light most accurately, for all angles of incidence, while small-amplitude perturbation theory is the least accurate. The only exception to this result occurs in p-polarization for angles of incidence near the Brewster angle, where the phase perturbation theory for the reflectivity displays a singularity. In that case self-energy perturbation theory appears to be most reliable.

The first experimental observation of the enhanced backscattering of p-polarized light from a one-dimensional, weakly rough, random metal surface caused by the coherent interference between multiply-scattered surface plasmon polariton paths, excited by the incident light through the roughness, and their time-reversed partners (A. R. McGurn, A. A. Maradudin, and V. Celli, Phys. Rev. B31, 4866 (1985)) was observed experimentally by C. S. West and K. A. O’Donnell (J. Opt. Soc. Am. A12, 390 (1995)), ten years after its theoretical prediction. This was accomplished by the use of (gold) surfaces fabricated on the basis of a power spectrum of the random roughness that is nonzero in only a narrow range of wave numbers about the wave number of the surface plasmon polariton supported by the surface at the frequency of the incident light. As a consequence, enhanced backscattering due to the surface plasmon polariton mechanisms is possible for only a limited range of values of the angles of incidence and scattering, but its intensity is greater by 2-3 orders of magnitude than that predicted when surfaces with the same rms height and rms slope fabricated on the basis of a Gaussian power spectrum are used, since the wave number of the surface plasmon polariton lies in the wings of the latter spectrum, leading to a weak roughness-induced excitation of that surface wave. In (11) the many-body perturbation theory of A. R. McGurn, A. A. Maradudin, and V. Celli (Phys. Rev. B31, 4866 (1985)), and the low-order perturbation theory of A. A. Maradudin and E. R. Méndez (Appl. Optics 32, 3385 (1993)), together with the West-O’Donnell power spectrum, are used to calculate the contribution to the mean differential reflection coefficient from the diffuse component of the scattered light. The good quantitative and qualitative agreement between the theoretical and
experimental results, with no fitting parameters, supports the conclusion of West and O'Donnell that their data demonstrate the existence of enhanced backscattering caused by the excitation of surface plasmon polaritons on a weakly rough random metal surface.

Finally, in (15) a numerical study of a model near-field optical microscope is carried out. Specifically, a two-dimensional model of the scattering of p- and s-polarized light, emitted by a coated tapered glass fiber, from a metal surface with a topographic or an optical defect, is investigated by numerical solutions of the integral equations of scattering theory. The intensity of the scattered electromagnetic field at the center of the bottom of the fiber, and the integrated intensity of the field scattered back through the fiber, as it is moved at constant height above the perturbed metal surface are calculated. It is found that the position and width of surface defects can be determined from these calculations with subwavelength (\(\lambda/26\)) resolution, although the intensities bear no simple relation to the surface scanned.

During the review period several experimental studies (3,9,16,17) were carried out to test the theoretical predictions presented in earlier work supported by the Army Research Office. In the first of these (3), it is shown that enhanced backscattering can occur in the scattering of p- and s-polarized light from a one-dimensional randomly rough vacuum-dielectric interface of a dielectric film deposited on a glass substrate, with a planar glass-vacuum interface. Since the scattering of p-polarized light from the one-dimensional random surface of a semi-infinite dielectric does not normally display enhanced backscattering (A. A. Maradudin, E. R. Méndez, and T. Michel, in *Scattering in Volumes and Surfaces*, eds. M. Nieto-Vesperinas and J. C. Dainty (North-Holland, Amsterdam, 1990), pp. 157-174), it is concluded that the enhanced backscattering observed in p-polarization is caused by the reflection of the light from the unilluminated planar dielectric-glass interface, which forces a double passage of the light through the random vacuum-dielectric interface. This scattering sequence interferes coherently with its time-reversed partner to produce the enhancement of the scattered intensity in the retroreflection direction. While the same mechanism plays a role in the enhanced backscattering of s-polarized light from this structure, it is less important, because the effect is already seen in the scattering of s-polarized light from the random surface of a semi-infinite dielectric medium (A. A. Maradudin, T. Michel, A. R. McGurn, and E. R. Méndez, Ann. Phys. (New
In (9) it is shown that enhanced backscattering is observed in the scattering of both p- and s-polarized light from a free-standing dielectric film whose illuminated surface is a one-dimensional random surface, while its back surface is planar. The mechanism responsible for the effect is the same one responsible for enhanced backscattering from a rough dielectric film on a glass substrate. It was predicted earlier by A. A. Maradudin, Jun Q. Lu, P. Tran, R. F. Wallis, V. Celli, Zu-Han Gu, A. R. McGurn, E. R. Méndez, T. Michel, M. Nieto-Vesperinas, J. C. Dainty, and A. J. Sant, Rev. Mex. Fis. 38, 343 (1992).

In (16) it was shown that enhanced backscattering is observed in the scattering of s-polarized light incident from a vacuum onto a one-dimensional, deterministic gold surface fabricated to approximate a surface whose profile is defined by the equation \( x_3 = \xi_0 \cos(ax_1 + bx_2^2) \). It is also present in the scattering from a dielectric film, whose vacuum-dielectric interface is defined by the same one-dimensional deterministic profile function, deposited on a planar glass substrate, when the structure is illuminated through the glass substrate. The experimental results are qualitatively and semi-quantitatively reproduced by the results of accompanying theoretical calculations. The prediction that enhanced backscattering should be observed in scattering from such deterministic surfaces was made earlier by A. A. Maradudin and E. R. Méndez (Optics Lett. 17, 1752 (1992)). It demonstrates that randomness is not necessary for the existence of this effect provided that the scattering occurs from a surface containing enough grooves with regularly varying widths that the resulting scattered field is an average over that number of grooves of the scattering from a single groove whose width is varied in a similar fashion, since Maradudin and Méndez showed that the latter system also gives rise to enhanced backscattering. They also showed that it is a multiple-scattering effect, not a single-scattering effect.

Finally, in (17) a critical evaluation of various theoretical approaches (small-amplitude perturbation theory, self-energy perturbation theory, phase perturbation theory, the Kirchhoff approximation, and numerical simulations) to the calculation of the reflectivity of one-dimensional randomly rough metallic (gold) surfaces for p- and s-polarized light was carried out. The wavelengths of the incident light were \( \lambda = 5.5\mu m \) and \( \lambda = 10.6\mu m \). The rms heights of the surfaces ranged from 0.38\( \mu m \) to 0.75\( \mu m \), and the trans-
verse correlation lengths of the surface ranged from 2.8\( \mu m \) to 15.0\( \mu m \). The results showed that phase perturbation theory has wider applicability than the other perturbation theories and the results of the Kirchhoff approximation. Given the experimental uncertainties, in all the cases studied the phase perturbation theory provided an accurate value for the reflectivity.

C. List of All Publications


D. Participating Scientific Personnel

Professor A. R. McGurn
Professor M. Nieto-Vesperinas
Professor V. Freilikher
Professor M. Kaveh
Professor G. A. Farias
Dr. E. R. Méndez
Dr. T. A. Leskova
Dr. J. A. Sánchez-Gil
Dr. I. Yurkevich
Dr. Jun Q. Lu
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Dr. Zu-Han Gu
Dr. A. D. Arsenieva
Dr. A. Martinez
Dr. R. M. Fitzgerald
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Dr. S. L. Cesar
Dr. M. Josse
Dr. A. G. Navarette
Mr. A. Madrazo
Mr. M. Pustilnik
Ms. Anne Sentenac
Mr. F. Pincemin
Mr. A. Mendoza-Suárez

Ms. Sentenac, Mr. Pincemin, and Mr. Mendoza-Suárez received Ph.D. degrees during the grant period.