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THE PREDICTION ALGORITHM FOR THE LANDING CRAFT AIR CUSHION (LCAC) SELECTION SYSTEM

D. J. Blower
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THE PREDICTION ALGORITHM FOR THE LANDING CRAFT AIR CUSHION (LCAC) SELECTION SYSTEM

D. J. Blower

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Abstract

This paper explains the prediction algorithm used by the Landing Craft Air Cushion Vehicle (LCAC) selection system. Five variables from a psychomotor test battery were combined to form a composite score. This composite score was then compared to a threshold score. If a candidate for an LCAC crew position achieved a composite score higher than the threshold score, that candidate was predicted to pass Phase I of LCAC training. Likewise, if a candidate scored lower than the threshold score he was predicted to fail training. The threshold score was determined by Statistical Decision Theory as interpreted from the Bayesian approach. Examples are given showing how the threshold scores can change as a function of the prior probabilities of pass or fail and the values attached to making correct and incorrect predictions.
Acknowledgements

I would like to mention my friends and colleagues in the LCAC project who have provided valuable assistance over the years: CDR Dan Dolgin, Dr. Tatree Nontasak, Mrs. Kathy Helton (née Travis) and Mr. Allen Chapman.
1 Introduction

The Naval Aerospace Medical Research Laboratory (NAMRL) first began research into a selection system for the Landing Craft Air Cushion (LCAC) Vehicle in 1987 with funding from the Naval Sea Systems Command. At that time, the attrition rate during Phase I training for the LCAC operator position was approximately 40%. Since NAMRL had been involved in aviator selection, it was thought that some of the same tests might help to reduce these high LCAC attrition rates. Even though the NAMRL selection test battery had been devised for officers in the aviation community, the tasks involved in piloting the LCAC might exhibit enough similarity to use them for the enlisted group targeted as LCAC operators.

Accordingly, the NAMRL selection test battery was modified to form an LCAC selection system, and research was begun to assess its efficacy in reducing attrition for the LCAC operator community. The selection system proved so successful that it was enlarged to encompass two other LCAC positions, the engineer and the navigator. The LCAC selection system is now routinely administered to about 100 candidates yearly for the operator and engineering positions at the Naval Aerospace and Operational Medical Institute. Modifications to the LCAC selection system to accommodate some unique requirements for the navigator position are currently being researched at NAMRL. The intent of this report is to document the prediction algorithm at the heart of the LCAC selection system. There are many other references [1-15] that describe the history of the research and the implementational details.

The LCAC selection system contains an algorithm for predicting the success or failure of an individual candidate who might possibly undergo training for the position of operator or engineer. The actual implementation of the algorithm in the software code is quite simple. Five variables from a psychomotor test battery are weighted and then combined to form a composite score. This composite score is assumed to be a good indicator of the more complex psychomotor skills relevant to success during LCAC training. Each candidate’s composite score on the test battery is then compared to a threshold score. If a candidate’s composite score is greater than or equal to the threshold score, the candidate is predicted to pass training, while a composite score below the threshold results in a prediction of failure for the candidate.

How were the variables and the associated weights that make up the composite score derived? Briefly, scores were constructed such that the distance between the mean score of those who eventually passed training and the mean score of those who eventually failed training was as far apart as possible. At the same time, the variance of the composite scores was kept to a minimum for each of these two groups. This goal was accomplished via the statistical technique of Discriminant Analysis.

While the actual implementation of the algorithm in the software code is relatively straightforward, the mathematical rationale for where to set the threshold that predicts the training outcome is a bit more complex. The development depends upon a branch of mathematics called statistical decision theory (SDT). SDT, in turn, hinges on a Bayesian treatment for making inferences when costs and prior probabilities can be specified.

If the costs of the correct and incorrect decisions and prior probabilities for the outcomes can
be specified, then SDT will provide the optimal placement of the threshold based on this input. Since these subjective and economic data are generally not disclosed to researchers involved in selection, it is left up to the developers of the LCAC selection system to interpret the desires of the LCAC community in making a determination of the optimal pass/fail threshold score. Arbitrarily choosing a threshold that is high will undoubtedly rule out the selection of a certain number of candidates that would otherwise be successful in training; however, if the costs associated with candidate failure during training are high, and outweigh the costs associated with eliminating some potentially successful candidates, then such a high pass/fail threshold might be desirable.

Even a successful selection test battery and associated prediction algorithm like the LCAC selection system cannot predict with complete certainty whether a candidate will pass or fail training. It should be understood that the LCAC selection system raises the probability for any candidate selected to successfully complete training. For example, before the selection system was implemented, the consequent random sampling of psychomotor skills might result in an overall probability of success equal to .50. With a selection system, by sampling some of the relevant psychomotor skills to successfully operate the LCAC, this probability might be raised to .80. This means that there will be less attrition during training, but does not mean that every candidate who passes the test battery will pass training. Any candidate chosen by the test battery has a higher probability of completing training than one chosen without the benefit of this information.

Any selection test that is not perfectly correlated with training outcome (and this includes all selection tests) will exhibit some recommendations that will later turn out to be in error. The correctness of decisions in trainee selection is related to the degree to which the selection test taps into the skills needed to perform critical tasks. For example, perhaps docking the LCAC aboard the mothership is a skill that is not sampled by the selection test. A candidate for training then can perform adequately on the psychomotor test battery and yet fail the training where this skill is needed. There are also those additional failures during training that the selection test could never have predicted because they are due to extraneous factors such as morale, motivation, unique situational occurrences, family concerns, personal life style changes, etc.

For all of these reasons, there will always be failures from the LCAC training program even after the selection test battery has given a stamp of approval. Raising the threshold can help reduce failures, but it is not a universal panacea. There comes a point where raising the threshold, even though it does provide you with a candidate with better psychomotor skills, cannot compensate for those other factors, mentioned above, that can cause failure. This is, of course, just another way of saying that the selection test battery cannot predict success or failure with a probability of 1.

Another consideration to bear in mind in connection with the probabilistic nature of the LCAC selection system is that even though we talk about raising the probability of success in training from, say, .50 to .80, the actual numbers passing or failing during training over a finite sample size will vary from the expectation dictated by these probabilities. Over a finite sample size of candidates, say 100 for an easy number, 50 will be expected to pass and 50 will be expected to fail without the selection test. But it could happen through random fluctuations over these 100 candidates that only 40 pass or perhaps 60 pass, while the underlying probability has not deviated from .5. Likewise, if the underlying probability has been raised through the use of the selection
system to .8, we would expect 80 to pass, but would not be surprised if only 72 passed or 86 passed. In both cases, these numbers are compatible with a heightened probability of success of .8 and reflect only the vagaries of the sample size. However, an extra benefit of the increased probability that use of the selection system affords is that, although sampling variability still exists, it is smaller at higher probabilities than at .5.

2 The Derivation of the Composite Scores

Five variables were chosen from the psychomotor test battery through discriminant analysis. These five variables were reduced to four by combining the last two variables. Further research may indicate other variables that could be included in a revised prediction algorithm, but at the present time these variables seem to perform well. The composite score (CS) was derived by multiplying these four variables by weights suggested by the discriminant analysis and adding a constant value. The composite scores used in the numerical examples for a candidate who was a predicted PASS and for the candidate who was a predicted FAIL are derived in the two tables shown below. The actual formula for the composite score can be expressed as follows,

\[ CS = \sum_{i=1}^{4} w_i S_i + C \]

\[ = (-1)S_1 + (-4)S_2 + (-1)S_3 + (+1)S_4 + 27 \]

\[ = 27 + S_4 - S_1 - 4S_2 - S_3 \]

As will be explained in the next section, the threshold composite score was calculated as +.14. If a candidate’s performance on the test battery as reflected by the composite score is above +.14, the candidate is a predicted PASS. If a candidate’s performance on the test battery is below +.14, the candidate is a predicted FAIL. The table below shows a candidate who scored fairly well on the test battery. Since the composite score of +2.00 is above +0.14, this candidate is a predicted PASS.

The variables, a typical value for each variable, their associated weights, and the resulting composite score are presented in Table 1 below.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Value</th>
<th>Weight</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stick and DLT</td>
<td>S_1</td>
<td>4.00</td>
<td>-1</td>
<td>-4.00</td>
</tr>
<tr>
<td>Stick and Rudder</td>
<td>S_2</td>
<td>4.50</td>
<td>-4</td>
<td>-18.00</td>
</tr>
<tr>
<td>Stick, Rudder, and Throttle</td>
<td>S_3</td>
<td>4.75</td>
<td>-1</td>
<td>-4.75</td>
</tr>
<tr>
<td>Horizontal Tracking and RT</td>
<td>S_4</td>
<td>1.75</td>
<td>+1</td>
<td>1.75</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>27.00</td>
<td>+1</td>
<td>27.00</td>
</tr>
<tr>
<td>Composite Score</td>
<td></td>
<td></td>
<td></td>
<td>+2.00</td>
</tr>
</tbody>
</table>
The tracking error scores shown in the first three rows of this table represent a logarithmic transformation of the actual raw pixel error in order to promote a more normal distribution for this kind of score. The fourth variable is itself a composite variable consisting of a derived measure of tracking while performing a digit cancellation task at the same time. An LCAC candidate with this kind of composite score would have a calculated posterior probability for PASS greater than .50 and therefore would have been a predicted PASS using the prediction algorithm. This statement is explained in subsequent sections dealing with the Bayesian nature of the prediction algorithm.

Table 2 shows a candidate who did not do as well on the test battery. Notice that $S_1$, $S_2$, and $S_3$ are error scores so that a higher score means worse performance. Lower scores on variable $S_4$ are associated with worse performance so that this subject also did not perform as well as the previous subject on Horizontal Tracking and RT. This subject achieved a composite score of -1.50 and is, therefore, a predicted FAIL since this composite score is below the threshold score.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Name</th>
<th>Value</th>
<th>Weight</th>
<th>Multiplication</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stick and DLT</td>
<td>$S_1$</td>
<td>4.50</td>
<td>-1</td>
<td>-4.50</td>
</tr>
<tr>
<td>Stick and Rudder</td>
<td>$S_2$</td>
<td>5.00</td>
<td>-4</td>
<td>-20.00</td>
</tr>
<tr>
<td>Stick, Rudder, and Throttle</td>
<td>$S_3$</td>
<td>4.90</td>
<td>-1</td>
<td>-4.90</td>
</tr>
<tr>
<td>Horizontal Tracking and RT</td>
<td>$S_4$</td>
<td>.90</td>
<td>+1</td>
<td>.90</td>
</tr>
<tr>
<td>Constant</td>
<td></td>
<td>27.00</td>
<td>+1</td>
<td>27.00</td>
</tr>
<tr>
<td><strong>Composite Score</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>-1.50</strong></td>
</tr>
</tbody>
</table>

### 3 Bayes’s Formula and Numerical Examples

The prediction algorithm for deciding whether to recommend a candidate for entrance into the LCAC training program is based on the composite score from the psychomotor test battery as just explained. However, this is just part of the story behind the prediction algorithm. The use of statistical decision theory, together with the assignment of relative costs to a decision-true state matrix, lies at the core of statistical decision theory as it is used here. This Bayesian approach seeks to update a prior state of knowledge about the probability of success in LCAC training through new information concerning skills thought to be relevant for training success.

An LCAC candidate is classified as a predicted PASS or FAIL by computing the posterior probability of belonging to the PASS or FAIL group, and then assigning the candidate to the group with the largest posterior probability. The extra information used to update the prior probabilities is the composite score derived from the psychomotor test battery. The general form of Bayes’s Formula used to calculate the posterior probability of group membership is

$$P(G_i|D) = \frac{P(D|G_i) \times P(G_i)}{\sum_{i=1}^{n} P(D|G_i) \times P(G_i)}$$  \hspace{1cm} (1)

where $P(G_i|D)$ is the posterior probability for Group $i$ after receipt of some test data, $P(D|G_i)$
is the likelihood of the test data given Group \( i \), and \( P(G_i) \) is the prior probability of Group \( i \).

In particular, for the two group case, which is the focus of our attention in the LCAC study, the formula reduces explicitly to

$$
P(\text{PASS}|D) = \frac{P(D|\text{PASS}) \times P(\text{PASS})}{[P(D|\text{PASS}) \times P(\text{PASS})] + [P(D|\text{FAIL}) \times P(\text{FAIL})]}
$$

(2)

$$
P(\text{FAIL}|D) = 1 - P(\text{PASS}|D)
$$

(3)

The likelihood terms, \( P(D|\text{PASS}) \) and \( P(D|\text{FAIL}) \)

are Gaussian,

$$
P(D|\text{PASS}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{PASS}}} e^{-1/2 \left( \frac{x - \mu_{\text{PASS}}}{\sigma_{\text{PASS}}} \right)^2}
$$

(4)

$$
P(D|\text{FAIL}) = \frac{1}{\sqrt{2\pi}\sigma_{\text{FAIL}}} e^{-1/2 \left( \frac{x - \mu_{\text{FAIL}}}{\sigma_{\text{FAIL}}} \right)^2}
$$

(5)

where \( x \) is the candidate's composite score, \( \mu_{\text{PASS}} \) is the group mean of the composite scores for the PASS group, \( \mu_{\text{FAIL}} \) is the group mean of the composite scores for the FAIL group, and \( \sigma_{\text{PASS}} \) and \( \sigma_{\text{FAIL}} \) are the standard deviations of the computed composite scores.

The discriminant analysis (DA) program from the statistical software package SPSSPC+ was used to derive an appropriate composite score. The DA program calculated \( \mu_{\text{PASS}} = .77239 \) and \( \mu_{\text{FAIL}} = .14135 \). \( \sigma_{\text{PASS}} \) and \( \sigma_{\text{FAIL}} \) are both equal to 1 by definition. Equation (2) can then be written as,

$$
P(\text{PASS}|D) = \frac{f_1}{f_1 + f_2}
$$

(6)

where

$$
f_1 = \exp\{-1/2 \left( x - \mu_{\text{PASS}} \right)^2\} \times P(\text{PASS})
$$

(7)

and

$$
f_2 = \exp\{-1/2 \left( x - \mu_{\text{FAIL}} \right)^2\} \times P(\text{FAIL})
$$

(8)

A numerical example using this formula for the calculation of the posterior probability of belonging to the PASS or FAIL group follows. Assume that a candidate takes the selection test battery and does reasonably well on the test. His composite score turns out to be equal to 2.00 just as in the example of the previous section. A summary of the variables needed to solve Equations (6)—(8) is presented in Table 3. The reason for these particular values of the prior probabilities will be explained later.
The composite score for the LCAC candidate | $x = 2.00$
---|---
Group mean of composite scores for PASS | $\mu_{\text{PASS}} = .77239$
Group mean of composite scores for FAIL | $\mu_{\text{FAIL}} = .14135$
Prior probability of PASS | $P(\text{PASS}) = .53$
Prior probability of FAIL | $P(\text{FAIL}) = .47$

$$f_1 = \exp\{-1/2 (2.00 - .77239)^2\} \times .53$$
$$= .4707 \times .53$$
$$f_2 = \exp\{-1/2 (2.00 - .14135)^2\} \times .47$$
$$= .1778 \times .47$$
$$P(\text{PASS}|D = 2.00) = \frac{.4707 \times .53}{(.4707 \times .53) + (.1778 \times .47)}$$
$$= .2495$$
$$= .7491$$

Therefore, the posterior probability of a PASS in LCAC training is .7491 given that a candidate achieved a composite score of 2.00. It follows that the posterior probability of a FAIL is .2509. The probability of belonging to the PASS group, given the data from the test battery, is about 75%. This candidate will be classified by the system as a PASS.

Another example of this formula is presented below; this time of an LCAC candidate with poorer performance on the test battery. Whereas in the first example the LCAC candidate scored a 2.00 and was classified as a PASS, in this example the candidate scores a -1.50 as a composite score. What is this candidate's predicted training outcome based on these data? As in the previous case, the relevant data are presented in Table 4.

The composite score for the LCAC candidate | $x = -1.50$
---|---
Group mean of composite scores for PASS | $\mu_{\text{PASS}} = .77239$
Group mean of composite scores for FAIL | $\mu_{\text{FAIL}} = .14135$
Prior probability of PASS | $P(\text{PASS}) = .53$
Prior probability of FAIL | $P(\text{FAIL}) = .47$
\[
\begin{align*}
f_1 &= \exp\left\{-1/2 \left(-1.50 - .77239\right)^2\right\} \times .53 \\
&= .0756 \times .53 \\
\end{align*}
\]

\[
\begin{align*}
f_2 &= \exp\left\{-1/2 \left(-1.50 - .14135\right)^2\right\} \times .47 \\
&= .2600 \times .47 \\
\end{align*}
\]

\[
P(\text{PASS}|D = -1.50) = \frac{.0756 \times .53}{(.0756 \times .53) + (.2600 \times .47)} \\
= \frac{.0401}{.0401 + .1222} \\
= .2470
\]

Therefore the posterior probability of a PASS in LCAC training, given that a candidate achieved a composite score of -1.50, is .2470. Again, it follows that the posterior probability of a FAIL for such a score is .7530. The posterior probability of belonging to the FAIL group is higher than the posterior probability of belonging to the PASS group, so this candidate is predicted to FAIL.

We do not have to actually carry out these calculations to make a prediction for a candidate. We simply have to find out beforehand the threshold score, i.e., the score such that anything above this score is a predicted PASS and anything below it is a predicted FAIL. Then the algorithm can make a prediction by simply comparing a candidate's composite score with the threshold score. The threshold score is that composite score where a predicted PASS and a predicted FAIL, based on the data from the test battery, are both equal to .50. With the prior probabilities set at \( P(\text{PASS}) = .53 \) and \( P(\text{FAIL}) = .47 \) in the above numerical examples, this threshold score is .14. This is observed to be the mean of the composite scores for the FAIL group. As a matter of fact, the prior probabilities were purposely manipulated so that the threshold score would be at this precise location.

Different prior probabilities would dictate a different threshold score where

\[
P(\text{PASS}|\text{Data}) = P(\text{FAIL}|\text{Data}) = .50
\]

In the Bayesian approach to statistical decision theory, the threshold score is determined by a multiplicative relationship between the costs associated with the available decisions and the prior probability. However, the DA program was coded in such a way that it was possible to enter only the prior probability, so it has to assume the entire burden for both of these factors. When a prior probability is specified, for example, \( P(\text{PASS}) = .53 \) and \( P(\text{FAIL}) = .47 \), this reflects both the historical frequency of passing and failing as well as the matrix of costs associated with the decisions.

After taking into consideration both the historical frequency of passing and failing and the costs associated with the predictions and the actual training outcomes, the threshold score was
set at the mean of the FAIL group. This decision, in turn, mandated the particular values for the prior probability parameters of the DA program that were used in the examples. Different historical frequencies and a different cost matrix would result in different numerical settings for these parameters. These different settings might reflect either a change in attrition rates or a desire to restrict or promote the flow of students into the training pipeline.

4 The Evaluation of Correct and Incorrect Decisions

In statistical decision theory, it is not enough to just provide the posterior probabilities for a candidate to pass or fail training. There must be some means for taking action based on such probabilities. Within the Bayesian framework, an incentive for taking action is cast quantitatively by assigning values to the matrix of all possible decisions with all possible true states of the world.

The construction of such a matrix is quite feasible in our problem because there are only two alternatives to be paired off with two possible decisions. The matrix will then be a 2 x 2 matrix with four cells. The two possible alternatives represent the true state of nature, i.e., whether the candidate actually passes or fails training. The two possible decisions are to predict a PASS or to predict a FAIL.

To complete the decision process, it is necessary to fill in these four cells with numbers that represent the relative value we ascribe to the two correct predictions and the two incorrect predictions. The convention employed in this report is to use positive numbers for the correct decisions and zero for the incorrect decisions. See Fig. 1 for such a 2 x 2 matrix with values specified generically as \( V_1 \) through \( V_4 \).

![Figure 1: The value matrix for the LCAC prediction algorithm. There are only two possible true states of the world and only two possible decisions. The two correct and the two incorrect decisions are labelled.](image)

The formula for reaching a decision about any individual candidate is now derived. In the literature, this is sometimes called “setting a response threshold.” The main concept from statistics
that is used in the following derivation is the definition of expectation, or more informally, the average. For a discrete variable, the expectation is defined as,

\[ E(X) = \sum_{i=1}^{n} p_i X_i \]  

(9)

where the \( p_i \) are probabilities for each \( X_i \), and the \( X_i \) are the values of the variable we are interested in. In our application, the \( p_i \) are posterior probabilities of passing or failing based upon data, and the \( X_i \) are the values assigned to the \( 2 \times 2 \) matrix that reflect our evaluation of the predictions and true outcomes.

The goal is to find the expected value of a PASS and compare it to the expected value of a FAIL. If the expected value of a PASS is greater than the expected value of a FAIL, then this provides the justification for acting and making a prediction of a PASS for a candidate. We only have to average over two \( X_i \) and two \( p_i \) to calculate the expected value of a PASS or the expected value of a FAIL. This is a simple application of the general rule of Bayesian decision making that calls for the minimization of the expectation of the loss function with respect to the posterior probability (Smith [16]).

4.1 Mathematical Derivation of Threshold Score

This derivation follows closely the one given by Coombs, Dawes and Tversky [17]. Let

\[ p_1 \equiv P(\text{Pass} | \text{Data}) \]
\[ p_2 \equiv P(\text{Fail} | \text{Data}) \]

\[ EV(\text{Pass}) = (V_1 \times p_1) + (V_3 \times p_2) \]
\[ EV(\text{Fail}) = (V_2 \times p_1) + (V_4 \times p_2) \]

If we let the expected value of a PASS be greater than the expected value of a FAIL in order to reach a decision about predicting a pass then,

\[ EV(\text{Pass}) \geq EV(\text{Fail}) \]

\[ (V_1 \times p_1) + (V_3 \times p_2) \geq (V_2 \times p_1) + (V_4 \times p_2) \]

\[ (V_1 \times p_1) - (V_2 \times p_1) \geq (V_4 \times p_2) - (V_3 \times p_2) \]

\[ p_1 (V_1 - V_2) \geq p_2 (V_4 - V_3) \]

\[ \frac{p_1}{p_2} \geq \frac{V_4 - V_3}{V_1 - V_2} \]

\[ \frac{P(\text{Pass} | \text{Data})}{P(\text{Fail} | \text{Data})} \geq \frac{V_4 - V_3}{V_1 - V_2} \]
Since the left-hand side of the final equation in the above derivation is the ratio of posterior probabilities, we plug in the likelihood times prior probability relationship from Bayes's Formula to find,

\[
\frac{P(\text{Pass}|\text{Data})}{P(\text{Fail}|\text{Data})} = \frac{P(\text{Data}|\text{Pass}) \times P(\text{Pass})}{P(\text{Data}|\text{Fail}) \times P(\text{Fail})}
\]

The denominator, \( P(\text{Data}) \), cancels out in forming the ratio. Substituting this relationship between the likelihood and the prior probability in the derivation of the expected value gives us,

\[
\frac{P(\text{Data}|\text{Pass}) \times P(\text{Pass})}{P(\text{Data}|\text{Fail}) \times P(\text{Fail})} \geq \frac{V_4 - V_3}{V_1 - V_2}
\]

At this point, we want to isolate the likelihood ratio on the left side of this inequality so the ratio of prior probabilities is moved to the right side resulting in,

\[
\frac{P(\text{Data}|\text{Pass})}{P(\text{Data}|\text{Fail})} \geq \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2}
\]

Notice that in transferring the ratio of prior probabilities from the left-hand side of the equation to the right-hand side, the ratio is inverted. These expressions on both sides of the equation have traditionally been labelled in the following manner: The expression on the left-hand side, the likelihood ratio, is given the notation,

\[
\mathcal{L}(x) = \frac{P(\text{Data}|\text{Pass})}{P(\text{Data}|\text{Fail})} \quad (10)
\]

while the expression on the right-hand side, consisting of the prior probabilities and the evaluation of the correct and incorrect decisions, is given the notation of \( \beta \),

\[
\beta = \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2} \quad (11)
\]

If \( \mathcal{L}(x) \geq \beta \) then \( EV(\text{Pass}) \geq EV(\text{Fail}) \)

So, in this condensed notation, the decision process is easily written down as summarized in Table 5.

| Predict PASS | \( \mathcal{L}(x) \geq \beta \) |
| Predict FAIL | \( \mathcal{L}(x) < \beta \) |
| Data from test battery | \( x \) |
| Threshold score | \( \beta \) |
Section 4.2: Numerical examples of setting the threshold score

Equation (12) shows that the score that divides the predicted PASS from the predicted FAIL is called the threshold score. The threshold score, \( \beta \), is seen from Equation (11) to be a function of two components, 1) the prior probabilities and 2) the values given to the correct and incorrect decisions. The first example illustrates the threshold calculation when the prior probabilities are equal and the values given to the correct and incorrect decisions are also equal. If the prior probabilities for only two possibilities are to be equal, they must equal \( 1/2 \). The values associated with the correct decisions are \( V_1 \) and \( V_4 \). They are given values of 1. The values associated with the incorrect decisions are \( V_2 \) and \( V_3 \). They are given values of 0.

\[
P(\text{Fail}) = 1/2 \\
P(\text{Pass}) = 1/2 \\
V_1 = 1 \\
V_2 = 0 \\
V_3 = 0 \\
V_4 = 1 \\
\beta = \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2} \\
= \frac{1}{2} \times \frac{1 - 0}{1 - 0} \\
= 1
\]

The threshold score, or cut-off score, is therefore set at 1. So whenever \( \mathcal{L}(x) \geq 1 \), the student is predicted to pass, and whenever \( \mathcal{L}(x) < 1 \), the student is predicted to fail. This placement of the threshold score results from making rational decisions about average cost with probabilities of cost determined by the posterior probabilities.

Figure 2 shows that the threshold score is located exactly where the distribution of test scores for the PASS and FAIL groups intersect. Of course, this intersection point is where,

\[
P(\text{Data|Pass}) = P(\text{Data|Fail})
\]

or equivalently where,

\[
\mathcal{L}(x) = \frac{P(\text{Data|Pass})}{P(\text{Data|Fail})} \\
= 1
\]

Therefore, any score to the right of this threshold score is predicted to pass, and any score to the left is
Figure 2: The threshold score is located where the likelihood ratio is equal to 1.

In this next example, we change one of the two components that make up $\beta$ to observe what effect this has on shifting the threshold score. We choose to manipulate the prior probability by increasing the prior probability of a FAIL from its previous setting at 1/2, but maintain the same value of the costs associated with the decisions.

\[
P(\text{Fail}) = 0.80
\]
\[
P(\text{Pass}) = 0.20
\]
\[
V_1 = 1
\]
\[
V_2 = 0
\]
\[
V_3 = 0
\]
\[
V_4 = 1
\]

\[
\beta = \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2}
\]
\[
= \frac{0.8}{0.2} \times \frac{1 - 0}{1 - 0}
\]
\[
= 4
\]
In this case, the threshold score is moved along the axis to the right. It must move to that point where the ratio of the densities,

\[
\frac{P(\text{Data|Pass})}{P(\text{Data|Fail})} = 4
\]

This movement of the threshold score when the prior probability is changed is shown schematically in Fig. 3. Because of the higher prior probability of a FAIL, irrespective of what the test data show, a student must score higher to move into the region where a PASS can be predicted.

![Figure 3: The threshold score has moved to the right because the prior probabilities have changed to favor a prediction of fail.](image)

If the situation were reversed so that the prior probability of a PASS were .80, then the threshold would move in the opposite direction to where \( L(x) = 1/4 \). Now the threshold score is lower, reflecting the symmetry of the movement in response to changes in the prior probability. If there is knowledge, independent of the score on the test battery, that a student will more likely pass than fail, this knowledge is captured in the prior probability. These last two examples illustrate that this knowledge has affected the threshold score placement in a logical fashion.

Having manipulated the first component, the ratio of prior probabilities, in the equation for setting the threshold score, we now observe what happens when the second component, the values associated with the decisions, are changed. In this example, the prior probabilities revert back to 1/2 for each group, but now it is more important to make a correct decision about a predicted PASS than about a predicted FAIL. To illustrate this numerically let,

\[
P(\text{Fail}) = .50
\]
\[
P(\text{Pass}) = .50
\]
\[ V_1 = 5 \]
\[ V_2 = 0 \]
\[ V_3 = 0 \]
\[ V_4 = 1 \]
\[ \beta = \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2} \]
\[ = \frac{.5}{.5} \times \frac{1 - 0}{5 - 0} \]
\[ = .20 \]

\( \beta \) moves to the left along the score axis as the value of a correct decision about a \textit{PASS} increases in relative value to a correct decision about a \textit{FAIL}. Figure 4 shows this movement of the threshold score for a change in the relative evaluation of decisions. If it is important to capture in the prediction most of the students who could pass without worrying as much about those predicted to PASS who eventually FAIL, then this movement of the threshold score makes sense. The symmetrical situation of having a costly environment for failures would cause the threshold score to move to the right and make it more difficult for a student to score above the cut-off score. This type of placement for the threshold score, of course, would appear in higher values for \( V_4 \) relative to \( V_1 \).

Figure 4: \textit{The threshold score moves to the left with a change in the evaluation of a correct predicted pass relative to the other decisions.}
The numerical examples in this section have shown the threshold score is sensitive to both the prior knowledge about success and failure and to the relative values that are attached to the predictions for success or failure. The threshold score moves in a direction justified by Bayes's Formula when either one or both of these components are changed.

5 Decision Theory Implications for LCAC

All the ingredients involved in the SDT treatment of the LCAC prediction algorithm have now been presented. As a summary, the calculation and figures for the actual LCAC situation are given in this section. Figure 5 shows the theoretical normal curve of the composite score data for the FAIL group with \( \mu_{\text{FAIL}} = .14135 \) and \( \sigma_{\text{FAIL}} = 1.00 \) and the corresponding normal curve for the PASS group with \( \mu_{\text{PASS}} = .77239 \) and \( \sigma_{\text{PASS}} = 1.00 \).

![Composite Score](image)

Figure 5: The theoretical normal curves for the composite scores of those LCAC candidates who passed training and those who did not.

The probability density function (the y-axis) of each of these curves goes into the calculation of the likelihood ratio, \( \mathcal{L}(x) \). The likelihood ratio at the threshold score is equal to \( \beta \). From the definition of \( \mathcal{L}(x) \),

\[
\mathcal{L}(x) = \frac{P(\text{Data} = .14135|\text{Pass})}{P(\text{Data} = .14135|\text{Fail})}
\]

The threshold score is located at the mean of the FAIL distribution. From a table of the normal
distribution, we can find that

\[ P(\text{Data} = .14135|\text{Fail}) = .3989 \]

To determine \( P(\text{Data} = .14135|\text{Pass}) \), we observe that a score of .14135 is \(-.631\) from the mean of the PASS distribution. The normal curve has a probability density of .3270 at this \( z \)-score. Therefore,

\[
\mathcal{L}(x) = \frac{.3270}{.3989} = .8198
\]

The actual historical frequency of the attrition rate before the selection system was implemented was about 40%. Therefore, \( P(\text{FAIL}) = .40 \) and \( P(\text{PASS}) = .60 \), and we can then determine what the costs of the decision must be for the threshold score set at the mean of the FAIL distribution. Establishing a scale by setting the value of the two incorrect decisions \( V_3 \) and \( V_2 \) at 0 and the value of the correct decision \( V_1 \) at 1, \( V_4 \) is found by

\[
\beta = \frac{P(\text{Fail})}{P(\text{Pass})} \times \frac{V_4 - V_3}{V_1 - V_2}
\]

\[
= \frac{.40}{.60} \times \frac{V_4 - V_3}{V_1 - V_2} = .8198
\]

\[ .8198 = .6667x \]

\[ x = 1.2296 \]

\[ x = \frac{V_4 - V_3}{V_1 - V_2} \]

\[ \frac{V_4 - V_3}{V_1 - V_2} = \frac{1.23 - 0}{1 - 0} \]

\[ V_4 = 1.23 \]

By valuing a correctly predicted FAIL slightly more than a correctly predicted PASS, we move the threshold to the right. Recall the discussion of the effects of the prior probabilities and the decision-true state matrix on the direction of movement in the threshold score. This is also the explanation promised in the previous section for why the ratio

\[
\frac{P(\text{Fail})}{P(\text{Pass})}
\]

was given the particular values in the numerical exercise.
As mentioned before, $V_4$ was arbitrarily manipulated to bring the threshold score to occur at the mean of the $P(\text{Data|FAIL})$ distribution. In an ideal world the prior probabilities and the decision-true state matrix would be known. These known values would then determine $\beta$, and the threshold score could be set. In the present scenario, this sequence of actions was reversed by pinning the threshold score at a precise location and then calculating what this implied for the value of the correct prediction of FAIL ($V_4$).

As always, the threshold score could be changed by different knowledge about either the prior probabilities of a PASS or FAIL, or by a different set of values for the correct and incorrect decisions. For example, if a correct prediction about a FAIL was valued even more highly than in the current example ($V_4 > 1.23$), then the threshold score would move to the right. It would then be harder for a candidate to pass the selection battery and there would be more predicted FAILS and fewer predicted PASSES. The down side of moving the threshold in this manner is that there would also be more incorrect decisions when predicting fail for candidates who would have passed. But this trade-off is implicitly accounted for within the factors making up $\beta$. The decisions ensuing from a particular placement of the threshold are, by definition, the optimal decisions given the prior probabilities and decision-true state matrix.
References


The Prediction Algorithm for the Landing Craft Air Cushion Vehicle (LCAC) Selection System

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This paper explains the prediction algorithm used by the Landing Craft Air Cushion Vehicle (LCAC) selection system. Five variables from a psychomotor test battery were combined to form a composite score. This composite score was then compared to a threshold score. If a candidate for an LCAC crew position achieved a composite score higher than the threshold score, that candidate was predicted to pass Phase I of LCAC training. Likewise, if a candidate scored lower than the threshold score he was predicted to fail training. The threshold score was determined by Statistical Decision Theory as interpreted from the Bayesian approach. Examples are given showing how the threshold scores can change as a function of the prior probabilities of pass or fail and the values attached to making correct and incorrect predictions.