SMALL-ANGLE SCATTERING OF NEUTRONS IN IRON

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Date of Manuscript: August, 1948
Date Declassified: October 28, 1948

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Technical Information Branch, Oak Ridge, Tennessee
AEC, Oak Ridge, Tenn., 4-6-49--850-A1347

Printed in U.S.A.
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In connection with a series of experiments on the polarization of neutrons, a new effect was found which complicated the polarization results for some time. When the single transmission effect (E, the percentage increase in transmission of an iron block upon magnetization) was measured under conditions of good geometry it was observed to increase appreciably. When the geometry was improved as much as possible with the equipment then in use (Figure 1 of Ref. 1) the measured value of E for a block 1 cm thick became 13% instead of the 3% to be expected from neutron polarization alone. By using iron blocks of smaller thickness, d, it was found that the excess single transmission effect became relatively even more predominant, being about 1% for a 0.1 cm block for which the real E (being proportional to d^2) would be expected to be only 0.03%. The spurious single transmission effect could also be obtained for small magnetizing fields H, unlike the true effect which requires extremely high magnetization.

It was thus quite clear that the new effect differed from the increased transmission caused by neutron polarization and should be eliminated in order to study the latter. Measurements with different geometries made it quite clear that if the geometry were made bad enough so that all neutrons scattered by as much as 1 degree were included in the transmitted beam then only the real transmission effect remained. It was then clear that the new effect was a small angle (less than 1 degree) scattering which took place in unmagnetized, but not in magnetized, iron. As the main interest at the time was in neutron polarization, the small angle scattering was not investigated in any more detail than was necessary to eliminate its influence on the polarization effects. After the polarization work was finished, however, the scattering was investigated in more detail in order to determine its cause.

THEORETICAL CONSIDERATIONS

The fact that the small angle scattering disappears with moderate magnetizing fields must mean that it is associated with the lining up of domains along the crystal axis (which takes place at low fields) and not with the rotation of the magnetic vector toward the applied field and away from the crystal axes (which takes place at high fields and which is necessary for neutron polarization). The cause of the scattering then, must lie in the domain structure of unmagnetized iron.

As neutrons pass through polycrystalline iron they are scattered when they encounter a microcrystal at a Bragg angle. For thermal neutrons, however, the Bragg scattering occurs at angles of the order of 30 degrees and greater and hence will have nothing to do with the small angle scattering. In addition, the neutron wave will be refracted at crystal boundaries and at domain boundaries. Of course, the crystal boundaries do not change with magnetization and hence refraction at them cannot contribute to the effect. Refraction at domain boundaries, however, could cause the scattering and deserves more detailed investigation.

The index of refraction of a crystal for neutron waves is given by

\[ n = 1 - \frac{\lambda^2 N_a}{2\pi} \] (1)

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where $\lambda$ is the neutron wavelength, $N$ the number of nuclei per cm$^3$, and $a$ the amplitude of coherent scattering. In the case of iron $a$ is partly nuclear and partly magnetic in origin, being given by

$$a = a_n \pm a_m$$  \hspace{1cm} (2)

where $a_n$ is the nuclear scattering amplitude (not spin-dependent) and $a_m$ is the magnetic scattering amplitude which adds to, or subtracts from, the nuclear amplitude, depending on the orientation of the neutron spin with respect to the domain magnetization.

Thus there will be two indices of refraction in the iron and the deviation experienced by a particular neutron at a domain boundary will depend on its orientation with respect to the magnetizations of the adjacent domains and on the orientation of the boundary. Upon magnetization, the boundaries and hence the deviations will disappear. The effect of the successive deviations will be to spread an initial collimated beam into a Gaussian whose width, $\sigma$, will increase with the square root of the thickness of iron traversed,

$$\sigma = \theta_{AV} \left( \frac{d}{\delta} \right)^{\frac{1}{2}}$$  \hspace{1cm} (3)

where $\theta_{AV}$ is the average deviation at a domain boundary and $\delta$ the domain size. The average deviation depends on the distribution of shape, orientation, and magnetization direction of the domains and it would be exceedingly difficult to compute. The maximum deviation, $\theta_{max}$, is of course given by the critical glancing angle for total reflection at a domain boundary where $a_m$ changes sign,

$$\theta_{max} = \left( \frac{N}{\pi} \right) \left( \frac{2a_m}{\lambda} \right)^{\frac{1}{2}}$$  \hspace{1cm} (4)

Using the experimental polarization data of Reference 1, which shows that $4\pi^2 a_n^2 = 10$ b and $4\pi(2a_n a_m) = 3.15$ b for a thermal neutron distribution, the value of $\theta_{max}$ can be calculated from Equation 4 as 5'.

$\theta_{AV}$ will depend on domain shape and orientation but will certainly be only a small fraction of $\theta_{max}$. The average deviation might be expected to be exceedingly small if the domains were orientated in some regular manner, say with boundaries nearly normal to the neutron motion. In general, the multiply refracted neutrons will be unpolarized because of the random nature of the deviations. Only in the special case of single scattering at the critical angle would the neutron spin states be separated.

(Scattering at the critical angle as a means of production of polarized neutrons has been considered.)$^3$

**MEASUREMENTS**

The first detailed investigations of the small angle scatterings were made with apparatus very similar to that used for the polarization experiments; that is, with neutron beams of cylindrical cross section. It was found that if the direct beam were blocked by a cadmium disk at the counter a negative single transmission resulted. However, attempts to measure the actual distribution of neutrons scattered outside the cadmium disc using annular cadmium rings as diaphragms showed only that the angles were much smaller than 1 degree. Further refinements pushed the upper limit down further until it was necessary to change to a slit geometry to obtain sufficient intensity for narrow angles.

The apparatus used to study the distribution of the scattered neutrons with slit geometry is diagrammed in Figure 1. A beam of neutrons from the thermal column of the Argonne heavy water pile is formed by two cadmium slits, 0.01 inch wide and 1 inch high, 290 cm apart. The horizontal angular distribution in this beam is measured with a proportional counter 284 cm past the second slit. The counter and a third 0.01 inch slit are moved in a horizontal direction by a micrometer screw to trace out the neutron distribution. The 0.01 inch slits were the smallest that could be used without getting into undue background trouble.

The neutron distribution was first checked with no iron in the beam to see how it compared with that expected from the slit geometry, the background being determined by placing cadmium over the central slit. Figure 2 shows the experimental points compared with the expected "theoretical"
Figure 1.

Figure 2.
The agreement is excellent and it is clear that negligible scattering is present with the slits alone. The width of the direct beam pattern (half-width about $\frac{1}{4}$ inch) will complicate the determination of the true scattering distribution if the latter is of the order of 1 inch or less. Contrary to early expectations, the scattering proved to be small enough so that the finite width of the direct beam complicated the analysis.

The distribution of the small angle magnetic scattering was measured by placing a block of cold rolled steel at the second slit between the poles of an electromagnet, as shown in Figure 1, and measuring the neutron intensity with the iron magnetized and unmagnetized. The results, with background subtracted, are given in Figure 3 for a 0.57 cm block. The points for H on fit the smooth curve which is that calculated for the shape of the direct beam. In other words, there is no change in the shape of a beam caused by passage through a magnetized iron block, any small angle scattering being negligible. About half the neutrons are scattered in the block, of course, but they all go into Debye-Scherrer rings at large angles and do not change the observed beam shape. In addition, there is a slight amount of incoherent scattering but the fraction contained in the small angles studied here is negligible.

The results for the unmagnetized iron, however, show a very definite spreading of the beam extending out to several minutes. It is quite clear that the small angle scattering is not much larger in magnitude than the geometrical spread of the direct beam, hence the shape of the curve of Figure 3 will depend both on the distribution of the scattering neutrons and on the shape of the direct beam. The intensity at the center of the pattern is much greater for H on than for H off. The ratio of these two intensities would be interpreted as the single transmission effect, related to neutron polarization, if the effect of small angle scattering were unknown. It was just this increased single transmission effect with good geometry that lead to the discovery of the small angle scattering. For the .57 cm block, for instance, for which the direct single transmission effect is about 1%, the good geometry of Figure 3 would have lead to an apparent single transmission effect of 100%. The errors shown on the points of Figure 3 are based only on the statistics of the number of counts observed. It is likely that there is some additional error caused by irregular changes in background, in view of the fact that a constant background was subtracted in plotting the points of Figure 3.

Even though it was difficult to obtain data well above background and sufficiently well separated from the direct beam at the same time, measurements were made as a function of block thickness $d$. In addition to the .57 cm block, thicknesses of 1.1 and 1.8 cm were used. Curves similar to Figure 3 were obtained for both the other blocks and it was qualitatively clear that the spread of the scattered neutrons increased with $d$. A more quantitative analysis of the different distributions will be given in the next section. Rough measurements made as a function of magnetizing field showed that the scattering disappeared with magnetizing fields of only a few hundred oersteds, thus verifying the earlier finding that the scattering disappeared as the domain walls disappeared.

DISCUSSION OF RESULTS

According to the theory that the scattering is caused by multiple refraction it should be possible to fit the results to a Gaussian distribution whose width increases with the square root of $d$. The comparison was made by assuming each part of the direct beam was split into a Gaussian and that the sum of these Gaussians would represent the scattered beam. In this way a series of curves for the scattered distribution was calculated, each corresponding to a particular width, $\sigma$, for the Gaussian. It was found that the scattered distribution for each thickness was consistent with the shape of the calculated curves and that $\sigma$ increased with $d$. The experimental points for each block are shown in Figure 4 with the calculated curves which best fit the results. Although the data are quite rough the calculated shape adequately accounts for the scattered distributions. The Gaussian width assumed for each block thickness is plotted as a function of the square root of $d$ in Figure 4 also. As the errors involved in fixing $\sigma$ for each curve are quite large it can only be said that the variation of $\sigma$ with $d$ is consistent with a $d^{1/2}$ relation and hence with the multiple refraction theory.
Figure 4.

- $d = 1.18\, \text{cm.}$
  - $\sigma = 0.7\, \text{minutes}$
- $d = 1.16\, \text{cm.}$
  - $\sigma = 0.44\, \text{minutes}$
- $d = 0.57\, \text{cm.}$
  - $\sigma = 0.34\, \text{minutes}$
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