Size and Deformation Limits to Maintain Constraint in $K_{ic}$ and $J_c$

Testing of Bend Specimens

Prepared by
K. C. Koppenhoefer, R. H. Dodds, Jr.

University of Illinois

Naval Surface Warfare Center

Prepared for
J.S. Nuclear Regulatory Commission
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ABSTRACT

The ASTM Standard Test Method for Plane–Strain Fracture Toughness of Metallic Materials (E399–90) restricts test specimen dimensions to insure the measurement of highly constrained fracture toughness values \( (K_{IC}) \). These requirements insure small-scale yielding (SSY) conditions at fracture, and thereby the validity of linear elastic fracture mechanics.

Recently, Dodds and Anderson have proposed a less restrictive size requirement for cleavage fracture toughness measured in terms of the \( J \)-integral \( (J_c) \), as given by \( a, b, B \geq 200 \frac{J_c}{\sigma_y} \). The size requirement proposed by Dodds and Anderson increases the applicability of fracture toughness experiments by expanding the range of conditions over which fracture toughness data meeting SSY conditions can be reliably measured. This investigation compares the proposed size requirement with that of ASTM Standard Test Method E399 and, by comparison with published experimental data for various alloys, provides validation of the new requirements.
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1. NOMENCLATURE

- \( a \) crack length, mm
- \( b \) length of uncracked ligament, mm
- \( B \) specimen thickness, mm
- \( B_0 \) normalizing thickness, mm
- \( \sigma_{ys} \) yield strength, MPa
- \( \sigma_{uts} \) ultimate tensile strength, MPa
- \( \sigma_0 \) flow strength (average of yield and ultimate strength), MPa
- \( E \) Young's modulus, MPa
- \( \nu \) Poisson's ratio
- \( r, \theta \) polar coordinates from crack tip
- \( T \) stress parallel to the crack, MPa
- \( \delta_{ij} \) Kronecker delta
- \( Q \) higher order term of an asymptotic series; a stress triaxiality parameter
- \( K_{corr} \) fracture toughness corrected for statistical thickness effects, MPa√m
- \( K_{min} \) threshold fracture toughness, MPa√m
- \( K_f \) experimental fracture toughness, MPa√m
- \( K_q \) provisional fracture toughness value, MPa√m
- \( K_{ic} \) specimen size independent fracture toughness value, MPa√m
- \( J_c \) experimental fracture toughness, kJ/m²
- \( J_{corr} \) fracture toughness corrected for statistical thickness effects, kJ/m²

2. INTRODUCTION

The ASTM Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials (E399-90) [1] restricts specimen dimensions relative to the deformation at fracture to insure that measured fracture toughness values (\( K_{ic} \)) correspond to highly constrained crack-tip conditions. These requirements are as follows:

\[
a, b, B \geq 2.5 \left( \frac{K_q}{\sigma_{ys}} \right)^2
\]

Satisfaction of Eq (1) insures small-scale yielding (SSY) conditions at fracture, and thereby validates the assumptions of linear elastic fracture mechanics. The approximate diameter of the plastic zone under conditions given by Eq (1),

\[
d_p \geq \frac{1}{3\pi} \left( \frac{K_q}{\sigma_{ys}} \right)^2
\]

is nearly 25 times smaller than relevant specimen dimensions. This degree of plastic zone confinement, set by the 2.5 multiplier in Eq (1), is based on experimental \( K_{ic} \).
data for many different metals. These data confirm that specimens satisfying Eq (1) produce equivalent (within scatter) fracture toughness values. However, different materials do not all indicate the need for a multiplier as severe as 2.5. Rolfe and Novak [2] and Facuher and Tyson [3] found that the 2.5 value could be reduced to as low as 1.0 for certain steel alloys (e.g. 18 Ni Maraging steel, micro–alloyed Lloyds LT–60). In contrast, Jones and Brown [4] presented data on titanium alloy 6Al–6Vn–2Sn in the aged condition demonstrating the need for the 2.5 value. To maintain a test standard independent of specific material, ASTM Committee E08 retains the more restrictive 2.5 value.

Recently, Dodds and Anderson [5] (hereafter referred to as DA) have proposed an alternative size requirement for cleavage fracture toughness measured in terms of the J–integral ($J_c$) which is less restrictive than the E399 requirement in many cases:

$$a, b, B \geq \frac{200 J_c}{\sigma_0}$$

(3)

This requirement derives from current research [6,7,8] examining the effects of constraint on fracture toughness. Experimental verification of Eq (3) would increase the applicability of measured fracture toughness values. For most metals, valid fracture toughness values can be obtained with smaller specimens. This paper re-examines the key data sets used to set the original 2.5 factor in the E399 requirement. By using $J_c$, rather than $K_{IC}$, as the measure of fracture toughness, the widely varying ratio of Young's modulus to yield strength is reflected in the requirements. For high strength–low modulus metals (e.g. titanium) Eq (1) and (3) are nearly identical. However, for lower strength–high modulus metals (e.g. structural steels), Eq (3) more closely agrees with the 1.0 multiplier in Eq (1). The comparisons here demonstrate that Eq (3) maintains the strict requirement of the E399 expression for materials originally used to set the 2.5 factor while correctly relaxing the size requirement for other metals, most notably structural and pressure vessel ferritic steels.

3. THEORETICAL BACKGROUND

Much recent work [6,8,9,10] in fracture mechanics focuses on quantifying the kinematic constraint against plastic flow at the crack tip to predict the effects of finite component size on fracture toughness. Two approaches of particular interest are the DA micromechanics constraint model, and the $J$–$Q$ theory to describe crack tip fields as developed by O'Dowd and Shih [8,9]. These approaches determine the level of loading, relative to specimen size, when global plasticity impinges on the small scale yielding (SSY) crack tip fields. Once global plasticity affects the near tip fields, the unique coupling between $J$, $K_I$ and the near tip fields is lost and specimen size (and geometry) influences the measured fracture toughness. The size requirements given in Eq (3) were first proposed by DA and, as will be shown here, are corroborated by the $J$–$Q$ methodology.

3.1 Dodds–Anderson Micromechanics Model

DA quantify the geometric effects on fracture toughness by coupling the global failure parameter ($J_c$) with a micromechanics based failure model. The model is designed for ferritic materials in the ductile to brittle transition region thereby limiting the fracture mechanism to transgranular cleavage. For this failure mechanism, several micromechanical models have been recently proposed [11, 12, 13]. These
models assume a favorably oriented particle (e.g. carbide or inclusion) initiates cleavage fracture. Failure of this particle creates a microcrack which triggers global fracture through a local Griffith instability. The sampling effects for a favorably oriented particle to create the initial microcrack suggests that the highly stressed volume of material ahead of the crack plays a dominant role. These features lead to adoption of the volume of material ahead of the crack over which the normalized principal stress \( \sigma_1 / \sigma_0 \) exceeds a critical value as the local failure parameter. In plane-strain, the volume is simply the area \( A \) within a principal stress contour \( \times \) the thickness \( B \). Dimensional analysis [5] demonstrates that

\[
A(\sigma_1 / \sigma_0) \propto J^2 / \sigma_0^2
\]

(4)

DA use nonlinear finite element analyses of plane strain models to calculate areas within principal stress contours ahead of a crack tip. The analyses reveal that as deformation applied to a single edge notch bend (SE(B)) specimen increases, the area within a stress contour ahead of the crack tip increases but at a lesser rate (due to constraint loss) than the small-scale yielding (SSY) limit (Fig. 1). As is apparent from the nearly horizontal lines in Figure 1, the level of deviation from SSY is essentially independent of the critical principal stress contour until large amounts of deformation. These analyses define deformation levels beyond which specimen dimensions influence the relationship between applied-\( J \) and area within a principal stress contour which drives the cleavage fracture (i.e. the measured \( J_c \) values become a function of specimen geometry). The area ratio is recast in terms of \( J \) as,

\[
\frac{J_{SE(B)}}{J_{SSY}} = \sqrt{\frac{A_{SSY}}{A_{SE(B)}}}
\]

(5)
DA calculate the ratio of $J$ in the finite size specimen ($J_{SE(B)}$) to the $J$ under small-scale yielding conditions ($J_{SSY}$) which generates equivalent stressed areas in the SE(B) ($A_{SE(B)}$) and SSY ($A_{SSY}$) conditions. The ratio $J_{SE(B)}/J_{SSY}$ quantifies the deviation from SSY conditions. Figure 2 shows the variation of this ratio with applied load and strain hardening exponent and illustrates the basis for the size requirement on in-plane dimensions ($a$ and $b$) expressed by Eq (3). At low deformation levels, plasticity in the SE(B) specimen is well contained (i.e. small scale yielding); increases of $J_{SE(B)}$ generate the same stressed volume of material as in SSY. As deformation increases, global plasticity affects the near tip stresses, and $A_{SE(B)}$ increases at a substantially slower rate than $A_{SSY}$. As is apparent from Figure 2, the ratio $J_{SE(B)}/J_{SSY}$ begins to increase rapidly above unity at a non-dimensional deformation of 200. The crack length provides a meaningful length to scale the level of plastic deformation relative to the in-plane size of the specimen. 3D finite element analyses of SE(B) specimens by Narasimhan and Roskas [14], and Faleskog [15], indicate that thicknesses, $B$, satisfying Eq (3) also maintain SSY conditions.

3.2 $J$–$Q$ Theory

The $J$–$Q$ description of crack tip fields derives from consideration of the Modified Boundary Layer (MBL) solution [16] which expresses near tip stresses for linear elastic plane strain conditions in the form,
\[
\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} \tilde{T}_{ij}(\theta) + T\delta_{ij}\delta_{ij} \tag{6}
\]

where \( T \) is the non-singular stress parallel to the crack plane. The \( T \)-stress term does not affect \( K_I \) or \( J \); however, Larsson and Carlsson [17] demonstrate the second term significantly affects the plastic zone shape and size under SSY conditions. In finite-sized specimens the elastic \( T \)-stress, which varies proportionally with \( K_I \), becomes ambiguous under conditions of large scale yielding as \( K_I \) saturates to a constant value at limit load.

O'Dowd and Shih [8,9] use asymptotic and finite element analyses to develop an approximate two-parameter description of the crack tip fields without the limitations of the \( T \)-stress,

\[
\sigma_{ij} = \sigma_0 f_{ij}\left(\frac{r}{J/\sigma_0}, \theta; Q\right) \tag{7}
\]

\[
\epsilon_{ij} = \epsilon_0 g_{ij}\left(\frac{r}{J/\sigma_0}, \theta; Q\right) \tag{8}
\]

The second term, \( Q \), in Eqs (7,8) is the mechanism by which \( \sigma_{ij} \) and \( \epsilon_{ij} \) of an SE(B) differ from the SSY solution at the same applied-\( J \). O'Dowd and Shih [8,9] determined that, to a good approximation, \( Q \) represents a uniform hydrostatic stress in the forward sector ahead of the crack tip, \(|\theta| < \pi/2 \) and \( J/\sigma_0 < r < 5J/\sigma_0 \). Operationally, \( Q \) is defined as

\[
Q \equiv \frac{(\sigma_{0\theta})_{SE(B)} - (\sigma_{0\theta})_{SSY}}{\sigma_0}, \quad \text{at } \theta = 0, \ r = 2J/\sigma_0 \tag{9}
\]

where stresses in Eq (9) are evaluated from plane strain finite element analyses containing sufficient mesh refinement to resolve the fields within the process zone for ductile and brittle fracture. At low deformation levels, the finite body is under SSY conditions and \( Q \) remains very nearly zero; however, under large-scale yielding conditions stresses at the crack tip are substantially less than those in SSY at the same \( J \)-values. This difference leads to negative \( Q \) values once the SE(B) specimen deviates from SSY conditions (Fig. 3). For deep notch bend specimens \( Q \) remains slightly positive at deformation corresponding to \( a\sigma_0/J_c > 200 \).

The \( J-Q \) description of crack-tip stress and strain fields expressed in Eqs (7,8) provides the needed justification to apply the requirements of Eq (3) to materials that do not necessarily fracture by the purely stressed controlled, transgranular cleavage mechanism of the DA model. Satisfaction of the size/deformation requirements in Eq (3) insures that both the stress and strain fields at fracture correspond to SSY and are unaffected by the global response of the specimen. Consequently, the specific details of the fracture micromechanism (stress vs. strain controlled) become unimportant since \( J \) (or \( K_I \)) uniquely defines both fields.

3.3 Statistical Thickness Effects

Previous experimental and theoretical work [20,21] on cleavage fracture in ferritic steels demonstrates an absolute thickness effect on fracture toughness not related to constraint. Metallurgical variations in the material along the crack front require a statistical treatment of thickness in experimental fracture toughness data. Wallin
[21] employs weakest link statistics to obtain the following statistical correction for fracture toughness data for specimens of different thickness ($B$ and $B_0$) which fail by cleavage without previous ductile tearing,

$$K_{corr} = K_{min} + \left( K_q - K_{min} \right) \left( \frac{B}{B_0} \right)^{1/4}$$

(10)

Recasting Eq (10) in terms of $J$ yields,

$$J_{corr} = J_c \left( \frac{B}{B_0} \right)^{1/2}$$

(11)

The corrections given in Eqs (10,11) arise solely from the increased volume of material sampled along the crack front due to increased thickness. Each point along the crack front is assumed to be stressed at the same level. As the sampled volume increases, the probability of finding a metallurgical weak link increases. Because the failure of a weak metallurgical defect controls cleavage fracture, fracture toughness decreases with increasing probability of finding a defect.

The statistical assumptions employed to obtain Eqs (10,11) preclude application to materials which do not fracture by weakest link mechanisms. Consequently, the remainder of this presentation addresses only the deterministic effects of specimen size (i.e. constraint) on measured values of fracture toughness. Statistical treatment of fracture data, for example the thickness effect of sampled volume, should be applied.
plied only to data that first meet the deterministic requirements for specimen size that maintain constraint.

**Table 1—References for Experimental Data**

<table>
<thead>
<tr>
<th>Material</th>
<th>Reference</th>
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<tr>
<td>18Ni Maraging Steel</td>
<td>Rolfe and Novack, <em>ASTM STP 463</em>, 1970, pp 94*</td>
</tr>
</tbody>
</table>

4. EVALUATION OF SIZE REQUIREMENTS

4.1 Materials and basis of comparison

Five experimental data sets spanning a variety of metals are considered in the comparison. Table 1 lists the materials along with the original references for the data. To compare the current E–399 and proposed size requirements for these metals, it is necessary to express them using the same fracture toughness parameter. Equation (3) is converted into terms of 

\[ J = \frac{K^2}{E/(1 - \nu^2)} \]  

(12)

After converting Eq (3) to \( K \) and expressing \( \sigma_0 \) in terms of \( \sigma_{ys} \) and \( \sigma_{uts} \), the DA size requirement is expressed as

\[ L_{200} \geq \frac{400 \, K_0^2 \, (1 - \nu^2)}{E(\sigma_{ys} + \sigma_{uts})} \]  

(13)

\( L_{200} \) refers to the minimum specimen size (i.e. \( a, b, B \)). With both size requirements expressed using the same fracture toughness parameter, their ratio becomes a function of material properties,

\[ \frac{L_{200}}{L_{E399}} = \frac{160 \, (1 - \nu^2) \, \sigma_{ys}^2}{E \, (\sigma_{ys} + \sigma_{uts})} \]  

(14)

This ratio quantifies the change in minimum specimen size afforded by the proposed size requirement for a specific material. A value of \( L_{200} / L_{E399} \) less than unity indicates that the proposed size requirement is less restrictive than the current E399 requirement. Table 2 lists, in ascending order, this size ratio for the five metals. The decrease in specimen size requirement ranges from a factor of 16 for A36 steel to 1.4 for Ti 6Al–6V–2Sn. The proposed size requirement is less restrictive than the E399 for all metals considered in Table 1, but only slightly so for the titanium alloy.
Table 2—Material properties and size ratios for experimental data

<table>
<thead>
<tr>
<th>Material</th>
<th>Yield [MPa]</th>
<th>Ultimate [MPa]</th>
<th>Modulus [GPa]</th>
<th>Poisson’s ratio</th>
<th>$L_{200} / L_{E399}$</th>
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</thead>
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<tr>
<td>A36 Steel</td>
<td>248</td>
<td>460</td>
<td>207</td>
<td>0.3</td>
<td>0.06</td>
</tr>
<tr>
<td>A533B Class 1 Steel</td>
<td>407</td>
<td>559</td>
<td>207</td>
<td>0.3</td>
<td>0.12</td>
</tr>
<tr>
<td>18Ni Maraging Steel</td>
<td>1323</td>
<td>1379</td>
<td>207</td>
<td>0.3</td>
<td>0.46</td>
</tr>
<tr>
<td>4340 Steel (399°C Temper)</td>
<td>1468</td>
<td>1538</td>
<td>207</td>
<td>0.3</td>
<td>0.49</td>
</tr>
<tr>
<td>Ti 6Al–6V–2Sn</td>
<td>1200</td>
<td>1269</td>
<td>117</td>
<td>0.32</td>
<td>0.71</td>
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4.2 Experimental data

The five experimental data sets are examined in the order given in Table 2. Fracture toughness is plotted against the relevant specimen dimension. Two lines designated $L_{200}$ and $L_{E399}$ appear on each plot and represent the size requirements (deformation limits) for E399 (solid line) and DA (dashed line). Fracture toughness values below (and to the right of) each line satisfy the corresponding size/deformation limit. Single and double arrows appear on the $L_{200}$ line in each plot for emphasis. Data points on the single arrow side of the $L_{200}$ line require a constraint correction as proposed by DA [22]; data points on the double arrow side satisfy the $L_{200}$ size/deformation limit but may require a statistical thickness correction. Double arrows appear on the $L_{E399}$ line to emphasize the region over which data satisfies the E399 criterion.

The A36 data set [23] consists of SE(B) specimens with a variety of crack depth, thickness, and width-to-thickness ($W/B$) ratios tested at $-76^\circ C$. The $J$ at cleavage, $J_c$, is given for two thickness ($B = 12.7$ and 31.75 mm). Figure 4 provides this data. Both thicknesses contain specimens with three different $W/B$ ratios as indicated by the different symbols. This material has the largest difference between $L_{E399}$ and $L_{200}$; application of the E399 size requirement indicates the entire data set is specimen size dependent. All of the $B = 31.75$ mm data and several of the data points with $B = 12.7$ mm meet the proposed size requirement of DA. The total data set shows a significant increase in toughness with decreasing thickness; however, the $L_{200}$ criterion successfully separates data points which show an increase in fracture toughness due to large scale yielding effects from specimen size insensitive data. Figure 5 shows the variation of fracture toughness with crack depth for the same data set. The $L_{200}$ criterion successfully indicates $J_c$ values dependent on crack depth; the E399 criterion indicates that all data values are size/deformation dependent (which does not appear to be correct for this data set).

Figure 6 shows fracture toughness values for an A533B Class 1 steel. The data includes 1/2T, 1T, 2T and 4T C(T) specimens tested at $-75^\circ C$. For this data set, the fracture toughness is plotted using $K_{Jc}$ values obtained by converting measured $J_c$ values using Eq (12). The proposed size requirement again indicates data points which cause the data set to show an increase in fracture toughness with decreasing thickness.

Deep notch SE(B) specimens of two thicknesses ($W = 102$ and 152 mm) provide fracture toughness data for 18 Ni maraging steel (Fig. 7). Rolfe and Novak use this data to argue for a reduction of the multiplier in E399 from 2.5 to 1.0. Fracture toughness values are clearly specimen size independent for thickness greater than
approximately $B = 10$ mm. The thickness requirement given by the $L_{200}$ curve agrees with the recommendations of Rolfe and Novak.

Fracture toughness values for a 4340 steel shown in Figures 8, 9 and 10 were obtained from a series of tests conducted on specimens removed from a 25.4 mm thick, hot-rolled and annealed plate. The specimen blanks were heat treated in a neutral salt bath at 843°C for 1/2 hour, oil quenched, and tempered at 399°C for one hour. The SE(B) specimens comprised three different widths ($W = 56, 25.4,$ and 14 mm) each having initial $a/W = 0.5$. Only the $W = 14$ mm data set reveals significant variations in $K_Q$ with thickness (Fig. 10). The rapid decrease in toughness with decreasing thickness which is observed in this data set may be due to the very thin specimens (e.g. $B = 3.8$ mm). Once the specimen thickness decreases beyond a critical point, fracture toughness decreases due to the reduction of material available for plastic energy dissipation. The DA size requirement indicates all data points showing specimen size dependency.

The high yield strength coupled with the low value of Young's modulus for Ti 6Al–6V–2Sn causes the $L_{200}/L_{E399}$ ratio to be significantly nearer to unity for this material than for the other four materials listed in Table 1. The titanium data (Fig. 11) shows a rapid increase in fracture toughness with decreasing thickness; this rapid upswing in toughness caused Jones and Brown [4] to argue (successfully) for the

![Graph](image)

**Specimen Thickness [mm]**

FIG. 4—Variation of fracture toughness with specimen thickness for A36 steel at -76°C.
more restrictive 2.5 multiplier in the E399 size requirement. The proposed size limit designates as size insensitive an additional data point beyond the E399 limit.

5. SUMMARY AND DISCUSSION

This paper offers experimental verification of the DA size requirements for brittle fracture given in Eq (3). DA originally proposed these requirements for materials that fracture by transgranular cleavage. Subsequent development of the $J$–$Q$ methodology generalizes the work of DA by removing the restriction of a stress–controlled, cleavage mechanism. The proposed size requirements are shown, using finite element analyses, to quantify the deformation limits under which conditions of small–scale yielding ($T = 0$) exist at the crack tip with both stress and strain fields uniquely characterized by $J$.

The proposed size requirements are examined for five existing data sets of fracture toughness which span properties between low strength–high modulus (A36) and high strength–low modulus (titanium). The proposed requirements successfully indicate toughness values in each data set which exhibit size dependency due to a loss of kinematic constraint against plastic deformation. The new size requirement is much less restrictive than the current E399 size requirement for materials with a low strength and high modulus, e.g., common structural and pressure vessel steels. For materials with a higher strength but lower modulus, e.g., the titanium alloy, the new requirement is just marginally less restrictive (the titanium alloy examined here played a key role in setting the E399 factor of 2.5). By expressing the fracture

![Graph](image)

**FIG. 5**—Variation of fracture toughness with crack depth for A36 steel at $-76^\circ$C.
FIG. 6—Variation of fracture toughness with specimen thickness for A533-B at -75°C.

FIG. 7—Variation of fracture toughness with thickness for 18 Ni maraging steel.
FIG. 8—Variation of fracture toughness with specimen thickness for 4340 steel $a_0 = 28$ mm, $W = 56$ mm.

FIG. 9—Variation of fracture toughness with specimen thickness for 4340 steel $a_0 = 12.7$ mm, $W = 25.4$ mm.
toughness in terms of $J$, the strong influence of Young's modulus relative to strength is correctly reflected in the proposed size requirements.

Recent work by Faleskog [15], and work-in-progress by the authors suggests that the size requirements might be reduced to

$$a, b, B \geq \frac{100 J_c}{\sigma_0}$$

for deeply cracked SE(B) specimens of materials having a low yield strength and high Young's modulus which includes most structural and pressure vessel steels. A similar reduction in size requirements for alloys possessing high yield strength to Young's modulus ratios, such as Titanium 6Al-6V-2Sn, may not be possible. Three-dimensional finite element analyses reveal that the centerplane in SE(B) specimens (with $B=W; B=W/2$) and standard C(T) specimens maintains small-scale yielding conditions at deformation levels greater than the plane-strain limit of Eq (3). Away from the centerplane, crack-tip conditions become less constrained which introduces the complexity of defining an "equivalent" thickness to quantify constraint levels. Additional experimental data from Wallin [21] on pressure vessel steels also supports size requirements suggested by Eq (15). Nevertheless, it is clear that the proposed size requirements in Eq (3) are conservative for these materials and specimen geometries and that on-going work may provide sufficient justification to adopt Eq (15) for ferritic materials.

**FIG. 10**—Variation of fracture toughness with specimen thickness for 4340 steel, $a_0 = 6.9$ mm, $W = 14$ mm.
FIG. 11—Variation of fracture toughness with thickness for Ti 6Al–6V–2Sn.
6. REFERENCES


The ASTM Standard Test Method for Plane-Strain Fracture Toughness of Metallic Materials (E399-90) restricts test specimen dimensions to insure the measurement of highly constrained fracture toughness values ($K_{IC}$). These requirements insure small-scale yielding (SSY) conditions at fracture, and thereby the validity of linear elastic fracture mechanics.

Recently, Dodds and Anderson have proposed a less restrictive size requirement for cleavage fracture toughness measured in terms of the J-integral ($J_C$), as given by $a, b, B \geq 200 J_C/\sigma_0$. The size requirement proposed by Dodds and Anderson increases the applicability of fracture toughness experiments by expanding the range of conditions over which fracture toughness data meeting SSY conditions can be reliably measured. This investigation compares the proposed size requirement with that of ASTM Standard Test Method E399 and, by comparison with published experimental data for various alloys, provides validation of the new requirements.