ANALYSIS OF THE "JOGGLE-LAP" JOINT FOR AUTOMOTIVE APPLICATIONS

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Analysis of the "Joggle-Lap" Joint
for
Automotive Applications

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Abstract

An analytical model is developed to describe the response of the "joggle-lap" joint to both tensile and bending loads. The model consists of a non-linear beam analysis which calculates stress profiles through the adherent thickness. A plane stress finite-element model was incorporated into the analysis to correctly determine the stress field in the adhesive zone where it was shown that beam analysis was less accurate. Elastic response of the "joggle-lap" joint due to tensile loads was verified through experimental testing and ultimate loads were accurately predicted within experimental error. Maximum adherent flexural stress was found to determine joint failure. A parametric study was undertaken by using the verified analytical model and the results were recorded as a series of design curves.
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Nomenclature

a  
cross-sectional area
b  
adhesive bond thickness
C_0, C_1, C_2  
constants
DEFLA  
deflection
DUDSA  
angular rotation
e  
eccentricity
e_2  
elongation
E  
modulus of elasticity for an isotropic material
E_f  
modulus of elasticity of fibers
E_m  
modulus of elasticity of matrix
E_n  
modulus of elasticity normal to the fiber plane
E_x', E_y', E_z  
modulus of elasticity of a general anisotropic body
F  
applied force
F_i i=1-5  
force components
h_i i=1-5  
nodal points
H_i i=1-5  
axial force of SEG_i
I  
moment of inertia about neutral surface
I_{eq.}  
equivalent moment of inertia

l  
length of SEG_i
M_i i=1-5  
applied moment at SEG_i
M_{corr}  
correcting moment
M(S)  
moment distribution
Nomenclature (Cont'd)

\begin{align*}
R & \quad \text{radius of curvature} \\
SEGi & \text{beam elements of the "joggle-lap" joint} \\
S & \quad \text{specific gravity} \\
S_{i} & \quad \text{ultimate shear strength} \\
t & \quad \text{adherent thickness} \\
\bar{u} & \quad \text{distance between neutral axis and centroidal axis} \\
u_0 & \quad \text{deflection at the end of SEG1} \\
u_i, s_i & \quad \text{local coordinate system corresponding to individual beam element} \\
v_f & \quad \text{volume fraction of fiber} \\
v_m & \quad \text{volume fraction of matrix} \\
V & \quad \text{shear force on SEG1} \\
W & \quad \text{weight fraction} \\
X, Y & \quad \text{global coordinate system} \\
Y & \quad \text{radial coordinate in curved beam members} \\
X^T & \quad \text{ultimate tensile strength} \\
X^c & \quad \text{ultimate compressive strength} \\
\alpha & \quad \text{angle measure} \\
\Delta & \quad \text{infinitesimal difference} \\
\varepsilon_{x}, \varepsilon_{y}, \varepsilon_{z} & \quad \text{strain components} \\
\varepsilon_{ult} & \quad \text{ultimate strain} \\
\theta & \quad \text{angle measure} \\
\lambda & \quad \text{linear measure}
\end{align*}
Nomenclature (Cont'd)

\( \nu_{ij} \)  
Poisson's ratio

\( \pi \)  
3.14159...

\( \sigma_1, \sigma_2, \tau_{12} \)  
plane stress components

\( \sigma_x, \sigma_y, \sigma_{xy} \)  

\( \sigma_{ult} \)  
ultimate strength

\( \phi \)  
angle measure
I. Introduction

Recent government regulations for increased gasoline mileage requirements have induced automobile manufacturers to seek light weight replacement material systems for existing metal parts. Since the automotive industry is a high volume operation, sheet molding compound (SMC) parts offer a feasible answer to the problem. The SMC molding time of from 1 to 3 min/piece depending on the size and thickness of the part is compatible with automotive assembly line production.

International Harvester et al are currently employing SMC molded body components on their vehicles to replace former sheet metal parts. This new direction has brought with it several problems, one of which is the design of adhesive joints. The joint must accommodate high rate fabrication techniques and provide optimum strength and durability. In addition, the joint must satisfy certain cosmetic requirements such as adjacent flush edges. With these criteria in mind, the "joggle-lap" joint has been chosen for detailed study and analysis. This joint configuration is shown in Figure 1. Since a joint of this type experiences a variety of loading conditions in practice, it was decided to model the joint
Figure 1: The Joggle Lap Joint Subject to Tensile and Bending Loads.
in pure tension and pure bending. By superposition, it is apparent that any combination of these two loading conditions may then be constructed.

This work focuses on the development of an analytical model to describe the behavior of the "joggle-lap" joint due to both tensile and bending loading conditions. The first section utilizes small deflection beam theory for both straight and curved beam elements to obtain a solution for the displacement and stress fields of the joint. Included in this analysis is the derivation of the governing differential equations for the deflection of the curved beam.

The second section utilizes a finite-element model to reveal localized stress concentrations in the adhesive zone. Boundary conditions for the finite element model are obtained from a transformation of stresses in the deformed geometry to equivalent stresses in the undeformed geometry. This transformation of stresses is performed via a computer routine for ease of calculation.

Finally, experimental verification of the analytical predictions is reported along with a description of testing procedures. The maximum flexural stress is shown to correlate strength data and failure analysis. Also, the microstructure of the joint was examined as a possible explanation of the failure mode.
II. Background

A. Adherent Materials

The adherents of the proposed "joggle-lap" joint were composed of a random-fiber composite known as SMC-25. SMC is defined as a sheet molding compound that contains reinforcements with an average fiber length of approximately 1 inch (2.54 cm) with random orientation in the plane. The number 25 indicates that the composite is 25 percent glass fibers by weight. The major constituents of SMC are E-glass fibers and a styrenated polyester resin in the form of a paste. It is quite common to use mineral fillers during the manufacture of the paste to facilitate flow when molding or to obtain certain characteristics from the molded part such as a high resistance to flame or increased stiffness. Another prime reason for using fillers is the fact that they are much cheaper than the polyester resin itself and thus reduce the cost of materials. At times, chemical additives may also be introduced into the paste to serve as catalysts during the molding cycle.

The process of SMC manufacturing is a highly innovative one which is completely automated. Figure 2 (taken from Owens/Corning Fiberglass SMC Review) depicts
a typical process currently in use by a competitive supplier of SMC. The first step of the procedure is to distribute the resin onto a polyethylene carrier film as shown. Continuous glass fibers are then chopped into lengths of less than three inches and distributed in a random fashion on the wetted film. A second layer of resin-coated polyethylene film serves as a top layer to the sandwich-like sheet. Several rows of rollers act to insure that the glass fibers are fully impregnated with the polyester resin thus yielding consistency in moldability of the SMC. Finally the product is directed to a take-up roll for ease of handling during shipping and storage.

SMC is usually placed in a constant temperature room while storing to allow maturation to take place. Maturation is nothing more than allowing the SMC to increase
in viscosity to enhance relative ease of handling of the sheet. Maturing the SMC sheet for extended periods of time greatly reduces the flow characteristics of the product while molding. Recommended shelf-life for SMC stored at 10-15° C is about 2 weeks, however in general it may often be used up to 2 months after the date of its manufacture.

Once the SMC sheet has reached maturity, it is ready for molding. Upon removing the protective polyethylene film, the molding compound is cut to size and strategically placed in the mold. This procedure is known as charging the mold. The so-called strategic locations of the mold are those positions that allow the SMC to flow to all parts of the mold and maintain uniform part thickness. To date these locations have been determined by trial and error coupled with experience.

Compression molding combines both temperature and pressure to induce an exothermic reaction which serves to cure the part in the mold. Figure 3 (taken from ref [3]) is an example of a typical curing cycle showing the temperature of the part as a function of time. It should be noted that platen temperatures of 200° C are usually sufficient for SMC molding and may be achieved with superheated steam. Another important fact seen from the figure is the overall cure time. Average cure times are generally
FIGURE 3: TYPICAL CURE CYCLE OF SMC
1-3 minutes (depending upon the thickness of the part) which lends itself to production line applications inherent in the automotive industry. Figure 4 (taken from ref [3]) shows the effect of pressure upon a typical cure cycle. Note that the peak pressure and maximum temperature correspond to the initiation of the exothermic reaction. The key to successful molding is to acquire fine control of the application of pressure to the cure cycle.

The main feature of SMC is the ability of the glass fibers to flow with the paste during the molding process. Since the fibers are transported to all parts of the mold, it is possible to produce a geometrically complicated part with quasi-constant mechanical properties. It has been shown by Pipes and Taggart [ref 5], that in areas of intensified flow, the fibers tend to align themselves with the direction of flow and thus produce areas of varying mechanical properties. It is therefore beneficial to understand the flow characteristics within the mold to produce a part with controlled and/or uniform mechanical properties. Taggart et al have determined the properties of SMC-25 to be those found in Table 1. Some scattering in the data was reported due to the inherent local variations in the material. To determine the normal modulus (modulus normal to the plane of the fibers), the relationship shown may be

-8-
FIGURE 4: PRESSURE VARIATION OF A TYPICAL CURE

- **Pressure (MPa)**
- **Time (minutes)**

- **Maximum pressure due to thermal expansion**
- **Mold closes**
Table 1
Properties of SMC-25

**Tension**

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension Modulus</td>
<td>$E_{\text{tension}}$</td>
<td>GPa (Msi)</td>
<td>14.48</td>
<td>(2.1)</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>$\nu_{\text{tension}}$</td>
<td></td>
<td>.3</td>
<td></td>
</tr>
<tr>
<td>Ultimate Stress</td>
<td>$\sigma_{\text{tension ult}}$</td>
<td>MPa (ksi)</td>
<td>90</td>
<td>(13.1)</td>
</tr>
<tr>
<td>Ultimate Strain</td>
<td>$\varepsilon_{\text{tension ult}}$</td>
<td>(µ in/in)</td>
<td>11,400</td>
<td></td>
</tr>
</tbody>
</table>

**Compression**

<table>
<thead>
<tr>
<th>Property</th>
<th>Symbol</th>
<th>Unit</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compression Modulus</td>
<td>$E_{\text{compression}}$</td>
<td>GPa (Msi)</td>
<td>12.41</td>
<td>(1.8)</td>
</tr>
<tr>
<td>Poisson's Ratio</td>
<td>$\nu_{\text{compression}}$</td>
<td></td>
<td>.28</td>
<td></td>
</tr>
<tr>
<td>Ultimate Stress</td>
<td>$\sigma_{\text{compression ult}}$</td>
<td>MPa (ksi)</td>
<td>204</td>
<td>(29.6)</td>
</tr>
<tr>
<td>Ultimate Strain</td>
<td>$\varepsilon_{\text{compression ult}}$</td>
<td>(µ in/in)</td>
<td>20,600</td>
<td></td>
</tr>
</tbody>
</table>
used. This relationship resembles the well-known rule of mixtures for continuous fibrous composites.

\[
\frac{1}{E_n} = \frac{v_f}{E_f} + \frac{v_m}{E_m}
\]

(1)

where \( E_n \) = normal modulus of elasticity of the composite 
\( v_f \) = volume fraction of fiber 
\( v_m \) = volume fraction of matrix 
\( E_f \) = modulus of glass fiber 
\( E_m \) = modulus of matrix

Table 2 provides the needed data for determining the normal modulus of elasticity. By definition, SMC is composed

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Polyester Resin</th>
<th>E-Glass Fiber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Elasticity (10^6 psi)</td>
<td>.5</td>
<td>10</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>1.28</td>
<td>2.54</td>
</tr>
</tbody>
</table>

of 25% fiber by weight. Utilizing the equation written below

\[
v_f + v_m = 1
\]

(2)
allows one to solve for \( v_f \) where \( v_m \) may be rewritten as

\[
v_m = v_f \left[ \frac{S_f}{S_m} \right] \cdot \left[ - \frac{W_m}{W_f} \right]
\]

\( S_f \) = specific gravity of fiber  
\( S_m \) = specific gravity of matrix  
\( W_m \) = weight fraction of matrix  
\( W_f \) = weight fraction of fiber

Making the appropriate substitutions, Eq. (2) becomes

\[
\frac{.75}{.25} \left[ \frac{2.54}{1.28} \right] v_f + v_f = 1
\]

Thus the corresponding volume fraction of fiber and matrix are .14 and .86 respectively. From Eq. (1) the value of \( E_n \) is now calculated to be \( 0.58 \times 10^6 \) psi.
B. Adhesive Materials

The adhesive system chosen for the "joggle-lap" joint was developed by the Adhesives Division of Goodyear Chemicals. The Pliogrip 6000 series is a general purpose structural adhesive with a polyurethane base. Currently available as a two-part system, Pliogrip 6000 exhibits both high flexibility and resilience. With the proper selection of curatives, the working time of the adhesive may be accurately controlled between 1-6 minutes.

In order to utilize this adhesive system only minimal surface preparation is necessary. The two surfaces to be bonded are prepared with a plastic wash primer (Pliogrip 6033/6034 Wash Primer) that is applied with a cloth. No sand blasting or surface stripping is necessary. To maintain reliably bonded parts, Pliogrip 6000 must be mixed at a precise ratio of 4 parts resin to 1 part curative by weight or volume. Deviations from this standard will yield resin-rich areas of uncured adhesive. The actual mixing of the two components must be carried out without the introduction of air into the system, thus the need for specialized equipment. Without this precaution, entrapped air bubbles in the cured adhesive would yield voids and greatly affect the performance of the bond. Curing this adhesive system can be accomplished at room temperature, however the use of heated fixtures will reduce cure times.
Recommended clamping pressures of heated fixtures range from 20 to 40 psi.

An important criterion in the design of bonded joints is that of the adhesive thickness. It has been shown that adhesive properties vary inversely with adhesive thickness. Thus the bulk properties of the adhesive are distinctly different from those in the film state. So the question is posed as to the optimum bond thickness as a function of shear strength. Figure 5 (taken from Pliogrip technical data, Goodyear Adhesives) shows the effect of glue line thickness on bond joint strength. A bond line thickness of 0.030 inches was chosen as optimal even though thicknesses less than 0.030 inches yield greater bond strengths. It was felt that bonding thicknesses less than 0.030 inches are not capable of being fabricated with consistency under production operations. (i.e. molded FRP parts will inherently not fit together with reliable precision).

To achieve uniform bond lines, one of two procedures is generally used. Adherents may have a small raised button of 0.030 inches in thickness which acts as a spacer for the joint to insure a uniform bond. Another procedure is to introduce small glass spheres (0.030 inches dia.) directly into the adhesive to achieve similar spacing. The effect of these spheres on joint strength has not been determined but it is argued that the variation from the norm is negligible.
FIGURE 5: EFFECT OF GLUE LINE THICKNESS ON BOND JOINT STRENGTH

Glue Line Thickness (inches)

Shear Strength (ksi)
III. Methods of Analysis

A. Tensile Loading
   a. Beam Model

   Recently, Adkins [ref 2] investigated the response of a scarf joint to simple tensile loadings. It was found that the scarf joint exhibits flexural deformation under tensile loading due to the misalignment between the neutral surface and the loading axis. This eccentricity induces a moment distribution along the joint (see Figure 9) which acts to align the neutral surface with the loading axis.

   The analysis of the "joggle-lap" joint, shown previously in Figure 1, is an extension of the concept discussed above. Again it is clear that under tension the joint will experience a lateral deflection as the neutral axis attempts to align with the applied force. To analyze the joint behavior under tensile loading conditions, it was decided to divide the joint into five segments. The obvious places to divide the joint are illustrated in Figure 6 along with the corresponding identifying labels and global coordinate system. Reference to beam segments via their identifying numbers will be utilized throughout the remainder of this analysis.

   In general, the goal of the analysis will be to determine the displacements of the neutral axis as measured perpendicularly from the undeformed neutral surface. Once
FIGURE 6: PIECEWISE REPRESENTATION
OF THE JOGGLE LAP JOINT

global coordinate system
the deflections are known, one may calculate a moment
distribution along the joint and thus determine the stress
distribution at any given cross-section.

The initial intention of such an investigation was
to develop a closed form solution for the stresses within
the joint. This effort was soon thwarted by the non-linearities
encountered in the governing equations for the beam elements.
These non-linearities result from a coupling between the
moment and deflection solutions, as will be evident later.
As an alternative solution, the displacement field was
obtained via numerical integration routines.

Linear elastic beam theory states that for a beam
under general loading conditions, the local radius of
curvature is given by

\[ R = \frac{EI}{M} \quad (3) \]

where

- \( R \) = radius of curvature
- \( E \) = modulus of elasticity
- \( I \) = moment of inertia about the neutral surface
- \( M \) = applied moment

The radius of curvature may be written in terms of
the lateral deflection as given by Eq. (4)

\[ \frac{1}{R} = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \quad (4) \]
Realizing that under the assumptions made with regard to small deflection theory, the term \( (d^2y/dx)^2 \) will be negligible when compared to unity. Thus one arrives at the governing equation for straight beam elements.

\[
\frac{d^2y}{dx^2} = \frac{M}{EI} \quad (5)
\]

Since the material system is relatively stiff, it is assumed that small deflection beam theory will yield sufficiently accurate results. Thus, one may write a governing differential equation for each segment of the joint. By matching boundary conditions of deflection and slope at each interface, the deflection of the entire joint may be obtained as a function of distance along the neutral axis. Details of the analysis may be referenced in Appendix B.

To enhance one's understanding of the joint behavior under applied tensile loadings, Figures 7 through 10 show deflection, slope, moment, and shear diagrams respectively at a load of 200 lbs. Many of the discontinuities found in the plots arise from a shift in the neutral axis which is a common occurrence among lap joints.

It was stated previously that analyzing the "joggle-lap" joint under tension was a non-linear problem. This was seen by the fact that the moment was a function of the deflection. Another way to view the non-linearities of the
FIGURE 9: MOMENT ALONG NEUTRAL SURFACE AT THE FAILURE LOAD
joint behavior is to investigate the response of the joint to varying tensile loads. Figures 11 and 12 provide a clear indication of the deviation from linearity even for small values of load. Both the deflection (Figure 11) and moment (Figure 12) were recorded at the beginning of SEG3. (i.e. \( S_3 = 0 \))
RESPONSE OF THE JOGGLE-LAP JOINT AT $S_3 = 0$

**FIGURE 11**

**FIGURE 12**
b. Finite Element Model (tension)

Anticipating the shortcomings of a beam bending model in the adhesive zone, defined to be the area of actual bonding, it was decided to model this area using finite-element methods. One of the underlying assumptions of small deflection beam theory is that plane sections remain plane during pure bending action. Clearly the validity of this assumption is questionable in the bonded area. Another reason for employing the finite element technique was to uncover any local stress concentrations that may not be revealed in a beam analysis. The finite-element mesh, consisting of 7 material types, is shown in Figure 13. Boundary conditions in the form of concentrated loads were applied to each of the finely meshed ends. Loading conditions were applied away from the adhesive layer at a distance of 1.5 times the thickness in an effort to minimize the effects of the end loads upon the stress solution. An explanation of how these boundary conditions were determined will follow shortly. A plane stress analysis was utilized to calculate the displacement and stress fields. Figures 15 through 17 are the result of a plotting routine which displays lines of constant stress. The figures should be interpreted in the same manner as that of a topographical map. Adjacent lines spaced closely together indicate areas of high stress gradients and possible sites for structural failure. The
Maximum $\tau_{12}$ Stress = 3896 psi
Minimum $\tau_{12}$ Stress = -3105 psi

FIGURE 17: THE ADHESIVE ZONE IN TENSION - CONTOURS OF $\tau_{12}$ STRESS
figures are labeled according to the component of stress being displayed. All three plots are the result of loading the specimen at the tensile failure load and are representative of the deformed geometry.

The limitations of the beam bending model are clearly displayed in Figure 15 and reveal the justification for the finite-element model. Shown in the figure is a smooth transition of stress across a change in cross-sectional area, (i.e. shift of the neutral axis) as calculated by the finite-element method. Experimental results have shown this to be a correct representation of the stresses. Beam analysis would have shown a sharp discontinuity in the stress profile where such a shift in the neutral axis occurs. Since the moment is nearly constant throughout SEG4 (see Figure 9) beam analysis would calculate $\sigma_1$ stress contours parallel to the adhesive layer. The $\sigma_1$, $\sigma_2$, and $\tau_{12}$ stress components are global oriented stresses as opposed to those that can vary according to element orientation. Marked on each figure are those areas where the assumptions made via beam analysis quite appreciably affect the accuracy of a correct solution.

Many analyses of lap joints assume a condition of constant shear stress in the adhesive layer itself. This would indeed be the case if the adherents were infinitely stiff as compared to the adhesive and also if the existence of a load transfer area was prohibited. Shear stress data
from the finite element model is plotted in Figures 18 and 19 and the indication is clear that the shear stress is not a constant in the load transfer area. The case of constant shear stress found toward the center of the adhesive zone, however, reveals the linear nature of the displacement function through the adhesive thickness in this area.
FIGURE 18: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE
FIGURE 19: SHEAR STRESS VARIATION THROUGH THE THICKNESS OF THE ADHESIVE ZONE
Boundary Conditions for the Finite-Element Model

The boundary conditions for the finite-element model are determined by applying the stress distribution as directed by the beam bending model to the finely meshed ends of the undeformed geometry of the finite element model. In other words, the stresses in the deformed geometry (beam model) must be moved through a distance to their equivalent point of application in the undeformed geometry (finite-element model). The reason for this difficulty with boundary conditions is that we are currently utilizing a linearized finite element routine, SAP V\(^2\), to solve a non-linear problem. Justification of such a procedure will hopefully become lucid with time.

To facilitate the derivation of a transformation routine, Figures 20 and 21 illustrate the following sign conventions. Figure 20 depicts a stress distribution for the left hand face of the finite element model with tension being taken as positive and compression being negative. Note that the neutral axis is not coincident with the centroidal axis inherent in the analysis of a curved beam. As mentioned previously, this fact yields a hyperbolic stress distribution which slightly complicates the computations. (SEE derivations of governing equation for stresses in a curved beam, Appendix A)

\(^2\)Structural Analysis Program V; University of Southern California, Department of Civil Engineering, Oct. 77.
FIGURE 20

Compression -
Tension +

Hyperbolic Stress Distribution

Centroidal Axis
Neutral Axis

FIGURE 21: SIGN CONVENTION
Figure 21 reveals a planar view of the deformed and undeformed sections. It is assumed in this derivation that the section of the beam can at most undergo a translation and a rotation. Translations are measured via the parameter DEFLA and are positive radially outward as shown. Small deflection theory also allows the rotations to be written as a change in slope. This parameter is DUDSA and is positive counter-clockwise.

With these sign conventions clearly in mind the stress distribution of the deformed geometry may now be resolved into concentrated force components. Representing the hyperbolic stress distribution as equivalent point forces and point couples acting at nodal points labeled 1 through 5 on Figure 21 corresponds mathematically to an integration of the stress distribution between fixed limits.

\[
F_{ni} = \frac{M}{\bar{u}} \int_{h_i}^{h_i-1} \frac{u}{R-u} \, du + \int_{h_i}^{h_i-1} F \cos (\theta + DUDSA) \, du \quad (6)
\]

where \( i = 1-5 \)

- \( F_{ni} \) = nodal force component
- \( M \) = moment
- \( \bar{u} \) = distance between neutral and centroidal axes
- \( a \) = cross-sectional area
- \( R \) = radius of curvature
- \( F \) = load
- \( \theta \) = angle subtended by SEG3

DUDSA = local slope of deformed neutral axis
The first term of Eq. (6) represents the contribution from the hyperbolic stress distribution. The second term acts to superimpose the component of force due to longitudinal loading.

A correcting moment is calculated for each node to equilibrate the two representations of stress on the section.

\[ M_{corr} = \int_{h_{i-1}}^{h_i} \sigma(u) u \, du - F_{ni} u \]  

(7)

The need for the correcting moment is due to the fact that a distributed force is now represented by a point force as shown in Figure 22.

The next step follows from a translation of the point forces. Elementary statics dictates that a point force may be equivalently represented by the same point force and an added moment to account for the translation from the original line of action.

After carrying out a similar procedure for the stresses at the right hand side of the finite element model, the entire system is set in equilibrium by accounting for the shear acting on each face of the model. The values of shear are obtained directly from the beam bending model. Thus a correct set of boundary conditions has been determined for the finite-element model of the adhesive zone. A computer routine designated by CONVERT was written to calculate appropriate boundary conditions and may be found
in Appendix C.

FIGURE 22: ILLUSTRATION OF THE CORRECTIVE MOMENT
Methods of Analysis

B. Flexure Loading

a. Beam Model

The bending behavior of the "joggle-lap" joint was also studied. It was found that the theoretical analysis was far simpler than that encountered for tensile loading. Each segment of the joint (see Figure 6) was modeled as if it were in pure bending. Stresses in the straight beam numbers were calculated via the flexure formula while for the curved beams the formula

\[
\sigma_y = \frac{My}{(R-y)ya}
\]

was used.

In order to compute bending stresses in SEG4 (layered beam) it is necessary to introduce the notion of equivalent sections. In this method we assume all materials to have the same modulus of elasticity. By replacing the actual section with a mechanically equivalent one allows
the flexure formula to be used as a means of computing stresses. The width of the sections are varied so that the new width equals the ratio of the old modulus of the material to the new modulus of the material times the old width as shown in Figure 23. Computing $I_{eq}$ for the specimen geometry,

$$I_{eq} = \sum_{i=1}^{3} \left( \frac{1}{12} b_i h_i^3 + a_i d_i^2 \right)$$

$b = \text{length of base}$

$h = \text{length of side}$

$a = \text{area}$

$d = \text{distance between element neutral axis and overall section neutral axis}$

it is apparent that the effect of the adhesive layer on overall section stiffness is negligible. Using the flexure formula and the relation

$$(\sigma_x')_{\text{actual}} = \frac{E_{\text{old}}}{E_{\text{new}}} (\sigma_x')_{\text{equiv}}$$

the stresses in SEG4 may easily be calculated.
\[ E_{1,3} = 2.1 \times 10^6 \text{ psi} \]
\[ E_2 = 1.0 \times 10^5 \text{ psi} \]

**Actual End Section View**

**Equivalent Section**

FIGURE 23: METHOD OF EQUIVALENT SECTIONS
b. Finite Element Model (flexure)

The boundary conditions of the finite-element model may be changed to accommodate pure bending. By utilizing couples at the finely meshed ends of the model, stresses in the adhesive zone may be monitored where it has been shown that the results from beam theory are less accurate. Figures 24 through 26 display $\sigma_1$, $\sigma_2$, and $\tau_{12}$ stress contours respectively within the "joggle-lap" joint in pure bending.
FIGURE 24: THE ADHESIVE ZONE IN BENDING - CONTOURS OF $\sigma_1$ STRESS

Maximum $\sigma_1$ Stress = 2846 psi
Minimum $\sigma_1$ Stress = -2834 psi
Maximum $\sigma_2$ Stress = 695 psi
Minimum $\sigma_2$ Stress = -898 psi

FIGURE 25: THE ADHESIVE ZONE IN BENDING - CONTOURS OF $\sigma_2$ STRESS
Maximum $\tau_{12}$ Stress = 1800 psi
Minimum $\tau_{12}$ Stress = -891 psi

FIGURE 26: THE ADHESIVE ZONE IN BENDING - CONTOURS OF $\tau_{12}$ STRESS
IV. Experimental Results

A. Tension

As set forth in the objectives of such a study, an emphasis was to be placed upon developing joint geometries which will accommodate high rate fabrication techniques. In an effort to meet this criterion experimentally, it was necessary to utilize a joint configuration currently being molded in industry. The time and expense of developing in-house molding capabilities proved to be beyond the scope of the research at hand. Thus, test sections were cut from premolded panels of SMC which were later bonded together to form the joint.

The bonding operation was also directed toward high fabrication procedures. All test specimens were adhesively joined at Goodyear Adhesives Division, Ashland, Ohio, via production adhesives application techniques. It was felt that by using these sophisticated application procedures optimum adhesive properties could be obtained.

In general, SMC is defined to be an anisotropic material because of the substantial difference between in-plane and out-of-plane properties. Referring to the coordinate system of Figure 1, the constitutive relations

-48-
FIGURE 27: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES
FIGURE 28: ELASTIC RESPONSE DUE TO TENSION - TOP FIBER STRESSES
FIGURE 29: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES
FIGURE 30: ELASTIC RESPONSE DUE TO TENSION - BOTTOM FIBER STRESSES
that the joint invariably strained beyond the small-deflection range at considerably small loadings. It was therefore a rather arduous task to approximately determine the experimentally applied moment to the joint. The correlation between the theoretical and experimental data may be referenced in Figures 31-34. As in the case of tensile loading, it should be noted that the stresses in SEG1 are again considerably higher than those predicted by theory, which is attributable to the molded geometry.
FIGURE 31: TOP FIBER STRESSES DUE TO BENDING
FIGURE 32: TOP FIBER STRESSES DUE TO BENDING
FIGURE 33: BOTTOM FIBER STRESSES DUE TO BENDING
FIGURE 34: BOTTOM FIBER STRESSES DUE TO BENDING
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<th>Contact Area (in²)</th>
<th>Loading Condition</th>
<th>Failure Load (LBS)</th>
<th>Failure Mode*</th>
</tr>
</thead>
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<td>tension</td>
<td>191</td>
<td>flexure</td>
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<tr>
<td>4_8097</td>
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<td>1.75</td>
<td>tension</td>
<td>200</td>
<td>flexure</td>
</tr>
</tbody>
</table>

*after failure initiation, it was observed that the crack was propagated via interlaminar shear*
V. Failure Analysis

One of the most important parameters to predict in a study of this type is the ultimate loading conditions. This in essence dictates the choice of a failure criterion. The maximum stress theory will be employed in this report because of its simplicity in application and execution. Other popular failure criteria, such as the Tsai-Wu criterion were deemed inappropriate due to the limiting assumptions made in accordance with beam theory.

Maximum stress criterion states that the material will fail when any component of stress exceeds the corresponding material strength. In general, the above statement may be written in equation form as

\[ \sigma_i \geq X_i^T \quad (\sigma_i > 0) \quad i = 1-3 \quad (14) \]
\[ |\sigma_i| \geq X_i^C \quad (\sigma_i < 0) \quad i = 1-3 \quad (15) \]
\[ |\sigma_i|^2 \geq S_i \quad i = 4-6 \quad (16) \]

where
- \( X_i^T \) = ultimate tensile strength
- \( X_i^C \) = ultimate compressive strength
- \( S_i \) = ultimate shear strength

These equations simplify to those listed below after employing the local coordinate nomenclature for the
"joggle-lap" joint.

\[ \sigma_u \geq x^T \quad (\sigma_u > 0) \]  \hspace{2cm} (17)

\[ |\sigma_u| \geq x^C \quad (\sigma_u < 0) \]  \hspace{2cm} (18)

\[ |\sigma_{us}| \geq S_i \]  \hspace{2cm} (19)

Applying this failure criterion to the model, it was found that the bottom fiber tensile stresses (see Figure 35) predicted the ultimate loading of the joint within experimental error. Thus the maximum flexural stress was utilized to predict failure.

All failures occurring as a result of tensile loading were initiated along the bottom surface of SEG3. Crack initiation was observed to be of the net tension mode, while propagation appeared to be due to "interlaminar shear". There was a general consistency among the initiation and propagation of the crack for all tension tests.

It was thought at one time that the curved sections of the joint (SEG2, SEG3) were either fiber deficient or highly anisotropic yielding a potential low strength area. However, a photomicrograph of this cross-sectional area clearly shows no such tendencies. (See Plate 7)
FIGURE 35: BOTTOM FIBER STRESSES AT THE FAILURE LOAD
FIGURE 36: TOP FIBER STRESSES AT THE FAILURE LOAD
VI. Conclusions

The response of the "joggle-lap" joint was investigated for both tensile and bending loads in this report. It was found that experimental data correlated rather well to the values of stress predicted by the analytical model. The results of the bending study were not as favorable, in that experimental verification proved to be more difficult.

A parametric study was undertaken for the "joggle-lap" joint subject to tensile loads in an effort to isolate the crucial design parameters. In Figures 37 through 40 a normalized stress value is plotted against one of four parameters - adherent thickness, inside radius, contact area, and load. From these design curves the following conclusions are inferred.

- If weight saving requirements are not stringent, the effect of increasing adherent thickness drastically reduces maximum flexural adherent stress.

- Increasing the radius of curvature has a negligible effect on reducing maximum adherent stress due to a trade-off between mechanisms.
Note: All stresses and failures were recorded in the adherent.

FIGURE 37: EFFECTS OF ADHERENT THICKNESS ON JOINT STRENGTH
Note: All stresses and failures were recorded in the adherent.

FIGURE 38: EFFECTS OF INSIDE RADIUS OF JOINT STRENGTH
Note: All stresses and failures were recorded in the adherent.
Note: All stresses and failures were recorded in the adherent.

FIGURE 40: EFFECT OF LOAD ON JOINT STRENGTH
Neglecting local stress concentrations, the effect of reducing the overlap length does not increase adherent stress significantly.

In the region of the failure load, the maximum adherent stress increases linearly with load.

An important parameter in joint design is that of joint efficiency. This parameter is defined to be the ratio of ultimate joint load divided by the ultimate load carried by the material if the joint were not present. The joint efficiency of the "joggle-lap" joint in tension is calculated to be 0.153.

The adhesive system employed in this report proved to be quite adequate from a structural point of view. For the given overlap length of 1 in (2.54 cm) there were no recorded failures in the adhesive layer. Failure loads were predicted using the maximum flexural stress as the limiting criterion.

This report would be incomplete if it did not offer several suggestions for future work as an outgrowth of this study. An obvious limitation to the work reported herein is the inability to extensively verify the analytical model by experimental testing of various joint geometries.
Further development in this area would greatly increase the reliability of the computer model.

More detailed work needs to be completed in the response of the "joggle-lap" joint to bending loads. This report included only a cursory investigation of bending behavior as a means of identifying the underlying problems associated with the experimental verification of theory.

It is felt that this report will provide a fundamental basis for future research concerning the "joggle-lap" joint.
VII. Acknowledgements

The authors wish to thank David W. Adkins and Joseph J. Quigley, graduate students at the University of Delaware, for their expertise and guidance throughout this research effort. Also, we wish to express our appreciation to Dr. Terry V. Baughn and Bill Englehart of International Harvester for their vested interest in the program and for supplying all of the test specimens. Special thanks are also directed to Larry Carapellotti and his staff at Goodyear Adhesives for their assistance in bonding the experimental specimens.
VIII. List of References


IX. Bibliography


Appendices

Appendix A  
Derivation of the Governing Equations for a Curved Beam

Consider the curved beam element shown in Figure 41.

The analysis begins by seeking an expression for the strain distribution perpendicular to the neutral axis. Assume that the curved beam, with an initial radius of curvature $R$, undergoes a small elastic deformation due to the applied moment. (It is important to note that the neutral axis of bending for a curved beam does not necessarily coincide with the centroidal axis of the beam.) Under the action of this moment it becomes apparent that segment cd rotates about the neutral axis.
to a new position c'd'. It is assumed here, as in classical beam analysis, that plane sections remain plane. It is readily seen that while the deformation of the beam varies linearly with the distance from the neutral axis, the strains do not. The reason is that the original length of all the fibers prior to the application of the moment are not constant.

Thus the following relation for the strain distribution is written below.

$$\varepsilon = \frac{e_L}{\Delta L} = \frac{-y\Delta \theta}{(R-y)\Delta \phi}$$ (20)

where  
\(e_L\) = elongation
\(y\) = radial coordinate (positive radially inward)
\(\Delta \theta\) = angle of deformation
\(\Delta \phi\) = angle subtended by curved beam

The above equation shows the strain to vary hyperbolically across the section. Using the plane stress constitutive relation, Eq. (20) becomes

$$\sigma = \frac{-Ey \Delta \theta}{(R-y)\Delta \phi}$$ (21)

Now it is appropriate to derive the formulas for flexural stress. First assume that the portion of the beam is in equilibrium. Following directly one may write the equations of equilibrium for an arbitrary section.
Making the appropriate substitutions for the stress, Eq. (22) becomes

\[ \int_A -Ey\Delta \theta da = 0 \]  \hspace{1cm} (23)

Assuming \( E, \Delta \phi, \) and \( \Delta \theta \) to be constants the integral is simplified as shown in Eq. (24).

\[ \int_A \frac{yda}{(R-y)\Delta \phi} = 0 \]  \hspace{1cm} (24)

It is possible to solve Eq. (24) for the radius of curvature and thus locate the neutral surface; however, it will suffice to let Eq. (24) stand as is for now.

Referring to Figure 42 and summing moments about the neutral axis, one finds that the stress distribution must also satisfy the equation below.
making the appropriate substitutions, Eq. (25) becomes

\[ M = - \int_A \sigma y \, da \]  

(25)

Notice the algebraic relation that permits the substitution of an equivalent expression into Eq. (27).

\[ \frac{y^2}{R-y} = \frac{Py}{R-y} - y \]  

(28)

Eq. (27) now becomes

\[ M = \frac{E \Delta \theta}{\Delta \phi} \left[ \int_A \frac{R y \, da}{R-y} - \int_A y \, da \right] \]  

(29)

and from the result of Eq. (24)

\[ M = \frac{E \Delta \theta}{\Delta \phi} \left( R(0) - a\bar{u} \right) \]  

(30)

where

\( a = \text{area} \)

\( \bar{u} = \text{distance between the neutral and centroidal axes} \)

Rearranging Eq. (30) yields

\[ \frac{\Delta \theta}{\Delta \phi} = \frac{-M}{E a\bar{u}} \]  

(31)
Comparing this equation with the well-known deflection equation for straight beams, it is apparent that

\[ \frac{d^2y}{dx^2} = \frac{M}{EI} \]  

the left hand side of Eq. (31) is not yet suitable. The ultimate goal of such an analysis is to seek an equation that relates the deflection of the neutral axis to the position along the neutral axis.

Consider Figure 43 shown below.

The beam is deflected as shown to illustrate the most general case of a non-constant moment. That is, the moment is a function of position. Now the deflection can be measured as the deviation between the undeformed neutral surface and the deformed neutral surface. For convenience just the neutral axis and appropriate parameters are drawn in Figure 44.
A coordinate system $u, s$ is defined and shown in the figure where $s$ traverses tangentially to the undeformed neutral axis and $u$ is defined to be perpendicular to that axis.

Enlarging the area of interest and focusing on the triangle of Figure 45, one finds that
Realizing that \( \tan \alpha \approx \alpha \) for small \( \alpha \), it follows that

\[
\Delta \lambda = \frac{(R+u)\Delta s}{R}
\]

and thus

\[
\theta(s) = \frac{R\Delta u}{(R+u)\Delta s}
\]

which may be written as

\[
\frac{\Delta u}{\Delta s} = \frac{(R+u)\theta(s)}{R}
\] (33)

Finally in the limit as \( \Delta s \to 0 \): Eq. (33) becomes

\[
\lim_{\Delta s \to 0} \frac{\Delta u}{\Delta s} = \frac{(R+u)\theta(s)}{R} = \frac{du}{ds}
\] (34)

From Eq. (31), several simplifications can be made with the proper substitutions.

\[
\frac{\Delta \theta}{\Delta \theta} = \frac{-M}{Eau}
\]

where \( \Delta \phi = \frac{\Delta s}{R} \)

\[
\lim_{\Delta s \to 0} 0 \frac{\Delta \theta}{\Delta s} = \frac{-M}{REau} = \frac{d\theta}{ds}
\] (35)

Differentiating Eq. (34) with respect to \( s \) yields
and substituting Eq. (35) into Eq. (36) yields the final results - a second order differential equation relating deflection to position in terms of the applied moment.

\[
\frac{d^2u}{ds^2} = \frac{(R+u) \, d\theta}{R \, ds}
\]  

(36)

\[
\frac{d^2u}{ds^2} = \frac{(R+u)M}{R^2 E a u}
\]  

(37)
Appendix B

Beam Bending Model of the "Joggle-Lap" Joint

SEGL may be modeled as a straight beam shown in Figure 46.

![Figure 46: SEGL Modeled as a Straight Beam](image)

In general, the moment experienced by any segment originates from two sources: eccentricity from geometry and eccentricity due to deflection. The preceding statement may be written algebraically as follows.

\[ M = F(e_{\text{geom}} + e_{\text{defl}}) \]  

(38)

where

- \( M \) = moment
- \( F \) = applied force
- \( e \) = eccentricity

It is readily seen that \( e_{\text{geom}} = 0 \) for SEGL. Writing Eq. (38) in the local coordinate system, the moment experienced by
this segment reduces to

\[ M = F u_1 \]  \hspace{1cm} (39)

where

\[ u_1 = \text{deflection in the local coordinate system} \]

Substituting Eq. (39) into Eq. (5) yields

\[ \frac{d^2 u_1}{ds_1^2} - \frac{F u_1}{EI} = 0 \]  \hspace{1cm} (40)

The corresponding boundary conditions are expressed below

\[ u_1(0) = 0 \]
\[ u_1(\lambda_1) = u_0 \]

where \( u_0 \) is yet undetermined.

The solution of Eq. (40) is of standard form and known to be

\[ u_1 = C_1 \sinh \sqrt{\frac{F}{EI}} \cdot s_1 + C_2 \cosh \sqrt{\frac{F}{EI}} \cdot s_1 \]  \hspace{1cm} (41)

Applying the boundary conditions to Eq. (41) determines the constants \( C_1 \) and \( C_2 \) to be

\[ C_2 = 0 \]
\[ C_1 = \frac{u_0}{\sinh \sqrt{\frac{F}{EI}} \cdot \lambda_1} \]

and thus
where $u_o$ is necessarily negative to correspond with the physical system. In other words, for a given tensile load it is expected that SEG1 will deflect downward. (Figure 6). Also

$$\frac{du_1}{ds_1} (l_1) = u_o \sqrt{\frac{F}{EI}} \cosh \frac{\sqrt{F/EI} \ l_1}{\sinh \sqrt{F/EI} \ l_1}$$

It should be noted that the deflection as given by Eq. (42) is not known explicitly in terms of the given parameters. $u_o$ is still unknown and it will be shown later how this value may be determined uniquely.

SEG2 is modeled as a curved beam and shown in Figure 47. The local coordinate system is a curvilinear coordinate system with the $s_2$ axis traversing the neutral axis as shown. Positive deflections are measured normal to the undeformed neutral axis in the direction of $u_2$.
From the derivation of the general case for a curved beam in pure bending (see Appendix A), the governing equation for the deflection is

\[
\frac{d^2 u_2}{ds^2} = \frac{(R + u_2)M}{R^2 E a \bar{u}} \quad (43)
\]

where

\begin{align*}
    s_2 &= \text{arc length} \\
    u_2 &= \text{deflection normal to neutral axis} \\
    M &= \text{moment} \\
    R &= \text{radius of curvature} \\
    E &= \text{modulus of elasticity} \\
    a &= \text{cross sectional area} \\
    \bar{u} &= \text{distance between neutral axis and centroidal axis and its value is necessarily negative}
\end{align*}

The moment may be written as the product of the applied load and the eccentricity, where the eccentricity in this case consists of both geometry and deflection considerations. At this point, it is appropriate to introduce the notion of extensional effects. It is realized that with the given loading conditions, the "joggle-joint" will undergo deflections parallel to the neutral axis as well. This fact would be of little concern if all beam segments of the joint configuration had their neutral axis aligned with the loading axis. If this were the case, the longitudinal displacement would not affect the eccentricity.
However, it is evident that the extensional strains in the curved beam segments give rise to an added component of eccentricity defined to be $e_{\text{ext}}$. To calculate the value of $e_{\text{ext}}$, one merely applies the criterion of force equilibrium to SEG2 (Figure 47) in the local coordinate system.

$$
\begin{align*}
\sum F_{u_2} &= 0 & H_2\cos\theta + V_2\sin\theta &= F \\
\sum F_{s_2} &= 0 & H_2\sin\theta &= V_2\cos\theta
\end{align*}
$$

thus

$$H_2 = F\cos\theta$$

where

$$\theta = \text{angle subtended by SEG2}$$

Employing the constitutive relationship

$$\sigma = E\varepsilon$$

where

$$\sigma = \text{stress}$$

$$E = \text{modulus of elasticity}$$

$$\varepsilon = \text{strain}$$

and considering only the $y$ (global coordinate) component of the extension we thus arrive with the expression for $e_{\text{ext}}$.

$$e_{\text{ext}} = \frac{F_2\cos(s_2/R)\sin(s_2/R)}{AE}$$

Eq. (44) must be added to the other terms which comprise the eccentricity due to deflection.

Therefore Eq. (43) becomes
\[
\frac{d^2 u_2}{ds_2^2} = \frac{-(R-u_2)F}{R^2 E au} [e_{\text{geom}} + e_{\text{defl}} + e_{\text{ext}}] \quad (45)
\]

where
\[
e_{\text{geom}} = R(1-\cos \left(\frac{s_2}{R}\right) + \bar{u})
\]
\[
e_{\text{defl}} = u_2 \cos \left(\frac{s_2}{R}\right)
\]
\[
e_{\text{ext}} = F s_2 \cos \left(\frac{s_2}{R}\right) \sin \left(\frac{s_2}{R}\right) \frac{aE}{s}
\]

Initial conditions for SEG2 are found by matching deflection and slope at the 1-2 interface.

\[
u_2(0) = u_0
\]
\[
\frac{du_2}{ds_2}(0) = u_0 \sqrt{\frac{F}{EI}} \cosh \sqrt{\frac{F}{EI}} \frac{\ell_1}{\sinh \sqrt{\frac{F}{EI}} \ell_1}
\]

Using a numerical integration routine to solve Eq. (45) the deflection \( u_2 \) may be marched out as a function of arc length \( s_2 \). A Runge-Kutta method based on Verners fifth and sixth order pair of formulas was used. An explanation of the integration routine DVERK may be referenced in Appendix C.

Figure 48 shows SEG3 modeled as a curved beam.

From Eq. (37) the governing differential equation for a curved beam in pure bending is

\[
\frac{d^2 u_3}{ds_3^2} = \frac{(R+u_3)}{R^2 E au} M \quad (46)
\]

where

\[
M = F(e_{\text{geom}} + e_{\text{defl}} + e_{\text{ext}})
\]
Through geometric considerations $e_{geom}$ can be shown to be

$$e_{geom} = \bar{u} + R(1 - \cos \theta) - 2\bar{u}\cos \theta + R\sin(\frac{\pi}{2} - \theta + \frac{s_3}{R})\cos \theta$$  \hspace{1cm} (47)

where

- $\theta =$ angle subtended by SEG3
- $\bar{u} =$ distance between centroidal and neutral axes
- $R =$ radius of curvature
- $s_3 =$ arc length along neutral surface of SEG3

Also

$$e_{defl} = M\cos(\frac{\pi}{2} - \theta + \frac{s_3}{R})$$  \hspace{1cm} (48)

From a similar argument developed earlier it may be shown that $e_{ext}$ for SEG3 is given by

$$e_{ext} = \frac{Fs_3\cos(\theta - \frac{s_3}{R}) \sin (\theta - \frac{s_3}{R})}{aE}$$  \hspace{1cm} (49)
Thus Eq. (46) becomes
\[
\frac{d^2 u_3}{ds_3^2} = -\frac{(R+u)}{R^2 E a y} F (e_{\text{geom}} + e_{\text{defl}} + e_{\text{ext}}) \tag{50}
\]
where \(e_{\text{geom}}\), \(e_{\text{defl}}\), and \(e_{\text{ext}}\), are given by Eqs. (47), (48), and (49) respectively. Matching boundary conditions at the 2-3 interface provides initial conditions to Eq. (50) which may be integrated numerically as before.

SEG4 is analyzed as a multi-layered beam and shown in Figure 49. Treating this segment to be composed of three linear elastic beam elements, the governing differential equation follows from Eq. (5) with a slight modification.

\[
\frac{d^2 u_4}{ds_4^2} = \frac{M}{\sum_{i=1}^{3} E_i I_i} \tag{51}
\]
where \(I_i = \text{moment of inertia of the } i\text{th section about the neutral axis}\)
\(E_i = \text{modulus of elasticity of the } i\text{th element}\)
\(\sum_{i=1}^{3} E_i I_i\) is referred to as an effective flexural stiffness and is merely a constant. The moment is defined in the usual manner as
\[M = F(e_{\text{geom}} + e_{\text{defl}})\]
FIGURE 49: SEG4 MODELED AS A LAYERED BEAM

FIGURE 50: SEG5 MODELED AS A STRAIGHT BEAM
where \( e_{\text{geom}} = 0.5(t+b) \)

\[ e_{\text{defl}} = u_4 \]

Thus Eq. (51) becomes

\[
\frac{d^2 u_4}{ds^2_4} = \frac{F}{3} \frac{(0.5(t+b) + u_4)}{\sum_{i=1}^{n} E_i I_i}
\]

Initial conditions are found by equating the deflection and slope at the 3-4 interface. Following in the usual manner, Eq. (52) is integrated to obtain an expression for the deflection of SEG4 as a function of arc length in the local coordinate system.

Finally SEG5 is shown in Figure 50 modeled as a straight beam member. The governing differential equation is the same as Eq. (5)

\[
\frac{d^2 u_5}{ds^2_5} = \frac{M}{EI}
\]

where \( M = Fu_5 \)

and the initial conditions are obtained by matching the deflection and slope at the 4-5 interface. Upon integration of Eq. (53) the deflection SEG5 will be a known function of the abscissa \( s_5 \) of the local coordinate system. Therefore the deflection and slope at point P of Figure 50 are also known. But it should be apparent that the values of the deflection
and slope at this point must be zero or at least within certain tolerance limits. This in fact is the final boundary condition to the problem that is needed to uniquely determine the value of \( u_0 \) which was previously assumed to be arbitrary. Thus, through an iterative process, a correct value of \( u_0 \) may be calculated by assuring that the deflection and slope of point \( P \) of SEG5 is sufficiently close to zero. To avoid confusion, it should be noted that by specifying zero deflection at point \( P \) we will force the slope to zero by the nature of the deflection function of SEG5. So in fact this is a well-posed problem, whereby we specify only enough boundary conditions as there are unknowns. The process for correctly determining \( u_0 \) is shown schematically in Figure 51.

\[^3\text{It was found that reliable results were obtained by using the tolerance limits listed here.}\]

\[
|\text{deflection (P)}| < .00001 \\
|\text{slope (P)}| < .00005
\]
FIGURE 51: ITERATIVE PROCESS FOR DETERMINING $u_0$
Appendix C

Computer Programs

a. JOGGLE

To facilitate ease of calculation, a computer routine identified as JOGGLE was developed and may be referenced below. Essentially this program calculates a correct value of $u_0$ and proceeds to determine a solution for the deflection while calculating stress profiles along the joint configuration. These stress profiles are linear in the straight beam members (SEG1, SEG5) and hyperbolic in the curved beams (SEG2, SEG3).
ANALYTICAL BEAM BENDING MODEL
FOR A Joggle LAP JOINT

DEVELOPED BY: RICHARD C. GIVLER
UNIVERSITY OF DELAWARE
SEPT 78 - JAN 79

PARAMETERS AND NOMENCLATURE

MATERIAL THICKNESS
THICK = .1

LONGITUDINAL MODULUS OF ELASTICITY OF SMC IN PSI
ESMC = 2.1E+06

MODULUS OF ELASTICITY OF ADHESIVE IN PSI
EADH = 1.0E+05

LOAD IN LBS
PLOAD = 200.

INSIDE RADIUS IN INCHES
RADI = 2.5*THICK

OUTSIDE RADIUS IN INCHES
RADO = 3.5*THICK

BONDING THICKNESS IN INCHES
BOND = .03

CONTACT WIDTH IN INCHES
CONTA = 1.0

SPECIMEN WIDTH
WIDTH = 1.0

LENGTH OF SEGMENT 1
SEGA = 3.5

LENGTH OF SEGMENT 5
SEGB = 4.0

LEFT INTERVAL LIMIT FOR ITERATION
AINT = .04

RIGHT INTERVAL LIMIT FOR ITERATION
BINT = .01

TOLERANCE LIMIT ON INITIAL DISPLACEMENT
ERR(1) = .00001

FILE: \( \text{FILE} \) KIND = REMOTE, MAXRECSIZE = 22
DIMENSION PR0D(3), ERR(I), T(2), E(3), YPRIME(2)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
       , BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR

SET AUTOBIND
RESET FREE

DIMENSION PROD(3). ERR(I). T(2). E(3). YPRIME(2)
COMMON R. PLOAD. ESMC. WIDTH. THICK. YBAR. THETA. EADH. PI
       . BOND. J. TORC. TORCS. TRAC. TRACE. SHEAR

PARAMETERS AND NOMENCLATURE

MATERIAL THICKNESS
THICK = .1

LONGITUDINAL MODULUS OF ELASTICITY OF SMC IN PSI
ESMC = 2.1E+06

MODULUS OF ELASTICITY OF ADHESIVE IN PSI
EADH = 1.0E+05

LOAD IN LBS
PLOAD = 200.

INSIDE RADIUS IN INCHES
RADI = 2.5*THICK

OUTSIDE RADIUS IN INCHES
RADO = 3.5*THICK

BONDING THICKNESS IN INCHES
BOND = .03

CONTACT WIDTH IN INCHES
CONTA = 1.0

SPECIMEN WIDTH
WIDTH = 1.0

LENGTH OF SEGMENT 1
SEGA = 3.5

LENGTH OF SEGMENT 5
SEGB = 4.0

LEFT INTERVAL LIMIT FOR ITERATION
AINT = .04

RIGHT INTERVAL LIMIT FOR ITERATION
BINT = .01

TOLERANCE LIMIT ON INITIAL DISPLACEMENT
ERR(1) = .00001
C-----STEP SIZE FOR NUMERICAL INTEGRATION
STEP=50.
PI=3.141592654
C-----TRACING CONSTANTS
TRAC=1.0
TRACE=1.0
C-----NOTE:DEFLECTIONS ARE MEASURED NORMAL TO THE UNDEFORMED
C-     NEUTRAL AXIS
C
YO=(AINT+BINT)/2.
C-----ITERATIVE CALCULATION OF YO TO FORCE ZERO DEFLECTION
C- AND SLOPE AT END OF SEG5
C- YO MUST LIE BETWEEN THE PROPOSED LIMITS
C OF AINT AND BINT
C
ITERATIVE CALCULATION OF YO TO FORCE ZERO
C AND SLOPE AT END OF SEG5
C
C MUST LIE BETWEEN THE PROPOSED LIMITS
C OF AINT AND BINT
C
C NUMERICAL INTEGRATION VIA LIBRARY ROUTINES DVERK AND UERTST
C
101 CONTINUE
DO 200 X=0,SEGA+.005,SEGA/STEP
PROD(1)=SQRT(LOAD/(ESMC*(1.0/WIDTH+THICK+3./12.)))
PROD(2)=.5*(EXP(PROD(3)*SEGA)-EXP(-PROD(3)*SEGA))
T(1)=PROD(1)/PROD(2)
T(2)=YO*PROD(1)/PROD(2)*(.5*(EXP(PROD(3)*SEGA)+EXP
C-(-PROD(3)*SEGA))

200 CONTINUE
DIMENSION C(24), Y(2), W(2.9)
EXTERNAL FCN1
C------CALCULATION OF THE RADIUS OF CURVATURE FOR CURVED MEMBERS
R=WIDTH+THICK/(ALOG(RADO/RADI))
NW=2
C------CALCULATION OF THE ANGLE SUSTENDED BY SEG2 AND SEG3
THETA=ARCOS((5.*THICK-BOND)/(6.*THICK))
N=2
C------CALCULATION OF THE DISTANCE BETWEEN NEUTRAL AXIS AND
C CENTROIDAL AXIS OF CURVED MEMBERS
Y3AR=RADI+THICK/2.-R
X=0.0
Y(1)=-T(1)
Y(2)=-T(2)
TOL=.000001
IND=1
DO 300 Z=0.0,R*THETA+.001,R*THETA/STEP
XEND=FLOAT(Z)
CALL DVERK(N,FCN1,X,Y,IND,C.NW,W,IER)
IF(IND.LT.0.OR.IER.GT.0) Go To 20

300 CONTINUE
20 CONTINUE
RINT=RNEW
EXTERNAL FCN2
X=0.0
Y(1)=-Y(1)
Y(2)=-Y(2)
NW=2
N=2

IND=1  
DO 250 M=1,24  
C(M)=0.0  
250 CONTINUE  
DO 350 Z=0.0,R*THETA+.001,R*THETA/STEP  
XEND=FLOAT(Z)  
CALL DVERK(N,FCN2,X,Y,XEND,TOL,IND,C,NW,IER)  
IF(IND.LT.0.OR.IER.GT.0) GO TO 70  
350 CONTINUE  
70 CONTINUE  
EXTERNAL FCN3  
X=0.0  
NW=2  
N=2  
IND=1  
DO 290 M=1,24  
C(M)=0.0  
290 CONTINUE  
DO 400 Z=0.0,CONTA+.005,CONTA/STEP  
XEND=FLOAT(Z)  
CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,IER)  
IF(IND.LT.0.OR.IER.GT.0) GO TO 80  
400 CONTINUE  
80 CONTINUE  
EXTERNAL FCN4  
X=0.0  
NW=2  
N=2  
IND=1  
DO 291 M=1,24  
C(M)=0.0  
291 CONTINUE  
DO 246 Z=0.0,SEGB+.005,SEGB/STEP  
XEND=FLOAT(Z)  
CALL DVERK(N,FCN4,X,Y,XEND,TOL,IND,C,NW,IER)  
IF(IND.LT.0.OR.IER.GT.0) GO TO 81  
246 CONTINUE  
81 CONTINUE  
IF(ABS(Y(1)).LT.ERRM) GO TO 88  
IF(Y(1).LT.0) GO TO 100  
IF(Y(1).GT.0) GO TO 89  
100 CONTINUE  
BINT=YO  
YO=(AIN+BIN)/2.  
GO TO 101  
89 CONTINUE  
AINT=YO  
YO=(AIN+BIN)/2.  
GO TO 101  
88 CONTINUE  
C-                    CALCULATION OF DEFLECTION AND SLOPE AS A FUNCTION  
C-                   OF X FOR SEGMENT 1  
C-  
WRITE(6,500) YO  
500 FORMAT(\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
502 FORMAT(1X, 'DEFLECTION AND SLOPE FOR SEGMENT 1')
WRITE(6, 2)
2 FORMAT(1X, 'DEFLECTION AND SLOPE FOR SEGMENT 2')
WRITE(6, 25)
9 FORMAT(1X, 'DEFORMATION', 5X, 'SHEAR', 7X, 'STRESS', 8X, 'STRESS', 8X, 'STRESS')
DO 505 X = 0.0, 0.005, 0.05/STEP
PROD(1) = SIGNUM(PLOAD/ESWC(/WIDTH*THICK**2.0, 12.0))
PROD(1) = SIGNUM(PLOAD/ESWC(/WIDTH*THICK**2.0, 12.0))
PROD(2) = SIGNUM(PLOAD/ESWC(/WIDTH*THICK**2.0, 12.0))
T(1) = PROD(1)/PROD(2)
(2) = PROD(3)/PROD(2) + 0.5 * (EXP(PLOAD/ESWC(/WIDTH*THICK**2.0, 12.0)))
T(I) = PROD(1)/PROD(2)
IF(INT(J/2).NE.J/2.0) GO TO 199
WRITE(6, 10) X, T(I), T(2), SIGT, PLOAD*T(I)
WRITE(6, 11) SIGB
199 J = J + 10
IF(INT(J/10).NE.J/10.0) GO TO 299
WRITE(6, 10) X, Y, XEND, TOL, INDIRECT(N, L, IER)
WRITE(6, 11) SIGB
24 FORMAT(1X, 'DEFORMATION', 5X, 'SHEAR', 7X, 'STRESS', 8X, 'STRESS', 8X, 'STRESS')
DO 510 Z = 0.0, R*THETA+.001, R*THETA/STEP
N = N + 1
X = 0.0
Y(1) = -T(1)
Y(2) = -T(2)
TOL = 0.00001
IND = 1
J = 0
IF(INT(J/10).NE.J/10.0) GO TO 410
WRITE(6, 11) SIGB
410 FORMAT(1X, 'DEFORMATION', 5X, 'SHEAR', 7X, 'STRESS', 8X, 'STRESS', 8X, 'STRESS')
DO 510 Z = 0.0, R*THETA+.001, R*THETA/STEP
XEND = FLOAT(Z)
CALL OVERK(-FNC1, X, Y, XEND, TOL, INDIRECT(N, L, IER))
IF(INT(LT.0. OR. IER.GT.0) GO TO 511
J = J + 10
WRITE(6, 30) X, Y(1), Y(2), TOL, SIGB
30 FORMAT(1X, 'DEFORMATION', 5X, 'SHEAR', 7X, 'STRESS', 8X, 'STRESS', 8X, 'STRESS')
YLOGB = -X**3.0, R*THETA, R*THETA**2.0, R*THETA, TOL/10.
C---- ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
AREA = WIDTH*THICK
BSIGX = TOL, SIGB**2.0, R*THETA**2.0, R*THETA, TOL/10.
C---- ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
AREA = WIDTH*THICK
BSIGX = TOL, SIGB**2.0, R*THETA**2.0, R*THETA, TOL/10.
C---- ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
AREA = WIDTH*THICK
BSIGX = TOL, SIGB**2.0, R*THETA**2.0, R*THETA, TOL/10.
WRITE(6,600) YGLOB, BSIGX
600 FORMAT(52X,F4.2,1X,E11.4)
YGLOB=YGLOB+THICK/10.
660 CONTINUE
299 J=J+1
510 CONTINUE
511 CONTINUE
C------CALCULATION OF DEFORMATION AND SLOPE FOR SEGMENT 3
C- AS A FUNCTION OF ARC LENGTH
C-
WRITE(6,27)
27 FORMAT('DEFLECTION AND SLOPE FOR SEGMENT 3'//)
WRITE(6,26)
26 FORMAT('10X,'ARC LENGTH',5X,'DEFLECTION',7X,'SLOPE',10X,'STRESS'
+12X,'MOMENT',10X,'SHEAR'//)
EXTERNAL FCN2
X=0.0
Y(1)=-Y(1)
Y(2)=-Y(2)
NW=2
N=2
IND=1
DO 515 M=1.24
C(M)=0.0
515 CONTINUE
J=0
DO 520 Z=0.0,R-THETA STEP
XEND=FLOAT(2)
CALL DVERKN.FCN2.X,Y,XEND.TOL,IND,C,NW,WIER)
IF(IND.LT.0.R.IER.GT.0) GO TO 525
UW=J+1
IF(INT(JW/10).NE.JW/10.0) GO TO 349
WRITE(I6.600) X, Y(1), Y(2), TSRC, SHEAR
60  FORMAT(10X ,3(E1: .4,3X) ,20X.E11.4)
YGL0B=YGL0B+THICK/10.
770 CONTINUE
C------STRESSES FROM THE MOMENT DISTRIBUTION SUPERIMPOSED
C- ON TENSILE STRESSES AS A FUNCTION OF BEAM THICKNESS
CSIGX=-TORCS'H/(R-H)*WIDTH*THICK*YBAR)+PLOAD*COS(THETA-X/R)/
-(WIDTH*THICK)
WRITE(6,700) YGLOB, CSIGX
700 FORMAT(52X,F4.2,1X,E11.4)
YGLOB=YGLOB+THICK/10.
770 CONTINUE
349 J=J+1
520 CONTINUE
525 CONTINUE
C------CALCULATION OF DEFORMATION AND SLOPE AS A FUNCTION
C- OF X FOR SEGMENT 4
C-
WRITE(6,28)
28 FORMAT('DEFLECTION AND SLOPE FOR SEGMENT 4'//)
WRITE(6,29)
29 FORMAT('10X,'X DISTANCE',5X,'DEFLECTION',7X,'SLOPE'//)
EXTERNAL FCN3
X=0.0
NW=2
N=2
IND=1
DO 530 M=1.24
530 CONTINUE
C(M)=0.0

CONTINUE

U = 0

DO 535 Z=0.0,CONTAB+.005,CONTAB/STEP
XEND=FLOAT(Z)
CALL DVERK(N,FCN3,X,Y,XEND,TOL,IND,C,NW,IER)
IF(IND.LT.0.OR.IER.GT.0) GO TO 540
U=W+10
IF(INT(JW/10).NE.JW/10.0) GO TO 399
WRITE(6,62) X, Y(1), Y(2)

CONTINUE

IF(JW.GE.15) GO TO 244
IF(INT(JW/10).NE.JW/10.0) GO TO 245
WRITE(6,65) X, Y(1), Y(2), PLOAD@Y(1)

DO 880 H=THICK/2..THICK/10.
WRITE(6,68) H, ESIGX
880 CONTINUE

CONTINUE

END

C-***SUBROUTINES**

C-*********************************************************

SUBROUTINE FCN1(N,X,Y,YPRIME)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EACH, PI
* BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
DIMENSION Y(2), YPRIME(2)
YPRIME(1) = Y(2)

C-- ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS
YEC = PLOAD *[cos(X/R) * sin(X/R)] / [WIDTH * THICK * ESMC]
TORC = PLOAD *[YEC - TRAC * YBAR + R * (1 - cos(X/R)) - Y(1) * cos(X/R)]
SHEAR = PLOAD *[PLOAD / WIDTH * THICK * ESMC] * (X/R - cos(X/R))**2
      + sin(X/R) * (X/R) * (sin(X/R) + cos(X/R)) * sin(X/R) + Y(1) / R
      + sin(X/R)]
SHEAR = PLOAD *[PLOAD / WIDTH * THICK * ESMC] * (X/R - cos(X/R))**2
      + sin(X/R) * (X/R) * (sin(X/R) + cos(X/R)) * sin(X/R) + Y(1) / R
      + sin(X/R)]
SHEAR = PLOAD *[PLOAD / WIDTH * THICK * ESMC] * (X/R - cos(X/R))**2
      + sin(X/R) * (X/R) * (sin(X/R) + cos(X/R)) * sin(X/R) + Y(1) / R
      + sin(X/R)]
YPRIME(2) = (R + Y(1)) * TORC / (R**2. + ESMC * WIDTH * THICK * YBAR)
RETURN
END

C-- NUMERICAL INTEGRATION OF SEG3
SUBROUTINE FCN2(N, X, Y, YPRIME)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
* BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
DIMENSION Y(2), YPRIME(2)
YPRIME(1) = Y(2)

C-- ECCENTRICITY DUE TO GEOMETRY
AGEO = YBAR + R * (1 - cos(THETA)) - YBAR * cos(THETA)

C-- ECCENTRICITY DUE TO GEOMETRY
BEGEO = R * (sin(X/R + PI/2. - THETA) - sin(PI/2. - THETA))

C-- ECCENTRICITY DUE ONLY TO EXTENSIONAL EFFECTS
EEXT = PLOAD *[sin(PI/2. - THETA + X/R) * sin(THETA - X/R) * X /
      (WIDTH * THICK * ESMC)]

C-- ECCENTRICITY DUE TO DEFLECTION
EDEFL = Y(1) * cos(THETA - X/R)
T0RCS = PLOAD * AGE0 + BEGEO + EDEFL + EEXT + TRAC
SHEAR = PLOAD *[cos(X/R + PI/2. - THETA) * Y(1) / R * sin(PI/2. - cos(X/R) + 
      THETA + X/R) / WIDTH * THICK * ESMC] * (X/R) / WIDTH * THICK * ESMC * cos
      (PI/2. - THETA + X/R))
YPRIME(2) = (R + Y(1)) * TORC / (R**2. + ESMC * WIDTH * THICK * YBAR)
RETURN
END

C-- NUMERICAL INTEGRATION OF SEG4
SUBROUTINE FCN3(N, X, Y, YPRIME)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
* BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
DIMENSION Y(2), YPRIME(2), EI(3)
EI(1) = ESMC * (1./12. * THICK**3 * WIDTH + THICK**3 + .5 * THICK + BOND
      - 2./2.)**2.
EI(2) = EADH**1.*12. * THICK**3 * WIDTH
EI(3) = ESMC * (1./12. * THICK**3 + WIDTH * THICK**3 + .5 * THICK + BOND
      - 2./2.)**2.
YPRIME(1) = Y(2)

C-- NUMERICAL INTEGRATION OF SEG4
SUBROUTINE FCN3(N, X, Y, YPRIME)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI
* BOND, J, TORC, TORCS, TRAC, TRACE, SHEAR
DIMENSION Y(2), YPRIME(2), EI(3)
EI(1) = ESMC * (1./12. * THICK**3 * WIDTH + THICK**3 + .5 * THICK + BOND
      - 2./2.)**2.
EI(2) = EADH**1.*12. * THICK**3 * WIDTH
EI(3) = ESMC * (1./12. * THICK**3 + WIDTH * THICK**3 + .5 * THICK + BOND
      - 2./2.)**2.
YPRIME(1) = Y(2)
DEN = EI(1) + EI(2) + EI(3)
YPRIME(2) = PLOAD * (.5 * THICK + BOND / 2 + Y(1)) / DEN
RETURN
END

C----- NUMERICAL INTEGRATION OF SEGS
C-
SUBROUTINE FCN4(IN.X.Y.YPRIME)
COMMON R, PLOAD, ESMC, WIDTH, THICK, YBAR, THETA, EADH, PI, R, BOND
DIMENSION Y(2), YPRIME(2)
YPRIME(1) = Y(2)
YPRIME(2) = PLOAD * (Y(1)) / (ESMC * WIDTH * THICK**3 / 12.)
RETURN
END
b. CONVERT

The program CONVERT essentially performs the tedious calculations involved in computing the boundary conditions for the finite-element model. Stresses dictated by the beam bending model are converted to equivalent point forces which are then applied to the finely meshed ends of the finite-element structure. In converting the stress distribution from deformed to undeformed geometry the program insures that the model be maintained in equilibrium through the introduction of a correcting moment.

The important parameters utilized in the routine are defined in the nomenclature section of the program. Frequent comment cards are intended to assist the user in the utilization of the program.
C. STRESS TRANSFORMATION PROGRAM

DEVELOPED BY: RICHARD C. GIVLER
UNIVERSITY OF DELAWARE
OCT 78 - JAN 79

C. CLEAR FREE

DIMENSION ZOM(5), RES(5), PART(4), ZOMX(5), RESB(5)
.* RESXB(5), RESYB(5)
C.-----PARAMETERS AND NOMENCLATURE
C.-----MATERIAL THICKNESS
THICK=.1
C.-----MATERIAL WIDTH
WIDTH=1.
C.-----ROTATION OF LEFT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)
DUOA=-0.01642
C.-----ROTATION OF RIGHT HAND FACE FROM UNDEFORMED GEOMETRY (RAD)
DUOB=.008420
C.-----TOTAL MOMENT ON LEFT HAND FACE (IN. LBS.)
TOTMA=10.64
C.-----SHEAR ON LEFT HAND FACE
SHEARA=132.6
C.-----TOTAL MOMENT ON RIGHT HAND FACE (IN. LBS.)
TOTMOB=-1.968
C.-----OUTSIDE RADIUS IN INCHES
RADO=3.5*THICK
C.-----INSIDE RADIUS IN INCHES
RAD=2.5*THICK
C.-----RADIUS OF CURVATURE OF CURVED MEMBERS
R=WIDTH*THICK/(ALOG(RADO/RAD))
C.-----DIFFERENCE BETWEEN NEUTRAL AXIS AND CENTROIDAL AXIS (IN.)
YBAR=RADI+THICK/2.-R
C.-----ADHESIVE BOND THICKNESS IN INCHES
BOND=.03
C.-----ANGLE SUBTENDED BY CURVED MEMBERS IN RADIANS
THETA=ARCCOS((5.*THICK+BOND)/(6.*THICK))
C.-----TENSILE LOAD
PLOAD=200.
C.
C------NOTE: DEFLECTIONS ARE MEASURED NORMAL TO THE NEUTRAL AXIS
C------DEFLECTION OF LEFT HAND FACE (IN.)
  DEFLA= .01547
C------DEFLECTION OF RIGHT HAND FACE (IN.)
  DEFLB= .007872
C------
C----------------------------------------------------------------------
C------RESOLVING STRESS DISTRIBUTION ON LEFT HAND FACE
C----------------------------------------------------------------------
C------
C------CALCULATION OF RESULTANT POINT FORCES FROM THE STRESS DISTRIBUTION
C----------------------------------------------------------------------
H=R-RADI
DO 100 N=1.5
  PART(1)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))
  +PLOAD/AREA*COS(THETA+DUDSA)*H
  H=H-.02
  PART(2)=TOTMOA/(YBAR*AREA)*(R-H-R*ALOG(R-H))
  +PLOAD/AREA*COS(THETA+DUDSA)*H
  RES(1)=PART(1)-PART(2)
100 CONTINUE
C------
C------CALCULATION OF ACTUAL MOMENTS FROM THE STRESS DISTRIBUTION
C----------------------------------------------------------------------
H=R-RADI
DO 200 N=1.5
  PART(3)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2
  -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2
  H=H-.02
  PART(4)=TOTMOA/(YBAR*AREA)*(-1.)*(.5*(R-H)**2-2.*R*(R-H)+R**2
  -*ALOG(R-H))+.5*PLOAD/AREA*COS(THETA+DUDSA)*H**2
  ZOM(N)=PART(3)-PART(4)
200 CONTINUE
C------
C------CALCULATION OF CORRECTION MOMENT DUE TO THE
C------REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT
C------FORCES
C----------------------------------------------------------------------
H=R-RADI-.01
DO 300 N=1.5
  ZOMX(N)=ZOM(N)-RES(N)*H
  H=H-.02
300 CONTINUE
C------
C------CALCULATION OF MOMENT DUE TO TRANSLATION OF THE STRESS
C------DISTRIBUTION THROUGH SPACE FROM THE DEFORMED GEOMETRY
C------TO THE UNDEFORMED GEOMETRY
C------
C------
C------CALCULATION OF MOMX FOR INPUT INTO THE FINITE ELEMENT
C------MODEL
C----------------------------------------------------------------------
H=R-RADI-.01
DO 400 N=1.5
  ZOMA(N)=RES(N)*((H+DEFLA)*COS(DUDSA)-H)
  ZOMX(N)=ZOMX(N)+ZOMA(N)
  H=H-.02
400 CONTINUE
CONTINUE

CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE ELEMENT MODEL

WRITE(6,25)
25 FORMAT('/','BOUNDARY CONDITIONS FOR LEFT HAND SEGMENT'/' )
WRITE(6,30)
30 FORMAT('/','NODE'.'FY','13X','FZ',12X,'MOMX'/' )
DO 500 N=1,5
RESX(N)=-RES(N)*COS(THETA+UDS过得)+SHEARA/5.*SIN(THETA)
RESY(N)=-RES(N)*SIN(THETA+UDS过得)+SHEARA/5.*COS(THETA)
WRITE(S,2)N,RESX(N),RESY(N),-ZOMX(N)
500 CONTINUE

RESOLVING STRESS DISTRIBUTION ON RIGHT HAND FACE

H=THICK/2.
DO 1000 N=1.5
PART(1)=.5*TOTMOB*H**2/(1./12.*WIDTH*THICK*3)+PLOAD/AREA*H
H=H-.02
PART(2)=.5*TOTMOB*H**2/(1./12.*WIDTH*THICK*3)+PLOAD/AREA*H
RESB(N)=PART(1)-PART(2)
1000 CONTINUE

CALCULATION OF RESULTANT POINT FORCES FROM THE STRESS DISTRIBUTION

H=THICK/2.
DO 1100 N=1.5
ZOMXB(N)=BMOM(N)+RESB(N)*H
H=H-.02
1100 CONTINUE

CALCULATION OF ACTUAL MOMENTS FROM THE STRESS DISTRIBUTION

H=THICK/2.
DO 1200 N=1.5
PART(3)=1./3.*((TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)
--.5*PLOAD/AREA*H**2
H=H-.02
PART(4)=1./3.*((TOTMOB)*H**3/(1./12.*WIDTH*THICK**3)
--.5*PLOAD/AREA*H**2
BMOM(N)=PART(3)-PART(4)
1200 CONTINUE

CALCULATION OF CORRECTION MOMENT DUE TO THE REPRESENTATION OF THE STRESS DISTRIBUTION BY POINT FORCES

H=THICK/2-.01
DO 1300 N=1.5
ZOMXB(N)=BMOM(N)+RESB(N)*H
H=H-.02
1300 CONTINUE

CALCULATION OF THE MOMENT DUE TO TRANSLATING THE STRESS DISTRIBUTION FROM DEFORMED TO UNDEFORMED GEOMETRY
C ---------CALCULATION OF MOMENT MX FOR INPUT INTO THE FINITE
C ---------ELEMENT MODEL
C---------
H=THICK/2.-.01
DO 1300 N=1,5
ZOMB(N)=RESB(N)*((H-DEFLB)*COS(DUDSB)-H)
ZOMXB(N)=ZOMXB(N)+ZOMB(N)
H=H-.02
1300 CONTINUE

C---------CALCULATION OF FY AND FZ FOR INPUT INTO THE FINITE
C---------ELEMENT MODEL
C---------
WRITE(6,40)
40 FORMAT(/,'BONDARY CONDITIONS FOR SEGB',/)
WRITE(6,45)
45 FORMAT(/,'NODE',6X,'FY',13X,'FZ',12X,'MOMX',/)
DO 1400 N=1,5
RESXB(N)=RESB(N)*COS(DUDSB)
RESYB(N)=RESB(N)*SIN(DUDSB)
WRITE(6,20) N, RESXB(N), RESYB(N), ZOMXB(N)
20 FORMAT(5X,F2.0,3X,E10.4,5X)
1400 CONTINUE
END
SUBROUTINE DVERK (N, FCN, X, Y, XEND, TOL, IND, C, NW, W, IER) DVEK0010

FUNCTION
DVERK S
DVEK0020

USAGE
DVEK0030
PARAMETERS
N
DVEK0040
FCN
DVEK0050

• LIBRARY
DVEK0060
DIFFERENTIAL EQUATIONS OF THE FORM
DVEK0070
DY/DT = F(X,Y) WITH INITIAL CONDITIONS.
DVEK0080
AND SIXTH ORDER PAIR OF FORMULAS IS USED.
DVEK0090

- CALL DVERK(N, FCN, X, Y, XEND, TOL, IND, C, NW, W, IER) DVEK0100
- NUMBER OF EQUATIONS. (INPUT)
DVEK0110
- NAME OF SUBROUTINE FOR EVALUATING FUNCTIONS.
DVEK0120

- THE SUBROUTINE ITSELF MUST ALSO BE PROVIDED
DVEK0130
BY THE USER AND IT SHOULD BE OF THE
DVEK0140
FOLLOWING FORM
DVEK0150
SUBROUTINE FCN(N, X, Y, YPRIME)
DVEK0160
DIMENSION Y(N), YPRIME(N)
DVEK0170

- FCN MUST APPEAR IN AN EXTERNAL STATEMENT IN
DVEK0180
THE CALLING PROGRAM AND N, X, Y(1),..., Y(N) DVEK0190
MUST NOT BE ALTERED BY FCN.
DVEK0200
- INDEPENDENT VARIABLE. (INPUT AND OUTPUT)
DVEK0210
ON INPUT, X SUPPLIES THE INITIAL VALUE.
DVEK0220
ON OUTPUT, X IS REPLACED WITH XEND UNLESS
DVEK0230
ERROR CONDITIONS ARISE. SEE THE DES-
DVEK0240
CRIPITION OF PARAMETER IND.
DVEK0250

- DEPENDENT VARIABLES. VECTOR OF LENGTH N.
DVEK0260
(INPUT AND OUTPUT)
DVEK0270
ON INPUT, Y(1),..., Y(N) SUPPLY INITIAL
DVEK0280
VALUES.
DVEK0290
ON OUTPUT, Y(1),..., Y(N) ARE REPLACED WITH
DVEK0300
AN APPROXIMATE SOLUTION AT XEND UNLESS
DVEK0310
ERROR CONDITIONS ARISE. SEE THE DES-
DVEK0320
CRIPITION OF PARAMETER IND.
DVEK0330

- VALUE OF X AT WHICH SOLUTION IS DESIRED.
DVEK0340
(INPUT)
DVEK0350
XEND MAY BE LESS THAN THE INITIAL VALUE OF
DVEK0360
X.
DVEK0370

- TOLERANCE FOR ERROR CONTROL. (INPUT)
DVEK0380
THE SUBROUTINE ATTEMPTS TO CONTROL A NORM
DVEK0390
OF THE LOCAL ERROR IN SUCH A WAY THAT THE
DVEK0400
GLOBAL ERROR IS PROPORTIONAL TO TOL.
DVEK0410
MAKING TOL SMALLER IMPROVES ACCURACY AND
DVEK0420
MORE THAN ONE RUN, WITH DIFFERENT VALUES
DVEK0430
OF TOL, CAN BE USED IN AN ATTEMPT TO
DVEK0440
ESTIMATE THE GLOBAL ERROR.
DVEK0450
IN THE DEFAULT CASE (IND=1), THE GLOBAL
DVEK0460
ERROR IS
DVEK0470
MAX(ABS(E(1)),...,ABS(E(N)))
DVEK0480
WHERE E(K) = Y(K) - YT(K)/MAX(1, ABS(Y(K))
DVEK0490
YT(K) IS THE TRUE SOLUTION, AND
DVEK0500
Y(K) IS THE COMPUTED SOLUTION AT XEND.
DVEK0510
FOR K=1,2,..., N.
DVEK0520
OTHER ERROR CONTROL OPTIONS ARE AVAILABLE. DVEK0530

- SUBROUTINE FOR EVALUATING FUNCTIONS.
DVEK0540
FCN SHOUL
SEE THE DESCRIPTION OF PARAMETERS IND AND D/EK0520 C/BELaw.

IND - INDICATOR, (INPUT AND OUTPUT)
ON INITIAL ENTRY IND MUST BE SET EQUAL TO EITHER 1 OR 2.
IND = 1 CAUSES ALL DEFAULT OPTIONS TO BE USED AND ELIMINATES THE NEED TO SET SPECIFIC VALUES IN THE COMMUNICATIONS VECTOR C.
IND = 2 ALLOWS OPTIONS TO BE SELECTED.

THE SUBROUTINE WILL NORMALLY RETURN WITH IND = 3, HAVING REPLACED THE INITIAL VALUES OF X AND Y WITH, RESPECTIVELY, THE VALUE XEND AND AN APPROXIMATION TO Y AT XEND.

THE SUBROUTINE CAN BE CALLED REPEATEDLY WITH NEW VALUES OF XEND WITHOUT CHANGING ANY OF THE OTHER PARAMETERS.

THREE ERROR RETURNS ARE ALSO POSSIBLE. IN WHICH CASE X AND Y WILL BE THE MOST RECENTLY ACCEPTED VALUES.
IND = -3 INDICATES THAT THE SUBROUTINE WAS UNABLE TO SATISFY THE ERROR REQUIREMENT. THIS MAY MEAN THAT TOL IS TOO SMALL.
IND = -2 INDICATES THAT THE VALUE OF HMIN (MINIMUM STEP-SIZE) IS GREATER THAN HMAX (MAXIMUM STEP-SIZE), WHICH PROBABLY MEANS THAT THE REQUESTED TOL (WHICH IS USED IN THE CALCULATION OF HMIN) IS TOO SMALL.
IND = -1 INDICATES THAT THE ALLOWED MAXIMUM NUMBER OF FCN EVALUATIONS HAS BEEN EXCEEDED. THIS CAN ONLY OCCUR IF OPTION C(7), AS DESCRIBED BELOW, HAS BEEN USED.

C - COMMUNICATIONS VECTOR OF LENGTH 24. (INPUT IF IND.NE.1. AND OUTPUT)
C IS USED TO SELECT OPTIONS AND TO RETAIN INFORMATION BETWEEN CALLS, THE USER NEED NOT BE CONCERNED WITH THE FOLLOWING DESCRIPTION OF THE ELEMENTS OF C WHEN DEFAULT OPTIONS ARE USED (IND=1).
HOWEVER, WHEN IT IS DESIRED TO USE IND=2 AND SELECT OPTIONS, A BASIC UNDERSTANDING OF DVERK IS REQUIRED. THE FOLLOWING PARAGRAPH DESCRIBES, BRIEFLY, THE BASIC TERMS. FOR MORE DETAILS, SEE THE REFERENCE.

DVERK ADVANCES THE INDEPENDENT VARIABLE X ONE STEP AT A TIME UNTIL XEND IS REACHED. THE SOLUTION IS COMPUTED AT XTRIAL = X+HTRIAL ALONG WITH AN ERROR ESTIMATE EST. IF EST IS LESS THAN OR EQUAL TO TOL (SUCCESSFUL STEP), THE STEP IS ACCEPTED AND X IS ADVANCED TO XTRIAL. IF EST IS GREATER THAN TOL (FAILURE), HTRIAL IS ADJUSTED AND THE SOLUTION IS RECOMPUTED. HMAG = ABS(HTRIAL) IS NEVER ALLOWED TO EXCEED HMAX NOR IS IT ALLOWED TO BECOME SMALLER THAN HMIN. THE FIRST TRIAL STEP IS MSTART. DURING THE COMPUTATION, A COUNTER (C(23)) IS
INCREMENTED EACH TIME A TRIAL STEP FAILS
TO PROVIDE A SOLUTION SATISFYING THE ERROR TOLERANCE. ANOTHER COUNTER (C(22)) IS
USED TO RECORD THE NUMBER OF SUCCESSFUL STEPS. AFTER A SUCCESSFUL STEP, C(23) IS
SET TO ZERO.

OPTIONS. IF THE SUBROUTINE IS ENTERED WITH
IND=2, THE FIRST 9 COMPONENTS OF THE
COMMUNICATIONS VECTOR MUST BE INITIALIZED
BY THE USER. NORMALLY THIS IS DONE BY
FIRST SETTING THEM ALL TO ZERO, AND THEN
THOSE CORRESPONDING TO PARTICULAR OPTIONS
ARE MADE NON-ZERO.

C(1) - ERROR CONTROL INDICATOR.
The subroutine attempts to control a norm
of the local error in such a way that the
global error is proportional to TOL. The
definition of global error for the
default case (IND=1) is given
in the description of parameter TOL. The default
weights are 1/MAXI ABS(Y(K)). When IND=2
is used, the weights are determined
according to the value of C(1).
IF C(1)=1, the weights are 1
(absolute error control).
IF C(1)=2, the weights are 1/ABS(Y(K))
for K=1,2,...,N.
(relative error control).
IF C(1)=3, the weights are
1/MAX(ABSIC(1),ABS(Y(K)))
for K=1,2,...,N.
(relative error control, unless
ABS/Y(K) is less than the floor
value, ABS(C(2))).
IF C(1)=4, the weights are
1/MAX(ABS(C(K+30)),ABS(Y(K)))
for K=1,2,...,N.
(Here individual floor values
are used). In this case, the dimension of C
must be greater than or equal to
N+30 and C(31), C(32),...,C(N+30)
must be initialized by the user.
IF C(1)=5, the weights are 1/ABSIC(K+30)
for K=1,2,...,N.
In this case, the dimension of C
must be greater than or equal to
N+30 and C(31), C(32),...,C(N+30)
must be initialized by the user.
For all other values of C(1), including
C(1)=0, the default values of
the weights are taken to be
1/MAX(1,ABS(Y(K)))
for K=1,2,...,N.

C(2) - Floor value, used when the indicator C(1)
has the value 3.

C(3) - HMIN specification. If not zero, the sub-
routine chooses HMIN to be ABS(C(3)).
Otherwise if uses the default value
10*MAX(DWARF,RE Reb*MAX(NORM(Y)/TOL,ABS(X)))
where DWARF is a very small positive machine
number and RE Reb is the relative roundoff.
ERROR BOUND.

C(4) - HSTART SPECIFICATION. IF NOT ZERO, THE SUBROUTINE WILL USE AN INITIAL HMAG EQUAL TO ABS(C(4)). EXCEPT OF COURSE FOR THE RESTRICTIONS IMPOSED BY HMIN AND HMAX, OTHERWISE IT USES THE DEFAULT VALUE HMAX*(TOL)^(1/6).

C(5) - SCALE SPECIFICATION. THIS IS INTENDED TO BE A MEASURE OF THE SCALE OF THE PROBLEM. LARGER VALUES OF SCALE TEND TO MAKE THE METHOD MORE RELIABLE. FIRST BY POSSIBLY RESTRICTING HMAX (AS DESCRIBED BELOW) AND SECOND, BY TIGHTENING THE ACCEPTANCE REQUIREMENT. IF C(5) IS ZERO, A DEFAULT VALUE OF 1 IS USED. FOR LINEAR HOMOGENEOUS PROBLEMS WITH CONSTANT COEFFICIENTS, AN APPROPRIATE VALUE FOR SCALE IS A NORM OF THE ASSOCIATED MATRIX. FOR OTHER PROBLEMS, AN APPROXIMATION TO AN AVERAGE VALUE OF A NORM OF THE JACOBIAN ALONG THE TRAJECTORY MAY BE APPROPRIATE.

C(6) - HMAX SPECIFICATION. FOUR CASES ARE POSSIBLE. IF C(6).NE.0 AND C(5).NE.0, HMAX IS TAKEN TO BE MIN(ABS(C(6)),2/ABS(C(5))). IF C(6).NE.0 AND C(5).EQ.0, HMAX IS TAKEN TO BE ABS(C(6)). IF C(6).EQ.0 AND C(5).NE.0, HMAX IS TAKEN TO BE 2/ABS(C(5)). IF C(6).EQ.0 AND C(5).EQ.0, A DEFAULT VALUE OF 2 IS GIVEN.

C(7) - MAXIMUM NUMBER OF FUNCTION EVALUATIONS. IF NOT ZERO, AN ERROR RETURN WITH IND = -1 WILL BE CAUSED WHEN THE NUMBER OF FUNCTION EVALUATIONS EXCEEDS ABS(C(7)).

C(8) - INTERRUPT NUMBER 1. IF NOT ZERO, THE SUBROUTINE WILL INTERRUPT THE CALCULATIONS AFTER IT HAS CHOSEN ITS PRELIMINARY VALUE OF HMAG, AND JUST BEFORE CHOOSING HTRIAL AND XTRIAL IN PREPARATION FOR TAKING A STEP (HTRIAL MAY DIFFER FROM HMAG IN SIGN, AND MAY REQUIRE ADJUSTMENT IF XEND IS NEAR). THE SUBROUTINE RETURNS WITH IND = 4, AND WILL RESUME CALCULATION AT THE POINT OF INTERRUPTION IF RE-ENTERED WITH IND = 4. IND MAY BE CHANGED BY THE USER IN ORDER TO FORCE ACCEPTANCE OF A STEP (BY CHANGING IND FROM 6 TO 5) THAT WOULD OTHERWISE BE REJECTED, OR VICE VERSA.

C(9) - INTERRUPT NUMBER 2. IF NOT ZERO, THE SUBROUTINE WILL INTERRUPT THE CALCULATIONS IMMEDIATELY AFTER IT HAS DECIDED WHETHER OR NOT TO ACCEPT THE RESULT OF THE MOST RECENT TRIAL STEP, WITH IND = 5 IF IT PLANS TO ACCEPT, OR IND = 6 IF IT PLANS TO REJECT. Y(*) IS THE PREVIOUSLY ACCEPTED RESULT. WHILE W(*,1) IS THE NEWLY COMPUTED TRIAL VALUE, AND W(*,2) IS THE UNWEIGHTED ERROR ESTIMATE VECTOR, THE SUBROUTINE WILL RESUME CALCULATION AT THE POINT OF INTERRUPTION ON RE-ENTRY WITH IND = 5 OR 6.

NW - THE FIRST DIMENSION OF W AS IT APPEARS IN THE CALLING PROGRAM. (INPUT)
-113-

C WORKSPACE MATRIX.
C THE FIRST DIMENSION OF W MUST BE NW AND THE
C SECOND MUST BE GREATER THAN OR EQUAL TO 9.
C IER - ERROR PARAMETER. (OUTPUT)
C TERMINAL ERRORS
C IER = 129. NW IS LESS THAN N OR TOL IS LESS
C THAN OR EQUAL TO ZERO.
C IER = 130. IND IS NOT IN THE RANGE 1 TO 6.
C IER = 131. XEND HAS NOT BEEN CHANGED FROM
C THE PREVIOUS XEND VALUE.
C IER = 132. THE RELATIVE ERROR CONTROL
C OPTION (C(1)=2) WAS SELECTED AND
C ONE OF THE SOLUTION COMPONENTS
C IS ZERO.
C PRECISION - SINGLE
C REQD. IMSL ROUTINES - UERTST
C LANGUAGE - FORTRAN
C LATEST REVISION - DECEMBER 1976

D/EK2480
D/EK2490
D/EK2500
D/EK2510
D/EK2530
D/EK2540
D/EK2550
D/EK2570
D/EK2580
D/EK2590
D/EK2600
D/EK2610
D/EK2620
D/EK2630
D/EK2640
D/EK2650
D/EK2660
D/EK2670
D/EK2680
D/EK2700
D/EK2710
D/EK2720
D/EK2730
D/EK2740
D/EK2750
D/EK2760
D/EK2770
D/EK2780
D/EK2790
D/EK2800
D/EK2810
D/EK2820
D/EK2830
D/EK2840
D/EK2850
D/EK2860
D/EK2870
D/EK2880
D/EK2890
D/EK2900
D/EK2910
D/EK2920
D/EK2930
D/EK2940
D/EK2950
D/EK2960
D/EK2970
D/EK2980
D/EK2990
D/EK3000
D/EK3010
D/EK3020
D/EK3030
D/EK3040
D/EK3050
D/EK3060
D/EK3070
D/EK3080
D/EK3090
BEGIN INITIALIZATION. PARAMETER CHECKING. INTERRUPT RE-ENTRIES

IER = 0
ABORT IF IND OUT OF RANGE 1 TO 6

IF (IND LT.1. OR. IND. GT.6) GO TO 295
CASES - INITIAL ENTRY, NORMAL
RE-ENTRY, INTERRUPT RE-ENTRIES

GO TO (5,5.40.145.265.265). IND
CASE 1 - INITIAL ENTRY (IND. EQ. 1)
CASE 2 - ABORT IF N.GT.NW OR TOL.LE.ZERO

5 IF (N.GT.NW. OR. TOL.LE.ZERO) GO TO 295
IF (IND.EQ.2) GO TO 15

INITIAL ENTRY WITHOUT OPTIONS (IND.
EQ. 1) SET C(1) TO C(9) EQUAL TO

DO 10 K=1,9
C(K) = ZERO
10 CONTINUE
GO TO 30

SUMMARY OF THE COMPONENTS OF THE COMMUNICATIONS VECTOR
PRESCRIBED AT THE OPTION OF THE USER

C(1) ERROR CONTROL INDICATOR
C(2) FLOOR VALUE
C(3) HMIN SPECIFICATION
C(4) HSTART SPECIFICATION
C(5) SCALE SPECIFICATION
C(6) HMAX SPECIFICATION
C(7) MAX NO OF FCN EVALS
C(8) INTERRUPT NO 1
C(9) INTERRUPT NO 2
C(10) RRREL ROUNDOFF ERROR BND
C(11) DWARF (VERY SMALL MACH NO)
C(12) WEIGHTED NORM Y
C(13) HMIN
C(14) HMAG
C(15) SCALE
C(16) HMAX
C(17) XTRIAL

DETERMINED BY THE PROGRAM
C(18) HTRIAL
C(19) EST
C(20) PREVIOUS XEND
C(21) FLAG FOR XEND
C(22) NO OF SUCCESSFUL STEPS
C(23) NO OF SUCCESSIVE FAILURES
C(24) NO OF FCN E vals
IF C(11) = 4 OR 5, C(31).C(32)....
C(N+30) ARE FLOOR VALUES
15 CONTINUE
INITIAL ENTRY WITH OPTIONS (IND .EQ. 0)
D/EK3720
D/EK3730
D/EK3740
D/EK3750
D/EK3760
D/EK3770
D/EK3780
D/EK3790
D/EK3800
D/EK3810
Appendix D

Material Property Data

It was necessary to perform a series of elastic modulus determination tests to characterize this adherent material. Slight variations in material properties can be evident in molding compounds even manufactured by the same supplier. In a separate, extensive study concerning material property data, Taggart reported the elastic modulus of SMC-25 to be $2.1 \times 10^6$ psi and the results shown in Table 4 are in close agreement.

Table 4

<table>
<thead>
<tr>
<th>Specimen #</th>
<th>Modulus (PSI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SPEC1</td>
<td>$2.21 \times 10^6$</td>
</tr>
<tr>
<td>SPEC2</td>
<td>$2.26 \times 10^6$</td>
</tr>
<tr>
<td>SPEC3</td>
<td>$2.18 \times 10^6$</td>
</tr>
</tbody>
</table>

Plate 6 shows a typical test specimen used for modulus determination. The data for these tests may be found on the following pages.
PLATE 2: Adhesive Zone of the "Joggle-Lap" Joint
PLATE 4: "Joggle-Lap" Joint Subject to Tension
PLATE 5: "Joggle-Lap" Joint Subject to Bending
PLATE 6: Tensile Coupons for Modulus Determination