Decision Problems Related to Structural Induction for Rings of Petri Nets with Fairness*

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Abstract Structural induction is a technique for proving that a system consisting of many identical components works correctly regardless of the actual number of components it has. Previously the authors have obtained conditions under which structural induction goes through for rings that are modeled as a Petri net satisfying a fairness requirement. The conditions guarantee that for some $k$, all rings of size $k$ or greater exhibit “similar” behavior. The key concept is the similarity between rings, where rings $R^k$ and $R^l$ of sizes $k$ and $l$, respectively, are said to be similar if, intuitively, (1) none of the components in either ring can tell whether it is in $R^k$ or $R^l$, and (2) none of the components (except possibly one) can tell its position within the ring to which it belongs. A ring satisfying this second property is said to be uniform. In this paper we prove the undecidability of various basic questions regarding similarity and uniformity. Some of the questions shown to be undecidable are: (1) Is there $k$ such that $R^k$ and $R^{k+1}$ are similar? (2) Is there $k$ such that all rings of size $k$ or greater are mutually similar? (3) Is there $k$ such that $R^k$ is uniform? (4) Is there $k$ such that $R^k, R^{k+1}, R^{k+2}, \ldots$ are all uniform?

1 Introduction

Given a system consisting of many identical finite state components that are connected in some regular topology, how can we determine whether it works correctly regardless of the actual number of components it has? Conventional theorem provers based on state-space search cannot be used directly to answer this question, since infinitely many instances of the system are involved. In fact, this problem is known to be unsolvable in general, even if the topology of the system is a unidirectional ring [1] [10]. The works reported in [2] [3] [5] [6] [7] [13] are some of the efforts to find a sufficient condition for such a system to be correct regardless of the number of components.

Inspired by [5] [13], the authors have considered the analysis problem stated above for rings of identical components given as a Petri net satisfying a fairness requirement, and obtained structural induction theorems that can be used to formally infer the correctness of a ring of any large size from the correctness of a ring having only a few components [6] [7]. Petri nets (see, for example, [9]) are widely used for modeling and analysis of concurrent processing systems. Fairness is an important property of concurrent systems often studied in the context of temporal logic [8]. The combination of Petri nets and temporal logic has been found to be extremely useful for formal analysis of such systems [4] [11] [12].

One of the key concepts in the authors’ induction theorems is the “similarity” between two rings. For $k \geq 2$, let $R^k$ be the ring consisting of $k$ identical copies $C_0, C_1, \ldots, C_{k-1}$ of a component $C$. We assume that all components except possibly $C_0$ have the same initial state. Intuitively, we say that rings $R^k$ and $R^l$ are similar if (1) none of the components in either ring can tell whether it is in $R^k$ or $R^l$, and (2) none of the components (except possibly $C_0$) can tell its position within the ring to which it belongs. (A ring satisfying this second property is said to be uniform.) The induction theorems reported in [6] [7] provide sufficient conditions for all rings of size $k$ or greater to be mutually similar for some $k$. Note that if $R^k$ is correct in some sense and all rings of size $k$ or greater are mutually similar, then we can conclude that $R^l$ is correct for all $l \geq k$. The theorems have been applied to the formal analysis of token-passing mutual exclusion, a simple producer-consumer system, and demand-driven token-circulation.

The goal of this paper is to prove the undecidability of the following basic questions regarding similarity and uniformity (as well as “weak similarity” defined later):

1. Is there $k$ such that $R^k$ and $R^{k+1}$ are similar?
2. Is there $k$ such that all rings of size $k$ or greater are mutually similar?
3. Is there $k \geq 3$ such that $R^k$ is uniform?
4. Are $R^3, R^4, \ldots$ all uniform?
5. Is there \( k \geq 3 \) such that \( R^k, R^{k+1}, R^{k+2}, \ldots \) are all uniform?

These negative results might seem somewhat expected in view of the results in [1] [10] and the undecidability results reported in [13] regarding the existence of “network invariants” that are needed for carrying out induction. However, similarity and uniformity of rings are stronger requirements than the existence of network invariants, and hence the proofs of the undecidability results reported here require certain unique arguments.

As is noted in [13] for the invariant method, despite these negative results we still expect that for many interesting, practical ring systems, the induction theorems of [6] [7] can be an effective tool for formal analysis.

The rest of the paper is organized as follows. Section 2 reviews the basic terminology of Petri nets. Section 3 introduces the basic concepts regarding components and rings. In Section 4 we introduce similarity and related concepts. The undecidability results are presented in Section 5. The concluding remarks are found in Section 6.

2 Petri Nets

We review the standard terminology of Petri nets [9].

A Petri net is a directed graph with two types of nodes, called transitions and places, and weighted arcs from a node of one type to a node of the other type. Formally, it is given as a triple \( N = (P, T, F) \), where \( P \) is a finite set of places, \( T \) is a finite set of transitions, and \( F : (P \times T) \cup (T \times P) \rightarrow \{0, 1, 2, \ldots\} \) is a weight function. A place \( p \in P \) is called an input place (or output place) of a transition \( t \in T \) if \( F(p, t) \geq 1 \) (or \( F(t, p) \geq 1 \)). Any function \( M : P \rightarrow \{0, 1, 2, \ldots\} \) is called a marking. A place \( p \) is said to have \( M(p) \) tokens at a marking \( M \). A transition \( t \in T \) is said to be fireable at \( M \) if \( M(p) \geq F(t, p) \) for every \( p \in P \). If \( t \) is fireable at \( M \), then it may fire and yield another marking \( M' \) such that \( M'(p) = M(p) - F(p, t) + F(t, p) \) for every \( p \in P \). We denote this by \( M \rightarrow_t M' \). This relation is extended by

1. \( M \rightarrow^\lambda M \) and
2. \( M \rightarrow^a M' \) iff there exists \( M'' \) such that \( M \rightarrow_a M'' \) and \( M'' \rightarrow_t M' \)

for all \( M, M', a \in T^+ \) and \( t \in T \). If \( M \rightarrow_a M' \) then \( M' \) is said to be reachable from \( M \) by a finite firing sequence \( a \). \( L(N, M) \) denotes the set of all finite firing sequences from \( M \). An infinite sequence \( \alpha \in T^\omega \) is an infinite firing sequence from \( M \) if \( \beta \in L(N, M) \) for every finite prefix \( \beta \).

We draw a Petri net using a circle and a square to represent places and transitions, respectively. An arc with weight \( F(p, t) \) (or \( F(t, p) \)) is drawn from \( p \) to \( t \) (or from \( t \) to \( p \)) if \( F(p, t) \geq 1 \) (or \( F(t, p) \geq 1 \)). The weight is omitted if it is 1. A marking \( M \) is represented by drawing \( M(p) \) dots in (the circle representing) \( p \).

When describing a system using a Petri net \( N = (P, T, F) \), we may designate a subset \( T' \subset T \) of transitions such that every transition \( t \in T' \) must be fired fairly, i.e., if \( t \) becomes fireable infinitely often, then it must fire infinitely often. Furthermore, to examine whether net \( N \) with initial marking \( M \) has certain liveness or eventuality properties (e.g., “if \( t_1 \) fires then eventually \( t_2 \) fires”), we may only be interested in those firing sequences \( \alpha \) that are either infinite, or finite and terminating in the sense that there is no transition \( t \) such that \( \alpha \in L(N, M) \) with \( \alpha \) terminating.

1. \( \alpha \) is infinite and satisfies the fairness requirement on the transitions in \( T' \), or
2. \( \alpha \) is finite and terminating.

3 Components and Rings

To save space, we introduce the necessary concepts informally, using examples. The reader is referred to [6] [7] for a formal discussion.

A component is a Petri net \( C = (P, T, F) \) in which the set \( T \) of transitions can be partitioned into three groups, the left interface transitions, the internal transitions and the right interface transitions, where the number of left interface transitions must equal the number of right interface transitions. See Figure 1(a) for an example of a component having two left interface transitions \( u_1 \) and \( u_2 \), three internal transitions \( v_1, v_2 \) and \( v_3 \), and two right interface transitions \( w_1 \) and \( w_2 \). We connect two or more copies of \( C \) to form a ring, as shown in Figure 1(b), by merging the respective interface transitions of adjacent copies of \( C \). For any \( k \geq 2 \), the ring consisting of \( k \) copies of \( C \) is denoted \( R^k \), and the copies of \( C \) in \( R^k \) are referred to as \( C_0, C_1, \ldots, C_{k-1} \), where \( C_{k+1} \) is the right neighbor of \( C_i \). (Subscripts are taken modulo \( k \) when we discuss \( R^k \).) Formally, the transitions and places of \( C \) must be renamed in each \( C_i \) (e.g., \( v_2 \) of \( C_1 \) might be renamed “\( v_{1,2} \)" in the example given above) so that they all have distinct names in \( R^k \). However, for convenience, in Figure 1(b)
we use their original names in each copy of $C$, and thus assign two names to every interface transition.

We assume that for all $k \geq 2$, the initial marking $M^k$ of $R^k$ is such that for some fixed markings $M$ and $M'$ of $C$, $C_0$ has $M$ and all other $C_i$'s, $i \geq 1$, have $M'$. (It is often necessary to break symmetry by giving $C_0$ an initial marking different from that of the other components.) Furthermore, we assume that for all $k \geq 2$, fairness (when required) should be imposed on an identical set of transitions in all components of $R^k$.

In the following, sets $L^\infty(R^k, M^k)$ and $L(R^k, M^k, T^k)$ are simply written as $L^\infty(k)$ and $L(k)$, respectively, where $T^k$ the set of transitions of $R^k$ (consisting of identical selections from all components) on which fairness is required.

Let $\alpha \in L^\infty(k)$ be a firing sequence of $R^k$. Let $C_i$ be a component in $R^k$. We define the local history of $C_i$ in $\alpha$, denoted $\langle(C_i)\rangle_\alpha$, to be the sequence obtained from $\alpha$ by deleting all transitions except those that belong to $C_i$. Here, a transition $t$ belongs to $C_i$ iff $t$ is either a left interface transition, a right interface transition, or an internal transition of $C_i$. (So, an interface transition between $C_i$ and $C_j$ is either a left interface transition, a right interface transition, or an internal transition of $C_i$.) Sequence $\langle(C_i)\rangle_\alpha$ is thus the portion of $\alpha$ that occurs in $C_i$.

As we mentioned above, formally in $R^k$ the transitions (and places) are renamed so that all transitions have distinct names. Therefore, to be able to compare local histories of different components (that may even belong to different rings $R^k$ and $R^\delta$), when describing $\langle(C_i)\rangle_\alpha$ we use the original transition names given in $C$, rather than the names in $R^k$ after the renaming (e.g., use "v2" instead of "v1,2" when describing $\langle(C_1)\rangle_\alpha$).

Example 1 Consider ring $R^3$ shown in Figure 1(b). Suppose that in the initial marking, only $C_0$ has a token, in place $p_1$, as shown in the figure. Consider the firing sequence $\alpha$ that moves the token through $C_1$ and $C_2$ and back to $C_0$, as indicated by the dashed arrow in Figure 1(b). Then the local history of $C_1$ in $\alpha$ is $\langle(C_1)\rangle_\alpha = u_1 v_1 w_1 u_2 v_2 w_2$. Similarly, the reader can verify $\langle(C_0)\rangle_\alpha = v_1 w_1 v_2$ and $\langle(C_2)\rangle_\alpha = u_1 v_1 u_2 v_2$. □

Let $M$ be a marking of $R^k$. The firability of an interface transition $t$ of $C_i$ at $M$ is determined by the token counts of the input places of $t$, and such places may belong to $C_{i-1}$, $C_i$ or $C_{i+1}$. We define the firability vector of $C_i$ at $M$ to be a column vector of token counts of all input places (in $C_{i-1}$, $C_i$ and $C_{i+1}$ of all interface transitions of $C_i$. (The order in which the token counts of these places appear must be the same for all components.)

Let $\alpha$ be a firing sequence of $R^k$. Note that during the execution of $\alpha$, a firing of a transition in $C_{i-1}$, $C_i$ and $C_{i+1}$ may change the firability vector of $C_i$. To describe the changes in the firability of its interface transitions. Of course, if $\langle(C_i)\rangle_\alpha = \langle(C_i)\rangle_\beta$ for firing sequences $\alpha$ and $\beta$, then $C_i$ cannot distinguish $\alpha$ and $\beta$. On the other hand, the information on the behavior of the "environment" of $C_i$ is not included in local histories $\langle(C_i)\rangle_\alpha$ and $\langle(C_i)\rangle_\beta$.}

![Figure 1: (a) Component C. (b) Ring $R^3$ consisting of three copies of C.](image-url)
4 Similarity and Uniformity

We now review the main concepts introduced in [6] [7] for stating the induction theorems. Recall that \( L(k) \) and \( L(\ell) \) are the set of firing sequences of ring \( R^k \) and \( R^\ell \), respectively, that use to determine their correctness.

**Definition 1** [6] Two rings \( R^k \) and \( R^\ell \) are similar, denoted \( R^k \sim R^\ell \), if

1. \( \{(C_0)_\alpha | \alpha \in L(k)\} = \{(C_0)_\alpha | \alpha \in L(\ell)\} \) and
2. \( \{(C_1)_\alpha | \alpha \in L(k)\} = \{(C_1)_\alpha | \alpha \in L(\ell)\} \) for any \( 1 \leq i \leq k-1 \) and \( 1 \leq j \leq \ell-1 \).

**Definition 2** [7] Two rings \( R^k \) and \( R^\ell \) are weakly similar, denoted \( R^k \sim^w R^\ell \), if

1. \( \{(C_0)_\alpha | \alpha \in L(k)\} = \{(C_0)_\alpha | \alpha \in L(\ell)\} \) and
2. \( \{(C_1)_\alpha | \alpha \in L(k)\} = \{(C_1)_\alpha | \alpha \in L(\ell)\} \) for any \( 1 \leq i \leq k-1 \) and \( 1 \leq j \leq \ell-1 \).

Intuitively, if \( R^k \sim R^\ell \), then none of the copies of \( C \) knows which of \( R^k \) and \( R^\ell \) it is in, and none of the copies of \( C \) other than \( C_0 \) knows which copy of \( C \) it is. \( C_0 \) might behave differently from others, since its initial marking may not be the same as that of others.) Weak similarity is identical to similarity, except that the firability vectors of components are not considered. By definition, \( R^k \sim R^\ell \) implies \( R^k \sim^w R^\ell \).

As is explained in [6][7], one way to prove the correctness of \( R^a \) for all values of \( n \geq k \) for some \( k \) is to show that

1. \( R^k \sim R^{k+1} \sim R^{k+2} \sim \ldots \),
2. \( S \subseteq \{(C_0)_\alpha | \alpha \in L(k)\} \subseteq S' \), where \( S \) and \( S' \) are sets of firing sequences of \( C \) describing the correctness requirements for \( C_0 \), and
3. \( S'' \subseteq \{(C_1)_\alpha | \alpha \in L(k)\} \subseteq S''' \) for any one \( j \), \( 1 \leq j \leq k-1 \), where \( S'' \) and \( S''' \) are sets of firing sequences of \( C \) describing the correctness requirements for all \( C_i \), \( i \geq 1 \).

Then \( C_0 \)'s in all \( R^k, R^{k+1}, \ldots \) have identical properties and \( C_0 \) of \( R^a \) is correct. Thus \( C_0 \) is correct in all \( R^k, R^{k+1}, \ldots \). Similarly, all \( C_i \)'s, \( i \geq 1 \), in all \( R^k, R^{k+1}, \ldots \) have identical properties and that particular \( C_j \) of \( R^k \) (mentioned above) is correct. So all these \( C_i \)'s in all \( R^k, R^{k+1}, \ldots \) are also correct. The main result in [6] is a sufficient condition for \( R^2 \sim R^3 \sim R^4 \sim \ldots \) (hence \( R^2 \sim R^3 \sim R^4 \sim \ldots \) to hold, and [7] presents a sufficient condition for \( R^k \sim R^{k+1} \sim R^{k+2} \sim \ldots \) to hold for the given \( k \geq 5 \). (We do not review these sufficient conditions since doing so requires additional definitions. The interested reader is referred to [6][7].) The concept of "uniformity" introduced next is used to state the condition in [7].

**Definition 3** [7] \( R^k, k \geq 3, \) is uniform if \( \{(C_i)_\alpha | \alpha \in L(k)\} = \{(C_j)_\alpha | \alpha \in L(k)\} \) for any \( 1 \leq i, j \leq k-1 \). \( i \neq j \).

Note that if \( R^k \) is uniform, then none of the components \( C_1, C_2, \ldots, C_{k-1} \) can determine its position in the ring.

**Example 3** We can show that the "environment" looks identical to all \( C_i, i \geq 1, \) in any \( R^k, k \geq 3, \) constructed from \( C \) of Figure 1(a). On the other hand, \( R^2 \) and \( R^3 \) are slightly different. In \( R^2, u_2 \) of \( C_1 \) becomes fireable when \( C_1 \) fires \( u_2 \), since its left and right neighbors are both \( C_0 \). This cannot happen in \( R^3, \) since the left and right neighbors of \( C_1 \) are different. If we ignore the firability vectors, of course, \( R^2 \) and \( R^3 \) become indistinguishable to \( C_1 \). Using an argument along this line, we can prove \( R^2 \not \sim R^3 \sim R^4 \sim \ldots \), \( R^2 \not \sim R^3 \sim R^4 \not \sim \ldots \), and that \( R^3 \) is uniform for all \( k \geq 3 \).

In [6][7], the argument outlined above has been used to prove the correctness of rings for token-passing mutual exclusion, a simple producer-consumers system, and demand-driven token-circulation.

5 Undecidability Results

As we discussed in Section 4, similarity, weak similarity and uniformity can be a basis for proving that a ring system is correct regardless of its size. In this section we prove that the basic questions regarding these concepts posed in Section 1 are undecidable in general.

**Theorem 1** Given \( C, M \) and \( M' \), the following problems are undecidable in general, even if fairness is not required on any transition.

1. Is there \( k \) such that \( R^k \sim R^{k+1} \)?
2. Is there \( k \) such that \( R^k \sim^w R^{k+1} \)?
3. Is there \( k \) such that \( R^k \sim R^{k+1} \sim R^{k+2} \sim \ldots \)?
4. Is there \( k \) such that \( R^k \sim R^{k+1} \sim R^{k+2} \sim \ldots \)?

**Proof** We first prove the second and fourth claims that do not involve the changes in the firability vectors of the components. The basic idea is to simulate the given Turing machine \( A \) (that does not halt in two steps) on the semi-infinite blank tape for \( (up \ to) n \) steps using a uniform ring \( R^n \) of size \( n \), in such a way that (1) if \( A \) halts
in $n$ steps, then $R^n \sim R^{n+1} \sim R^{n+2} \ldots$, and (2) if $A$ never halts, then $R^k \not\sim R^\ell$ for all $k \neq \ell$. In the sense that we use $R^n$ to simulate a Turing machine or a two-counter automaton for $n$ steps, the proof is similar to those found in [10] and [14]. But here the proof is technically more involved, since weak similarity is a fairly strong condition.

Given a Turing machine $A$, we construct a component $C$ and markings $M$ and $M'$ of $C$ such that $R^n$ simulates the computation of $A$ on the semi-infinite blank tape for (up to) $n$ steps. We will first describe the components as a finite state machine, and then later explain how we can represent them by a Petri net. Number the tape cells 0, 1, . . ., starting with the leftmost one. For each $0 \leq i \leq n - 1$, $C_i$ maintains cell $i$. Initially, the cells are all blank, and $C_0$ has the tape head (i.e., the tape head is reading cell 0). The component that has the tape head also remembers in its finite control the current state of $A$. To simulate a single step of $A$, the component, say $C_i$, that currently has the tape head (1) rewrites the symbol in cell $i$, (2) determines the next state $q$ of $A$, and then (3) sends $q$ to either $C_{i-1}$ or $C_{i+1}$ depending on whether the tape head is moved left or right. The neighbor of $C_i$ that receives $q$ knows that it now has the tape head and simulates the next step of $A$.

To ensure that the simulation stops in $n$ steps and we do not run out of space ($R^n$ has exactly $n$ tape cells), we use a flag COUNT (initially false) in each component and two special symbols called NEWSTEP and EXECUTE in the following manner. Intuitively, we set one COUNT in the ring to true before each step of simulation. Specifically, suppose $C_i$ wishes to simulate a step of $A$. If its own COUNT is still false, then it sets COUNT to true and sends EXECUTE to the right and waits for EXECUTE to arrive from the left. If its own COUNT is true, then it sends NEWSTEP to the right and waits for EXECUTE to arrive from the left. In either case, $C_i$ simulates a step of $A$ as described above when EXECUTE arrives. A component that does not have the tape head always passes EXECUTE to the right. A component that receives NEWSTEP for the first time (in this case COUNT is still false) changes its COUNT to true and sends EXECUTE (instead of NEWSTEP) to the right. A component that receives NEWSTEP when its COUNT is true passes NEWSTEP to the right.

The simulation ends when either (1) component $C_i$ that has the tape head simulates one step of $A$ and finds that the next state is a halting state, or (2) $C_0$ receives either NEWSTEP or a state of $A$ from the left. In the first case, $A$ has halted within $n$ steps, and $C_i$ circulates a special symbol HALT once around the ring and halts. In the second case, $A$ did not halt within $n$ steps, and $C_0$ circulates a special symbol SUSPEND once around the ring and halts. In either case, the entire ring comes to a halt after the circulation of HALT or SUSPEND.

Since each component has only a finite number of states, constructing $C$ as a Petri net is straightforward. Transfers of various symbols (NEWSTEP, EXECUTE, HALT, SUSPEND, and a state of $A$) can be represented by different interface transitions. The fact that initially $C_0$ has the tape head can be reflected in its initial marking $M$ different from $M'$ of other components. Fairness is not imposed on any transition.

Now, we modify $C$ slightly to make $R^n$ uniform. That is, before the simulation starts, we give any component in $R^n$ a chance to "act" as $C_0$ in the simulation. We achieve this by letting (the real) $C_0$ nondeterministically do one of the following: (1) start the simulation, (2) send a token to the left through a special interface transition, and (3) send a token to the right through another special interface transition. A component that receives a token through one of the special interface transitions can, nondeterministically, either start the simulation acting as $C_0$ (as described above) or simply pass the token to the neighbor on the opposite side. This modification guarantees that $R^n$ is uniform. A side effect is that the simulation of $A$ may never start, but, this only adds two (identical) sequences (one for each direction in which the token is passed forever) to the sets of local histories of all $C_i$'s, $i \geq 1$, and two sequences to the set of local histories of $C_0$. So whether or not two components have identical sets of local histories is not affected.

Suppose that $A$ never halts. Then for any $n$, (the component acting as) $C_0$ of $R^n$ receives EXECUTE exactly $n$ times. Thus for any distinct $k$ and $\ell$, the two conditions of Definition 2 do not hold, and hence $R^k \not\sim R^\ell$.

Suppose that $A$ halts within $n \geq 3$ steps. Then for any $k \geq n$, in $R^k$, (the component acting as) $C_0$ and its $n - 1$ right neighbors (call them $C_1, \ldots, C_{n-1}$ for convenience) perform the actual simulation (the simulation itself does not depend on the value of $k$) and each of $C_n, \ldots, C_{n-1}$ simply pass EXECUTE $n$ times. (In addition, all components pass HALT once.) This, together with the fact that all rings are uniform, implies that for any $k, \ell \geq n$, the two conditions of Definition 2 are satisfied, and hence $R^k \sim R^\ell$. This completes the proof of the second and fourth claims.

As for the first and third claims of the lemma, it is easy to see that component $C$ described above can be constructed in such a way that each interface transition $t$ has exactly one input place, say $p$, and once $p$ receives a token it loses the token only when $t$ fires. Then given any firing sequence $a$ of $R^2$, the changes in the firability vectors during the execution of $a$ can be determined completely. So the conditions of Definition 1 are satisfied iff those of Definition 2 are satisfied. Therefore the claims we made on the relation between the behavior of $A$ and weak similarity among rings hold also for similarity among rings. This completes the proof of the first and third claims. \(\square\)
Theorem 2 Given C, M and M', the following problems are undecidable in general, even if fairness is not required on any transition.

1. Is there \( k \geq 3 \) such that \( R^k \) is uniform?
2. Are \( R^3, R^4, \ldots \) all uniform?
3. Is there \( k \geq 3 \) such that \( R^k, R^{k+1}, R^{k+2}, \ldots \) are all uniform?

Proof We modify the construction given in the proof of Theorem 1. Note that the real \( C_0 \) can transfer its role as the initiator of the simulation to other components, but even after the transfer, it can still "remember" that it is the real \( C_0 \). So, when the real \( C_0 \) receives SUSPEND, (in addition to passing it as before) we let it fire a special right interface transition. Then the local history of the real \( C_1 \) (the right neighbor of \( C_0 \)) becomes different from those of any other component, since that interface transition never fires at other components. So if \( A \) never halts, then the special transition is always fired, and thus \( R^n \) is not uniform for any \( n \). On the other hand, if \( A \) halts within \( n \geq 3 \) steps, then \( R^k \) is uniform for all \( k \geq n \). This completes the proof of the first claim. To prove the second and third claims, all we need to do is to change the behavior of the real \( C_0 \) so that it fires that special right interface transition when it receives HALT, instead of SUSPEND. Then if \( A \) halts in \( n \geq 3 \) steps, then \( R^k \) is not uniform for any \( k \geq n \). If on the other hand \( A \) never halts, then that special transition is never fired, and thus \( R^n \) is uniform for all \( n \). This completes the proof of the second and third claims. \( \Box \)

6 Concluding Remarks

Since any finite alphabet can be encoded as binary strings, the negative results presented above remains true even if a component has only four interface transitions on each side, two for sending symbols and two for receiving symbols. It would be interesting to investigate the decidability of analogous problems for the case when the components have exactly one interface transition on each side, and for some restricted classes of Petri nets.

References