The workshop "Stochastic Systems and Their Applications" was held in Newport, RI on April 15 & 16, 1994. The main topics of the conference were asymptotic methods in stochastic systems theory, and related applications. The goal of the conference was to review recent advances in asymptotic methods and expose some important new application areas where those methods might be useful.
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STOCHASTIC SYSTEMS AND THEIR APPLICATIONS

Newport Marriott Hotel
25 America's Cup Ave.
Newport, RI 02840
April 15 & 16, 1994

Conference sponsored by Air Force Office of Scientific Research

Grant Number: F49620-93-1-0279
The workshop “Stochastic Systems and Their Applications” was held in Newport RI on April 15 & 16 of 1994. The main topics of the conference were asymptotic methods in stochastic systems theory, and related applications. The goal of the conference was to review recent advances in asymptotic methods and expose some important new application areas where these methods might be useful.

Each day was divided into two parts. In the morning talks of a more theoretical nature were given, while the afternoon was devoted to applications. The main application areas discussed were from communications and manufacturing.

On the more theoretical side there were talks by T. G. Kurtz and A. D. Wentzell on approximations via stochastic averaging and asymptotic expansions of functionals of Markov processes, respectively. M. Zakai spoke on error bounds in nonlinear filtering, while R. Khasminskii reported new work on methods of stabilizing solutions to stochastic differential equations.

The talks by I. Karatzas, M. Reiman and L. Ljung were of a more applied nature. Karatzas described recent work on dynamic allocation when the problem had been formulated as a bandit problem. Reiman discussed various heavy traffic limits for Markov-modulated queues, which are the sort that appear in current models of digital communication networks, while Ljung gave a general talk on recent work on the tracking ability of adaptive algorithms.

The conference was rounded out by two talks on applications that are currently of great interest. D. Mitra discussed asymptotic methods in the analysis, design, and control of high-speed communication networks. P. R. Kumar considered approximation methods for queueing systems. The highlight of his talk was a new method for controlling re-entrant line queues, and an application of the methods to semi-conductor manufacturing.
# SCHEDULE FOR CONFERENCE

**FRIDAY**

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SATURDAY

Morning session:

10:00  Some bounds on the filtering error in nonlinear filtering  Moshe Zakai

11:15  Some robust problems in stability of SDE  Rafail Khasminskii

Afternoon session:

2:30  Tracking ability and accelerated convergence in adaptive algorithms  Lennart Ljung

3:45  Dynamic allocation and multi-armed bandit problems  Ioannis Karatzas

7:30  Reception/party

The Chart House
22 Bowens Wharf
Newport, RI
Averaging stochastically perturbed Hamiltonian systems

A classical problem in the study of mechanical systems is concerned with the behavior of such systems in the presence of nearly rigid constraints. A system of this type can be modeled as follows. Let \( U, W \in C^2(\mathbb{R}^d) \). Assume that \( W \geq 0 \) and that \( M = \{ x \in \mathbb{R}^d : W(x) = 0 \} \) is an \( m \)-dimensional manifold with \( m < d \). The model is then given by the Hamiltonian system

\[
\dot{X}_n = -\nabla U(X_n) - n \nabla W(X_n)
\]

where the parameter \( n \) is assumed to be large and represents the strength of the nearly rigid constraint. The main problem is to characterize the limit of \( X_n \) as \( n \to \infty \). We consider the more general model

\[
d\dot{X}_n = d\eta_n - n \nabla W(X_n)
\]

where \( \{ \eta_n \} \) is a sequence of semimartingales. For example, \( \eta_n \) might be given by

\[
d\eta_n = \sigma(X_n) dW - \nabla U(X_n) dt
\]

where \( W \) is standard Brownian motion. The classical limit results are extended to the stochastic setting and ways of exploiting the stochasticity to avoid some of the nonuniqueness problems of the deterministic setting are explored.

Asymptotic expansions in limit theorems for stochastic processes

For some families of stochastic processes \( \eta(t), 0 \leq t \leq T \), one can prove limit theorems on weak convergence:

\[
EF(\eta[0,T]) = EF(\eta^0[0,T]) + o(1)
\]
as $\epsilon \downarrow 0$ for all bounded continuous functionals $F$. One can try to find asymptotic expansions for $o(1)$ for some narrower classes of functionals:

$$EF(\eta'[0,T]) = EF(\eta^0[0,T]) + k(\epsilon)C_1 + \ldots + k(\epsilon)^mC_m + o(k(\epsilon)^m),$$

where $k(\epsilon) \to 0$ ($\epsilon \downarrow 0$). Some theorems of this kind have been obtained, and in them the coefficients $C_i$ are expressed in the form

$$C_i = A_iEF(\eta^0[0,T]),$$

where $A_i$ are linear operators acting on functionals that are expressed in terms of characteristics of the stochastic processes $\eta'$; in some natural classes of cases, infinite-dimensional differential operators.

Possible applications to Monte-Carlo computation are described.

Debasis Mitra  
AT&T Bell Laboratories  
MH-2C-125  
Murray Hill, NJ 07974

Some recent directions in the analysis of high-speed communication networks

The talk will review recent results on (i) stochastic fluid models for burst-scale effects in ATM, (ii) loss and service priority systems, (iii) admission control and, time permitting, (iv) feedback-based adaptive allocation of bandwidth.

Martin I. Reiman  
AT&T Bell Laboratories, RM 2C-117  
600 Mountain Ave.  
Murray, NJ 07974

Some observations on scaling for Markov modulated queues in heavy traffic
Queues with Markov modulated input, where the arrival rate depends on the state of a "background" Markov process, are of practical interest in communications systems, where the state of the background Markov process is the number of active sources. The most common input process models in this context (given the state of the background process) are a deterministic "fluid" or a Poisson process. Although much has been done in the area of obtaining efficient numerical solutions for these models, there is still an interest in asymptotics. One particular asymptotic regime that has received a lot of attention is heavy traffic, where the average arrival rate is close to the service rate. Since there are many parameters in this model, there are many ways to achieve heavy traffic. For example, the number of "sources" can remain fixed or grow to infinity. The achievable limit processes include: reflected Brownian motion, (finite state) Markov modulated Brownian motion, Ornstein-Uhlenbeck modulated fluid, and Ornstein-Uhlenbeck modulated reflected Brownian motion. We describe each of these limits and the parameter scalings which give rise to them.

P. R. Kumar  
University of Illinois  
Coordinated Science Laboratory  
1308 West Main Street  
Urbana, IL 61801

Scheduling queueing networks: Stability, performance analysis and design

Queueing networks are a useful class of models in many application domains, e.g., manufacturing systems, communication networks, and computer systems. Control is typically exercised over such systems by the use of scheduling policies.

However, if one ventures outside a certain special class of systems for which the steady state distribution has a product form, very little is known concerning their performance or even stability. In the first half of talk, we present new theoretical developments on stability analysis and performance evaluation for queueing networks and scheduling policies. In the second half,
we address the problem of scheduling a class of queueing networks called re-entrant lines which model semiconductor manufacturing plants. We propose a new class of efficient scheduling policies based on smoothing all the flows in the system.

Moshe Zakai  
Electrical Engineering Department  
Technion – I. I. T.  
Haifa 32000, Israel

Some bounds on the filtering error in nonlinear filtering

The first part of the talk will be devoted to a general introduction to Wiener, Kalman-Bucy and nonlinear filtering. The main results for the optimal filter and the associated minimal mean squared error will be discussed. The second part will be devoted to the presentation of a bounding technique for the lower bounding of the filtering error and a particular case will be evaluated explicitly.

Rafail Khasminskii  
Department of Mathematics  
Wayne State University  
Detroit, MI 48202

Some robustness problems in stability of stochastic differential equations.

The problems of robustness of asymptotic stability, instability, Lyapunov exponents and stability index for linear and homogeneous of degree one stochastic differential equations with respect to small nonlinear perturbations of coefficients are considered. Sufficient conditions for the robustness of stability index are proposed in particular.
Tracking ability and accelerated convergence in adaptive algorithms

Basic stochastic approximation has been a great source of inspiration for all kinds of adaptative and tracking algorithms used in many technical applications. To understand and develop the algorithms it is necessary to have good insight into their performance properties. In this presentation we will overview approaches to performance analysis, in particular the ability of different algorithms to track changing phenomena. This includes some quite recent results with explicit expressions and bounds for the performance measures.

The presentation will also cover some aspects of the recent development in accelerated convergence (with a “second round of averaging”). The focus is here on whether the ability to track changing properties is also improved in a similar way.

Dynamic allocation and multi-armed bandit problems

Recent results are presented on the optimality of Gittins index-type strategies for very general dynamic allocation problems of the “multi-armed bandit” form, in both discrete and continuous time. In particular, it is shown that no Markovian assumptions are necessary, and that the independence conditions on the evolution of the different projects or “arms” can be relaxed considerably. (This is joint work with Professor Nicole El Karoui.)