NUMERICAL INVESTIGATION OF ONE-DIMENSIONAL HEAT-FLUX CALCULATIONS

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FOREWORD

This report addresses one aspect of the one-dimensional surface heat-flux calculations performed on wind tunnel model surface thermocouple data at the Navy's Hypervelocity Wind Tunnel No. 9 (Tunnel 9). These calculations have been used on Tunnel 9 data since the early 1980s when coaxial surface thermocouples began to replace gardon gauges as the standard means of obtaining heat-flux data. A one-dimensional heat flow assumption allows practical computation of heat-flux from a single thermocouple output. However, this assumption will break down under two- and three-dimensional heat conduction. Quantitative information on the limits of the one-dimensional assumption is needed by project engineers planning wind tunnel tests. This report addresses issues associated with the one-dimensional assumption and should be consulted prior to making heat transfer measurements in Tunnel 9. Analysis on other aspects of the heat transfer testing in Tunnel 9 is ongoing.

The authors acknowledge Michael Metzger and Leonard Zentz of the Weapons Dynamics Division for their helpful instructions during the execution of the ABAQUS finite element code.

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INTRODUCTION

Heat transfer measurements are a major component of the testing conducted in the Navy's Hypervelocity Wind Tunnel No. 9 (Tunnel 9) located at the White Oak, MD site of the Dahlgren Division, Naval Surface Warfare Center. Measurements made in Tunnel 9 have employed the use of coaxial surface thermocouples since the early 1980s. These transducers are commonly used for obtaining transient surface heat-flux measurements on wind tunnel models. Details on the use of the coaxial thermocouples are found in References 1 and 2. Advantages of the coaxial surface thermocouple technique, copied in part from Reference 2, include the following:

a) durability
b) small size (0.031" or 0.061" typical diameters available)
c) easy installation, flush mounting to virtually any model contour
d) fast response time (<50 μsec)
e) no power supply required
f) no calibration required

The main drawback of the coaxial thermocouple technique is that heat-flux, the desired quantity, is calculated from the time history of the coaxial thermocouples output temperature. A model must be constructed in order to calculate the heat-flux. In the majority of Tunnel 9 tests, good heat-flux results are obtained from coaxial thermocouples with relatively simple models. However, a single simple model will not work under all conditions. Leading edges, nose tips, and other complex geometries as well as spatially varying heating rates can require different solution techniques. Understanding the solution technique and its assumptions and limitations is very important.

The two common techniques used to calculate heat-flux from thermocouple outputs are the semi-infinite slab assumption (see Reference 1), and a finite thickness one-dimensional (1D) finite difference approximation. The finite difference approximation assumes the model wall thickness is finite and allows the user to define a back-face boundary condition. Since the finite difference technique is more adaptive to wind tunnel model configurations, it is the standard technique used for heat-flux measurements made in Tunnel 9. QCALC is the name of the subroutine which contains the finite difference scheme used for Tunnel 9 data.
QCALC SUBROUTINE

QCALC is a FORTRAN subroutine which solves the transient 1D heat equation in Cartesian coordinates (Equation 1). Equation 2 shows the second order Euler-explicit finite difference approximation used in QCALC. Subscripts i and j refer to the time and space steps respectively. Temperature versus time data from the model surface are input into QCALC which uses Equation 2 to solve for the temperature distribution at each node through the model wall thickness for each time step. Heat-flux is obtained from a second order approximation to the derivative of the temperature profile evaluated at the model surface.

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]  

(1)

\[ \frac{T_{i1,j} - T_{i,j}}{\Delta t} = \alpha \frac{(T_{i,j+1} - 2T_{i,j} + T_{i,j-1})}{(\Delta x)^2} \]  

(2)

The solution begins at time zero (i=1) with all nodes set to a uniform initial temperature. QCALC solves for the temperature at successive time steps by solving for $T_{i1,j}$ from Equation 2. The outer surface boundary condition is the measured surface temperature. The back-face temperature is calculated by assuming an insulated condition. As an option, the back-face temperature can be measured. Heat-flux is computed at the completion of each time step. Appendix A contains the QCALC subroutine FORTRAN code. Major assumptions and techniques used in the QCALC model are summarized below.

a) one-dimensional heat conduction equation in Cartesian coordinates
b) second order Euler-explicit finite difference approximation
c) constant and homogeneous material properties
d) uniform initial temperature
e) front-face boundary condition measured by coaxial thermocouple
f) back-face boundary assumed insulated (optional temperature measurement)
g) heat-flux is obtained from a second order approximation to the temperature gradient at the model surface
h) negligible radiation effects

Figure 1A illustrates a wind tunnel model with a conical geometry and a spatially varying heat-flux. The local approximation made by QCALC is illustrated in Figure 1B. Model surface curvature and spatial variations in heat-flux are assumed negligible for the small measurement region. This assumption will begin to break down when the local spatial variation of the heating rate becomes substantial and/or the geometry of the model is not locally flat. Evaluating the limiting factors of these two assumptions is the objective of this report.
Node spacing and the time step ($\Delta t$) must be chosen based on the stability criteria for this technique. The error in the second order technique used in QCALC is proportional to the node spacing squared ($\Delta x^2$) and can be improved by decreasing the value of $\Delta x$. Using too few nodes increases $\Delta x$ and will lead to larger errors. For this report the authors used a 3/8 inch model wall with 50 nodes and a time step of 1/500 second. The thermal properties of constantan are used for this analysis and can be found in the QCALC code in Appendix A.

**INVESTIGATION APPROACH**

The approach for this investigation involved testing the assumption of 1D conduction in a Cartesian coordinate system when applied under two- and three-dimensional conditions. Two possible conditions are considered; what happens when the surface is not locally flat, and what happens when the heating rate varies spatially to the extent that it cannot be considered locally uniform. The authors considered various geometries subjected to spatially uniform heating loads to test the locally flat condition. Spatially varying heating loads applied to flat plates were analyzed briefly in support of Tunnel 9 testing and those results are included. All analysis assumed homogeneous constant property materials.

**MODEL GEOMETRIES**

Typical models tested in Tunnel 9 include many varieties of cones as well as hypersonic airframes. Most of the surfaces are not flat. Three geometries were identified for evaluation which locally represent many wind tunnel model surfaces better than the flat plate while remaining simple enough for the present study. The geometries selected for evaluation were several sizes of cones, cylinders, and spheres.

**Conical Section**

The conical section is a small element of a seven degree half-angle cone. Directly applicable to cone models, this geometry could also represent many areas on the fuselage of hypersonic aircraft. Figure 2A illustrates a conical element. Radius of curvature and temperature values are referenced to the center of the element.

---

* For stability, $\Delta t \leq (\Delta x)^2/(2 \alpha)$ where $\alpha$ is the thermal diffusivity.
Cylindrical Section

The cylindrical section is simply a portion of a circular cylinder. This model is representative of many local sections of an airframe fuselage as well as rounded leading edge geometries. Figure 2B illustrates a cylindrical element.

Spherical Section

A spherical section is illustrated in Figure 2C. This geometry is typical of blunted nose tips but could represent other areas of wind tunnel models as well.

PROCEDURE

The analysis tested QCALC’s ability to compute heat-flux applied to the geometries illustrated in Figure 2. Temperature rise data input to QCALC were obtained from finite element solutions* with known heat-flux inputs. Heat-flux calculated by QCALC was compared to the heat-flux used for inputs to the finite element solutions. This numerical approach was chosen to isolate the effects of the 1D heat transfer assumptions in the computation of heat-flux on complex models. No experimental measurements were attempted. The analysis steps are summarized below.

Generation of Temperature Data

Temperature versus time data were generated using the ABAQUS finite element code. For each case, one of the model geometries described above was subjected to a known heat-flux which varied with time. The inner wall (back face) was insulated (heat-flux = 0) to coincide with the assumption used in the QCALC code. Several finite element solutions of various element sizes and time steps were obtained for each case to ensure convergence of the solution. Once a converged solution was generated, the surface temperature at each time step was put into a computer file for subsequent use by the QCALC code. The applied heat-flux at each time step was also stored in a computer file for later comparison with the QCALC results.

Computation of Heat-flux

The temperature histories generated by the finite element code were used as input to the QCALC code. QCALC, as described above, calculated the surface heat-flux for each time step. The insulated back wall condition was used in QCALC to coincide with the finite element

* ABAQUS finite element code Version 4.9, Hibbit, Karlson & Sorenson, Inc.
solutions' back-face boundary condition. Heat-flux obtained from QCALC was stored in a computer file for comparison to the heat-flux used to generate the temperature histories above.

QCALC RESULTS

SPATIALLY UNIFORM HEATING ON VARIOUS GEOMETRIES

Each of the geometries illustrated in Figure 2 was used for analyzing the QCALC code. The flat plate geometry was used as a reference condition. The heating loads applied were time dependent and spatially uniform over the modeled geometry.

Flat Plate

The technique was applied to a flat plate geometry to coincide with the assumptions in the QCALC code. Results are shown in Figures 3 and 4 for a step and a half sine wave input of heat-flux. QCALC accurately predicts the heat-flux in the case of the flat plate model.

Figure 3 illustrates the normalized output from QCALC when the temperature distribution arises from a flat plate model subjected to a step input of heat-flux. Results were independent of the magnitude of the step input. A finite response time can be seen as QCALC responds to the step input. Only a small deviation in the calculated heat-flux can be seen during the first 50 milliseconds of Figure 3. Within 10 milliseconds from the initiation of the step, the QCALC output value is within 3 percent of the expected value and after 25 milliseconds the output is within 1 percent. These deviations are considered small in comparison to typical heat transfer uncertainties which are greater than 6 percent (Reference 1). It should be noted that a step increase of this type is not representative of Tunnel 9 testing. In addition, the node spacing and sampling period could be decreased to improve these results (Reference 5).

Figure 4 illustrates QCALC's ability to follow a smoothly varying heat-flux. This 1/2-cycle-per-second sine wave variation in heat-flux with a peak at 25 BTU/ft²/sec is typical of the time scale and magnitude of the heat-flux obtained during many Tunnel 9 tests. In this case, QCALC predicted the applied heat-flux to within 0.12 BTU/ft²/sec.

The preceding two examples of QCALC results from flat plate models illustrate the ability of the QCALC code to accurately predict heat-flux when the geometry is locally flat.
Cylindrical Section

Cylindrical geometries (Figure 2B) of radius 1/2, 1, 2, 4, and 8 inches were subjected to spatially uniform heating loads. A step and a half sine input of heat-flux were applied in the same manner as the flat plate analysis.

Figure 5 illustrates the normalized output from QCALC diverging from the flat plate result when a step input of heat-flux is applied to each of the investigated model geometries. The flat plate results (radius = ∞) are included from Figure 3 as a reference. The over-prediction of heat-flux increases with time and decreasing radius of curvature.

These results can be explained by comparing the cylindrical geometry to the flat plate geometry assumed by QCALC. For the flat plate, heat flows perpendicular to the surface with constant temperature lines parallel to the surface. The cross section through which the heat flows remains unchanged. The cylindrical geometry tends to funnel the heat flow into an ever decreasing cross sectional area as the heat flows inward from the surface. This decreasing cross sectional area makes this geometry more resistive to conducting heat away from the surface and hence a larger surface temperature rise is observed (when compared to the flat plate). The larger surface temperature rise will give rise to a larger calculated heat-flux from QCALC. For increased heating times, the heat penetrates deeper into the model and the problem becomes more pronounced.

Figure 6 shows results for a half sine input of heat-flux applied to several cylindrical elements of various geometries. Results from Figure 4 (radius = ∞) are included as a reference. It is noted that the errors grow with time and with decreasing radius of curvature. These trends are similar to those noted from Figure 5 above.

Conical Section

Seven degree half-angle cone elements (Figure 2A) of central radii 1/2, 1, 2, 4, and 8 inches were subjected to spatially uniform heating loads. Figure 5 shows the normalized output from QCALC diverging from the expected result. The seven degree cone models yielded results with negligible differences from the results generated from the cylindrical models of the same radius.

Spherical Section

Spherical geometries (Figure 2C) of radii 1, 2, 4, 8, and 16 inches were subjected to spatially uniform heating loads. A step input of heat-flux was applied in the same manner as the cone and cylinder analyses. Figure 5 shows the normalized output from QCALC for the spherical sections subjected to a step input of heat-flux. The results from a spherical section of a given
radius are equivalent to results from a cylindrical section of half the radius. For a given radius, heat-flux results for the spherical sections diverge faster from the expected results than the heat-flux data from the cone/cylinder sections. The faster divergence can be explained by comparing the spherical geometry to the cylindrical geometry. As heat moves inward, away from the surface, the cross section through which the heat travels decreases at a higher rate for the spherical elements when compared to cylindrical elements of the same radius. This will create a higher surface temperature rise for the spherical element. A higher surface temperature rise will cause a higher predicted heat-flux from QCALC.

SPATIALLY VARYING HEAT-FLUX ON A FLAT PLATE

The QCALC code was tested under the conditions of a spatially varying heat-flux (Reference 6). Two-dimensional (2D) solutions were generated for a heat-flux which varied linearly in space along the face of a flat plate element. The QCALC code was used to compute the heating load at one point on the surface. Deviations of the 1D QCALC solution compared to the 2D solution are given in Figure 7 which was obtained from Reference 6. The error is shown to increase with time and with the magnitude of the spatial gradient of heat-flux. These results can be used to show the reliability of the 1D heat-flux calculation when used for spatially varying heating loads. As shown in Figure 7, extreme spatial variations in the heat-flux will lead to errors in the calculated heat-flux. Typical values of \( (dQ/ds)/Q^* \) for Tunnel 9 testing are much less than 1.

QCALC MODIFICATIONS

The QCALC code works well when used on geometries which locally can be considered flat and when the spatial gradient of the heat-flux is not too large. It has been shown in the previous section that the QCALC code will diverge from the expected results when these conditions are not met. Modified codes have been developed to solve the 1D heat equation in cylindrical and spherical coordinates under spatially uniform heating loads. These codes are useful when model geometry can locally be represented by cylindrical/conical or spherical elements.

\[ Q \] is the local heat-flux value and \( s \) is the direction along the model surface.
CYLINDRICAL COORDINATES (QCYL)

For cases where model geometry is best described as a cone or cylinder, the QCALC code was modified to cylindrical coordinates. The governing 1D Cartesian heat equation (Equation 1) was replaced with its 1D representation in cylindrical coordinates (Equation 3). A second order Euler-explicit finite difference scheme was used to approximate Equation 3. This new code is referred to as QCYL and is available for use in support of Tunnel 9 testing. Modifications made to the QCALC code to create the QCYL code are contained in Appendix B.

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(3)

The QCYL program is based on cylindrical model surfaces which locally represent many of the models tested in Tunnel 9. A comparison of QCALC results with QCYL results is shown in Figure 8 for a cylindrical shell model with 1" radius subjected to a step input of heat-flux. Results for several radii were tested with similar results. The QCYL program takes into account the cylindrical geometry of the element and does not over-predict the heat-flux. As mentioned earlier, the conical and cylindrical geometries yield results which are essentially the same. The QCYL program is therefore applicable to cases where model geometry can be described as a cone* or cylinder.

The response of the QCYL program to a step input of heat-flux on a cylindrical model is slightly slower than the response of QCALC to a step input of heat-flux on a flat plate. Figure 9 shows the results of QCALC and QCYL for a step input of heat-flux on a flat plate and 1 inch radius cylindrical shell respectively. Both calculations were executed with the same time step and node spacing. The QCALC code responds slightly faster to the step input when compared to the cylindrical QCYL program. It is assumed by the authors that the extra term in Equation 3 \((1/r \frac{\partial T}{\partial r})\) adds some numerical dissipation to the QCYL solution. This factor should be considered when using the QCYL program to measure rapid changes in heating loads. The response shown in Figure 9 is considered adequate for most Tunnel 9 testing since typical model surface heat-flux values vary much slower than the evaluated step input. Increasing the number of nodes used as well as the sampling frequency would allow the QCYL program to respond faster.

SPHERICAL COORDINATES (QSPH)

Next the QCALC code was modified to allow the input of radius of curvature for the spherical element case. The governing 1D Cartesian heat equation (Equation 1) was replaced with its representation in spherical coordinates (Equation 4). A second order Euler-explicit finite

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* Data from a seven degree half-angle cone. Larger angles would require further study.
difference scheme was used to approximate Equation 4. This new code is referred to as QSPH and is available for use in support of Tunnel 9 testing. Modifications made to the QCALC code in the development of the QSPH code are contained in Appendix C.

\[
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{1}{\alpha} \frac{\partial T}{\partial t}
\]  

(4)

It was noted from Figure 5 that a sphere radius twice as large as a given cylindrical radius will have the same effect on the 1D QCALC results. Equation 4 shows the origins of this relationship. The second term in Equation 4 is simply twice the second term from the cylindrical coordinates equation (Equation 3). This second term is twice as important in the equation for spherical geometries.

Figure 10 illustrates the ability of the QSPH code to predict the heat-flux applied to a spherical element with 1 inch radius. The data in Figure 10 are not as smooth looking as the data presented earlier. This result is due to the numerical format of the input temperature data for this individual case and not the result of the QSPH program. Original input data were deleted and only a copy of the input temperature data, truncated to three decimal positions, was available for analysis. This truncation leads to the slight variations in the heat-flux since it is obtained through taking derivatives.

CONCLUSIONS

The validity of the assumptions in the QCALC code should be considered when analyzing any heat-flux data from Tunnel 9. In particular, the 1D conduction (Cartesian) assumption begins to break down when the model surface curvature becomes too small and/or the heating load spatial gradient becomes too large. Longer run times lead to greater deviations in both of these cases. The QCALC program should be used for most of the measurements made in Tunnel 9. However, the QCYL (or QSPH) program will do a better job of computing heat-flux on model configurations when the local cylindrical (or spherical) radius of curvature is small. Spherical effects become important at radii twice as large as the radii where cylindrical effects become important. It is also noted that more nodes would be required in the QSPH and QCYL programs to obtain the response time characteristics of the QCALC code.
RECOMMENDATIONS

When considering results in this report, the method of analysis must be considered. One idealized model is being compared to a second idealized model. No physical measurements were made.

The finite element solutions modeled homogeneous material sections. Most of the surface temperature measurements made in Tunnel 9 involve the use of coaxial thermocouples. For this type of gauge, the measurement section is not homogeneous. A typical coaxial gauge consists of a 0.012-inch diameter constantan wire with a 0.0005-inch thick insulative coating protecting it from the 0.0607-inch diameter chromel outer jacket. This gauge is cemented into a 0.0625 inch hole in stainless steel. The effects of having three different metals, the cement, and possible contact resistances between the metals were not considered in this report. Further study of these factors along with experimental measurements are recommended to further understand the heat transfer results obtained from coaxial thermocouple gauges.
spatially variable heating load $q(s,t)$

(A) TYPICAL WIND TUNNEL MODEL WITH VARIABLE HEATING

Conditions for 1D heat equation solution

Initial Condition:
$T(x,t) = \text{initial temperature (constant and uniform)}$

Boundary Conditions:

- $X = 0$ $\quad T = T_{\text{measured, surface thermocouple}}$
- $X = d$ $\quad \frac{dT}{dx} = 0$, assumed insulated

or
- $T = T_{\text{initial, assumed constant}}$

or
- $T = T_{\text{measured, backface thermocouple}}$

(B) IDEALIZED LOCAL GEOMETRY AND BOUNDARY CONDITIONS

FIGURE 1. ILLUSTRATION OF LOCAL APPROXIMATION MADE BY QCALC
FIGURE 2. GEOMETRIC SECTIONS USED FOR ANALYSIS
FIGURE 3. NORMALIZED QCALC OUTPUT FOR STEP INPUT APPLIED TO FLAT PLATE

FIGURE 4. QCALC COMPARISON FOR SINUSOIDAL HEAT-FLUX APPLIED TO FLAT PLATE
FIGURE 5. NORMALIZED QCALC OUTPUT FOR STEP INPUT APPLIED TO MODEL GEOMETRIES
FIGURE 6. QCALC RESULTS FOR SINUSOIDAL HEAT-FLUX APPLIED TO CYLINDRICAL MODELS
FIGURE 7. HEATING RATE UNCERTAINTY UNDER SPATIALLY VARYING HEAT-FLUX
FIGURE 8. COMPARISON OF NORMALIZED QCALC AND QCYL RESULTS
FIGURE 9. COMPARISON OF QCALC AND QCYL RESPONSE TIMES TO STEP INPUT
FIGURE 10. COMPARISON OF NORMALIZED QCALC AND QSPH RESULTS
REFERENCES


APPENDIX A

QCALC FORTRAN CODE

SUBROUTINE QCALC(TIME, T1, T2, QOUT, NTHI, NODES, THICK)
    CCCC   call qcalc(tim,co(1,k),bface,qout,nptg,nodes,thickft)
C     Now the standard version... 2/88
C

PURPOSE

THIS SUBROUTINE USES THE FINITE (FORWARD) DIFFERENCE METHOD TO
DETERMINE SURFACE HEAT TRANSFER FROM THE CALCULATED TEMPERATURE PROFILE.
IT STARTS AT SECOND POINT IN THE T ARRAYS AND CALCULATES THE HEAT
FLUX, QDOT, FOR NTHI PTS.

EXPLANATION

STABILITY CONSTRAINTS OF THE FINITE DIFFERENCE METHOD MUST BE SATISFIED
(THET <= .5, ie. THE TIME INTERVAL MUST BE SMALL ENOUGH),
IF THE RECORDED TIME INTERVAL IS TOO LARGE, THEN EACH
INTERVAL IS DIVIDED INTO SUBINTERVALS.

FORWARD FINITE DIFFERENCE DIAGRAM

   t: time
   T: temperature
   x: distance

   t(j-1) ! T1  T2  T3
   t(j)  !

   -------------------------
   x(i-1)  x(i)  x(i+1)

   temperatures T1,T2,T3 are used to calculate the
   temperature T4

SUBROUTINES CALLED

                         (NONE)

VARIABLE GLOSSARY

ADJUST = AMOUNT TO SUBTRACT FROM THE BACKSIDE THERMOCOUPLE
         TEMPERATURES IN 'T2'
COND = CONDUCTIVITY (BTU/FT-SEC-DEGF)
CONDODX = CONDUCTIVITY/DX
PARAMETER( MAXNODE=50 )
DIMENSION TIME(1),T1(1),T2(1),QOUT(1),T(MAXNODE,2)

C-----PRELIMINARY CALCULATIONS C

IF(NODES.EQ.0) NODES = 10
IF(NODES.LE.MAXNODE) GO TO 10
PRINT *, ' SUBROUTINE QCALC... NUMBER OF NODES GREATER THAN DIMEN
1SION; NUMBER OF NODES SET EQUAL TO', MAXNODE
NODES = MAXNODE
10 NODEM1 = NODES - 1
HIDT = TIME(2) - TIME(1)
DX = THICK / NODEM1

C assume properties of steel model, and chromel-constantan TCP's are close
C enough to use constantan properties
C
C These are the CONSTANTAN properties used for data reduction
CP=.094 ! (BTU/Lb-DEGF)
RHO=.322*172'1 ! (Lb/in^3) * 1728 in^3/ft^3
COND=(0.2676E-03)*12 ! (BTU/in sec F) * 12 in/ft

CONODX = COND / DX
THETOT = COND / (CP * RHO * DX**2)

C----- THET MUST BE LESS THAN 0.50 TO MAKE SOLUTION PROCEDURE STABLE C

THET = 0.5
DTLO = THET / THETOT
NTLO = HIDT / DTLO
IF (NTLO .LT. 0) THEN

A-2
NTLO = IABS(NTLO)
ENDIF
NTLO = NTLO + 1
DTLO = HDT / NTLO
THET = THETOT * DTLO
B = 1.0 - (2.0 * THET)
C if semi-infinite slab is assumed, then in main program set T2 array of
backface temps equal to first point of T1 array of frontface temps
ADJUST = T2(I) - T1(I)  ! backface(I) - co(I,k)
DO 52 I = 1,NTHI  ! I=1,nptq
   T2(I) = T2(I) - ADJUST  ! backface temps
52 CONTINUE
C DO 102 I = 1,NODES
   T(I,1) = T1(I)  ! co(I,k) thru all nodes
102 CONTINUE
C L1 = 1
L2 = 2
C-----HI TIME LOOP (loop through time intervals)
C DO 302 IHI = 2,NTHI ! Ihi=1,nptq
C-------LO TIME LOOP (loop through time subtintervals)
C DO 252 ITLO = I,NTLO
C--------CALCULATE TEMPERATURE FOR CURRENT TIME AT ALL NODES
DO 202 INODE = 2,NODEM1
   T(INODE,L2) = THET * (T(INODE+1,L1) + T(INODE-1,L1))
   + B * T(INODE,L1)
202 CONTINUE
TFAC = FLOAT(ITLO) / FLOAT(NTLO)
T(1,L2) = T1(IHI-1) + (T1(IHI) - T1(IHI-1)) * TFAC
C Longer times sc can't assume semi-infinite slab equation
C Qdot = 0 at backface rather than a constant
T(NODES,L2) = T(NODES-1,L2) - (T(NODES-2,L2) - T(NODES-1,L2))/3.  ! SD5
CCC CCC CCC CCC CCC
T(NODES,L2) = T2(IHI-1) + (T2(IHI) - T2(IHI-1)) * TFAC  ! semi-inf
C LHOLD = L1
L1 = L2
L2 = LHOLD
252 CONTINUE
CCC CCC
C--------NEW WALL TEMPERATURES NOW AT L1 (ie. at the input recorded time,
C--------TIME(IHI))
C CCC CCC CCC QOUT(IHI) = CONODX * (T(1,L1) - T(2,L1))
CCC CCC CCC QOUT(IHI) = CONODX * (- 2. * T(1,L1) + 3. * T(2,L1)
C + - T(3,L1) )
C + -T(3,L1))
302 CONTINUE
RETURN
END
APPENDIX B

QCYL FORTRAN CODE MODIFICATIONS

SUBROUTINE QCYL(TIME, T1, T2, QOUT, NTHI, NODES, THICK, R)

R = Radius of curvature of surface (Ft)
Gage is normally mounted on outside of model and local radii will decrease from 1 to nodes.
If gage is mounted on inside of model, then flag code by inputting Radius as a negative value. Local radii will increase from 1 to nodes.

DIMENSION TIME(l), T1(l), T2(l), QOUT(1), T(MAXNODE,2),
+ RC(MAXNODE)

C---- Calculate the local radial value for each node
C If R is input as negative, then curvature is outward

DO I = 1, NODES
  DELR = (I-1) * DX
  RC(I) = R - DELR ! radius of curvature vector (ie 1,.9,.8,...,
ENDDO
IF(R .LT. 0.) THEN ! outward curvature , convex
  DO I = 1, NODES
    RC(I) = - RC(I) ! (ie 1.0, 1.1, 1.2,...)
  ENDDO
ENDDO

C--------- Calculate temperature for current time at all nodes
TFAC = FLOAT(ITLO) / FLOAT(NTHI)
T(1,L2) = T1(IHI-1) + (T1(IHI) - T1(IHI-1)) * TFAC
DO INODE = 2, NODES
  TPART = THET * (T(INODE+1,L1) + T(INODE-1,L1)) +
          B * T(INODE,L1)
  T(INODE,L2) = TPART - (T(INODE+1,L1) - T(INODE-1,L1)) *
                THET * DX / RC(INODE) / 2.0 ! cyl coords 1D
ENDDO
APPENDIX C

QSPH FORTRAN CODE MODIFICATIONS

SUBROUTINE QSPH(TIME, T1, T2, QOUT, NTHI, NODES, THICK, R)

C R = Radius of curvature of surface (Ft)
C Gage is normally mounted on outside of model and local
C radii will decrease from 1 to nodes.
C If gage is mounted on inside of model, then flag code
C by inputting Radius as a negative value. Local radii
C will increase from 1 to nodes.

DIMENSION TIME(1), T1(1), T2(1), QOUT(1), T(MAXNODE,2),
+ RC(MAXNODE)

C---- Calculate the local radial value for each node
C If R is input as negative, then curvature is outward

DO I = 1,NODES
  DELR = (I-1) * DX
  RC(I) = R - DELR
  ! radius of curvature vector (ie 1.9, 0.8, ...)
ENDDO
IF(R .LT. 0.) THEN
  ! outward curvature, convex
  DO I = 1,NODES
    RC(I) = - RC(I)
  ! (ie 1.0, 1.1, 1.2, ...)
ENDDO
ENDIF

C------- Calculate temperature for current time at all nodes
TFAC = FLOAT(ITLO) / FLOAT(NTLO)
T(1,L2) = T1(IHI-1) + (T1(IHI) - T1(IHI-1)) * TFAC
DO INODE = 2,NODEM1
  TPART = THET * (T(INODE+1,L1) + T(INODE-1,L1)) +
  + B * T(INODE,L1)
  T(INODE,L2) = TPART - (T(INODE+1,L1) - T(INODE-1,L1)) *
  + THET * DX / RC(INODE) ! sph coords 1D
ENDDO

C-1
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Numerical Investigation of One-Dimensional Heat-Flux Calculations

C. F. Boyd and A. Howell

Naval Surface Warfare Center (Code K24)
10901 New Hampshire Avenue
Silver Spring, MD 20903-5640

Two new computer codes, which assume 1D conduction in cylindrical and spherical coordinates, accurately predict 1D heat-flux applied to cylindrical, spherical, and conical geometries. These two codes are online for use as a tool in evaluating heat transfer from Tunnel 9 data.
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