At millimeter frequencies, it has been found that substrate anisotropy plays an important role in determining the characteristics of ICs. Regardless whether anisotropy occurs naturally or is purposely implanted, it must be considered. Until now, the analysis of unilateral fin-line resonators has been devoted using isotropic substrate as the supporting medium. This paper presents an approach based on the spectral domain technique which can be used to study fin-line resonators on biaxial substrates.

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ANALYSIS OF UNILATERAL FIN-LINE RESONATORS ON BIAXIAL SUBSTRATES
VIA SPECTRAL DOMAIN TECHNIQUE

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Introduction

At millimeter frequencies, it has been found that substrate anisotropy plays an important role in
determining the characteristics of ICs [1-3]. Regardless whether anisotropy occurs naturally or is
purposely implanted, it must be considered. Until now, the analysis of unilateral fin-line resonators
has been devoted using isotropic substrate as the supporting medium. This paper presents an
approach based on the spectral domain technique which can be used to study fin-line resonators
printed on biaxial substrates.

Formulation

Fig. 1 shows the geometry of the unilateral fin-line resonator. The biaxial layer is arbitrarily
located inside the rectangular waveguide. The resonator is formed by a slot printed on one side of the
supporting medium. The material permittivity is characterized by the following dyadic tensor

\[ \varepsilon = \varepsilon_x \hat{x}\hat{x} + \varepsilon_y \hat{y}\hat{y} + \varepsilon_z \hat{z}\hat{z} \]  

(1)

\[ a \]

Since the slot extends in the direction of propagation, the Fourier transform of field quantities in terms
of \( z \) and \( y \) is considered. The Fourier transform of quantity \( \Phi \) is defined as

\[ \tilde{\Phi} (\alpha, \beta) = \int_{-b/2}^{b/2} \int_{-d/2}^{d/2} \Phi (y, z) e^{i\alpha y} e^{i\beta z} dy \, dz \]  

(2)

where \( \alpha \) and \( \beta \) are the transformed variables with respect to \( y \) and \( z \). Note that \( \alpha \) is discrete and \( \beta \) is
continuous.

In the biaxial region with no sources, the tangential electric fields satisfy the fourth order
differential equation

\[ 0-7803-1246-5/93/$3.00 © 1993 IEEE. \]
\[ \frac{d^2 \tilde{E}_{x,z}}{dx^2} + Q_1 \frac{d^2 \tilde{E}_{y,z}}{dx^2} + Q_2 \tilde{E}_{y,z} = 0 \]  

(3)

where coefficients \( Q_1 \) and \( Q_2 \) are functions of the permittivity and transformed variables. In the isotropic regions, the fields in general can be written in terms of Hertz potential functions [3]. The solutions for the \( z \) component of the electric field in the three regions after applying the boundary conditions are:

\[ \tilde{E}_z^1 = A_n \sin \gamma_1 (x + a_x) + B_n \sin \gamma_1 (x + a_x) \]

\[ + C_n \cos \gamma_1 (x + a_x) + D_n \cos \gamma_1 (x + a_x) \]  

(4)

\[ \tilde{E}_z^2 = \left[ \frac{\omega \mu_0}{\beta} \gamma_2 A_n - j \alpha B_n \right] \sin \gamma_2 (x - a_w) \]  

(5)

\[ \tilde{E}_z^3 = \left[ \frac{\omega \mu_0}{\beta} \gamma_3 C_n - j \alpha D_n \right] \sin \gamma_3 (x + a_t) \]  

(6)

where \( A_n, B_n, C_n, D_n, A_n, B_n, C_n, \) and \( D_n \) are unknown constants. To solve for these unknowns, we apply the boundary conditions at the interfaces of \( x = 0 \) and \( x = -a_x \).

\[ \begin{bmatrix} \tilde{E}_y^1 = \tilde{E}_y^2 & \tilde{E}_z^1 = \tilde{E}_z^2 \\ \tilde{H}_y^1 = \tilde{H}_y^2 & \tilde{H}_z^1 = \tilde{H}_z^2 \end{bmatrix} \]

(7a)

\[ \begin{bmatrix} \tilde{E}_y^1 = \tilde{E}_y^3 \\ \tilde{E}_z^1 = \tilde{E}_z^3 \\ \tilde{H}^1 = \tilde{H}_y^3 & \tilde{H}_z^1 = \tilde{H}_z^3 \end{bmatrix} \]

(7b)

Systematically simplifying equation (7) yields a set of algebraic equations that relates the current densities \( \tilde{J}_y \) and \( \tilde{J}_z \) on the conducting metal fins with the electric fields \( \tilde{E}_y \) and \( \tilde{E}_z \) in the slot.

\[ \begin{bmatrix} \tilde{\mathbf{Y}}_y(k_o) & \tilde{\mathbf{Y}}_{yz}(k_o) \\ \tilde{\mathbf{Y}}_{yz}(k_o) & \tilde{\mathbf{Y}}_{zz}(k_o) \end{bmatrix} \begin{bmatrix} \tilde{E}_y \\ \tilde{E}_z \end{bmatrix} = \begin{bmatrix} \tilde{J}_y \\ \tilde{J}_z \end{bmatrix} \]

(8a)

where the dyadic Green's function elements are given by

\[ \tilde{\mathbf{Y}}_y = \frac{1-P_4}{P_6} \left\{ \frac{H_5 (k_o^2 - \beta^2)}{\gamma_2 \tan \gamma_2 a_w} - H_z + \frac{\alpha \beta H_3}{\gamma_2 \tan \gamma_2 a_w} \right\} \]

\[ + \frac{1-P_1}{P_3} \left\{ \frac{H_6 (k_o^2 - \beta^2)}{\gamma_2 \tan \gamma_2 a_w} - H_z + \frac{\alpha \beta H_4}{\gamma_2 \tan \gamma_2 a_w} \right\} \]

\[ \tilde{\mathbf{Y}}_{yz} = \frac{P_5 - P_2}{P_6} \left\{ \frac{H_5 (k_o^2 - \beta^2)}{\gamma_2 \tan \gamma_2 a_w} - H_z + \frac{\alpha \beta H_3}{\gamma_2 \tan \gamma_2 a_w} \right\} \]

\[ - \frac{s_1 + P_2}{P_3} \left\{ \frac{H_6 (k_o^2 - \beta^2)}{\gamma_2 \tan \gamma_2 a_w} - H_z + \frac{\alpha \beta H_4}{\gamma_2 \tan \gamma_2 a_w} \right\} \]

(8b)

(8c)
\[ \ddot{Y}_{xy} = \frac{1 - P_4}{P_6} \left\{ \frac{H_z \cot y_{z} a_w (\gamma_2 k_0^2 + \alpha^2 \beta^2) + H_y + \frac{\alpha \beta H_5}{\gamma_2 \tan y_{z} a_w}}{\gamma_2 (k_0^2 - \beta^2)} + 1 + \frac{P_1}{P_3} \left\{ \frac{H_z \cot y_{z} a_w (\gamma_2 k_0^2 + \alpha^2 \beta^2) + H_y + \frac{\alpha \beta H_5}{\gamma_2 \tan y_{z} a_w}}{\gamma_2 (k_0^2 - \beta^2)} \right\} \right\} \]

\[ \ddot{Y}_{zz} = \frac{P_5 - S_2}{P_6} \left\{ \frac{H_z \cot y_{z} a_w (\gamma_2 k_0^2 + \alpha^2 \beta^2) + H_y + \frac{\alpha \beta H_5}{\gamma_2 \tan y_{z} a_w}}{\gamma_2 (k_0^2 - \beta^2)} \right\} - \frac{S_1 + P_2}{P_3} \left\{ \frac{H_z \cot y_{z} a_w (\gamma_2 k_0^2 + \alpha^2 \beta^2) + H_y + \frac{\alpha \beta H_5}{\gamma_2 \tan y_{z} a_w}}{\gamma_2 (k_0^2 - \beta^2)} \right\} \]

with \( S_1, S_2, H_1, \ldots, H_8, P_1, \ldots, P_5, \) and \( P_6 \) representing modal terms.

The parameters \( \dot{J}_y \) and \( \dot{J}_z \) in equation (8a) can now be eliminated using Galerkin's procedure and Parseval's relation. After expanding the electric field distributions in the slot in terms of basis functions, the inner product is performed to obtain the characteristic equation

\[ \begin{bmatrix} \sum_{m=1}^{N} \sum_{n=-m}^{m} \int \hat{e}_y \hat{Y}_{yy}(k_0) \hat{e}_y(\beta) \, d\beta \\ \sum_{m=1}^{N} \sum_{n=-m}^{m} \int \hat{e}_y \hat{Y}_{yz}(k_0) \hat{e}_y(\beta) \, d\beta \\ \sum_{m=1}^{N} \sum_{n=-m}^{m} \int \hat{e}_z \hat{Y}_{yz}(k_0) \hat{e}_z(\beta) \, d\beta \\ \sum_{m=1}^{N} \sum_{n=-m}^{m} \int \hat{e}_z \hat{Y}_{zz}(k_0) \hat{e}_z(\beta) \, d\beta \end{bmatrix} \begin{bmatrix} \xi_m \\ \xi_m \\ \xi_m \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \]

which can be used to compute the resonant frequency by setting the determinant of the coefficient matrix equal to zero. The matrix equation (9) is exact. However, the accuracy of the solution depends on the accuracy with which the basis functions represent the true field distribution in the slot.

When the width of the slot is very small compared with the wavelength, the longitudinal component of the electric field makes insignificant contributions to the final solution. In this case, it can be ignored. As a result, equation (9) becomes a one-term equation with \( \hat{e}_y \) given in [4]

\[ \hat{e}_y = \hat{e}_y(\alpha) \hat{e}_y(\beta) \]

The basis function \( \hat{e}_y(\beta) \) must satisfy the boundary condition that its amplitude at both ends of the slot is zero. Additionally, this function also takes into account the presence of the waveguide evanescent modes. The expression for \( \hat{e}_y(\alpha) \) is identical to the one used for computing infinitely long fin-lines.

**Verification**

To verify the theoretical results, consider a special case. The permittivity tensor is reduced from biaxial to a simple isotropic case with \( \varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz} = 2.22 \), along with \( a = 7.112 \) mm, \( b = 3.556 \) mm, \( a_x = 0.127 \) mm, \( a_y = 2.927 \) mm, \( a_w = 4.185 \) mm, and \( d_2/b = 0.1 \). Fig. 2 shows the resonance characteristics of the fin-line resonator operating at Ka-band. The slot is at the center of the waveguide. As the resonator length \( d_1 \) varies from 4.733 mm to 3.102 mm, the resonant frequency increases from 28.0 GHz to 35.0 GHz. The numerical data from reference [4] is reproduced and also presented in this figure for comparison. The two sets of data seem to match very well within the computed range.
Conclusions

The analysis of the unilateral fin-line resonators printed on biaxial substrates is presented. The admittance Green's function is derived through a fourth order formulation. This technique can be further extended to include the magnetic substrate problems where the permeability is a second rank tensor. Even though this analysis is directed toward a single uniform slot, arbitrarily shaped resonators can also be analyzed using this method, as long the correct field expansion functions are provided.

References


Fig. 2. Variation of length $d_1$ versus resonant frequency $f_0$. 

This method
--- Data from [4]