**ADAPTIVE MATHEMATICAL MORPHOLOGY FOR RANGE IMAGERY**

**Author(s)**
J.G. Verly; R.L. Delanoy

**Performing Organization**
Lincoln Laboratory, MIT
P.O. Box 73
Lexington, MA 02173-9108

**Sponsoring/Monitoring Agency**
Advanced Research Projects Agency
3701 N. Fairfax Drive
Arlington, VA 22203

**Abstract**
We explore the application of adaptive (i.e., data-dependent) mathematical morphology techniques to range imagery, i.e., the use of structuring elements that automatically adjust to the grayscale values in a range image in order to deal with (e.g., extract or eliminate) features of known physical sizes.

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Adaptive Mathematical Morphology for Range Imagery
Jacques G. Verly and Richard L. Delanoy

Abstract—We explore the application of adaptive (i.e., data-dependent) mathematical morphology techniques to range imagery, i.e., the use of structuring elements that automatically adjust to the grayscale values in a range image in order to deal with (e.g., extract or eliminate) features of known physical sizes.

I. INTRODUCTION

Often, the output of a mathematical-morphology (MM) operation [1] (either binary or grayscale) describes how well a shape, called a structuring element (SE), either fits or does not fit inside a local image feature.

Whereas the SE’s used in most applications remain constant as they probe an image, there are situations where SE’s must change their size, orientation, and/or shape during probing. If these changes are determined a priori, independently of the data, the SE’s can be said to be space variant. If the change is made on the basis of the value(s) of one or more pixels around the pixel being probed, the SE’s can be said to be data dependent or adaptive. We are aware of only a few papers dealing with space-variant SE’s, e.g., [2] (also in [3]) and [4], and of none dealing with adaptive SE’s.

Since range imagery [5] contains significant shape information and since MM deals with shape in images, it is also surprising that so few papers have been written on the application of MM to range imagery (examples are [6]–[10]).

Since “angle-angle” range images are subject to the usual perspective distortions, the apparent length of a feature in such images is a function of the feature range. With range available at each pixel, one can simultaneously process (e.g., extract or eliminate) all the differently-scaled instances of the feature or object of interest by adapting the sizes of the SE’s to the local range.

For simplicity, we will assume that all SE’s of interest become fully determined when their scalable x size and y size are specified (x and y, respectively, refer to the horizontal and vertical image axes). In other words, we assume that adapting an SE to range simply amounts to finding the appropriate x and y sizes. Examples of useful SE’s that have such properties are the rectangular and ellipsoidal SE’s (either binary or grayscale). Note that, for this class of SE’s, changing the x and y sizes can in some cases cause a spatial compression/expansion of the pattern (either binary or grayscale). The goal of this paper is to develop a systematic, practical methodology for designing and applying adaptive SE’s for range imagery under the constraints just described.

II. PRINCIPLES

Let us consider a range image R of a scene S. The grayscale value \( r \) associated with each pixel is related to the distance \( r_m \) (say, in meters) between the corresponding patch of S and the sensor by

\[
r = \left[ \frac{r_m - r_0}{b} \right]
\]

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The authors are with the Machine Intelligence Technology Group, MIT Lincoln Laboratory, Lexington, MA 02173-9108. IEEE Log Number 9206911.

where \( r_0 \) is a range offset corresponding to the grayscale value of 0, \( b \) is the range quantization value, and \( \lfloor x \rfloor \) is the integer part of \( x \).

The length \( n_x \), in units of pixels of a horizontal feature F (in R) that has an actual length of \( x_m \) meters (in S) and is located at a distance of \( r_m \) meters from the imaging sensor is given by the small-angle approximation

\[
n_x = \frac{x_m}{r_m \alpha_x}
\]

where \( n_x \) is the horizontal angular width (in radians) of a pixel. This expression can also be used to find the x dimension \( n_x \) (in pixels) of the SE required to detect F in R (for convenience, we can temporarily focus on the case of horizontal, rectangular, 1-pixel high SE’s). Notice that we use primed symbols like \( n_x' \) to denote arbitrary lengths in units of pixels, and unprimed symbols like \( n_x \) for lengths that are an integer number of pixels.

From the functional dependency of \( r_m \) upon \( n_x' \) for fixed \( \alpha_x \) and \( x_m \) (e.g., as in Fig. 1), we observe the following.

First, consider SE’s that must fit inside F in R. Such SE’s will be said to be of type \( MF \). For example, a SE used in an opening with the intent of preserving a protrusion must be of type \( MF \); ditto for closing and intrusion. A fixed-length SE designed to fit inside F at a given range will fit in the same F at a shorter range, but not at a longer range. Since the localization of F based on a fixed-length SE becomes less sharp as the range decreases, one should switch to the next larger size SE as soon as its design range is reached. In summary, in the case of \( MF \)-type SE’s, the range interval where a given SE can be used is from (and including) its associated range (i.e., the range at which it fits exactly in F) down to (but excluding) the design range of the next larger SE. For example, when all integer-length SE’s are available, the semi-open range interval for the SE corresponding to \( n_x = 1 \) is obtained from the solid curve segment labeled 1 in Fig. 1 (the interval is shown along the \( r_m \)-axis). Pixels with ranges corresponding to \( n_x = 0 \) should not be processed at all.
A. Selection of Necessary SE's

can find the limits of the span where \( i \) is a reminder that these numbers are integers. Using (1), one
design range of the next smaller SE (Fig. 2). \( r \)
(i.e., the range at which it fits exactly in
a given SE can be used is from (but excluding) its associated range
opening with the intent of
will be said to be of type MNF. For example, a SE used in an
3,IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 2, NO. 2, APRIL
of integers
where \( K_i \) is
set
(in meters) in
Figs. 1 and 2 show the range intervals and the corresponding SE sizes
necessary to cover the total range interval of \( R \). The dashed lines in
Fig. 2. Range intervals associated with integer-length SE's when the SE's
must not fit inside feature.

Second, consider SE's that must not fit inside \( F \) in \( R \). Such SE's
will be said to be of type MNF. For example, a SE used in an
opening with the intent of eliminating a protrusion must be of type
MNF; ditto for closing and intrusion. Here, the range interval where a
given SE can be used is from (but excluding) its associated range
(i.e., the range at which it fits exactly in \( F \)) up to and including the
design range of the next smaller SE (Fig. 2).

In practice, it may not be feasible to use all the sizes \( n_x \) the: arc
necessary to cover the total range interval of \( R \). The dashed lines in
Figs. 1 and 2 show the range intervals and the corresponding SE sizes
for the hypothetical case where only the odd sizes \( n_x \) are available.

III. IMPLEMENTATION

Denote the minimum and maximum values of \( R \) by \( r_A, \) and \( r_B, \)
where \( i \) is a reminder that these numbers are integers. Using (1), one
can find the limits of the span [\( r_A, r_B, \)] of actual range values
(in meters) in \( R, \) i.e.,
\[
\begin{align*}
& r_A, m = r_0 + r_A, i \times b, \\
& r_B, m = r_0 + (r_B, i + 1) \times b.
\end{align*}
\]

A. Selection of Necessary SE's

Using (2), one finds that the required interval of sizes \( n_x \) is
\( [n_x, \min, n_x, \max], \) with
\[
\begin{align*}
& n_x, \min = \frac{x_m}{r_B, m}, \\
& n_x, \max = \frac{x_m}{r_A, m}.
\end{align*}
\]

Assuming that the only SE sizes available are those in the ordered
set
\( N_F = \{ n_x, k \mid k = 1, K_F; n_x, 1 = 0; n_x, k < n_x, k+1 \} \)
where \( K_F \) is the number of available sizes, the corresponding interval
of integers \( n_x \) is \( [n_x, \min, n_x, \max] \) where the limits are as follows.

1. For SE's of type MF:
\[
\begin{align*}
& n_x, \min = \text{largest integer in } N_F^\ast \text{ less than or equal to } n_x', \min, \\
& n_x, \max = \text{largest integer in } N_F^\ast \text{ less than or equal to } n_x', \max.
\end{align*}
\]

2. For SE's of type MNF:
\[
\begin{align*}
& n_x, \min = \text{smallest integer in } N_F^\ast \text{ strictly greater than } n_x', \min, \\
& n_x, \max = \text{smallest integer in } N_F^\ast \text{ strictly greater than } n_x', \max.
\end{align*}
\]

Thus the set of SE's that can be called upon is the ordered subset
\( N_x = \{ n_x, k \mid k = 1, K_x; n_x, k \in N_F^\ast \} \)
\( n_x, \min \leq n_x, k \leq n_x, \max; n_x, k < n_x, k+1 \}
of \( N_F^\ast, \) where \( K_x \leq K_F \) is the maximum index \( k \) needed.

B. Selected-SEs' Range Intervals in Meters

Denoting the actual range (in meters) associated with each \( n_x, k \)
in \( N_x \) by \( r_x, k, m, \) with
\[
r_x, k, m = \frac{x_m}{n_x, k, n_x}
\]
it can be shown that the lower and upper limits (in meters) of the
semi-open interval \( [r_x, L, k, m, r_x, H, k, m] \) are as follows.

1) For SE's of type MF:
\[
\begin{align*}
& r_x, L, k, m = r_x, k+1, m \text{ if } k < K_x, \\
& r_x, H, k, m = r_x, k, m \text{ if } k = K_x.
\end{align*}
\]

2) For SE's of type MNF:
\[
\begin{align*}
& r_x, L, k, m = r_x, k, m \text{ if } k > 1, \\
& r_x, H, k, m = r_x, k-1, m \text{ if } k = 1.
\end{align*}
\]

C. Selected-SEs' Range Intervals in Range Integers

Expressing interval limits in terms of range integers (instead of
meters) can speed up the repeated comparison of pixel grayscale
levels to the various intervals. Introducing
\[
r_x', k, i = \frac{r_x, k, m - r_0}{b}
\]
one can show that the upper and lower limits (in range integers) of the
closed interval \( [r_x, L, k, i, r_x, H, k, i] \) associated with each \( n_x, k \)
in \( N_x \) are as follows.\(^1\)

1) For SE's of Type MF:
\[
\begin{align*}
& r_x, H, k, i = \text{largest integer smaller than or equal to } r_x', k, i, \\
& r_x, L, k, i = \begin{cases} 
  r_x, H, k, i + 1 & \text{if } k < K_x, \\
  r_A, i & \text{if } k = K_x.
\end{cases}
\end{align*}
\]

2) For SE's of type MNF:
\[
\begin{align*}
& r_x, L, k, i = \text{smallest integer strictly greater than } r_x', k, i, \\
& r_x, H, k, i = \begin{cases} 
  r_x, L, k, i - 1 & \text{if } k > 1, \\
  r_B, i & \text{if } k = 1.
\end{cases}
\end{align*}
\]

D. General Implementation Strategy

So far, we have focused on the \( x \) dimension of an SE and on
the particular case of a 1-pixel high horizontal SE. Of course, the
argument can be identically repeated for the \( y \) dimension and for a

\(^1\)The limits \( r_x, L, k, i \) and \( r_x, H, k, i \) are derived independently of their
counterparts \( r_x, L, k, m \) and \( r_x, H, k, m \).
l-pixel wide vertical SE. In the case of an arbitrary SE, one must deal with both dimensions simultaneously.

The general strategy for efficiently implementing adaptive MM algorithms is to (a) determine the sets \( X_1 \) and \( X_2 \) of applicable SE sizes, (b) find the interval \( I_{1,2} \) of range values (either in meters or in range integers) each SE size \( a, \leq n, \leq X_1, a, \leq n, \leq X_2 \) applies to, (c) denote by \( I_{1,2} \) the (possibly empty) range interval that is common to \( I_{1,2} \) and \( I_{1,2} \) and associate with it the available SE with sizes \( a, \leq n, \leq X_1, a, \leq n, \leq X_2 \), and (d) perform the desired operation on the image of interest by selecting the appropriate available SE at each pixel based upon the range at that pixel. This last step may be implemented in a variety of ways, including in parallel, e.g., with one processor per applicable SE. Once again, one should stress that the processing will depend upon the type of each SE of interest, i.e., \( MH \) or \( MVF \).

IV. Application

Adaptive MM is routinely used by the authors for the extraction of vehicles of known sizes from \textit{real} low-resolution (2-6 meters) tactical laser-radar imagery. It is one of the techniques used in the XTRS system described in [11]. Here, to better demonstrate the potential of adaptive MM, we use a 2-m resolution \textit{synthetic} image of a scene consisting of a ground plane, a background vertical wall at 1,700 m, and 3 objects: a 3.6 m-wide tank and a 2.4 m-wide box, both at 700 m, and a replica of the first tank at 1,500 m (Fig. 3(A)).

In images taken roughly at ground level, vehicles that have width \( a, \leq n, \leq X_1, a, \leq n, \leq X_2 \) and length \( l, \leq n, \leq X_1, l, \leq n, \leq X_2 \) produce observable silhouettes whose lengths should be in the approximate interval \( \left( a, \leq n, \leq X_1, a, \leq n, \leq X_2 \right) \). For the tank in Fig. 3(A), the limits of this interval for the tank body are approximately 3 m and 7 m.

An image \( B \) that highlights the tank bodies in the image \( R \) of Fig. 3(A) can be created by using the \textit{difference-of-closings}

\[
B = R^{H_{\leq n}} - R^{H_{\leq n}}
\]

where \( H_{\leq n} \) is a \( MH \)-type adaptive SE designed for a feature length \( a, \leq n, = 7.0 \) m and \( H_{\leq n} \) is a \( MVF \)-type adaptive SE designed for \( a, \leq n, = 3.0 \) m (both SEs are 1-pixel high). The result is shown in Fig. 3(D). First, note that both tank bodies are correctly highlighted in spite of their different apparent widths.

This results from the fact that the visible 3.6 m physical widths of the bodies are within the allowed 3-7 m physical-size range for the visible widths of such bodies. Second, note that the box in front of the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{image.png}
\caption{Extraction of tank bodies from a synthetic range image: (A) range image \( R \), (B) closing \( R^{H_{\leq n}} \), (C) closing \( R^{H_{\leq n}} \), (D) output image \( B = R^{H_{\leq n}} - R^{H_{\leq n}} \).}
\end{figure}
The farthest tank is not detected even though its apparent width is between those of the tank bodies. Of course, the reason is that the physical width of the box is outside the allowed range for physical body sizes. Clearly, the exact same technique would extract any number of tanks (e.g., in column), provided the tanks do not significantly overlap.

The success of our simple object extraction procedure is not only due to the well-known "sieving" properties of successive closings (or openings) with differently sized SE's [1], but also to the use of range-adaptive SE's properly designed with respect to the $MF/MNF$ dichotomy.

V. CONCLUSIONS

In this work, we have pointed out the role of adaptive MM in the processing of range imagery. We have emphasized the importance of correctly distinguishing between, and implementing, adaptive (i.e., data dependent) SE's that either "must fit" or "must not fit" within the feature(s) of interest. It should be clear that the choice among the elements of the $MF/MNF$ dichotomy is based on the problem at hand, and is not an intrinsic property of the MM operation used. In other words, there is no such conclusion as saying that MM operation X (e.g., dilation, erosion, opening, closing, etc.) should always be used with an adaptive SE designed so that it fits (or, similarly, does not fit) inside the feature of interest.

Although we have discussed adaptive MM in the specific case of range imagery, this technique is applicable to any type of image for which the distance to a scene element is available for each pixel. Such a situation occurs in laser-radar imaging where pixel-registered range data is available from the range image. Clearly, the opportunity exists for developing a more general theory and methodology for designing and applying adaptive SE's to range imagery. Furthermore, the whole question of how to efficiently implement adaptive MM should be investigated in detail.

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