A theory of a grating stimulated echo (GSE) is developed. The GSE involves the sequential excitation of atoms by two counterpropagating traveling waves, a standing wave, and a third traveling wave. It is shown that the echo signal is very sensitive to small changes in atomic velocity, much more sensitive than the normal stimulated echo. Use of the GSE as a collisional probe or accelerometer is discussed.

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I. INTRODUCTION

Echo phenomena are an effective means for probing relaxation processes in the free-field region between the excitation pulses. In the usual two-pulse echo [1], the signal reflects relaxation of the atomic dipole coherence, while in the stimulated echo (SE), it is atomic state populations that can be monitored. Although the SE is an effective method for studying decay rates for atomic state populations, it is not a particularly sensitive probe of relaxation associated with changes in atomic velocity. Increased sensitivity to small velocity changes can be achieved by using the grating stimulated echo (GSE) described in this paper.

The difference between the SE and GSE can be understood in terms of the Doppler phase diagram for the GSE shown in Fig. 1. The ordinate on this graph represents the Doppler phase diagram for the GSE shown in this paper. The phase difference between the SE and GSE is shown in the inset. This phase change is of order \( |\mathbf{k} \cdot \delta \mathbf{v} T_2| \leq |\mathbf{k} \cdot \delta \mathbf{v} / \gamma | \), where \( \gamma \) is the rate at which the atomic coherence decays.

FIG. 1. Doppler phase associated with various density matrix elements leading to a grating stimulated echo (GSE). The four input pulses \( E_i \), \( i = 1-4 \), give rise to an echo \( E_f \) at \( t = T_4 \). The field propagation vectors are \( \mathbf{k}_1 = -\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}_2 \). There is a second contribution to the echo signal (not shown) for which \( \mathbf{k}_2 = -\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}_2 \); that contribution is obtained by reflection about the \( \mathbf{r} \) axis and involves density matrix elements \( \rho_{21}, \rho_{12}, \rho_{11}, \rho_{22} \). The corresponding diagram for the stimulated echo (SE) has \( E_i = 0 \) and \( \mathbf{k}_1 = -\mathbf{k}_1 = -\mathbf{k}_2 = \mathbf{k}_2 \); the Doppler phase for the SE is constant in the time interval \( T_{42} \). The atomic energy-level diagram is shown in the inset. Each level decays to an external reservoir at rate \( \gamma \) and the overall decay rate of level 3 is \( \Gamma = \Gamma_0 + 1/\gamma \). The incident fields drive only the \( 1 \rightarrow 3 \) transition.
was first discussed by Mossberg et al. [2].

In Sec. II, an expression for the GSE is derived. The long-lived ground-state grating exists only if the sum of ground- plus excited-state population is not conserved [3,4]; consequently, the atomic model we choose is one in which the ground state consists of two sublevels. In Sec. III, two applications of the GSE are discussed. It is shown that the GSE can be used as a sensitive probe of collisions occurring between ground-state “active” atoms and perturbers. Furthermore, it is shown that the GSE can serve as the basis for an atom accelerometer. Such an accelerometer is similar to those proposed by Borde [5] and demonstrated experimentally by Riehle et al. [6] and Kasevich and Chu [7]. An essential distinction is that the GSE does not require a long-lived excited state, as in [6], and is not connected with the establishment of a coherence between ground-state sublevels, as in [7].

II. ECHO FORMATION

Let us consider a gas of three-level atoms interacting with a radiation field \( E(r,t) \). The upper atomic level 3 decays to level 1 at rate \( \Gamma_{13} \) and to level 2 at rate \( \Gamma_{32} \) (see Fig. 1). The field

\[
E(r,t) = \frac{1}{\hbar} \exp[-i\omega t] \sum_j E_j g_j(t - T_j)
\]

consists of a number of pulses, each having central frequency \( \omega \). Pulse \( j \) \((j=1,2,3)\) has amplitude \( E_j \), and is characterized by a pulse envelope function \( g_j(t - T_j) \) centered at \( t = T_j \), having a temporal width of order \( \tau_j \). Pulse intervals \( \tau_k \) are defined by \( \tau_k = T_i - T_{i-1} \) and \( T_1 = 0 \) is set equal to zero. All pulses are polarized along the \( x \) axis. The propagation directions are chosen as \( k_1 = -k_2 = k \) and \( \beta_4 = \pm k \), where \( k = \Omega/c \).

We wish to calculate the echo signal generated at times \( t > T_4 \). The master equation for the density matrix \( \rho \) (in the interaction representation) is

\[
\rho_{33} - (i/\hbar)[H, \rho]_{33},
\]

\[
\rho_{11} = \Gamma_{31} \rho_{33} - (i/\hbar)[H, \rho]_{11} + \gamma_{1} W_{1}(v),
\]

\[
\rho_{22} = \Gamma_{32} \rho_{33} + \gamma_{1} W_{2}(v),
\]

\[
\rho_{13} = - (i/\hbar)[H, \rho]_{13},
\]

where \( d/dt = \partial/\partial t + \mathbf{v} \partial/\partial x \), \( \mathbf{v} \) is the \( x \) component of the atomic velocity, \( \mathbf{v} = -\mathbf{E}(r,t) \) is the atom-field interaction Hamiltonian, \( \mathbf{v} \) is the atomic dipole moment operator, \( \Gamma = \Gamma_{31} + \Gamma_{32} + \gamma_{1} \) is the upper level decay rate, \( \gamma_{1} \ll \Gamma \) is some state-independent, effective loss rate for the atoms, \( \gamma = \Gamma/2 \) at the rate at which the coherence \( \rho_{13} \) decays, and \( \mathbf{W}_{i}(v) \) \((i=2,3)\) is the level \( i \) equilibrium velocity distribution in the absence of the fields.

Equations (2) are solved assuming that \( |\Delta \tau|_j \gg 1 \), \( \Gamma_{13} < 1 \), and \( k \tau_j \ll 1 \), where \( \Delta = \Omega \omega_{13} \) is the detuning of the field frequency relative to that of the atomic transition \( 3 \rightarrow 1 \) and \( u \) is the most probable atomic speed. Furthermore, the traveling-wave pulses are treated to lowest order in perturbation theory. These restrictions are imposed for mathematical simplicity, they are in no way critical to the effect to be derived herein.

It is convenient to carry out the calculation in the atomic rest frame. The transformation between the laboratory and atomic frames is given by \( x_0 = x - ct \), where \( x_0 \) is the atomic position at \( t = 0 \). Following a rather straightforward calculation [1,8,9], one arrives at the density matrix for \( t > T_4 \). Keeping only that part of the density matrix responsible for echo formation at \( t = \tau \), (see Fig. 1), one finds

\[
\rho(t) = \rho(\tau) \exp[-\gamma(T_{21} + \tau) - \gamma(T_{13})] W_1(v) \theta_4 \]

\[
\times \sum (\theta_4) [\cos(\omega_0 + \tau T_4) + \cos(\omega_0 - \tau T_4)] W_2(v) \theta_4 \]

\[
\times \cos|\theta_4| [\cos(\omega_0 + \tau T_4)] |\beta_4| |\beta_4| + \text{H.c.}
\]

\[
\rho_{33}(t) = \rho(\tau) \exp[-\gamma(T_{21} + \tau) - \gamma(T_{13})] W_1(v) \theta_4 \]

\[
\times \sum (\theta_4) [\cos(\omega_0 + \tau T_4) + \cos(\omega_0 - \tau T_4)] W_2(v) \theta_4 \]

\[
\times \cos|\theta_4| [\cos(\omega_0 + \tau T_4)] |\beta_4| |\beta_4| + \text{H.c.}
\]

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2,
\]

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2,
\]

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2.
\]

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2.
\]

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2.
\]

where

\[
P_{\text{echo}}(t) = \frac{n}{\hbar} \exp[i\omega t/(1/4)] \int_0^L dt e^{-i\omega t} \int_{-\infty}^\infty dt e^{i\omega t} |\rho(t)|^2.
\]
\[ P = \frac{k}{2\pi n} \sum_{m=0}^{\infty} (-1)^m J_{2m}(\|\theta_1\|) \int_0^L dx_0 \]
\[ \times \int_{-\infty}^{\infty} dt' W_1(t') \exp[i x_0(k_4 + k_2 - k_3 + 2mk - k_1) + \delta \{ T_{32}(k_4 + 2mk - k_1) + T_{34}(k_4 - k_1) \}] \]
\[ + \delta \{ T_{31}(k_4 + k_2 + 2mk - k_3) - k_3 \} \]  
(5b)

and \( P' \) is obtained from (5b) with the replacements \( k_1 \rightarrow -k_1, k_2 \rightarrow -k_2, k_3 \rightarrow k_3, k_4 \rightarrow -k_4 \).

Let us consider expression (5b) for \( P \). The spatial phase must vanish for a coherent contribution to the echo field. This condition, together with the fact that \( k_1, k_2, k_3, k_4 \) leads to a value for the propagation constant. \( k_1 = k_3 + 2(m - 1)k \). Since \( kuT_{32} \gg 1 \) and \( T_{12} \gg T_{21} \), it is not possible to get a nonvanishing result after averaging over \( \tau \) unless \( T_{41} \approx T_{24}((m - 1) \). Integral values \( m \approx 2 \) are allowed. It then follows that the phase-matching condition \( k_1 = k_3 + 2(m - 1)k \) can be satisfied only for \( m = 2(T_{41}) = T_{32} \), \( k_4 = -k \), and \( k_3 = k \) (echo propagation in the \( +x \) direction). In this case, the echo amplitude is

\[ E_{\delta}(\tau) = -(\pi kL_0) \exp\left[\frac{1}{2} ku(k_4 + k_2 - k_3) \right] \exp\left[-\gamma(T_{21} + \tau) - 2\gamma_1 T_{21} + 2\gamma_2 T_{21} + T_{41} \right] \]  
(6)

where \( F_1(t) = \int_{-\infty}^{\infty} dt' W_1(t') e^{-i\delta t} \).

The corresponding result for the \( P' \) term of Eq. (5a) is \( P' = T_{32}, k_4 = k \), and \( k_3 = -k \). In this case, the echo signal propagates in the \(-x\) direction, having amplitude \( E_{\delta}(\tau) = \theta_4 E_{\delta}(\tau) / \theta_3 \exp[2i\Delta \tau] \).

### III. DISCUSSION OF THE ECHO SIGNAL: APPLICATIONS

1. The structure of the echo depends on the homogeneous width \( \gamma \) and inhomogeneous width \( ku \). Two limiting cases of experimental interest are \( ku \gg \gamma \) and \( ku \ll \gamma \). The first condition is typical for atoms at room temperature while the latter typical for sub-Doppler, laser cooled atoms.

For \( ku \gg \gamma \) one finds that the GSE has a temporal width of order \( k \gamma^{-1} \) and is centered at \( t = T_{21}, T_{32} \) assuming \( T_{32} = T_{21} \). The backward-directed echo contains Ramsey fringe structure, with a factor of \( \exp[2i\Delta T_{32}] \) in the echo amplitude, which is typical for echoes induced by the separated counterpropagating waves [10]. Such a factor is absent for the forward directed echo; however, it is possible to create Ramsey fringes for the forward-directed echo by introducing a difference \( \Delta T = T_{32} - T_{21} \) between the pulse intervals. If \( \Delta T \ll \gamma^{-1} \), one does not destroy the echo signal, which is centered at \( t = T_{41} + T_{21} - \Delta T \). The forward directed echo amplitude contains a factor of \( \exp[2i\Delta \tau] \) which gives rise to Ramsey fringe structure.

When \( ku \ll \gamma \), the time scale of the echo experiment must be much larger than \( \gamma^{-1} \) to allow for inhomogeneous relaxation of the different atomic velocity subgroups. Consequently, the standard two-pulse excitation scheme (two pulses separated by a time interval of order \( \gamma^{-1} \) [1]) does not lead to echo formation. Excitation pulse sequences similar to those of the SE or GSE, with \( T_{41} \ll \gamma^{-1} \), are needed for echo formation in the limit \( ku \ll \gamma \). The temporal structure of the echo is very different than that for \( ku \gg \gamma \). When \( ku \gg \gamma \), the echo signal appears as soon as the pulse at \( t = T_{41} \) is applied, and has a duration of order \( \gamma^{-1} \). In the SE, the echo propagates in the same direction as both the last excitation pulse and the free decay signal following this pulse.

Only the specific dependence on the delay \( T_{12} \) would permit one to extract the SE contribution to the total signal. In contrast, the GSE propagates in a direction opposite to that of the last excitation pulse and can be spatially separated from both the driving pulse and the free decay signal. Thus the GSE offers a distinct advantage over the SE for the observation of such a signal.

2. Aside from the directionality properties, the major advantage of the GSE over the SE is its sensitivity to small velocity changes. The GSE echo amplitude, \( E'(\text{GSE}) \), is proportional to \( \tau \) (see Fig. 1) \( \exp\{2\gamma_1 T_{21}(\tau - \tau)\} \exp\{2\gamma_2 T_{21}(\tau - \tau)\} \) while the corresponding SE amplitude, \( E(\text{SE}) \). (with \( k_1 = k_3 = k_4 = -k \) and \( F_1 = 0 \)) is proportional to \( \exp\{2\gamma_1 T_{21}(\tau - \tau)\} \), reflecting the fact that the population part of the phase diagram (Fig. 1) is back-reaction of \( T_2 \) and \( T_4 \) for the SE. Any small change \( \delta \tau \) in \( \tau \) results in a phase shift of order \( 2\gamma_1 T_{21}(\tau - \tau) \) for the GSE, but only \( \delta \tau \) for the SE. Since \( T_{32} \ll T_{21} \), the sensitivity to small \( \delta \tau \) is much larger for the GSE than the SE. This feature has been exploited by Kasevich and Chu in constructing an atom interferometer based on the Ramsey effect [7].

The sensitivity to small \( \delta \tau \) makes the GSE an ideal tool for probing atomic collisions. Consider the limiting case when \( k \delta T_{32} \ll 1 \) and \( T_{12} \ll 1 \), where \( T_{12} \) is the collision rate associated with the atomic coherence \( \rho_{13} \). Assuming, moreover, that \( T_{12} \ll 1 \), \( \gamma_1 \ll \Gamma \) (\( \gamma_1 \) is the collision rate for atoms in states \( i = 1,2,3 \)) and that the collision kernel \( W_i(r',r) \) (\( i = 1,2,3 \)), defined as the probability density per unit time that an atom in state \( i \) changes its velocity from \( r' \) to \( r \), can be written as \( W_i(r',r) = \Gamma_i f_i(r - r') \) [11], where \( f_i(r) = f_i(r - \epsilon) \) and \( \int f_i(r) dr = 1 \). one can show that the SE amplitude is given by [12]

\[ A(\text{SE}) = \Gamma_{12} + \Gamma_{13} \frac{k^2 T_{21}^2}{\Gamma} \left( (\delta \tau^{-1})_1 - (\delta \tau^{-1})_2 \right) \]  
(7)

where

\[ A(\text{SE}) = \Gamma_{12} + \Gamma_{13} \frac{k^2 T_{21}^2}{\Gamma} \left( (\delta \tau^{-1})_1 - (\delta \tau^{-1})_2 \right) \]  
(7)

\[ (\delta \tau^{-1})_1 = \int f_i(r) \delta r, \quad \text{and} \quad T = T_{32} = T_{21} \]  
(8)

The corresponding expression for the GSE is
where \( A_1(t) = \Gamma_1 \left[ 1 - \int f_1(\epsilon) \exp(-i\epsilon \omega_0) d\epsilon \right] \) and \( H_1(t) = \gamma_1 \int \left[ f_1(\epsilon) \exp(2ik\epsilon T) \right] \exp(-2\gamma_1 T) \exp[-2\Gamma_1 T] \left[ 1 - H_1(T) \right] \). Note that, if \( \Gamma_1 = \Gamma_2 \), \( f_1(\epsilon) = f_2(\epsilon) \), and \( \Gamma_2 = 0 \), population is conserved for each velocity subgroup (except for an overall decay rate) and the SE and GSE amplitudes vanish [4].

In comparing the SE and GSE signals, one notes the following differences: (1) The SE amplitude depends only \( (\delta v^2) \), and not on the specific form of the kernel, while the GSE amplitude could, in principle, be used to extract the form of \( f_1(\epsilon) \). (2) The minimum velocity change in a single collision detectable using the GSE is of order \((2kT)^{-1} \geq \gamma_2/2k\), whereas it is \((2T/T_{21}) \geq 1\) times larger for the SE. (3) The minimum collision rate \( \Gamma_1 \) detectable using the GSE is of order \((2T)^{-1} \geq \gamma_2/2\), whereas it \((2k^2\gamma_2^2/\Gamma_2 T_{21})^{-1} \geq 1\) times larger for the SE. Thus, the GSE is a much more sensitive probe of collisional effects than the SE.

For thermal samples, one can take \( \gamma_2 = 10^4 \text{ cm/s} \) to simulate some effective lifetime of the atoms in the atom-field interaction region. In that case, one can use the GSE to measure \( \delta v^2 \) as small as 0.1 cm/s (taking \( k = 10^3 \text{ cm}^{-1}) \) and collision rates \( \Gamma_1/2\pi \) as small as \( 10^3 \text{ Hz} \) [13]. For cold atoms the sensitivity increases linearly with the increased atomic lifetime in the interaction zone.

(3) Inertial effects can significantly modify the phase of the GSE. We calculate the effect of a gravitational acceleration along the \( k \) direction on the amplitude of the GSE, taking into account the temporal phase: \( \phi_1 \), \( \phi_2 \), \( \phi_3 \), and \( \phi_4 \) of the excitation pulses. We find that the phase of the echo signal \( \phi_e \) can be expressed as

\[
\phi_e = \phi_4 \pm \left( \phi_2 - \phi_1 \right) - \frac{1}{2} k g T_2^2 \tag{9}
\]

where \( g \) is the acceleration of gravity and \( + \) and \( - \) refer to the \( P \) and \( P' \) terms, respectively. We have ignored terms of order \((k g T_{21}^2) + k g T_2 T_4 \). The phase \( \phi_2 \), may be measured by a homodyne technique using a reference laser as both the source of the excitation pulses and as a local oscillator for the echo phase detection. It is clear from Eq. (9) that the reference laser need be phase coherent only over a time of order \( T_{21} \) and not over the duration of the complete echo sequence \( T_4 \). Since, according to Eq. (9), \( T_{21} \) may be made arbitrarily short without loss of sensitivity to \( g \), a highly stabilized laser is not required for the measurement. By comparison, the gravitational phase shift for the SE is \[\phi_e = \pm \left( \phi_2 - \phi_1 \right) - k g T_{42} \leq 1/\gamma T_{42} \]

To achieve high precision with the SE, a long-lived excited state is required as well as a very stable laser source.

As an example, we calculate the sensitivity to gravitational acceleration of the GSE applied to a gas of atoms at room temperature. Assuming an uncertainty in the phase measurement of \( \delta \phi_2 \), the corresponding relative uncertainty in a measurement of \( g \) is \( \delta g / g = 2 \delta \phi_2 / k g T_2 \). For \( k \approx 10^3 \text{ cm}^{-1} \) and \( T_2 \approx 10^{-2} \text{ s} \) (limited by the atoms' lifetime in the interaction volume), one finds \( \delta g / g \approx 2 \delta \phi_2 \). Thus, for a phase sensitivity of \( 10^{-3} \), one could measure the gravitational force on a gas of room-temperature atoms to a precision of about 0.1%. For a gas of laser cooled atoms, one should be able to achieve the same sensitivity \( \delta g \) as that reported in Ref. [7].

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[11] Such a kernel does not obey detailed balance; nevertheless it accurately models weak velocity-changing collisions for a time scale smaller than those needed to reach equilibrium.
[13] For \( \delta v \leq 1.0 \text{ cm/s} \), a fully quantized theory should be used to properly account for atomic recoil on absorption or emission of radiation.