NONLINEAR ANALYSIS, SCIENTIFIC COMPUTATION, AND CONTINUUM MECHANICS APPLIED TO THE SCIENCE OF MATERIALS

CENTER FOR NONLINEAR ANALYSIS
Final Report

Principal Investigators: Morton E. Gurtin
William O. Williams

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This grant enabled the department to form the Research Group in Mathematical Materials Science in 1990, a group that formed the nucleus of the Center for Nonlinear Analysis, established in 1991, by the ARO. The Center has created a vigorous environment for collaboration among mathematicians and allied scientists. Within the international mathematics community the Center has assumed a leadership role, especially for questions related to materials science. The major research effort has focused toward developing, analyzing, and unifying mathematical models that characterize material behavior at a phenomenological level. The main thrust is applied nonlinear analysis, nonlinear continuum physics, and scientific computation. The educational goals have been to train young scientists, and to train and involve female and minority students in the sciences.
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1. SUMMARY

This grant enabled the department to form the Research Group in Mathematical Materials Science in 1990, a group that formed the nucleus of the Center for Nonlinear Analysis, established in 1991 by the ARO. The Center has created a vigorous environment for collaboration among mathematicians and allied scientists. Within the international mathematics community the Center has assumed a leadership role, especially for questions related to materials science. The major research effort has focused toward developing, analyzing, and unifying mathematical models that characterize material behavior at a phenomenological level. The main thrust is applied nonlinear analysis, nonlinear continuum physics, and scientific computation. The educational goals have been to train young scientists, and to train and involve female and minority students in the sciences. Highlights include five major conferences, seven research workshops (highly focused workshops bringing together small groups of varied backgrounds to identify and attack major outstanding problems), and a four-week Undergraduate Mathematics Institute targeted at minorities and women. The training of 4-5 postdocs and 7-8 graduate students each year has been a major activity. The Center has attracted scores of visitors, both nationwide and international. Three interdisciplinary courses were introduced, as was a program of interdisciplinary weekly seminars in nonlinear analysis. A major outcome has been the initiation of several cross-disciplinary collaborations. Much of the work developed in conjunction with the Center has: (I) led to a physically consistent framework that seems useful to applied scientists for the development of specific theories; (ii) led to physically well-motivated models that are studied by analysts; (iii) led to new techniques of analysis used by other mathematicians.

2. SPECIFIC ACTIVITIES


3. PARTICIPATING SCIENTIFIC PERSONNEL:

A. Graduate Students: Nenad Antonic (Ph.D., May 1992), Ana Barroso, Dan Burkett (Ph.D., August 1993), Chih-Wen Cheng (Ph.D., May 1993), Sophia Demoulini (Ph.D., May 1993), Jose Matias (Ph.D., May 1993), Dmitry Pugachevsky, Allan Struthers (Ph.D., May 1991), Stephen Watson, Gregor Weiske.


C. Visitors: R. Almgren (U. Chicago), M. Avellaneda (Courant), I. Babuska (U. Maryland), J. Ball (Heriot-Watt U.), H. Berestycki (U. Paris), M. Bertsch
EDUCATIONAL EFFORTS:

A. Workshops for undergraduate students from Historically Black Colleges and Universities: at hampton University, November 1991, with minicourses by W. Hrusa (Calculus of Variations), M. Soner (Evolving Curves); for the Consortium of HCBU's in Atlanta, October 1972, with a minicourse by W. Hrusa (Calculus of Variations). Both workshops included a panel discussion on preparing for the succeeding in graduate school.

B. Summer Undergraduate Mathematics Institute: This is a four-week program for students who have completed their junior year, with participation by women and minorities especially sought. The primary goal is to prepare students for and interest them in graduate work in applied mathematics. Six students were accepted for the inaugural program held during June 1992. The core of the program was two courses, one in real analysis and the other in numerical solutions of differential equations. Each course had one hour of lecture per day, and a two-hour, problem-working session once per week. In addition, there was a computer laboratory in which students learned to use the symbolic programming language Maple. Carnegie Mellon credit was awarded for successful completion of these
courses. In conjunction with the courses there was a regular series of seminars in which faculty presented applied mathematics research topics.

C. **Nonlinear analysis seminar**: A program of weekly seminars in nonlinear analysis with strong focus on applications.


**CENTER REPORT SERIES**

**Nonlinear Analysis Series**:¹

NAMS-1  F. Davi and M. E. Gurtin, On the motion of a phase interface by surface diffusion, ZAMP 41, 782-811.


¹This series originated as the Nonlinear Analysis and Materials Science Series (1990-1991) with reports labelled NAMS.
| NAMS-12 | M. Marcus, A variational problem arising from a model in thermodynamics. |
| NAMS-13 | A. Visintin, Model of pattern formation. |
| NAMS-18 | D. Kinderlehrer, Theory of magnetostriction. |
| NAMS-19 | J. B. McLeod, The wedge entry problem. |
| NAMS-20 | W. W. Mullins, A one dimensional nearest neighbor model of coarsening. |
| NAMS-21 | I. Fonseca and G. Parry, Remarks on variational problems for defective crystals. |
| NAMS-22 | N. B. Firoozye and R. V. Kohn, Geometric parameters and the relaxation of multiwell energies. |
| NAMS-25 | G. Kossioris, Formation of singularities for viscosity solutions of Hamilton-Jacobi equations in one space variable, Submitted to *Comp. PDE*. |
| NAMS-26 | G. Kossioris, Formation of singularities for viscosity solutions of Hamilton-Jacobi equations in higher dimensions, Submitted to *Comp. PDE*. |
| 91-NA-001 | P.L. Lions, Jacobians and Hardy spaces. |
91-NA-002  Y. Giga and M. H. Soto, Generalized interface evolution with the Neumann boundary condition.

91-NA-003  H. M. Soner and P. E. Souganidis, Uniqueness and singularities of cylindrically symmetric surfaces moving by mean curvature.


91-NA-006  M. E. Gurtin and P. Voorhees, Two-phase continuum mechanics with mass transport and stress.

91-NA-007  E. Fried, Non-monotonic transformation kinetics and the morphological stability of phase boundaries in thermoelastic materials.

91-NA-008  M. E. Gurtin, Evolving phase boundaries in deformable continua.


92-NA-003  L. Bronsard and D. Hilhorst, On the slow dynamics for the Cahn-Hilliard equation in one space dimension.


92-NA-005  N. Antonic, Memory effects in homogenization linear second order equation.


92-NA-012  S. Alama and Y. Li, On "multibump" bound states for certain semilinear elliptic equations.


92-NA-014  N. Antonic, H-Measures applied to symmetric systems.


92-NA-016  P. Pedregal, Jensen's inequality in the calculus of variations.

92-NA-017  I. Fonseca and S. Muller, Relaxation of quasiconvex functionals in $BV(\Omega,RF)$ for indefinite $f(x,u,u)$, *Arch. Rat. Mech. Anal.* Forthcoming.


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<td>92-NA-025</td>
<td>P. J. Swart and P. J. Homes</td>
<td>Energy minimization and the formation of microstructure in dynamic anti-plane shear.</td>
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<td>92-NA-030</td>
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<td>92-NA-034</td>
<td>Leo, Perry H. and Herng-Jeng Jou</td>
<td>Shape evolution of an initially circular precipitate growing by diffusion in an applied stress field, October 1992.</td>
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<td>92-NA-038</td>
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93-NA-004 James, Richard and Kinderlehrer, David, Theory of Magnetostriction with Applications to $Tb_{y_1-x}Fe_2$, February 1993.


93-NA-011 Fonseca, Irene, Kinderlehrer, David, and Pedregal, Pablo, Relaxation in $BV \times L^\infty$ of Functionals Depending on Strain and Composition, February 1993.
STOCHASTIC ANALYSIS SERIES:

