A Design of a Fast and Area Efficient Multi-input Muller C-element

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**Abstract:**
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A Design of a Fast and Area Efficient Multi-input Muller C-element

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Abstract

A multi-input Muller C-element has frequently been used for joining signal transitions or completion time detection in self-timed circuits. This paper presents an n-input Muller C-element design which uses the multi-level login design technique and has a symmetric format for any integer $n \geq 2$. In comparison with series-parallel MOS structure implementations and C-element tree implementations, our design has fewer restrictions in terms of $n$, less path delay, less delay variance from inputs to output, and less area consumption. Experimental validation in this paper is based on an industrial standard cell library.

1 Introduction

A Muller C-element [5] is used as a basic component in the design of speed-independent circuits. A C-element is functionally equivalent to an SR latch. Under the assumption of unbounded gate delays it is not possible to guarantee that S and R will not be 1 simultaneously. This problem does not arise with $\sim$ C-element [4]. The output of a two input C-element will equal the value of the inputs after both inputs have reached the same value; otherwise the output remains unchanged. That is, if $i_1$ and $i_2$ are the two inputs and $O$ is the output, then the defining equation of the C-element is $O = i_1 \cdot i_2 + O \cdot i_1 + O \cdot i_2$ [5]. A two input C-element can be viewed as a logical and of two events, where an event can be a 0-1 or 1-0 transition [7]. This behaviour is shown in Figure 1.

A C-element is commonly used for joining signal transitions to signal the completion of an operation [1, 2, 3, 4, 6]. For example, the 16 output computational block in Figure 2 will require a 16 input C-element to join all 16 completion signals in order to generate one signal transition to indicate the completion of the block.

The output of an n-input C-element is 1 if all the inputs are 1 and it is 0 if all the inputs are 0; otherwise its value remains unchanged [6]. The state diagrams for two-input and three-input C-elements are shown in Figure 3, where the state is labeled "inputs/output". The initial state of a C-element is having all inputs and its output zero. This is denoted by the double circle in the state diagram.
Figure 1: Timing diagrams for 2-input Muller C-elements

(a) A typical self-timed computational block

(b) Timing diagram of the completion/reset signals

Figure 2: Join of completion/reset signals
(a) Two-input C-element

(b) Three-input C-element

Figure 3: State diagrams for Muller C-elements

(a) Series-parallel MOS structure

(b) C-tree structure

Figure 4: Three-input series-parallel C-element designs
C-elements with large numbers of inputs are very useful and an efficient implementation in terms of area and speed is needed. Two designs of multi-input C-elements have been used: a series-parallel MOS structure [6] shown in Figure 4(a) and the C-element tree [4, 6] shown in Figure 4(b). Because the C-element is associative, the tree implementation uses \((n - 1)\) two-input C-elements to form an \(n\)-input C-element. The input-output delay of the series-parallel MOS structure is less than that of the tree structure. However, the series-parallel implementation is not feasible for large \(n\). For this reason, most existing designs employ the tree structure. The main disadvantages of the tree implementation are that it is very slow and the variance in the delays over different input-output paths is very large.

In this paper we present an efficient design of a multi-input C-element. Our design is symmetric and the variance in delay over different input-output paths is very small. In Section 2 we derive a symmetric form of an \(n\)-input C-element and provide estimates of delay and area. In Section 3 we demonstrate the advantages of our design by presenting experimental results for C-elements with inputs ranging from 2 to 128.

## 2 Design of a Multi-input C-element

The target technology of our design is CMOS. Since inverted logic is faster than non-inverted logic in CMOS, we will use and-or-invert (AOI) logic and inverters instead of and-or logic. The defining equation of an \(n\)-input C-element with a reset is given by

\[
\text{OUT} = ((I_1 \cdot I_2 \cdot \ldots \cdot I_n) + (I_1 + I_2 + \ldots + I_n) \cdot \text{OUT}) \cdot \text{RESET}
\]  

(1)

where \(I_i\), for \(i = 1, \ldots, n\), are inputs of the C-element, and \(\text{OUT}\) is the output of the C-element. Using DeMorgan's law we transform Equation (1) as follows:

\[
\begin{align*}
\text{OUT} &= ((I_1 \cdot I_2 \cdot \ldots \cdot I_n) + (I_1 + I_2 + \ldots + I_n) \cdot \text{OUT}) \cdot \text{RESET} \\
&= ((I_1 \cdot I_2 \cdot \ldots \cdot I_n) + (I_1 + I_2 + \ldots + I_n) \cdot \text{OUT}) \cdot \text{RESET} \\
&= (I_1 \cdot I_2 \cdot \ldots \cdot I_n) \cdot (I_1 + I_2 + \ldots + I_n) \cdot \text{OUT} + \text{RESET} \\
&= (I_1 \cdot I_2 \cdot \ldots \cdot I_n) \cdot ((I_1 + I_2 + \ldots + I_n) + \text{OUT}) + \text{RESET}
\end{align*}
\]

(3)

Equation (3) can be further decomposed to following equations.

\[
\begin{align*}
\text{NAND.TREE} &= (I_1 \cdot I_2 \cdot \ldots \cdot I_n) \\
\text{NOR.TREE} &= (I_1 + I_2 + \ldots + I_n) \\
\text{OUT} &= \text{NAND.TREE} \cdot (\text{NOR.TREE} + \text{OUT}) + \text{RESET}
\end{align*}
\]

(4)

(5)

(6)
Figure 5: C-elements design

Figure 6: Multi-level NAND_TREE implementation
The above decomposition is very useful when it is mapped to a CMOS cell-based implementation. In Figure 5 we show a C-element design consisting of 3 parts: a NAND_TREE, a NOR_TREE, and a OAO_PART (or-and-or), each implemented separately. The OAO_PART shown in Figure 5 remains the same for all values of n.

Therefore, we only need to find a proper NAND_TREE/NOR_TREE implementation. In the HP C34000 library [8], there are n-input NAND and NOR gates for 2 ≤ n ≤ 8. For larger n, NAND_TREE (NOR_TREE) can be further decomposed into two-level NAND-OR tree (NOR-AND tree). When a two-level structure is not sufficient, we can use more levels, i.e., use a nand-nor-nand three level structure to implement a NAND_TREE or a nor-nand-nor structure to implement a NOR_TREE. Figures 6 (a) and (b) show the two-level and the three-level implementations for NAND_TREE. In order to minimize the variance of the input-output delays, the structure of the NAND_TREE implementation is identical to the structure of the NOR_TREE implementation.

We now provide estimates of delay and area for our design and compare them with the tree implementation of a C-element. A comparison with a series-parallel MOS structure is unnecessary since such an implementation is not feasible for large numbers of inputs.

**Delay:** For an n-input C-element, the input-output delay of a C-element tree implementation is equal to the number of levels in the structure multiplied by the input-output delay of a 2-input C-element, where the number of levels is \([\log_2 n]\), i.e.,

\[
D_{tree}(n) = \lfloor \log_2 n \rfloor \times (\text{2-input Muller C-element delay}) \tag{7}
\]

\[
\approx \lfloor \log_2 n \rfloor \times (\text{2-input NAND/NOR delay} + \text{OAOI delay}) \tag{8}
\]

\[
\approx \lfloor \log_2 n \rfloor \times \text{OAOI delay} + \lfloor \log_2 n \rfloor \times \text{2-input NAND/NOR delay}
\]

The input-output of our design is

\[
D_{multi}(n) = \text{OAOI delay} + n\text{-input NAND_TREE/NOR_TREE delay} \tag{9}
\]

The n-input NAND_TREE/NOR_TREE can be implemented by \([\log_2 n]\) stages of a NAND2-NOR2 tree, and can be made faster by using \([\log_m n]\) stages of NANDm-NORm tree. Therefore,

\[
D_{multi}(n) \leq \text{OAOI delay} + [\log_2 n] \times \text{2-input NAND/NOR delay} \tag{10}
\]

Obviously, our design is much faster than C-element tree implementation for n > 2, although both \(D_{tree}(n)\) and \(D_{multi}(n)\) are \(O(\log(n))\).

**Delay Variance:** In order to have less delay variance among the input-output paths in our design, two sufficient conditions need to be met.

1. Transistors in the OAOI element of the OAO_PART must be sized so that the delay from one OR gate input to the output and the delay from one AND gate input to the output is the same.
2. the structure of NAND_TREE should be symmetric and identical to the structure of the NOR_TREE.

These two conditions are easily satisfied for cell-based designs, and they are a property of the HP standard cell library. However, the tree implementation of an n-input C-element is balanced only if $n = 2^{\lfloor \log_2 n \rfloor}$. For this reason, the variation in delays among the different input-output paths is very large.

Area: In comparing our design with the C-element tree implementation, the routing area is not considered. Similar to the delay comparison above, the C-element tree has a big overhead in terms of size due to the repeated use of the OAO_PART in every two-input C-element in the tree. An estimate of the area of a C-element tree implementation is

$$A_{\text{tree}}(n) \approx (n - 1) \times (A_{\text{NAND2}} + A_{\text{NOR2}} + A_{\text{AOIO}} + A_{\text{INV}})$$

$$\approx (n - 1) \times (A_{\text{AOIO}} + A_{\text{INV}}) + (n - 1) \times A_{\text{NAND2}} + (n - 1) \times A_{\text{NOR2}}$$

The estimate of area for our design is

$$A_{\text{multi}}(n) \approx A_{\text{AOIO}} + A_{\text{INV}} + A_{\text{n-input NAND_TREE}} + A_{\text{n-input NOR_TREE}}$$

The n-input NAND_TREE/NOR_TREE can be implemented by $(n - 1)$ number of NAND2-NOR2 elements, and it may have a similar sized implementation by using $\lceil \log_2 n \rceil$ stages of NANDm-NORm tree. Therefore,

$$A_{\text{multi}}(n) \approx A_{\text{AOIO}} + A_{\text{INV}} + (n - 1) \times A_{\text{NAND2}} + (n - 1) \times A_{\text{NOR2}}$$

Obviously, our design is much smaller than C-element tree implementation for $n > 2$, although both $A_{\text{tree}}(n)$ and $A_{\text{multi}}(n)$ are $O(n)$.

3 Experimental results

We present experimental results using the HP C34000 standard cell library [8]. The input-output path delay is obtained by simulating a Verilog™ model distributed with this cell library. Wiring capacitances are included in the model. For each n-input C-element, both the rise and fall delays for all input-output paths are simulated, i.e., $2n$ path delays are collected. The path delay of an n-input C-element is taken to be the average of these $2n$ delays. The delay variance among input-output paths is calculated by the difference between the maximum input-output path delay and the minimum input-output path delay of all input-output paths. The area of a design is computed as the sum of the area of all cells in the design. The results are shown in the Table 1. In this table, the name of Muller C-element suffixed with “A” is the series-parallel structure implementation, the name suffixed with “B” is our design, and the name suffixed with “AT” is the C-element tree implementation using “mullerC2A”, and the name suffixed with “BT” is the C-element tree implementation using “mullerC2B”.

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1 Verilog is a hardware description language, and it is a trademark of Cadence Design Systems, Inc.
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Table 1: Delay and delay variance and area comparisons of Muller C-element
Figure 7 shows the path delays in terms of $\log_2 n$. As expected, the path delay of the C-element tree is approximately equal to $[\log_2 n] \times K$, where $K$ is the delay of a 2-input C-element including wiring delay. Our design is much faster than the C-element tree implementation, and path delay grows very slowly as $n$ increases. For example, our 32-input C-element design is 3.89 to 4.43 times faster than C-element tree design. Figure 8 shows the delay variance among input-output paths in terms of $n$. The delay variance of our design is small for any number of inputs, but the delay variance of the tree implementation is only small for a balanced tree, i.e., where $n = 2^{[\log_2 n]}$. Finally, the area comparison between our design and the C-element tree implementation is shown in Figure 9. In both our design and C-tree implementation, the cell area grows linearly with $n$. However, the increasing area in our design is due to its NAND.TREE and NOR.TREE whereas the increasing area for the C-element tree implementation is due to the number of 2-input C-elements. Therefore, the rate of increase of area for our design is much smaller than for the tree implementation. For example, our 32-input design is 3.20 to 3.81 times smaller than the C-element tree implementation.

Figure 7: Comparison of path delay
Figure 8: Comparison of delay variance

Figure 9: Comparison of cell area
4 Conclusions

We have presented a new design for an n-input C-element, which is faster, has less delay variance among input-output paths, and is smaller than the C-element tree implementation. We have also demonstrated the advantages of our design using an industrial standard cell library.

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References


