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An Assessment of the Effects of Sound Speed Fluctuations on Sound Propagation in Shallow Water Using a Perturbation Method

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Abstract – Scintillations in the intensity of an acoustic signal are a common feature of propagation of sound in the sea, manifesting temporal variability in the index of refraction (sound velocity) of the medium. In this paper, a recently developed high-order perturbation method is described and applied to the problem of sound propagation in the sea. The method uses a "canonical" solution (sound speed profile) to form a set of basis functions that span the solution space and adequately represent the exact eigenvalue problem. The basis functions used in the calculations are derived from sound speed profiles obtained in an acoustic propagation experiment conducted in a shallow-water region of the Mediterranean. At particular source frequencies, calculations of modal functions and acoustic transmission loss were compared for the mean and several perturbed profiles. The results confirm the significant effects on acoustic transmission of seemingly minor variations in sound speed and, moreover, demonstrate the efficacy of the new perturbation method in handling such problems.

1. INTRODUCTION

The propagation of sound in the ocean is inevitably accompanied by fluctuations in the amplitude and phase of an acoustic signal received at large distances from the source. The fluctuations, or scintillations, are manifestations not only of changing patterns of interaction with the bottom and surface, particularly important in shallow-water propagation, but also passages of the wave through time-varying inhomogeneities in the ocean medium. The variability in acoustic propagation can be considered to arise from variations in the index of refraction, or sound velocity, of the medium, which, in turn, are induced by a variety of ocean processes covering a wide range of temporal and spatial scales. The fluctuations in the acoustic field are more than "merely" of academic interest, since they result in significant degradations in the performance of underwater acoustic systems.

Theoretical approaches to the propagation of sound in a realistic ocean environment are still evolving. Well-known deterministic numerical models include the parabolic equation (PE) method, fast field programs (FFP), the normal mode method, finite element methods, finite difference approaches, etc. Attempts to understand the non-deterministic component of acoustic intensity fluctuations have been centered around the theory of propagation through a random medium. Because the pertinent stochastic wave equation cannot be solved deterministically, solutions are obtained in terms of statistical moments of the acoustic pressure, such as the expected value (ensemble average), various correlation functions, and higher order moments. The fourth moment is of particular interest since this gives the intensity fluctuations. Much of the new work in extending the range of validity to large intensity fluctuations arising from multiple scattering is based on the non-deterministic version of the PE method. The PE method is a simplification of the wave equation for the case of scattering that is mostly in the forward direction. The two main methods of solving the non-deterministic PE are the Feynman path integral (used by Flatte, et al [1]) and the moment-equation (used by Uscinski, et al [2]). Both methods have had some recent success in attributing measured intensity fluctuations to the action of internal waves in deep water. The preceding success has been possible because of the availability of the required statistical description of the stochastic medium, viz, the Garrett-Munk (GM) empirical model [3,4]. In the upper ocean and in shallow water the GM model turns out to be inappropriate; instead, a quasi-deterministic explanation involving soliton propagation appears to be more relevant [5].

In this paper a recently developed high order perturbation technique is applied to the problem of sound propagation in a varying underwater acoustics environment. The method uses a "canonical" solution (sound speed profile) to form a set of basis functions that span the solution space and adequately represent the exact eigenvalue problem. This new "perturbation method" is far less limiting than conventional perturbation approaches since higher order terms are not neglected. The basis functions used in the calculations are derived from sound speed profiles obtained in an acoustic propagation experiment conducted in a shallow-water region of the Mediterranean. The question as to whether the varying sound speed profiles are manifestations of stochastic phenomena or quasi-deterministic events in the ocean environment is secondary to the purpose at hand. Simply stated, our primary objective is to demonstrate that the perturbation technique developed is eminently suitable for handling problems such as those treated here and, in fact, offers considerable advantages over conventional perturbation techniques.

The paper is organized as follows. In the following section the newly developed perturbation theory is summarized. This is followed by a brief description of the measured sound speed profiles and the environment in which they were obtained. Next, examples of the computed acoustic transmission loss for several mean and "perturbed profiles" are discussed. Finally, conclusions are provided.

2. THEORY

2.1 A New Full-Perturbation Method for Normal Modes

In this section we outline a new full perturbation approach to obtain solutions to the normal mode problem. This method proves to be very fast and particularly suitable for work on examining fluctuations in a waveguide, as will become apparent below. We write the exact solution for an unperturbed case as follows:

$$\frac{d^2\Psi_i}{dz^2} + \left[k_0^2 - \lambda_i^0\right]\Psi_i = 0\,, \qquad (1)$$

where, λ_i^0 are the horizontal wave numbers and ψ_i are the modal amplitudes from which the acoustic pressure field is obtained. To obtain the perturbed wave functions, U_i, we seek solutions to the equation:

$$\frac{d^2U_i}{dz^2} + \left[k^2(z) - \lambda_i\right]U_i = 0\,, \qquad (2)$$

which, for convenience we rewrite as follows:

$$\frac{d^2U_i}{dz^2} + \left[k_0^2 - \lambda_i^0\right]U_i = QU_i \qquad (3)$$

where

$$Q = k_0^2 - k^2(z) + \lambda_i - \lambda_i^0 = q + \Delta\lambda_i\,,$$

and

$$q(z) = k_0^2 - k^2(z) \text{ and } \Delta\lambda_i = \lambda_i - \lambda_i^0\,.$$

For the iso-velocity case we can rewrite Eq. 1 as:

$$\frac{d^2\Psi_i}{dz^2} + \alpha_i^2\Psi_i = 0 \text{ where } \alpha_i^2 = k_0^2 - \lambda_i^0\,.$$

We impose the following orthonormality conditions:

$$\left(\Psi_i\,,\frac{1}{\rho}\Psi_j\right) = \delta_{ij} \text{ and } \left(U_i\,,\frac{1}{\rho}U_j\right) = \delta_{ij}\,.$$

We assume closure so that we can express U_i:

$$U_i = \sum_{j=1}^{N} a_{ij}\Psi_j\,. \qquad (4)$$

We insert Eq. 4 into Eq. 3 to arrive at:

$$\sum_{j=1}^{N} a_{ij}\left(\alpha_i^2 - \alpha_j^2\right)\Psi_j = \sum_{j=1}^{N} a_{ij}\left(q(z) + \Delta\lambda_i\right)\Psi_j \qquad (5)$$

where $i = 1, 2, \ldots N$.

By integrating the overlap of the above expression with Ψ_i we obtain:

$$\lambda_i = \lambda_i^0 - q_{ii} - \sum_{i\neq j}^{N} \tilde{a}_{ij}\,q_{ij} \text{ where } \tilde{a}_{ij} = \frac{a_{ij}}{a_{ii}}\,. \qquad (6)$$

This yields the Eigenvalue Correction Equation

By integrating the overlap of Eq. 5 with Ψ_k we obtain:

$$\tilde{a}_{ik}\left(\alpha_i^2 - \alpha_k^2 - \Delta\lambda_i\right) - \sum_{j\neq i}^{N} q_{jk}\,\tilde{a}_{ij} = q_{ik}$$

where $k = 1, 2, 3 \ldots N$ and $i \neq k$.

This is an eigenvalue equation. We can use the expansion for the eigenvalue above to rewrite the equation as follows:

$$\tilde{a}_{ik}\left(\alpha_i^2 - \alpha_k^2 + H_{ik}\right) - \sum_{j\neq\{i,k\}}^{N} q_{jk}\,\tilde{a}_{ij} = q_{ik}\,.$$

Where $k = 1, 2, 3 \ldots N$ and $i \neq k$.

The diagonal terms are $\left(\alpha_i^2 - \alpha_k^2 + H_{ik}\right)$

where $H_{ik} = q_{kk} - \Delta\lambda_i$.

This will prove useful later. The H_{ik} terms contain the first and the higher order terms, some of which are negligible in many cases. The diagonal terms are almost always greater than the off-diagonal terms. Thus we may in some cases use the Gauss-Seidel method. The first iteration via Gauss-Seidel leads to:

$$\tilde{a}_{ik} = \frac{q_{ik}}{\alpha_i^2 - \alpha_k^2 + q_{kk} - \Delta\lambda_i} = \frac{q_{ik}}{\alpha_i^2 - \alpha_k^2 - H_{ik}}$$

where we rewrite $H_{ik} = q_{kk} - q_{ii} - \sum_{j\neq i}^{N} a_{ij}\,q_{ij}$.

This can be a big improvement over the ordinary perturbation term in that $H_{ik} > q_{ik}$ and thus we expect that generally we will have convergence if the master matrix is diagonally strong. We see readily that we can arrive at two improved perturbation theories by retaining the full form of H_{ik} or by excluding the last term (the higher order terms) of H_{ik}. The advantage to excluding the last term is that we need not employ a sophisticated perturbation approach to be discussed below. Further this expression shows that the old theory overestimated the expansion coefficient.

The new complete perturbation expansion for the eigenvalue is now:

$$\lambda_i = \lambda_i^0 - q_{ii} - \sum_{i\neq j}^{N} \frac{q_{ij}\,q_{ji}}{\alpha_i^2 - \alpha_j^2 + H_{ij}}\,.$$

This expression indicates that the first order correction is still the same but that the higher order corrections are over/under estimated in the old theory. The preceding equations can be reordered to form an eigenvalue problem. The details of the solution of this problem will be omitted here. Instead, we proceed directly to the results in a form most relevant for our present application, namely the expression for the acoustic field. The acoustic field can be readily computed as a sum of appropriate wavefunctions obtained by the usual method of separation of variables. This leads to the following expression for the acoustic pressure in an acoustic duct of water thickness h:

$$P = \rho\omega^2 \sum_{lm} a_{lm} \sin \alpha_m Z \frac{e^{ik_l r}}{\sqrt{k_l r}} e^{-\beta_l r} \quad 0 \le Z \le h .$$

where the a_{lm} and α_m are as defined previously, ρ is density, ω is radian frequency, $k_l = \sqrt{\lambda_l}$ is the horizontal wavenumber, and β_l is the attenuation coefficient.

3. THE MEASURED SOUND SPEED PROFILES

The sound speed profiles used here were measured during an acoustic propagation experiment conducted in shallow water (80 m) in the Mediterranean by the SACLANT Undersea Research Centre.

Figure 1 provides an example of the variation of sound speed with depth over a period of 25 hours (Ali et al [6]). Figure 1a plots in the usual manner the sound speed profiles over approximately hour intervals. In Figure 1b the same profiles are displayed side by side to clearly demonstrate the nature of the variation, while Figure 1c is shown to emphasize the sound speed fine-structure. From the surface down to about 20 m the region appears to be essentially isovelocity with sound speed of approximately 1538 m/s. The depth from approximately 25 to 35 m comprises the steepest portion of the thermocline. Fluctuations in sound speed are quite evident, particularly at a depth of about 25 m within the thermocline. The plot clearly indicates an oscillation in the thickness of the mixed layer (surface duct). Further, "kinks", typical features in the curve, appear to migrate vertically with time - a motion often suggesting the presence of internal waves (Gregg [7]). Of particular note is the fine-structure evident in the expanded portion of the profile (Fig. 1c.). Generally, the fine-structure in this example was characterized by vertical dimensions of the order of a few centimeters to one or two meters. This fine-structure tends to scatter high frequency sound.

The most salient characteristics of the sound speed profiles of Figure 1 are the negative gradients of the thermoclines. As a result, the sound fields will be dominated by downwardly propagating energy, resulting in substantial interaction with the seafloor and, most likely, channeling into a bottom sound duct. However, the differences between the individual profiles, manifested by the apparent oscillation of the surface duct thickness, are likely to lead to distinguishable differences in acoustic transmission, even through the general characteristics of the propagation may be similar. It is well known that even a relatively small change in sound speed may lead to significant alterations in underwater acoustics propagation, particularly over long ranges.

4. RESULTS OF THE CALCULATIONS

In order to concentrate on the essential physics of the propagation in the water column, we have modelled the medium as an 80-m water column overlying an infinite half space bottom with the following geoacoustic properties: compressional speed and attenuation 1650 m/s and 0.8 dB/λ, respectively, and relative density 1.9. These parameters are typical of sand-silt bottoms. We have ignored shear effects in the bottom, which, for the relatively high frequencies we are considering here, play a negligible role on the propagation in the water column.

Calculations of the propagating acoustic field—in particular the mode shapes (wave functions) and transmission loss as a function of receiver depth in the water column — were made for several different frequencies and source depths. To demonstrate the effect on the acoustic field of differences in the sound speed profiles, we could have proceeded in several ways. One method is to compute the appropriate field for each measured profile and compare the results for the various profiles. An

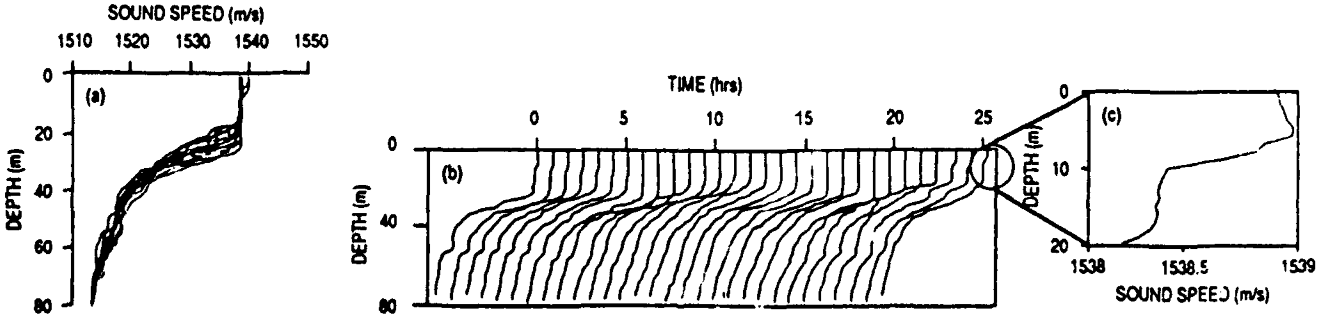

Figure 1. Measured sound speed profiles over approximately 25 hours.

alternative approach is to generate a depth-averaged profile from the measured set and compare the acoustics of this "mean profile" with that of each of the others. We chose this second approach for at least two reasons. The first is mathematical: a key strength of the mathematical model is the ease with which it handles a perturbation effect. Therefore, each of the measured profiles is considered a perturbation from the mean profile, with readily computable coefficients for the normal mode expansion. Second, comparison with a mean profile is sensible from a physical point of view. The mean profile, although it does not represent an actually measured profile, helps enhance our understanding of the physics of the propagation. For, the mean profile represents a "smoothed" version of the actual profiles: it eliminates finestructure and other "details" while retaining the essential large-scale features of the propagation problem. But, of course, it is these details which distinguish the various profiles; and these effects are more readily apparent if compared against a mean profile than against each other.

The results shown here were calculated for three profiles: the mean and two perturbed profiles, the latter chosen to represent extremes. The profiles are shown in Figure 2; Profile 1 is the first profile displayed in Figure 1, while Profile 2 is profile 28, almost at the end of the display. For each of the three profiles shown, a number of normal modes was calculated, for a source depth of 70 m and a frequency of 3 kHz. In Figure 3, several of the modes are compared. There are several noteworthy features. Independent of profile, the lowest mode (which corresponds to the smallest grazing angle for the equivalent ray) is more concentrated in the region of low sound speed below the thermocline, close to the bottom. The higher order modes (with larger grazing angles) occupy substantially greater portions of the water column. A part from the preceding behavior, expected for all the profiles, the modes reveal individual differences attributable to variations among the profiles. It is noted that, for the lower modes, Profile 1

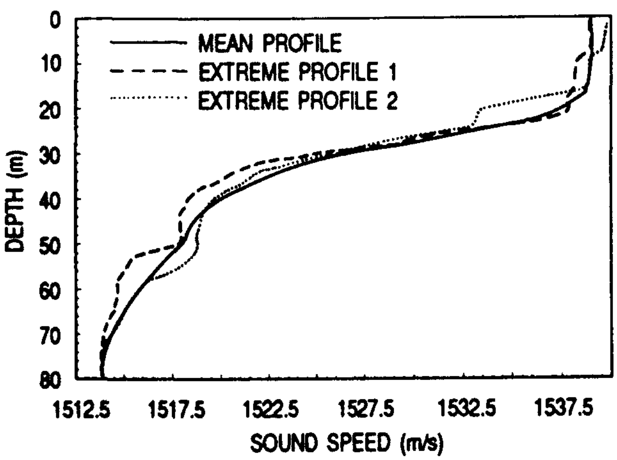

Figure 2. The depth-averaged mean and two extreme sound speed profiles.

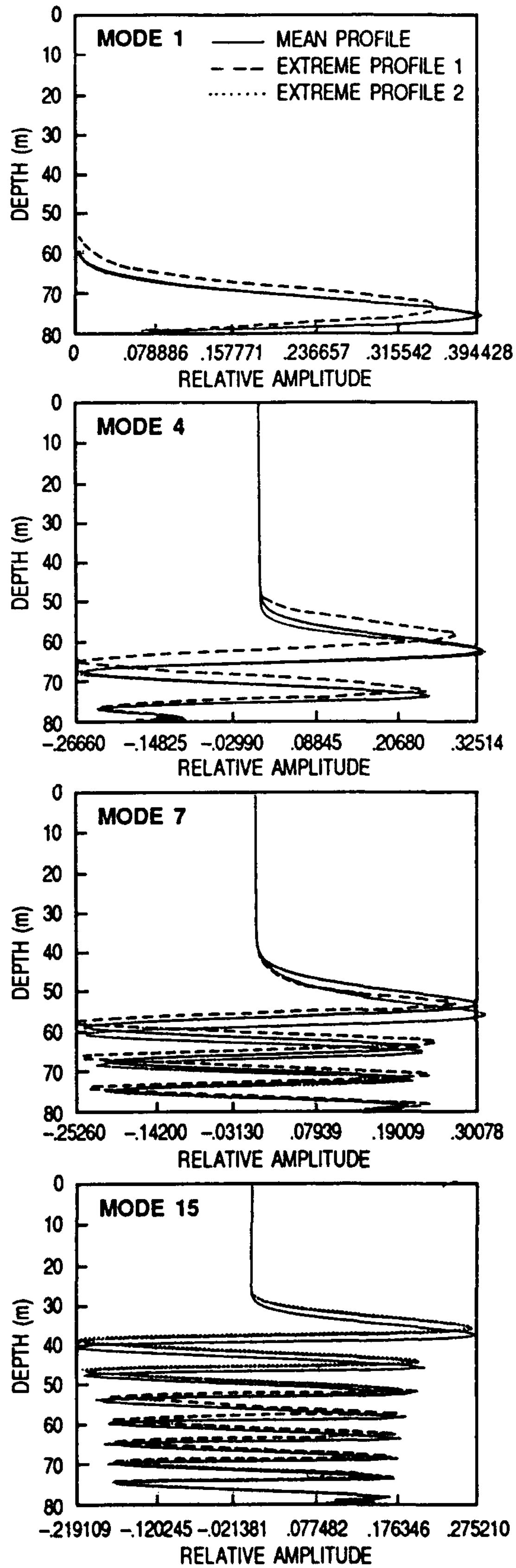

Figure 3. Selected modal functions at 3 kHz.

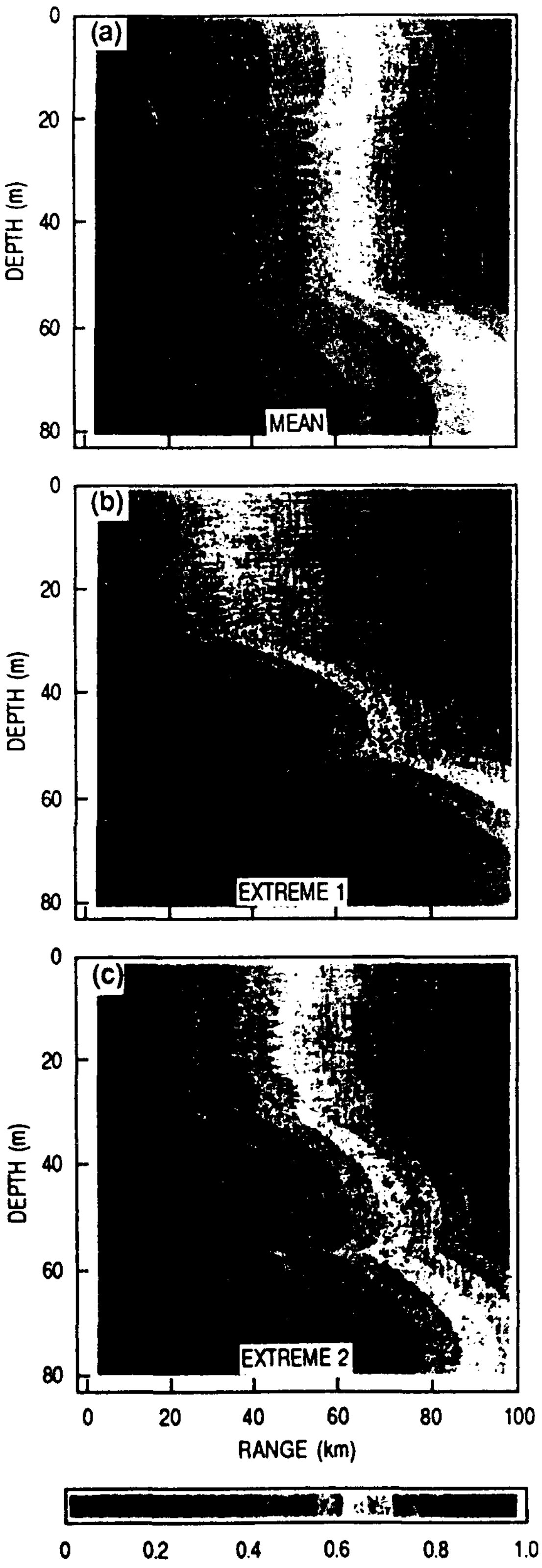

Figure 4. Transmission loss for a 3-kHz source at a depth of 70 m for (a) the mean profile, (b) extreme profile 1, and (c) extreme profile 2.

shows a greater deviation from the mean profile than does Profile 2. Though not shown here, this behavior is consistent for at least the first 6 modes. For higher modes, the trend is less predictable. Since the acoustic field is obtained in terms of a coherent sum of the modal functions, the behavior evidenced in Figure 3 suggests that Profile 1 will lead to propagation characteristics more noticeably different from the mean profile than will Profile 2. This supposition is based on the fact that the lower order modes will contribute more than the higher modes to the propagation, particularly at greater ranges, since the lower modes will be less degraded by interaction with the bottom.

On the whole, the transmission loss contours shown in Figure 4 support the preceding *a priori* contention. As seen, channeling along the bottom is the dominant feature of all three figures, and the degree of channeling (i.e., the energy levels) of the mean and Profile 2 cases are very similar. Profile 1 clearly leads to stronger coupling with the bottom channel. In retrospect, one could have expected this, based on the somewhat larger negative gradient seen in Profile 1 (Figure 2). An important common feature of the two perturbed sound speed profiles, not shared with the mean profile, is the formation of a second duct centered around 45-50 m depth. Clearly, this second channeling is attributable to the quasi-bilinear feature evident in the two profiles at approximately 50 m. On the other hand, the mean profile leads to a fairly well-defined surface duct centered around a depth of 10 m, a result whose plausibility follows from a perusal of Figure 2. A closer look at the propagation behavior at particular depths may be obtained by a conceptual horizontal slice of Figure 4. The results at two depths, 45 m and 75 m, are shown out to a range of 30 km in Figure 5. These figures show even more strikingly the increasing divergence in transmission loss arising from differences in the sound speed profiles.

One could continue in this vein, obtaining a good deal of useful and interesting information regarding the subtleties of sound propagation arising from variations in the sound speed. But the objective here was to demonstrate the utility of the newly-developed theoretical approach to handle problems of this nature.

5. SUMMARY AND CONCLUSIONS

A newly-developed high-order perturbation method has been described and applied to the problem of sound propagation in a time-varying ocean environment. Using a series of measured sound speed profiles a set of basis functions was generated and used to calculate the resulting sound propagation in a shallow-water environment. The results confirm the significant effects on acoustic transmission of seemingly minor variations in sound speed and, moreover, demonstrate the efficacy of the new perturbation method in handling such problems.

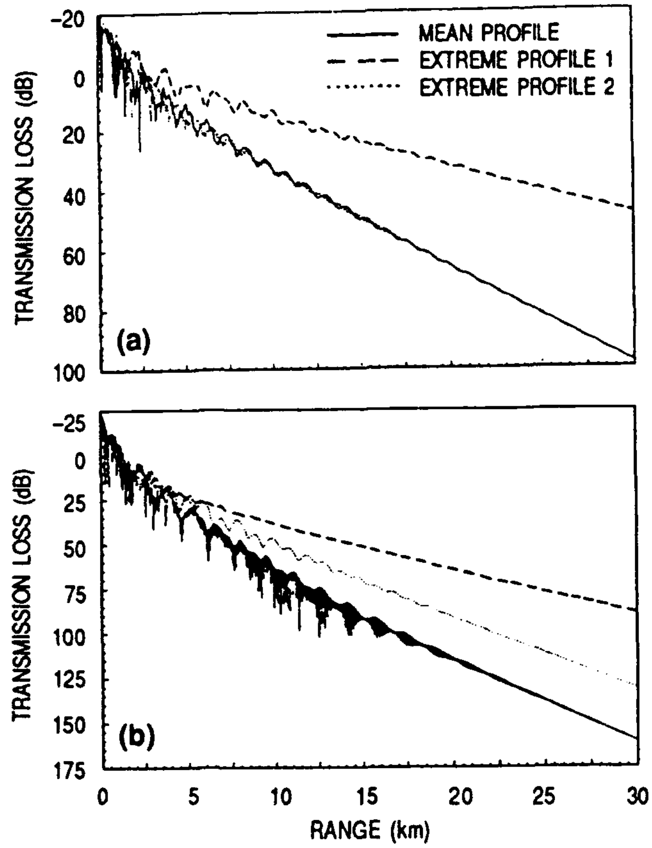

Figure 5. Transmission loss at receiver depths of (a) 75 m and (b) 45 m, for a 3-kHz, 70-m depth sound source.

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