
In the process of scattering from submerged elastic shells, it is possible to excite many types of resonances. Among these are the lowest order symmetric and antisymmetric Lamb modes, and the waterborne waves, such as the pseudo-Stoneley resonances and the higher order symmetric and antisymmetric Lamb modes $S_i$ and $A_i$ ($i = 1, 2, 3, ...$), the frequency at which these originate are referred to as critical frequencies. We establish simple rules to determine the frequencies at which the resonances originate as a function of shell thickness and material properties.
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Critical Frequencies in Scattering from Submerged Elastic Shells

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Abstract — In the process of scattering from submerged elastic shells, it is possible to excite many types of resonances. Among these are the lowest order symmetric and antisymmetric Lamb modes, and the waterborne waves, such as the pseudo-Stoneley resonances and the higher order symmetric and antisymmetric Lamb modes \( S_i \) and \( A_i \) for \( i = 1, 2, 3, \ldots \). The frequency at which these originate are referred to as critical frequencies. We establish simple rules to determine the frequencies at which the resonances originate as a function of shell thickness and material properties.

1. INTRODUCTION

The presence of resonances generated from acoustical signals impinging on submerged elastic shells has been known for some time. In particular, the presence of the symmetric or dilatational Lamb mode \( S_0 \) as well as the lowest order antisymmetric Lamb or Flexural mode \( A_0 \) are well known and frequently studied. Moreover, the existence of higher order symmetric \( S_i \) and antisymmetric \( A_i \) Lamb modes \( i > 0 \) manifest themselves with increasing frequency [1]. In addition, newly studied phenomena such as pseudo-Stoneley resonances [2-4] and pure waterborne waves [5] have received attention recently. All but the last phenomena have analogues for the infinite flat plate case which is fluid loaded on one side and evacuated on the other.

It is usual to associate resonances with vibrations, and the presence of the Lamb resonances on spherical shells can be associated with symmetric or antisymmetric vibrations that at discrete frequencies form standing waves on the object surface. These standing waves radiate into the fluid and add coherently with the specularly scattered signal producing a characteristic signature. The nature and appearance of the resonances just described are a function of material characteristics and shell thickness in addition to frequency. For very thin shells the lowest order resonance has a large amplitude and is in a region where there is a large recoil effect leading to both a large monopole term as well as the dipole term associated with the recoil effect. The subsequent symmetric Lamb modes are characterized by a sharp minimum followed by a sharp rise and then a return to a normal slowly varying back scattered return signal (form function). Flexural or antisymmetric resonances do not arise until the flexural phase velocity equals the speed of sound in the fluid [4, 6] (subsonic material waves are too heavily dampened to be observed); this value of frequency is referred to as coincidence frequency. At and a little below coincidence frequency another phenomenon enters the picture, namely sharply defined waterborne waves which have their analogue in flat plates, namely Stoneley waves. Thus, the resonances that arise from these waterborne waves are labeled pseudo-Stoneley resonances. They occur only in the frequency region about coincidence frequency and give rise to very sharp spikes superimposed on broadly overlapping flexural resonances. This effect can be very dramatic. Another dramatic effect arises from the \( S_1 \) symmetric resonance which is a separate topic presented by Werby and Gaunaurd [7, 8]. Interestingly the onset of all of the higher order Lamb resonances can be obtained from the simple expressions used to predict the critical frequencies for the flat plate case. We will demonstrate this effect by employing the residual partial wave analysis (the partial wave component minus the exact acoustical background for a shell). It is only possible to perform the correct partial wave analysis if one has the correct background for the elastic shell [9, 10].

2. ACOUSTIC SCATTERING FROM SUBMERGED ELASTIC SHELLS

2.1 The Form Functions for Aluminum and Steel for \( ka \) from 0 to 500

We illustrate in this section the form function for 5% thick Aluminum and Steel shells. Figure 1a–d represent backscatter from aluminum 1(a) from \( ka = 0 \) to 250, 1(b) the residual results (with background subtracted) from 0 to 250, 1(c) for aluminum from 250 to 500, and 1(d) the residual for aluminum from 250 to 500. Figure 2a–d represent backscatter from steel 2(a) from \( ka = 0 \) to 250, 2(b) the residual results from 0 to 250, 2(c) for steel from 250 to 500, and 2(d) the residual for steel from 250 to 500. It is clear that a great deal of detail is present in each of the plots. The low frequency large returns with the sharp spikes for both materials are due to a superposition of the pseudo-Stoneley resonances with the weaker broadly overlapping flexural resonances. The higher frequency resonances (about \( ka = 250 \) in both cases) are due to the onset of the \( S_1 \) Lamb mode. We have indicated in the plots the onset of each of the modes.
2.2 Discussion of Pure Waterborne Waves

We have earlier discussed pseudo-Stoneley waves [3, 4]. There is another phenomenon that corresponds to waves that have a phase velocity that is about the speed of sound in water. They are not, however, sharply defined in partial wave space, nor are they associated with the flexural wave or coincidence frequency. They are associated with the density of the material, and the thickness (really just the mass of the target) and the frequency. Their importance increases with frequency and they do not manifest themselves as sharp resonances in the form function but rather wash out other resonances such as \( S_0 \) and \( A_0 \) resonances. Thus, for light material and thin shells such as aluminum and at high frequency one does not observe sharp resonances due to this washout effect. We will not discuss this effect here.

2.3 A Partial Wave Analysis

If one subtracts the correct background [9, 10] from the elastic response then by definition one is left with the "pure" resonance response. Resonances excited on bodies of canonical shape usually correspond to circumferentially excited waves which for spheres have a unique wave number. To be sure, this fact can be obscured by, for example, broadly overlapping partial waves; but none the less plotting the residual partial wave components—which is here referred to as a partial wave analysis—can be very revealing. There are two ways to perform a partial wave analysis: one can fix the mode number \( N \) and plot the residual response with respect to \( k_a \). On the other hand one can fix \( k_a \) and plot the partial wave function with respect to \( k_a \). The first of these approaches is the most commonly used.

Figure 1a–c illustrates the PWA for 5% thick aluminum shells out to a \( k_a \) of 500 for modes 1, 2, and 10. We have listed the onset of the different Lamb modes in Table 1 and indicated with arrows in the plots here the critical frequencies for each case. The same has been done for steel in Figure 1a–c. It is clear that the simple expressions listed in Table 1 and the computed values agree with the onset of the higher order Lamb modes.
Figure 2. (a) Backscatter from 5% steel shell from ka = 0 to 250; (b) residual backscatter for case 1a; (c) backscatter from 5% steel shell from ka = 250 to 500; and (d) residual backscatter for case 1c.

TABLE 1

<table>
<thead>
<tr>
<th>CRITICAL FREQUENCIES FOR THE HIGHER ORDER LAMB MODES</th>
<th>kash = π(ν,a)h/k h IS % THICKNESS OF SHELL.</th>
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<tbody>
<tr>
<td>ALUMINUM</td>
<td>STEEL</td>
</tr>
<tr>
<td>A1 129.3</td>
<td>A1 140.2</td>
</tr>
<tr>
<td>S1 258.5</td>
<td>S1 243.9</td>
</tr>
<tr>
<td>S2 269.1</td>
<td>S2 280.3</td>
</tr>
<tr>
<td>A2 387.8</td>
<td>A2 420.5</td>
</tr>
<tr>
<td>A3 403.7</td>
<td>A3 487.9</td>
</tr>
<tr>
<td>S3 517.1</td>
<td>S3 560.6</td>
</tr>
<tr>
<td>S4 538.2</td>
<td>S4 731.9</td>
</tr>
</tbody>
</table>

2.4 Phase Velocity Plots

We have included in this work the phase velocities for the steel shell illustrated in Figure 5. Here we include the pseudo-Stoneley resonance (Fig. 5a), the pure waterborne wave (Fig. 5b), the A0 resonance (Fig. 5c), the S1 resonance (Fig. 5d), the A1 resonance (Fig. 5e), the S2 resonance (Fig. 5f), the S2 resonance (Fig. 5g), the A2 resonance (Fig. 5h), and the A3 resonance (Fig. 5i). Note that the onset of each of the higher order Lamb resonances conforms to the values listed in Table 1. Further note that the S1 resonance has a phase velocity that in effect decreases at some point (early on) then increases and then decreases again.

3. CONCLUSION

This is only a preliminary study of a large ongoing study of Lamb resonances. It is encouraging that most effects are easily understood in terms of flat plate theory.
Figure 3. Partial wave for aluminum: (a) mode 1, (b) mode 2, and (c) mode 10.

Figure 4. Partial wave for steel: (a) mode 1, (b) mode 2, and (c) mode 10.
and that the critical frequencies can be predicted by such simple expressions. Further some of the more dramatic effects such as the pseudo-Stoneley resonances and those due to the $S_1$ resonance can be interpreted.

4. ACKNOWLEDGMENTS

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5. REFERENCES

Figure 5. Phase velocity for steel (cont.) (f) $S_1$ resonance; (g) $S_2$ resonance; (h) $A_2$ resonance; and (i) $A_3$ resonance.


