This report presents a summary of 14 papers published in archival literature, dealing with the issues of nonlinear dynamics of finitely deformed space structures, mixed time finite element methods for large rotational motions of constrained multibody systems, and wave propagation in lattice structures.
INNOVATIVE METHODS IN
THE DYNAMICS OF
LARGE SPACE STRUCTURES

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In this final report, a descriptive summary of research accomplished under AFOSR Contract No. F49620-87-C-0064 is presented. Technical details of research, mathematical formulations, their computational implementations, and their verifications are presented in the archival papers listed at the end of the report.

In [1] the problem of transient dynamics of highly flexible three-dimensional space-curved beams, undergoing large rotations and stretches, is treated. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is considered. Using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangian description of motion, the mass-matrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotation parameters and attendant accelerations; the stiffness matrix depends nonlinearly on the deformation; and an 'apparent' damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semi-discrete finite element equations in time. Several examples of transient dynamic response of highly flexible beam-like structures, including those in free flight, are presented to illustrate the validity of the theoretical methodology developed in this paper.

The following topics are discussed in [2]: (i) some of the recent advances in formulating finite deformation (large rotations as well as stretches) plate and shell theories, and attendant mixed finite element formulations based on symmetric variational statements; (ii) finite element/boundary element formulations based on unsymmetric variational statements, Petrov-Galerkin methods, and the use of fundamental solutions in infinite space, for the highest-order differential operator of the problem, as test functions in solving nonlinear plate and shell problems; and (iii) algorithms for solving the problems of control of nonlinear dynamic motion of plates and shells.

Article [3] deals with nonlinearities that arise in the study of dynamics and control of highly flexible large-space-structures. Broadly speaking, these nonlinearities have various origins: (i) geometrical: due to large deformations and large rotations of these structures and their members; (ii) inertia: depending on the coordinate systems used in characterizing the overall dynamic motion as well as elastic deformations; (iii) damping: due to nonlinear hysteresis in flexible joints; viscoelastic coatings, etc., and (iv) material: due to the nonlinear behavior of the structural material. The geometrical and material nonlinearities affect the "tangent stiffness operator" of the structure; the inertia nonlinearities affect the "tangent inertia operator".

To study the nonlinear transient dynamic response and control of flexible space-structures, one may think of: (i) semi-discrete approximation methods, and (ii) space-time methods. In the former class of methods, an appropriate spatial discretization is employed through weak-formulations (finite-element and field/boundary
element) in space, and thus a set of coupled nonlinear ordinary
differential equations (O. D. E.) is derived. These O. D. E.s are
solved often through temporal integration techniques of the finite
difference-type. The semi-discrete methods are not ideally suited
for traveling-wave type propagating disturbances. The second
category of methods, viz., the space-time methods, wherein weak
formulations in both space and time are employed, are somewhat
better suited for wave-propagation type problems. In this article,
attention is primarily focused on semi-discrete methods.

Depending on the scale of the response that is required to be
studied, a large-space-structure may either be modeled as an
equivalent continuum, or as a lattice structure with the details of
each member being accounted for. The spatial discretization in
either case is required to be of the least-order possible so that
the control algorithms may be meaningfully implemented. The
reduced-order-modeling of the "tangent stiffness" operator of
either a continuum model, or a lattice model of a space-structure
is treated in some detail in this paper, for structures undergoing
large dynamic deformations.

The control of dynamic motion of space-structures is currently
envisaged to be through either active processes, passive processes,
or some combinations thereof. One of the concepts of active
control that is considered in detail in [3], and by other authors,
is the use of piezo-ceramic actuators that are bonded to the truss
and frame members of the space-structure in various locations. The
controlling shear stress transmitted by the actuator to the truss
by frame member depends on the axial force, transverse shear
forces, and bending and twisting moments, in the member itself, as
well as the excitation voltage applied to the piezo-actuator. This
problem of mechanical coupling between the structural member and
actuators is discussed in some detail in this work.

The problem of control of nonlinear dynamic motion is
addressed in [3]. The problem is posed in the form of determining
the feedback gain matrix and the attendant control force vector,
such that the response as predicted by a semi-discrete system of
coupled nonlinear ordinary differential equations, subject to a set
of arbitrary initial conditions, is damped out in a pre-set time.

In the first part of article [3], continuum models of space-
structures are analysed. These include models of the space-beam
type as well as the shallow shell type. In the case of space-beams,
the problem of nonlinear dynamic response, when the beam undergoes
large overall rigid as well as elastic motion, is discussed. The
beam is assumed to undergo large rotations as well as stretches. A
simple finite element algorithm to predict the response is
presented. When a shallow-shell type continuum model is used, a
field-boundary element approach based on nonlinear integral
equations is presented as a means to create a reduced-order dynamic
model of the semi-discrete type. A simple algorithm to control the
response predicted by these nonlinear semi-discrete equations is
discussed.

In the second part of [3], detailed models of the lattice type
space-structures are discussed. Each member of the structural
lattice is assumed to be either a "truss member", or as a "frame
member." The "truss member" is assumed to carry only an axial load,
and has three displacement degrees of freedom at each node. The "frame member" is assumed to carry an axial force, transverse shear forces, bending moments, and a twisting moment; and is assumed to have three displacement and three rotational degrees of freedom at each node. Explicit expressions for the tangent stiffness matrices of both "truss" type and "frame" type members, which undergo arbitrarily large displacements, arbitrarily large overall rigid rotations, and moderate local (relative) rotations, are derived. In all cases, each member (truss or frame type) is modeled by a single finite element, in the entire range of large deformations. Several examples are presented to illustrate the efficiency and on-board computational feasibility of these reduced-order models for lattice structures. In each instance, remarks on needs for future research are made.

The problem of transient dynamics of highly-flexible 3-dimensional space-beams, undergoing large rotations and stretches, is treated in [4]. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. Based on the present governing equations, a general mixed variational principle for the static problem is presented. Furthermore, using the three parameters associated with a conformal rotation vector representation of finite rotations, a well-defined Hamilton functional is established for the dynamic problem of a flexible beam undergoing finite rotations and stretches. This is shown to lead to a symmetric tangent stiffness matrix at all times. In the present total Lagrangian description of motion, the mass-matrix of a finite element depends linearly on the linear accelerations, but nonlinearly on the rotational parameters and attendant angular accelerations; the stiffness matrix depends nonlinearly on the deformation; and an "apparent" damping matrix depends nonlinearly on the rotations and attendant velocities. A Newmark time-integration scheme is used to integrate the semi-discrete finite element equations in time. An example of transient dynamic response of highly flexible beam-like structures in free-flight is presented to illustrate the validity of the theoretical methodology developed in this paper.

A scheme for active control of nonlinear vibration of space-structures, wherein each member is modeled as a beam-column, is presented in [5]. The expressions for shear stresses transmitted to the structural member by the distributed segmented piezo-electric actuators, which are bonded on the surfaces of the member, are derived in the general case in which the structural member is subjected to moments, transverse shear forces and an axial force. Based on the weak form of the governing equations, and a complementary energy approach based on assumed stress fields, the viability of active control of nonlinear dynamic response of lattice-type space structures, using piezo-electric actuators, is studied. Four examples are given to demonstrate the feasibility of the approaches presented in this paper.

The deformation of a beam-column, the upper and lower surfaces of which are bonded in segments with piezo-ceramic liners, is studied in [6] for the purpose of obtaining appropriate expressions for the force transferred to the structural member by the piezo-actuator. This concept may be employed for the control of large
dynamic deformations of a lattice-type flexible space-structure. The present model, which is based upon a static analysis, accounts for the effects of transverse shear and axial forces in addition to a bending moment on the beam in formulating the governing equilibrium equations. The present model provides more complete expressions for the force transmitted to the structural member than a model reported earlier in literature, in which the shear and axial forces are neglected.

The dynamic response of frame-type structures with hysteretic damping at the structural joints, resulting from slipping and nonlinear flexible connections, is investigated in [7]. The slipping at a structural joint is represented by the modified Coulomb joint model. The behavior of a nonlinear flexible connection is modeled by the Ramberg-Osgood function. A simple computational model for the dynamic analysis of frames with hysteretic damping is presented here. Several numerical examples are included to illustrate the usefulness of the approach in analyzing large space structures.

The prediction of transient response of structures, in the form of traveling waves, is very important for controlling the dynamic behavior of structures. It is well known that the standard semi-discrete form of the finite element method is not suitable for predicting the wave propagation, due to the inherent dispersion involved. In [8], an application of space-time finite element method to the wave propagation problem is discussed. The main concerns in such problems consist of developing a consistent and stable scheme and also of capturing a shock wave, without wiggling. At first, a weak form of the wave propagation problem is discussed, taking into account the jump condition associated with velocity and stress. A mixed finite element formulation plays an important role in evaluating the velocity explicitly. The application of present formulation to the linear wave equation shows that the present numerical results at the discontinuity give the mean values of jump. In the case of flexural wave propagations in a Timoshenko beam, the present method captures the wave front easily, as opposed to the semi-discrete method.

A novel theory and its computational implementation are presented in [9] for the analysis of strongly nonlinear dynamic response of highly-flexible space-beams that undergo large overall motions as well as elastic motions with arbitrarily large rotations and stretches. The case of conservative force loading, which may also lead to configuration-dependent moments on the beam, is treated. A symmetric tangent stiffness matrix is derived at all times even if the distributed external moments exist. An example of transient dynamic response of the beam is presented to illustrate the validity of the theoretical methodology.

Currently, there is renewed interest in the study of multi-body-dynamics and its application in many fields of engineering. The mathematical model of a rigid body is useful whenever the overall motion, involving large rigid rotation, is of interest. The nonlinear dynamic equations of motion, in their explicit form, appear quite complex due to the expression for the absolute accelerations. In [10], weak formulations of linear and angular momentum balance laws of a rigid body undergoing large overall
motion are stated a priori. Holonomic as well as nonholonomic constraints, that may exist on the motion of the rigid body, are introduced into this weak form in a fundamentally novel fashion here. Comments are made on the incremental form (and consistent linearization) of the weak formulation (with constraints), and the time-finite-element solutions thereof.

The paper referred to in [11] presents general variational formulations for dynamic problems, which are easily implemented numerically. The development presents the relationship between the very general weak formulation arising from linear and angular momentum balance considerations, and well known variational principles. Two and three field mixed forms are developed from the general weak form. The variational principles governing large rotational motions are linearized and implemented in a time finite element framework, with appropriate expressions for the relevant "tangent" operators being derived. In order to demonstrate the validity of the various formulations, the special case of free rigid body motion is considered. The primal formulation is shown to have unstable numerical behavior, while the mixed formulation exhibits physically stable behavior. The formulations presented in this paper form the basis for investigations into constrained dynamical systems and multi-rigid-body systems.

Constrained equations arise in the dynamics of mechanical systems whenever there is the need to restrict kinematically possible motions of the system. In practical applications, constraint equations can be used to simulate complex, connected systems. If the simulation must be carried out numerically, it is useful to look for a formulation that leads straightforwardly to a numerical approximation. In [12], the methodology of previous work is extended to incorporate the dynamics of holonomically and nonholonomically constrained systems. The constraint equations are cast in a variational form, which may be included easily in the time finite element framework. The development of the weak constraint equations and their associated "tangent" operators is presented. This approach to constraint equations may be employed to develop time finite elements using a quaternion parametrization of finite rotation.

Weak formulations in analytical dynamics are developed, parallelizing the variational methods in elastostatics, and including a fundamental yet novel approach for treating constraints (both holonomic and nonholonomic). A general three-field approach is presented, in which the momentum balance conditions, the compatibility conditions between displacement and velocity, the constitutive relations and the displacement and momentum boundary conditions are all enforced in weak form. A primal, or kinematic, formulation is developed from the general form by enforcing the compatibility conditions and displacement boundary conditions a priori. The conditional stability of the kinematic formulation is the counterpart of the locking phenomenon in elastostatics and may be avoided, either by reduced order integration or by utilizing a mixed formulation. Toward this end, a two-field mixed formulation is presented, which follows from the general form, when the constitutive relations are satisfied a priori. A general set of the constraint equations is introduced into the kinematic and mixed
formulations, using a specific choice of multipliers, which results in modified variational principles. Several simple examples concerning rigid body dynamics are presented [13].

The article in [14] deals with the effect of non-linearly hysteretic joints on the static and dynamic response of space frames. It is shown that a complementary energy approach based on a weak form of the compatibility condition as a whole of a frame member, and of the joint equilibrium conditions for the frame, is best suited for the analysis of flexibly jointed frames. The present methodology represents an extension of the authors' earlier work on rigidly connected frames. In the present case also, an explicit expression for the tangent stiffness matrix is given when (i) each frame member, along with the flexible connections at its ends, is represented by a single finite element, (ii) each member can undergo arbitrarily large rigid rotations and only moderate relative rotations and (iii) the non-linear bending-stretching coupling is accounted for in each member. Several examples, with both quasi-static and dynamic loading, are included to illustrate the accuracy and efficiency of the developed methodology.
References:


