FOREIGN TECHNOLOGY DIVISION

BURN PHASE CORRECTIONS OF MANEUVER BALLISTIC Missiles WITH STAR LIGHT

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Abstract In this paper, the maneuver ballistic missile parametric correction in the burning phase with star light is studied, when the maneuver ballistic missile is launched under the condition that the initial position is not exactly defined and the inertial navigation platform is not completely leveled and aligned.

Key words star sensor, star light, star map.

I. FORWARD

Taking inertial techniques as the primary ones, and, in conjunction with this, taking star light, radio, and other similar techniques to be auxilliaries in the assembly of navigation and guidance technologies, from the 1970s onward, the development has been relatively fast. In conjunction with this, these have been recognized as feasible and effective technological means. In particular, as far as large model aircraft and spacecraft are concerned, for example, space planes [1,2], long range missiles [3], and so on, it is only necessary to opt for the use of precision composite navigational technologies. Only when this is done is it possible to guarantee the completion of this type of flight mission. Any single navigational and guidance technology, in all cases, has its advantages and weak points. Taking these single techniques and putting them together, one accentuates the advantages and avoids the weaknesses, mutually compensating one for the other. There is absolutely no doubt that this is a feasible path to follow. With the introduction of star light [4,5], it will cause a very, very great drop in the requirements on inertial guidance systems. At the same time, it is also possible to guarantee, in situations in which guidance precision is lowered, the causing of missile capability, under conditions in which starting states are not precisely determined, for the completion of launches and flight missions.

II. STAR LIGHT AND THE RECOGNITION OF PLATFORM MODEL PARAMETERS

1. Star Light Observations and Measurements of Platform Drift Angles

On the basis of status standards supplied by inertial guidance platforms, star sensor devices installed on platforms, in
predetermined star areas of space, point toward celestial navigation bodies. In conjunction with this, in the sensor components—in areas photographed by charge coupled components (CCD)—one sees presented a navigational star map. Going through recognition and calculations related to this star map, and, in conjunction with that, carrying out comparisons with theoretical star maps which are stored in the navigational computers, the differences are then the deviations from the status standards of the inertial guidance platform [5, 6].

Because the deviations are small angles, then

\[
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix} =
\begin{bmatrix}
  1 & \gamma & -\beta \\
  -\gamma & 1 & \alpha' \\
  \beta & -a & 1
\end{bmatrix}
\begin{bmatrix}
  x_e \\
  y_e \\
  z_e
\end{bmatrix}
\]

(1)

In this, \([x, y, z]^T\) is the theoretical value of the celestial body in the CCD coordinate system;

\([x_e, y_e, z_e]^T\) is the actually measured value of the celestial body in the CCD coordinate system;

\([a, \beta, \gamma]^T\) are the deviation angles from the status standards of the inertial guidance platform.

It is possible to clearly demonstrate that, in equation (1), as far as the required error angles \([a, \beta, \gamma]^T\) are concerned, the three equations are related to one another, that is, if only one star is received, then, the status deviation angle \([a, \beta, \gamma]^T\) is not precisely determined. Because of this, if one assumes that the CCD star sensor devices simultaneously receive a set of \(N\) stars, at that time, equation (1) can be changed to be written as

\[
\begin{bmatrix}
  x_i - x_{ei} \\
  y_i - y_{ei} \\
  \vdots \\
  x_N - x_{eN} \\
  y_N - y_{eN}
\end{bmatrix} =
\begin{bmatrix}
  0 & -z_{ei} & y_{ei} \\
  z_{ei} & 0 & -x_{ei} \\
  \vdots & \vdots & \vdots \\
  z_{eN} & 0 & -x_{eN}
\end{bmatrix}
\begin{bmatrix}
  a \\
  \beta \\
  \gamma
\end{bmatrix}
\]

Making use of minimum weighted squares, one obtains

\[
\begin{bmatrix}
  a \\
  \beta \\
  \gamma
\end{bmatrix} = (H^TH)^{-1}H^TQ
\]

\[
\begin{bmatrix}
  x_i - x_{ei} \\
  y_i - y_{ei} \\
  \vdots \\
  x_N - x_{eN} \\
  y_N - y_{eN}
\end{bmatrix}
\]
In this

\[ H = \begin{bmatrix}
0 & -z_{e1} & y_{e1} \\
z_{e1} & 0 & -x_{e1} \\
\vdots & \vdots & \vdots \\
z_{en} & 0 & -x_{en}
\end{bmatrix}, \]

\[ \dot{\psi}(t) = \psi(0) + E_0 + K\dot{\psi} + K_s\dot{\psi} + D_0 + \int_0^t \dot{\psi} dt + D_s + \int_0^t \dot{\psi} dt + V \]

In this \( \psi(t) = (a(t) \beta(t) \gamma(t))^T \) is the platform drift angle vector;

\[ E = (\epsilon, \epsilon, \epsilon)^T \] is the platform constant value drift speed vector;

\[ \dot{W} = [\dot{W}_x \dot{W}_y (\text{illegible}) \dot{W}_z]^T \] is the visual acceleration vector;

\[ \dot{W}_c (\text{illegible}) = [\dot{W}_x \dot{W}_y \dot{W}_z \dot{W}_x \dot{W}_y]^T \] is the crossed visual acceleration vector;

\[ V = [V_x V_y (\text{illegible}) V_z (\text{illegible})]^T \] is the random error vector;

\( K, K_1, D, \) and \( D_1 \) are all error coefficient matrices. They are respectively

\[ K = \begin{bmatrix}
0 & K_1 & K_2 \\
K_1 & 0 & K_3 \\
K_2 & K_3 & 0
\end{bmatrix}, \quad K_s = \begin{bmatrix}
K_1 & 0 & 0 \\
0 & K_2 & 0 \\
0 & 0 & K_3
\end{bmatrix} \]

\[ D = \begin{bmatrix}
D_1 & D_2 & D_3 \\
D_2 & D_3 & D_4 \\
D_3 & D_4 & D_5
\end{bmatrix}, \quad D_s = \begin{bmatrix}
D_1 & 0 & 0 \\
0 & D_2 & 0 \\
0 & 0 & D_3
\end{bmatrix} \]
Going through rewriting, one has the equation below:

\[ \psi'(t) = A(t)X + V' \]

In this

\[ A(t) = \left[ T \int_0^t \int \nu \psi^2 dt \right] \]

\[ T = [1 1] \]

\[ X = (\psi(0) e_{K K, D D},) \in R^{*}\]

If missiles, after flying out to a height of 30km, begin to make observations and measurements of celestial bodies, at around the instant this begins, one has the equations

\[ Y_1 = H_1 X + V_1' \]  \hspace{1cm} (2)

\[ Y_3 = H_3 X + V_3' \]  \hspace{1cm} (3)

In these

\[ Y_1 = (\psi(t_1) \psi(t_2) \ldots \psi(t_{m_1})) \in R^{*} \]

\[ H_1 = (A'(t_1) A'(t_2) \ldots A'(t_{m_1}))' \in R^{*} \]

\[ Y_3 = (\psi(t_1) \psi'(t_2) \ldots \psi(t_{m_3}))' \in R^{*} \]

\[ H_3 = (A'(t_1) A'(t_2) \ldots A'(t_{m_3}))' \in R^{*} \]

\[ V_1' = (V(t_1) V(t_2) \ldots V(t_{m_1})) \in R^{*} \]

\[ V_3' = (V(t_1) V'(t_2) \ldots V(t_{m_3})) \in R^{*} \]

Going through star sensor devices, \( Y_1 \) is capable of making observations and measurements as necessary. In order to correct ballistic burn phase parameters in the atmosphere, as well as specifying initial platform configurations, there is a need, on the basis of the observed and measured \( Y_1 \), to come to calculations of platform parameter matrix \( X \). As a result of this, one takes it and
substitues into (2), calculating out platform configurations inside 30km. It is necessary to go through observations and measurements of \( Y_1 \) in order to calculate \( Y_0 \). The conditions are such that it is necessary to have \( Y_0 \) included within the vector space that stretches out from \( Y_1 \). It is also necessary to have matrix \( P \) to make

\[
Y_0 = PY
\]

or

\[
Y_1 = PY \quad H_\cdot = P^H_\cdot, \quad P \in \mathbb{R}^{**}
\] (4)

If this is not the case, we will then not calculate out \( Y_0 \) from \( Y_1 \).

It is possible to clearly demonstrate that the necessary and sufficient conditions existing for equation (4) can be used in the equation below to express:

\[
H_\cdot H_\cdot H_\cdot = H_\cdot
\]

In this, \( H_1^+ \) is the generalized inverse. As a result, it is possible to select

\[
P = H_0 H_1^+
\]

As a result, one has

\[
Y_0 = PY_1 = H_0 H_1^+ Y_1
\]

On the basis of equation (2), \( H_1^+ Y_1 \) is precisely the parameter matrix \( \hat{X} \) (illegible) which we needed to solve for. After observations and measurements are made which directly receive the noise pollution, one obtains a parameter matrix which is nothing more than an optimum calculation with the significance of minimum squares, that is

\[
\hat{x} = H_1^+ Y_1
\]
Because of this, it is possible to calculate a step further for missiles within 30km. This is also nothing else than the platform status at the instant $t_m$

$$y'_o = H^+_0 H^+_1 y(\text{illegible})$$

III. PRECISE STAR LIGHT DETERMINATION OF MISSILE START POSITIONS

As far as making use of star light to measure the deviation angle of platforms to determine the selection of the ideal inertial coordinate system is concerned, if one selects an inertial coordinate system set up for missiles launched at ideal launch points, then, going through a derivation, it is possible to obtain the ideal coordinates of celestial bodies in this ideal inertial coordinate system, that is $(2, 5)$

$$
\begin{bmatrix}
  x \\
  y \\
  z
\end{bmatrix}
= 
\begin{bmatrix}
  \cos \theta, \sin \theta, 0 \\
  -\sin \theta, \cos \theta, 0 \\
  0, 0, 1
\end{bmatrix}
\begin{bmatrix}
  \cos \theta, & 0, & -\sin \theta, \\
  0, & 1, & 0 \\
  0, & \cos \theta, & \sin \theta
\end{bmatrix}
\begin{bmatrix}
  1 & 0 & 0 \\
  0 & \cos \alpha, & \sin \alpha, \\
  0, & -\sin \alpha, & \cos \alpha
\end{bmatrix}
\begin{bmatrix}
  \cos A, & 0, & \sin A \\
  0, & 1, & 0 \\
  -\sin A, & 0, & \cos A
\end{bmatrix}
\begin{bmatrix}
  \cos (t_0 + \lambda), & \sin (t_0 + \lambda) \\
  -\sin (t_0 + \lambda), & \cos (t_0 + \lambda) \\
  0, & 1
\end{bmatrix}
\begin{bmatrix}
  \cos \phi \cos \alpha \\
  \cos \phi \sin \alpha \\
  \sin \phi
\end{bmatrix}

$$

In this, $f$ is the star sensor device optical system focal distance, $a$ is the celestial right ascension, $\delta$ is the celestial declination, $\lambda$ is the longitude of the ideal missile launch point, $\phi$ is the latitude of the ideal missile launch point, $t_g$ is the Greenwich hour angle. $\alpha$ is the ideal missile launch angle. $\theta_x, \theta_y, \theta_z$ is the Euler angle between CCD star sensor device coordinate systems and inertial guidance platform coordinate systems.

It is possible to see that the radius vector for each celestial
body $r=[x \ y \ z]^T$ is, in all cases, a function of an ideal launch point. If missiles, due to ballistic launches, are caused to be so that the actual launch location is not positioned on the ideal launch location, then, one has

$$\dot{r}_* = \dot{r} + \frac{\partial \dot{r}}{\partial \Phi} \Delta \Phi + \frac{\partial \dot{r}}{\partial \lambda} \Delta \lambda + \frac{\partial \dot{r}}{\partial A} \Delta A + \frac{\partial \dot{r}}{\partial \lambda} \frac{\partial A}{\partial \lambda} \Delta \lambda$$

(5)

In this, $r$ is the ideal vector for celestial bodies in the inertial launch coordinate system. $r_*$ is the actual vector for celestial bodies in the inertial launch coordinate system.

If the missile, at the time of launch, has already had the platform leveled, then, at the instant of launch, one has

$$\dot{r}_* = C(\psi(t_*)) \dot{r}$$

$$C(\psi(t_*)) = \begin{bmatrix} 1 & r(t_*) & \beta(t_*) \\ -r(t_*) & 1 & \alpha(t_*) \\ \beta(t_*) & -\alpha(t_*) & 1 \end{bmatrix}$$

If one does not lose universality, it is possible to assume that the platform, at the moment of launch, had undergone no leveling. However, going through platform leveling signals, one obtains the Euler angle matrix between the platform coordinate system and the actual launch coordinate system.

$$\psi = \begin{bmatrix} 1 & \phi & -\phi \\ \phi & 1 & -\phi \\ -\phi & \phi & 1 \end{bmatrix}$$

In that case, one has
On the basis of equation (5), one has

\[ \tilde{r}_* = C \tilde{\psi} (I_c) \tilde{\Phi} \tilde{r} \]

If one observes and makes measurements of a set of stars, then, one has

\[ U = S \begin{bmatrix} \Delta \Phi \\ \Delta \lambda \end{bmatrix} \]

In this

\[ U = \begin{bmatrix} (C \Phi - 1) \tilde{r}_1 \\ (C \Phi - 1) \tilde{r}_2 \\ \vdots \\ (C \Phi - 1) \tilde{r}_* \end{bmatrix}, \quad S = \begin{bmatrix} \frac{\partial \tilde{r}_1}{\partial \Phi} + \frac{\partial \tilde{r}_1}{\partial \Phi} \frac{\partial A}{\partial \Phi} & \frac{\partial \tilde{r}_1}{\partial \lambda} + \frac{\partial \tilde{r}_1}{\partial \lambda} \frac{\partial A}{\partial \Phi} \\ \frac{\partial \tilde{r}_2}{\partial \Phi} + \frac{\partial \tilde{r}_2}{\partial \Phi} \frac{\partial A}{\partial \Phi} & \frac{\partial \tilde{r}_2}{\partial \lambda} + \frac{\partial \tilde{r}_2}{\partial \lambda} \frac{\partial A}{\partial \Phi} \\ \vdots & \vdots \\ \frac{\partial \tilde{r}_*}{\partial \Phi} + \frac{\partial \tilde{r}_*}{\partial \Phi} \frac{\partial A}{\partial \Phi} & \frac{\partial \tilde{r}_*}{\partial \lambda} + \frac{\partial \tilde{r}_*}{\partial \lambda} \frac{\partial A}{\partial \Phi} \end{bmatrix} \]

Making use of minimum squares, it is possible to obtain

\[ \begin{bmatrix} \Delta \Phi \\ \Delta \lambda \end{bmatrix} = (S^T S)^{-1} S^T U \]

Finally, it is possible to obtain missiles' actual launch position

\[ \Phi_* = \Phi + \Delta \Phi \]
\[ \lambda_* = \lambda + \Delta \lambda \]
Actual launch angles are

\[ A_* = A + \frac{\partial A}{\partial \alpha} \Delta \alpha + \frac{\partial A}{\partial \lambda} \Delta \lambda \]

IV. BURN PHASE NAVIGATIONAL PARAMETER CORRECTIONS

In the burn phase, due to the fact that missile flight times are relatively short and trajectory heights are relatively low, as a result, it is possible to take the gravitational field and see it as coming to have a central force field in conjunction with a vector from the missile to the earth taken as a first order approximation. It is possible to obtain, in the launch inertial coordinate system, the missile error equation

\[ \Delta \dot{x} = A \Delta \dot{X} + \Delta \dot{w} \]

In this

\[ \Delta X = (\Delta x \Delta y \Delta z \Delta \dot{x} \Delta \dot{y} \Delta \dot{z}) \]

\[ A = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-\frac{g_o}{R} & 0 & 0 & 0 & 0 & 0 \\
0 & -\frac{2g_o}{R} & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{g_o}{R} & 0 & 0 & 0 \\
\end{pmatrix} \]

\[ \Delta \dot{w} = \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
1 & -r(t) & \beta(t) \\
r(t) & 1 & -\alpha(t) \\
-\beta(t) & \alpha(t) & 1 \\
\end{pmatrix} \]

\[ \begin{pmatrix}
\Delta \dot{x} \\
\Delta \dot{y} \\
\Delta \dot{z} \\
\Delta \dot{\alpha} \\
\Delta \dot{\lambda} \\
\end{pmatrix} = \begin{pmatrix}
\dot{w}_x \\
\dot{w}_y \\
\dot{w}_z \\
\end{pmatrix} \]
In these equations, \( R \) is the distance from the launch point to the center of the earth. \( g_0 \) is the acceleration of gravity at the launch point. Solving error equations, it is possible to obtain

\[
\Delta X(t) = G(t, t_0) \Delta X(t_0) + \int_{t_0}^{t} G(t, \tau) \Delta \omega d\tau
\]

In this

\[
G(t, \tau) = \begin{bmatrix}
\cos(\omega_e(t - \tau)) & 0 & 0 \\
0 & \cosh(\sqrt{2} \omega_e(t - \tau)) & 0 \\
0 & 0 & \cos(\omega_e(t - \tau)) \\
\end{bmatrix}
\]

\[
\frac{1}{\omega_e} \sin(\omega_e(t - \tau)) \\
0 & \frac{1}{\sqrt{2} \omega_e} \sinh(\sqrt{2} \omega_e(t - \tau)) & 0 \\
0 & 0 & \frac{1}{\omega_e} \sin(\omega_e(t - \tau)) \\
\end{bmatrix}
\]

\[
-\omega_e \sin(\omega_e(t - \tau)) \\
0 & \sinh(\sqrt{2} \omega_e(t - \tau)) & 0 \\
0 & 0 & -\omega_e \sin(\omega_e(t - \tau)) \\
\end{bmatrix}
\]

\[
\frac{1}{\omega_e} \cos(\omega_e(t - \tau)) \\
0 & \frac{1}{\sqrt{2} \omega_e} \cosh(\sqrt{2} \omega_e(t - \tau)) & 0 \\
0 & 0 & \frac{1}{\omega_e} \cos(\omega_e(t - \tau)) \\
\end{bmatrix}
\]

\[
\omega_e = \sqrt{\frac{g_0}{R}}
\]

\( X(T_0) \) is, at the time of launch, the initial location calculated on the basis of latitudes and longitudes after corrections subtracting the initial locations calculated from ideal latitudes.

Carrying out corrections to missile navigational parameters for the instant \( t_m \), one has

\[ X^*(t_m) = X(t_m) + \Delta X(t_m) \]
After the instant \( t_m' \), taking instant \( t_m \) navigational parameters after they have been corrected as initial conditions, going through observations and measurements of celestial bodies in order to unceasingly carry out angular corrections on perceived accelerations, as a result, one causes the carrying out of navigational calculations in the precisely accurate inertial coordinate system in order to guarantee the navigational parameters required by the system.

REFERENCES

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