Studies of the Integer Cyclotron Resonances in a Modified Betatron Accelerator

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This paper briefly summarizes recent experimental results from the studies of the integer cyclotron resonances in the NRL modified betatron accelerator. Special emphasis is given to the $L=12$ resonance, because the NRL device has a twelve-fold symmetry. To test the importance of the $L=12$ resonance, we intentionally introduced an $L=12$ field error using twelve resonant coils. By activating these coils when the resonance is observed, the duration of x-rays produced by beam loss was reduced from 900 $\mu$s to 5.5 $\mu$s, while the amplitude of the signal increased from 0.5 to ~40 volts. The full width of the x-ray pulse at half maximum is inversely proportional to the current through the resonant coils and proportional to the risetime of the pulse.

Work is in progress with a set of twelve internal coils that have a risetime of 300-400 ns. Studies of the spatial distribution of beam losses at $L=12$ show that the beam is always lost in six well defined toroidal positions. In addition, it has been shown experimentally that when the ratio of the total toroidal magnetic field to the vertical field is not an integer, the cyclotron resonance is not excited.

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I. INTRODUCTION

The cyclotron resonance is due to the coupling, caused by a field error(s), of the cyclotron motions associated with the toroidal and vertical fields. The field error(s) that excites the resonance can be either vertical ΔB_z or axial (toroidal) ΔB_θ. In the case of a vertical field error and in the absence of acceleration and strong focusing field, the normalized transverse velocity β_⊥ and thus the Larmor radius of the transverse motion of the gyrating particles grows linearly with time\(^1\), provided that nonlinear effects associated with the particle velocity are neglected. When such effects are taken into account, β_⊥ exhibits cycloidal behavior.\(^2\)

In the presence of an accelerating field and a large vertical field error, β_⊥ increases proportionally to the square root of time, while γβ_θ saturates, i.e., the electrons lock-in to a specific resonance (lock-in state). When the amplitude of ΔB_z is below a threshold, β_⊥ exhibits Fresnel behavior, i.e., β_⊥ grows quickly for approximately 1 μsec and then saturates until the beam reaches the next resonance.\(^2\)

In the case of an axial field error and in the absence of acceleration, β_⊥ grows exponentially with time only for a very short period. Since β_⊥ increases at the expense of β_θ, the particles are kicked off resonance. Thus, β_⊥ varies cycloidally with time. Similarly, in the presence of an accelerating field β_⊥ behaves as in the case of the vertical field error.

The previous discussion is based on the assumption that the strong focusing field is zero. In addition to introducing new characteristic modes, the strong focusing field makes the expression for the regular cyclotron mode more complicated.\(^3\) However, it can be shown that for the parameters of the NRL device and provided ℓ ≫ 1, the strong focusing has only minor effect on the cyclotron resonance. This is also supported by extensive computer calculations.

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In this paper, we present results from several experiments related to the excitation of cyclotron resonances, for both vertical and toroidal field errors. The last Section addresses possible remedies for the suppression of the cyclotron resonances in the modified betatron accelerator.

II. EXPERIMENTAL APPARATUS

Detailed description of the NRL modified betatron accelerator can be found in previous publications\(^4-6\), where various experimental results, including observation of the electron cyclotron resonances were reported. There are three main external magnetic fields that provide confinement to the high current beam, namely, the toroidal \(B_\phi\), the strong focusing \(B_s\), and the vertical \(B_z\) fields. They are generated by pulsed, aircore electromagnets with risetimes equal to 2.3 ms, 1.0 ms, and 2.6 ms respectively. The peak value of \(B_\phi\) is \(-5\) kG and that of \(B_z\) is \(-2\) kG. The strong focusing windings can be operated at peak currents as high as \(-30\) kA. A photograph of the experiment is shown in Fig. 1.

III. EXPERIMENTAL RESULTS

In a modified betatron with strong focusing windings there are four characteristic transverse modes\(^3\) \(\omega_{\pm\pm}\). Integer resonances occur when the frequency of these modes, in the laboratory frame, over the relativistic cyclotron frequency of the vertical field \(\Omega_{z0}/\gamma\) on the minor axis is an integer, i.e., when

\[
\frac{\omega_{\pm\pm}}{(\Omega_{z0}/\gamma)} = k, \quad \text{where} \quad k = \pm 1, 2, \ldots
\]

(1)

The four characteristic modes \(\omega_{\pm\pm}\) are in general very complex. However, for modest beam and strong focusing currents, as those in the NRL device and when \(B_\phi/B_z \gg 1\) the modes are considerably simplified. Under these conditions the mode that is responsible for the cyclotron resonance becomes \(\omega_{\pm} \approx \Omega_{\theta0}/\gamma\), where \(\Omega_{\theta0}\) is the cyclotron frequency that corresponds to the toroidal magnetic field. Under these conditions Eq. (1) takes the very simple form \(B_{\theta0}/B_{z0} \approx \ell\), where \(\ell = 1, 2, \ldots\), and \(B_{\theta0}\) and \(B_{z0}\) are the toroidal and vertical fields on the minor axis. Therefore, the cyclotron resonance is due to the coupling, caused by a field error(s), of the cyclotron motions associated with the toroidal and vertical fields.
The x-rays are monitored by three collimated x-ray detectors (scintillator-photo-multiplier tube) that are housed inside lead boxes. In the results shown in Fig. 2, the x-rays enter the scintillator through a 1.94 cm-dia. tube and the detector is located 10.8 m from the vacuum chamber. As a rule, the shape of the x-ray signal recorded by all three detectors is spiky and the peaks always occur at the same value of $\frac{B_{\theta O}}{B_{x0}}$, independently of the current flowing in the stellarator windings.

In addition to the x-ray pulse, Fig. 2 shows the ratio of $\frac{B_{\theta O}}{B_{x0}}$ at the peaks of the signal. The solid circles are from the experiment and the crosses are from the resonance condition $\frac{B_{\theta O}}{B_{x0}} = \frac{(2\ell^2 - 1)}{2\ell}$, $\ell = 1, 2 \ldots$ of the cyclotron resonance. For $\ell = 8, 9$ and 10, experiment and theory are in good agreement. However, for the remaining $\ell$ values there is noticeable divergence between the theoretical predictions and the experiment.

The theory of the cyclotron resonance has been developed under the assumption that the beam is located on the minor axis. However, the experimental observations indicate that the electron ring starts to move off axis after 200-300 $\mu$sec. Recently, the theory of the cyclotron resonance has been extended to a beam that is located on the midplane but off the minor axis. For such a beam the resonance conditions becomes

$$\frac{r_0\Omega_{\theta O}}{c\gamma \beta_\theta} = \frac{2\ell^2 - 1}{2\ell},$$

where $r_0$ is the major radius, and $\beta_\theta$ is the normalized toroidal velocity. The agreement between the experimental results and the theory is improved when the radial motion of the beam is taken into account.

It is apparent from the resonant condition that when $\frac{B_{\theta O}}{B_{x0}}$ = constant $\neq$ integer, the cyclotron resonance is not excited. To test this supposition, we have installed 24 single-turn coils on the outside of the vacuum chamber, as shown in Fig. 3. These coils are powered by a capacitor bank and have a risetime of approximately 100 $\mu$sec. During this time period the coils generate a toroidal field ramp that increases linearly with time and the total toroidal field increases in synchronism with the betatron field. Results from the experiment are shown in Fig. 4, when the coils are energized at 800 $\mu$sec. Beam losses are suppressed for 100 $\mu$sec, i.e., as long as the condition $\frac{B_{\theta O}}{B_{x0}} \neq$ integer is satisfied.

The damage done to the beam at each resonance depends on the speed with which the resonance is crossed. By modulating the toroidal field with a rapidly varying ripple the resonance is crossed quickly and thus the damage inflicted to the beam is reduced. This
is the dynamic stabilization or tune jumping technique and requires a carefully tailored pulse to be effective over many resonances.

Results from the numerical integration of orbit equations are shown in Fig. 5. The left column shows the transverse velocity $\beta_\perp$, axial velocity $\beta_\theta$ and the product $\gamma \beta_\theta$ as a function of time when the coils that produce the ripple are not energized. As the beam crosses the $t=10$ resonance, its transverse velocity increases by a few percent while at $t=9$, $\beta_\perp$ reaches 18%. At the same time $\beta_\theta$ decreases. Even more importantly, the product $\gamma \beta_\theta$ saturates at $t=10$, i.e., all the energy from the accelerating fields goes to $\beta_\perp$ instead of $\beta_\theta$. This interesting phenomenon is due to nonlinear effects and will be called "lock-in" state. The corresponding results with the stabilizing coils on are shown in the right column of Fig. 5. At both $t=10$ and $t=9$ the transverse velocity of the beam increases modestly but $\gamma \beta_\theta$ does not saturate.

Results from the experiment are shown in Fig. 6. The toroidal field modulation is produced by the 24 coils that are mounted on the exterior surface of the vacuum chamber, two coils on each sector. The period of the under damped LRC circuit is 70 $\mu$sec. The decay of the current waveform is mainly due to the resistance of the vacuum chamber that plays the role of the secondary.

Figure 6a shows the x-ray waveform in the absence of modulation. The corresponding x-ray waveform when the coils are energized at 430 $\mu$sec and the first peak of the modulation is positive is shown in Fig. 6b. It is apparent that the x-ray signal is modulated with the period of the $\Delta B_\theta$ field and the two modulations are in phase. An interesting feature of these results is the drastic reduction of the x-ray signal following the initiation of the $\Delta B_\theta$ ripple. Typically, the x-ray signal goes to zero, temporarily, when the amplitude of $\Delta B_\theta$ exceeds a threshold. When the polarity of the $\Delta B_\theta$ field is reversed, the x-ray waveform exhibits similar characteristics as in the case of positive first peak and is shown in Fig. 6c. However, in contrast with the previous case, the beam losses are accelerated immediately after the initiation of the $\Delta B_\theta$ ripple.

The experimental results appear to be in agreement with the theoretical predictions and the computer calculations shown in Fig. 5. When the beam is in the lock-in state and the amplitude of the modulation is not large enough, $\gamma \beta_\theta$ remains locked and tracks the modulation, as required by the resonance condition of Eq. (1). For positive first peak, $\gamma \beta_\theta$ increases and thus $\beta_\perp$ is reduced. The smaller Larmor radius results in reduced

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particle losses. For negative first peak, $\gamma \beta$ is reduced and thus $\beta$ increases. The larger Larmor radius drives the electrons to the wall and the x-ray signal increases. It should be emphasized, however, that the above interpretation of the experimental results is not unique. There is at least one more model that is consistent with the results.

The basic periodicity in the NRL device is twelve-fold. There are twelve toroidal field coils, twelve sectors, and so on. For a long time, the $\ell=12$ resonance was the dominant resonance in the experiment. To test the importance of the $\ell=12$ resonance, we have intentionally introduced an $\ell=12$ axial field error using twelve resonant coils.

Initially, the set of twelve resonant coils was installed on the outside of the chamber (external coils). These coils are initiated at $t=430$ ms, i.e., when the $\ell=12$ condition is reached. A $\Delta B_\theta$ pulse with a quarter period risetime of 12 $\mu$sec and amplitude of 200 G is generated on the minor axis when the current in the coils is 9 kA. Figure 7a shows the x-ray pulse when the resonant coils are off and Fig. 7b when the coils are on. It is apparent from these results that the width of the x-ray signal is reduced from approximately 900 $\mu$sec to 8 $\mu$sec, i.e., by more than two orders of magnitude while its amplitude has increased by a factor of thirty. The width of the x-ray signal varies inversely with the current in the resonant coils as shown in Fig. 8 and also depends on the risetime of the resonant coil pulse.

By connecting the coils in parallel instead of in series, the risetime was reduced to 5 $\mu$sec, and even shorter risetime pulses have been produced with a set of internal coils. The internal coils are wound on a blue nylon housing and encapsulated with epoxy (Fig. 9) and are powered by new drivers with a risetime of 400 ns. Being faster and nearer to the sector flanges, the internal coils require substantially higher voltage and current to produce the required field perturbations. The dependence of the x-ray pulse width on the risetime of the current pulse is shown in Fig. 10.

To determine the spatial distribution of beam losses when the resonant coils are energized, several 400-$\mu$m-diameter optical fibers were mounted on the outside of the vacuum chamber. By the time the $\ell=12$ resonance is crossed, the electrons have acquired sufficient energy to penetrate the chamber. The light generated when the electrons strike the fiber is monitored with a photomultiplier tube. The results from scanning around the torus at 10° intervals are shown in Fig. 11. Only six peaks instead of the expected twelve are observed. This is most likely due to the beating between the $\ell=12$ and $\ell=6$ mode that
is excited by the six field periods of the stellarator windings. Computer simulation shows that the strong focusing modulates the $\ell=12$ peaks in such a way that only every other peak strikes the wall as shown in Fig. 12. It is still not clear at this time why three of the peaks are higher than the rest.

Under normal operating conditions the current that produces the strong focusing field is passively crowbarred and the fields decay with a long time constant $L/R$, where $L$ is mainly the inductance of a ballast inductor that is in series with the windings. To test the effect of the strong focusing field on the distribution of beam loss, the ballast inductor was removed and the circuits were actively crowbarred. The shape of the pulse is a half sine with a half period of $\sim 650 \mu$sec. The beam is injected near the peak of the pulse. Thus, the strong focusing field is practically zero when the resonant coils are energized. Under these conditions most of the beam is lost at a single toroidal position near $\theta = 90^\circ$. These experiments are currently in progress.

IV. REMEDY

Although the effective accelerating gradient in the present device is large, its actual accelerating gradient is low ($\sim 150V/m$). As a result the electrons have to perform a large number of revolutions around the major axis in order to obtain the desired peak energy of 20 MeV. Thus, the existing modified betatron is sensitive to field errors. Possible remedies for the problem of the cyclotron resonances are listed in Table 1. Computer calculations show that at the contemplated acceleration rate of $300 \, G/\mu$sec for the advanced devices the cyclotron resonance is not excited, provided that the various fields have been designed with an accuracy better than 0.1%.
### Table I

**Possible remedies for the cyclotron resonance**

<table>
<thead>
<tr>
<th>Remedy</th>
<th>Comments</th>
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<tbody>
<tr>
<td>Design the device carefully to avoid field errors</td>
<td>Increase cost.</td>
</tr>
<tr>
<td>Carry out accurate mapping of fields (≈ 0.1%). Use trim coils to eliminate field errors.</td>
<td>Tedious and time consuming. Standard technique in accelerators.</td>
</tr>
<tr>
<td>Keep ratio $B_\theta/B_z$ = const. $\neq$ integer during acceleration.</td>
<td>Powerful approach. Requires high $B_\theta$.</td>
</tr>
<tr>
<td>When $dB_z/dt$ is large the cyclotron resonance is not excited.</td>
<td>In contemplated advanced devices $dB_z/dt \approx 300 \text{ G/\mu sec}$. Thus, the cyclotron resonance will not be excited.</td>
</tr>
<tr>
<td>Modulate $B_\theta$ field to cross resonances quickly and thus to reduce damage. (Dynamic Stabilization or tune jumping)</td>
<td>Requires carefully tailored pulse to be effective over many resonances.</td>
</tr>
</tbody>
</table>
Fig. 1. Photograph of the NRL modified betatron accelerator.

Fig. 2. X-ray pulse and ratio of $B_{\theta 0}/B_{\theta 0}$ at the peaks of the x-ray signal.

Fig. 3. External resonant coils.
Fig. 4. X-ray signal. Between 800-900 µs
$B_{80}/B_{50}=$ constant ≠ integer

Fig. 5. Results from the numerical integration of orbit equations

Fig. 6. X-ray signals vs. time at reduced toroidal field.

Fig. 7. X-ray signal with the resonant coils off (a) and when the coils are activated: (b) full trace; (c) expanded.
Fig. 8. Width (at halfmax.) of the x-ray pulse vs. resonant coil current at $\ell=12$. Peak $\Delta B_\theta = 28G/kA$

Fig. 9. Internal coils

Fig. 10. Width (at half max.) of the x-ray pulse vs. current risetime.

Fig. 11. Distribution of beam losses.

Fig. 12. Computer results that show the radial displacement of the beam from the minor axis vs. toroidal angle.