Minimum Polarization Modulation: a Highly Bandwidth Efficient Coherent Optical Modulation Scheme

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Abstract

We propose and analyze a novel binary coherent optical transmission scheme based on continuous polarization modulation. Its bandwidth efficiency is better than that of both PSK and MSK. The signal spectrum is independent of the polarization transformations along the fiber. Its phase-noise tolerance is similar to FSK and its sensitivity is 3 dB better than that of single-filter FSK.

1 Introduction

The State Of Polarisation (SOP) of a lightwave can be exploited for digital transmission on optical fiber (POLSK, POLarisation Shift Keying systems). Both theoretical and experimental works have been published on this matter [1]-[7].

The high insensitivity to phase noise (FSK or ASK-like) and the possibility of compensating polarization fluctuations due to the optical channel by means of electronic processing of the received signal are among the most interesting features of these systems.

In [3, 5, 6] we have presented the theoretical analysis of detection, noise statistics and maximum likelihood decision rules for POLSK systems, and the exact (for shot and gaussian receiver noise) performance analysis of several binary and multilevel configurations.

In [7], for the first time, Continuous Polarisation Modulation (CPOLSK) has been proposed. Here we present a new scheme based on the same principle that we call MPM (Minimum Polarisation Modulation).

The spectral efficiency of MPM turns out to be better than both those of MSK and of the CPOLSK scheme proposed in [7].

Sensitivity is similar to that of dual-filter FSK, three dB better than ASK.

2 Modulation Principle

All the previously proposed binary POLSK schemes operate switching abruptly between two orthogonal SOP's. However, abrupt switching implies discontinuity of the signaling waveforms, leading to bandwidth broadening. MPM transmission, on the contrary, completely relies on continuous signaling waveforms.

A pictorial description of the modulation principle of MPM is shown in the Stokes parameter representation in Fig. 1.

Chosen a maximum circle on the Poincaré sphere, transmission of a 'zero' is accomplished by moving the vector \( \mathbf{Z} \), which represents the state of polarization of the transmitted light, along the circle with constant speed. The speed is such that after the bit time \( T \) a rotation equivalent to a quarter of a complete lap (90 degrees) has been performed. Transmission of a 'one' is done similarly, but the movement occurs in the opposite direction.

Formally, identifying the maximum circle through a pair of orthogonal unit vectors \( \hat{v}_1 \) and \( \hat{v}_2 \) (see Fig. 1), the transmission law can be written:

\[
\mathbf{Z}(t) = \sum_{k=0}^{\infty} \left[ \cos(2\omega_d[\sigma_k T + \alpha_k(t - kT)]) \hat{v}_1 + \sin(2\omega_d[\sigma_k T + \alpha_k(t - kT)]) \hat{v}_2 \right] u(t - kT)
\] (1)

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1 Apart from [7], that we call CPOLSK1 throughout the paper.
Figure 1: Transmission of one of the two symbols, in the Stokes parameter representation of SOP's.

with:

\[ \sigma_k = \sum_{i=0}^{k-1} \alpha_k \quad \sigma_0 = 0 \quad \omega_d = \frac{\pi}{4T} \]

The \( \alpha_k \)'s are independent stochastic variables taking on the values \{-1, 1\} with probability \( \frac{1}{2} \), representing binary data, while \( u_T(t) \) is a step function of duration \( T \).

The above representation translates into the following Jones vector representation, which gives the complex envelope of the transmitted light, decomposed with respect to two orthogonal fiber polarizations:

\[ \mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} \eta_0 + \mu_0 \nu_0 + 1 \\ \eta_0 - \mu_0 \nu_0 - 1 \end{bmatrix} \]

with:

\[ \eta_0 = \theta_0 = 0 \quad \mu_0 = \nu_0 = 1 \]

and:

- if \( (\eta_k = 2 \text{ and } \alpha_k \mu_k = 1) \) or \( (\eta_k = -2 \text{ and } \alpha_k \mu_k = -1) \)
  then \( \nu_k = -\mu_{k+1}; \)
  \[ \eta_{k+1} = \eta_k + \alpha_k \mu_k \]

- if \( (\theta_k = 0 \text{ and } \alpha_k \nu_k = 1) \) or \( (\theta_k = -4 \text{ and } \alpha_k \nu_k = -1) \)
  then \( \nu_k = -\nu_{k+1}; \)
  \[ \theta_{k+1} = \theta_k + \alpha_k \nu_k \]

The quantity \( V_{\text{set}} \) is the voltage required to bring the Mach-Zehnder from complete signal transmission to complete signal extinction.

The waveforms of the driving voltages \( v_1 \) and \( v_2 \) are continuous and therefore their bandwidth is very narrow. An eye diagram of \( v_1(t) \) is reported in Fig. 3 while in Fig. 4 its power spectrum is shown.

It can be seen that the spectral components above 0.7 \( \cdot \) \( R \), with \( R \) the bit rate, are extremely low. This suggests that the bandwidth of the modulator can be as low as 0.7 \( \cdot \) \( R \), and still deliver undistorted MPM signals.

Figure 3: Eye diagram of the modulating voltage \( v_1(t) \) over four bit intervals, with \( V_{\text{set}} = 1 \).

This hypothesis has been verified through simulations, as shown in Section 5.1.

A drawback of this solution is that 3dB are lost since at any time only half of the optical power gets through the modulator.

In general, any device capable of performing endless polarization modulation over a maximum circle of the optical power needs to be 'open' since this is not the decision signal, but just the modulating waveform.

\[ \eta_0 = \theta_0 = 0 \quad \mu_0 = \nu_0 = 1 \]

\[ \eta_{k+1} = \eta_k + \alpha_k \mu_k \]

\[ \theta_{k+1} = \theta_k + \alpha_k \nu_k \]

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The main effect due to the fiber is birefringence while dichroism or other transformations are in general far less important. However, non-negligible dichroism can be caused by optical components other than the fiber. In [6] we proposed a new receiver for polarization modulation, called Lorentz-Stokes receiver, that permits to recover the effects of both birefringence and dichroism in the optical channel. This receiver can be used for MPM too, with straightforward changes at the decoder level. The full treatment of the Lorentz-Stokes receiver is beyond the scope of this paper and we refer the reader to [6] for the details. For the time being we shall assume that $Q$ is a pure birefringence matrix.

In the space of the Stokes parameters, the effect of $Q$ consists in translating the maximum circle identified by $\alpha_1$ and $\alpha_2$ onto another maximum circle, identified by another pair of unit vectors $\hat{u}_1$ and $\hat{u}_2$, so that the expression of the received signal in the Stokes parameter representation becomes:

$$W(t) = \sum_{k=0}^{\infty} \left[ \cos(2\omega_d(\sigma_k T + \alpha_k(t - kT))) \hat{u}_1 + \sin(2\omega_d(\sigma_k T + \alpha_k(t - kT))) \hat{u}_2 \right] u_r(t - kT)$$

(5)

All POLSK receivers [2], [3] make use of a first stage that extracts the Stokes parameters of the incoming lightwave. A schematic of this front-end is reported in Fig. 5.

In general, $\hat{u}_1$ and $\hat{u}_2$ will not coincide with any of the axes $\hat{s}_1$, $\hat{s}_2$ or $\hat{s}_3$ of the Stokes space reference system used by the receiver (Fig. 6). Therefore, after Stokes parameter extraction, a processing stage is needed to compensate for this misalignment.
Figure 5: Schematic of the receiver Stokes parameter extracting stage.

Figure 7: Schematic of the baseband processing stage of the receiver.
Figure 6: Misalignment between the transmission axes and the Stokes receiver axes.

A possible solution is reported in Fig. 7. An estimation block extracts the coordinates of \( \hat{u}_1 \) and \( \hat{u}_2 \) with respect to \( s_1, s_2, s_3 \). These values are input to the multipliers so that the final outputs \( d_e \) and \( d_s \) turn out to be:

\[
d_e = \vec{W} \cdot \hat{u}_1 = \cos (2\omega_d [\sigma_b T + \alpha_b (t - kT)])
\]

\[
d_s = \vec{W} \cdot \hat{u}_2 = \sin (2\omega_d [\sigma_b T + \alpha_b (t - kT)]) \tag{6}
\]

The eye diagrams of \( d_e \) and \( d_s \) are reported in Figs. 8 and 9. These two signals are identical to the complex envelope of the in-phase and quadrature components of the signal of a synchronous MSK scheme [10, p. 241]. This special correspondence is one of the circumstances that suggested the denomination MPM. As a result, the same decision strategies which can be used for synchronous MSK can also be used for MPM\(^3\). In particular, MPM (like MSK) can be treated as a binary or a 4-ary (multi-level) transmission scheme.

From (6) or from Figs. 8 and 9 it can be seen that at time instants which are multiples of \( T \), one of the two decision variables takes on the values +1 or -1, while the other ideally is zero. After another \( T \) seconds, the roles are exchanged. Therefore a binary output stream can be recovered by deciding every \( T \) seconds, alternating decision on \( d_e \) and \( d_s \).

Otherwise, decision can be made every \( 2T \) seconds, as in a 4-ary system, sampling both decision variables after delaying one of a fixed amount \( T \).

\(^3\)The signal correspondence with synchronous MSK does not imply that MPM is synchronous too. MPM is not demodulated synchronously and is insensitive to phase noise, as discussed later.

Figure 8: Eye diagram over 4 bit periods of \( d_e \).

Figure 9: Eye diagram over 4 bit periods of \( d_s \).

The estimation block of Fig. 7 processes samples of the signals available at the output of the Stokes parameter extracting stage. The sampling rate can be much slower than the bit rate, since changes in the polarization transformations due to the fiber are very slow. In fact, the critical parameter for the estimator is the time needed to obtain the best estimate of \( u_1 \) and \( u_2 \), so that reception can start. Delays on the order of 100 \( \mu \)s seem to be achievable with silicon logic circuitry.

4 Analytical evaluation of spectral properties

The spectra of the MPM signals corresponding to two arbitrary orthogonal polarizations in the fiber are identical and independent of the choice of such polarizations. This result has no counterpart in any other POLSK scheme, for which spectral properties depend on the pair of ar-
The orthogonal polarizations used to analyze the signal is equivalent to saying that MPM spectra are invariant with respect to birefringence polarization transformations. In addition, the presence of dichroic elements in the channel can make the level but not the shape of the spectra vary according to the chosen pair of orthogonal polarizations.

The analytical expression of these power spectra, which also coincide with those of the IF signals $y_1$ and $y_2$ in the Stokes parameter extracting stage (Fig. 5) of the MPM receiver, is:

$$G(f) = A^2 \left\{ |X(f + fd)|^2 + |X(f - fd)|^2 + R[A(f)] \right\}$$

$$= (e^{j\frac{\pi}{2}} X(f + fd) X^*(f - fd) + e^{-j\frac{\pi}{2}} X(f - fd) - X^*(f + fd) + e^{-j\frac{\pi}{2}} |X(f + fd)|^2 + e^{j\frac{\pi}{2}} |X(f - fd)|^2) \right\}$$

where:

$$A(f) = \sqrt{\frac{1}{1 - \frac{\omega_d^2}{2} e^{-j\omega_d f}}} \left( \frac{1}{1 - \frac{\omega_d^2}{2} e^{-j\omega_d f}} - 1 \right)$$

$$X(f) = \frac{\sin(\pi f T)}{\pi f} e^{-j\omega_d f} \quad f_d = \frac{\omega_d}{2\pi}$$

and $R[.]$ stands for 'real part'.

$G(f)$ is depicted in Fig. 10. This analytical curve has been verified through simulation. The insensitivity to birefringence has been tested as well. A qualitative comparison with the PSK spectrum immediately shows that the MPM spectrum is much narrower.

Spectral performance is often evaluated through the fractional power containment, i.e. the normalized signal power $\epsilon$ contained in a given bandwidth $B$ around IF. A plot of $\epsilon$ versus the normalized bandwidth $\frac{B}{R}$ is given in Fig. 11 where, for comparison, the MSK, CPOLSK1 and PSK curves are reported too. The plot shows that most of the power of MPM is concentrated in a very small bandwidth around IF and that, to this respect, it performs better than the other transmission schemes.

In addition, for some relevant values of $\epsilon$, the corresponding full width bandwidths are shown in Table 4. To hold 99.9% of the signal power ($\epsilon = 99.9$), MPM needs a bandwidth of 1.42$R$ while MSK needs 2.73$R$.

### 5 Sensitivity

We first assume negligible phase noise. If treated as a binary system, decision for MPM occurs on either $d_0$ or $d_1$ in an alternated fashion. The statistics of noise for

<table>
<thead>
<tr>
<th>Power Fraction $\epsilon$</th>
<th>MPM</th>
<th>CPOLSK1</th>
<th>MSK</th>
<th>PSK</th>
</tr>
</thead>
<tbody>
<tr>
<td>95%</td>
<td>0.525</td>
<td>0.642</td>
<td>0.912</td>
<td>4.155</td>
</tr>
<tr>
<td>99%</td>
<td>0.9</td>
<td>1.013</td>
<td>1.183</td>
<td>20.58</td>
</tr>
<tr>
<td>99.9%</td>
<td>1.42</td>
<td>1.592</td>
<td>2.732</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 1: Full width bandwidths containing a given fraction of the signal power.

both these variables is the same as for binary POLSK, since both schemes make use of the same Stokes parameter extracting stage. Error probability was analytically assessed in [4] and [3] and evaluates to:

$$P(E) = \frac{1}{2} \exp \left( \frac{-E_b}{2\eta N_0} \right)$$

The quantity $E_b$ represents the energy per transmitted bit divided by the noise spectral density. For shot-noise limited demodulation this quantity equals the number of received photons per bit. The parameter $\eta$ represents the ratio between the noise equivalent bandwidth $B_N$ of the IF filter in the Stokes parameter extracting stage and the bit rate $R$:

$$\eta = \frac{B_N}{R}$$

For conventional binary POLSK, $\eta$ can be ideally as low as one, which corresponds to the case of ideal NRZ modulation and the use of a matched IF filter at the receiver, or to the case of raised cosine spectrum pulses. With zero roll-off, filtered through a rectangular filter of bandwidth $R$. In practice, these ideal situations can never be achieved and a certain penalty is always incurred. In the MPM case, it is not possible to establish a priori how tight the filtering can be, although the good performance of power confinement reported in Table 4 suggests that a very narrow filter can be used.

To obtain a performance estimate, we simulated the MPM chain and used a raised-cosine IF filter like the one shown in Fig. 12, where the relevant parameters are the extension of the flat part of the response $B_{flat}$ and the -6 dB bandwidth. The roll-off is then:

$$\rho = 1 - \frac{B_{flat}}{B_{-6dB}}$$

and the resulting noise equivalent bandwidth is:

$$B_N = B_{-6dB} \left( 1 - \frac{\rho}{4} \right)$$

We varied the filter parameters and obtained several eye diagrams, on which we measured the penalty due to...
Figure 10: Solid line: power spectrum of MPM; dashed line: power spectrum of PSK.

MPM can therefore get as close as 0.26 dB to the ideal performance of conventional binary POLSK. But while this ideal performance cannot be attained in practice for 2-POLSK, due to the unrealistic signal shape or filtering requirements, the above simulations show that this MPM result (in the absence of phase noise) is indeed achievable.

Hence, we can conclude that MPM shows a sensitivity performance that is similar to that of conventional binary POLSK. For comparison, this sensitivity is identical to...
Fractional Power Containment

![Fractional Power Containment Graph](image)

Figure 11: Fraction of the total signal power contained in a full bandwidth $B/R$ around IF.

dual filter FSK, 3 dB below DPSK and 3 dB better than ASK or single-filter FSK (all of these in the absence of phase noise and for optimum filtering).

Finally, we remark that the best result was obtained using a filter whose response is zero at $0.66R$ from the IF. This means that the IF can be very low and as a result the receiver bandwidth can be smaller than with other schemes.

5.1 Low modulator bandwidth

To assess the effectiveness of the continuous-waveform modulator described in Section 2 (Fig. 2), we simulated modulation using a modulator response of raised-cosine type (Fig. 12) with $B_{flat} = 0.7R$ and $\rho = 0.25$. The receiver IF filter was set so as not to add any further signal distortion. The resulting eye diagram is shown in Fig. 14. The virtually undistorted eye proves that the described approach permits to use relatively low-bandwidth modulators and still obtain a totally undistorted MPM signal. For comparison, in order to obtain relatively undistorted NRZ pulses, a bandwidth of approximately $3R$ is needed\(^3\).

\(^3\)It is often claimed that for NRZ pulses a bandwidth of $0.7R$ is enough. This refers to the whole transmission chain including all the filtering stages down to the decision variable. When non-linear demodulation is used, the topic becomes even more involved. But in any case, if a modulator bandwidth of $0.7R$ is used, then any further filtering causes ISI and therefore penalty. This is not true

![Eye diagram with a raised-cosine modulator response](image)

Figure 14: Eye diagram with a raised-cosine modulator response and $B_{flat} = 0.7R$, $\rho = 0.25$.

6 Phase noise induced penalty

In the presence of phase noise, tight IF filtering is no longer optimum. In [3] we proved that in the limit of wide IF filtering the Stokes parameter extracting stage cancels phase noise similarly to ASK or FSK systems. In [9] it was shown that for ASK and FSK the resulting

for MPM, that can keep all the receiver filtering stages unchanged since the signal shape is not affected by the limited modulator bandwidth.
poorer IF noise filtering can be compensated through the use of a tight post-detection filter, achieving relatively small overall penalties.

For MPM a similar behaviour can be inferred. To assess its performance in this context, we used again a simulation approach. We used raised cosine IF filters, with $\rho = 0.25$, and a postdetection integrate-and-dump filter that integrates over a time $2T$. The integration time is $2T$ because the waveforms of $d_c$ and $d_s$ have time-length $2T$, as can be seen in Figs. 8 and 9. This circumstance marks a peculiarity of MPM with respect to other transmission schemes. The use of a $2T$ integrate-and-dump filter never causes inter-symbol-interference (ISI). This is not true for instance for ASK or single filter FSK, where a bit-time integrator can be used as a post-detection filter only if the IF filter is so broad that it does not distort the NRZ pulse at all. Otherwise, the post-detection integration time must shrink. With MPM, the $2T$ post-detection filter could be used even with the minimum-width IF filter that yields the eye diagram of Fig. 13, without causing any ISI penalty. While it is not clear to what extent this circumstance helps to reduce the effects of gaussian noise, we found through simulation that it helps to reduce the residual effect of phase noise induced fluctuations on the baseband eye diagram.

We chose three specific values of the linewidth to bit rate ratio $\Delta$: 0.03, 0.1 and 0.3, and we assigned $B_N$ increasingly higher multiples of $R$ until negligible phase noise effects were found in the eye diagram. The $B_N$ necessary to achieve this result were $2R$, $3R$ and $5R$ respectively.

Since the noise statistics of the decision variables $d_c$ and $d_s$ is similar to that of dual filter FSK, we expect the results reported in [9] on the penalty due to loose IF filtering and tight postdetection filtering for that scheme to be a good estimate for MPM too. For an IF bandwidth of $5R$, [9] reports a penalty of only 1.5 dB, which means for MPM that laser linewidths as large as $0.3R$ can be tolerated with a relatively small power penalty.

These results confirm the very high insensitivity to phase noise that MPM can achieve.

7 MPM in an FDM environment

The fact that MPM spectral power confinement is excellent suggests that its behaviour in an FDM environment could be very good as well. To test it, we simulated three MPM channels, and demodulated the one in between. The IF filter was the one that yielded the best result of sensitivity ($\rho = 0.25$, $B_N = 1$). For a carrier spacing of $1.2R$ the resulting eye closure brings about an excess penalty of 0.3 dB. The IF power spectrum is shown in Fig. 15, while the eye diagram is reported\(^6\) in Fig. 16. For a channel spacing of $1.0R$ the penalty is about 1.5 dB.

These penalty figures stand out as compared to the results obtained using for instance CPFSK [8], which is reputedly one of the best solutions as far as FDM performance is concerned. Channel spacings 40% to 50% lower seem to be possible with MPM.

8 Conclusions

Higher bandwidth efficiency permits greater channel packing and the use of slower receiver electronics. In

\(^6\)The eye diagram of the actual simulation included 1000 symbols, while the one in Fig. 16 only shows 80 symbols. This is because this paper is fully computer typeset and retrieval and manipulation of the huge amount of data involved by the original eye diagram was impossible.
addition, it can be traded off with power efficiency by means of FEC or convolutional codes, up to several dB's. MPM achieves the best bandwidth efficiency among all the binary schemes so far proposed for coherent optical transmission. These features, together with its high phase-noise insensitivity, put it forward as a possible candidate for both bandwidth and power efficient optical communications.

References


Digital Transmission Theory.