There are two primary accomplishments directly due to the efforts of this project. The details of the techniques and outcomes were reported in the paper "The Underwater Solar Light Field: Analytical Model from a WKB Evaluation," contained in the SPIE conference proceedings volume 1537, *Underwater Imaging, Photography, and Visibility*.

1) Formal and approximate mathematical techniques were brought to bare to solve the radiative transfer equation in the WKB approximation. This is the first time this approximation technique has been used in ocean optics, and radiative transfer in general. The most unique outcome of the approach is that analytical solutions of the radiative transfer equation can be generated without using the small-angle assumption, and without assuming a fixed number of directions (streams) of propagation. Because of the particular approach used, the technique is applicable not just to the problem of solar lighting, but also to other types of lighting conditions, and to time-dependent pulse propagation problems as well. In fact, there is in progress an effort to transition the technique to 6.2 projects involving pulse propagation.

2) The WKB solution described above was used to obtain a semi-analytic solution for the radiance distribution in the ocean produced by solar lighting. The result was compared to Tyler's Lake Pend Oreille data, and shown in the figure below. At this point, the discrepancy between data and model appears to be due to the simplified treatment of the phase function in the coarse of the approximation. This treatment is not required for the WKB approximation however, and further effort to more accurately handle the phase function will almost certainly produce a much better agreement.

![Graph](image-url)
Accomplishments

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Figure: The radiance distribution in the plane of the sun as a function of angle from the vertical, for several depths. The dots are from Tyler's Lake Pend Oreille data, and the solid curves are the outcome of the WKB approximation.