Electron Interaction and Optical Gap of Conjugated Polymers

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# Electron Interaction and Optical Gap of Conjugated Polymers

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Electron Interaction and Optical Gap of Conjugated Polymers

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Abstract

The controversy over whether the optical gap of a polymer is increased or decreased by the electron-electron interaction can be clarified by studying the effect of the screened Coulomb interaction on the correlation function of the excited states. Since the competition between the diagonal and off-diagonal parts of the electron interaction depends on the screening, the screening becomes the decisive factor in determining the dependence of the optical gap of the electron interaction. Our theory shows that the electron interaction with weak or usual screening increases the optical gap, but if the screening is very strong, the optical gap can be reduced by the electron interaction.
1. Introduction

Experimentally, one of the most prominent features of the conjugated polymer is the finite excitation energy from the ground state to the lowest optically-allowed excited state, the optical gap. But, theoretically, how to understand the physical origin of the optical gap is one of the most controversial issues today in this field. There exists a sharp dispute about the effect of the electron-electron interaction on the optical gap. One school shows that the electron interaction increases the optical gap, some groups even think that the main origin of the optical gap is the electron interaction. However, the other school holds an opposite opinion that the electron interaction should reduce the optical gap, which is produced by the electron-phonon interaction. Their discrepancy comes from the different descriptions for the electron interaction. The former school takes the extended Hubbard model to describe the electron interaction. Starting from that model, many different theories including the perturbation [1], Gutzwiller variational [2], renormalization
group [3], Monte Carlo simulation [4], valence bond [5-8], configuration interaction [9], exact diagonalization[10] and others have been used to investigate the effect of electron interaction on the dimerization and optical gap. Although their methods and approximations are different, all these approaches reach the same result that the electron interaction increases the dimerization and optical gap. However, the latter school argues that the extended Hubbard model only contains the site-charge repulsion U and V, missed the bond-charge repulsion W. When W is added to the extended Hubbard model, they find the dimerization and optical gap is reduced [11]. Nevertheless, the former school is not convinced because the latter takes the extreme case with W = V and makes only first-order perturbation [12-14]. Thus, whether the electron interaction increases or decreases the optical gap becomes an open question.

It is not a puzzle that the conclusions of these two schools are completely opposite; the reason is their model Hamiltonians are different. The matter is to what extent these models are suitable. Apparently, the key to settle this
dispute is to analyze the limitations of these models and find a better description for electron interaction in polymers. We know that the parameters $U$ and $V$ in the extended Hubbard model are the diagonal matrix elements of the Coulomb interaction; and the bond-charge repulsion $W$, which is the exchange term, is one of the off-diagonal elements. There are also many other off-diagonal elements such as the site-bond repulsion $X$ etc. It has been pointed by D. Campbell and his co-workers that the parameters $U, V, W, X$ cannot be dialed at will [10]. The ratios between these parameters should be determined from the Coulomb interaction. On the other hand, as is well known, that the Hubbard model is a good approximation for those systems which bandwidths are narrow; then the off-diagonal elements are much smaller than the diagonal elements and they can be neglected. But, the bandwidth of the conjugated polymer is wide and even larger than the electron interaction. In such a case the off-diagonal elements are not always negligible comparing to the diagonal partner, especially when the screening is strong. Then, the results based on selecting some part of the interaction elements with
Artificial values are questionable. Therefore, it will be more reliable directly using screened Coulomb repulsion $v(r) \propto (1/r) \exp(-\beta r)$ to describe the electron interaction. In the second quantized representation this interaction includes all diagonal and off-diagonal elements. And in this description, the ratios between all interaction parameters U, V, W, X etc. can be determined correctly, avoiding any artificiality. This description is general, both the former model (extended Hubbard) and the latter model (KSSH [11]) are different approximations of this interaction. So, it can be expected that the results based on this interaction will provide a satisfying answer to the above dispute.

Such screened Coulomb repulsion has been successfully used to clear up the confusion about the effect of the electron interaction on the bond alternation of the polymer [15-18]. The bond alternation is a property which is only associated with the ground state of the polymer. But the optical gap involves the excited states. In this paper we are going to use the same interaction to study the effect of the electron interaction on the excited states and the optical gap.
of the polymer.

In the next section we first establish the integral equations for the correlation function of the excited states in an interacting electron system and find the relation between the optical gap and the correlation function of the excited state. Then, in the third section, the dependence of the optical gap on the electron interaction will be obtained by solving the integral equations numerically. In the discussion of the results it tells how the controversy about the optical gap can be settled.
II. Theoretical framework

By using the screened Coulomb repulsion to describe the electron interaction, the Hamiltonian of our system reads

\[ H = H_0 + \sum_{\alpha} + (k/2) \sum_{\alpha} (x_\alpha - x_\alpha^\prime)^2. \] (2.1)

\( H_0 \) describes the interaction between the electron and the lattice of the polymer.

\[ H_0 = \sum_x \left\{ -(\hbar^2/2m) \nabla^2 + V(x, X_{\alpha} \phi(x, X_{\alpha}^\prime) \right\}. \] (2.2)

where \( V(x, X_{\alpha}) \) is the potential produced by the atom at \( X_{\alpha} \) and exerting on the electron at \( x \). The eigenvalues and eigenfunctions of \( H_0 \) are \( \epsilon(k) \) and \( \phi_x \). \( H_{i, \alpha} \) is the electron interaction

\[ H_{i, \alpha} = \sum_j \nu(r_{i,j}). \] (2.3)

\[ \nu(r) = (U_0/r) \exp(-\beta r). \] (2.4)
where $U_0$ and $\beta$ are the strength and the screening factor of the electron interaction in the polymer. The last term in (2.1) is the elastic energy, and $K$ is the elastic constant.

The optical gap is the threshold of an interband optical absorption. In the system with electron interaction some excited state such as $2^1\Delta_s$ can exist within the gap, but it is dipole-forbidden from the ground state, which symmetry is $1^1\Delta_s$. The lowest many-body excited state in this paper has the symmetry $1^1\Xi$, which is dipole-allowed, and the transition from the ground state to this lowest excited state produces the optical gap [19].

The calculation of the optical gap for a system with an intermediate electron interaction is an important but difficult problem. The polymer has a wide bandwidth of about 10 eV, and the electron interaction in the polymer is about 5 eV. It means that the electron interaction is neither weak enough to use the perturbation nor able to be treated by the strong-limit approximation. It is an intermediate case. The
various methods mentioned in the Introduction have got some progress for this subject, but they are not able to deal with the Coulomb interaction. Meanwhile, the exact calculations methods for the optical gap suffered from the finite-size effect. In this paper we use the Feenberg-Jastrow variational method [20], which can work for the Coulomb repulsion and for any length of the polymer chain. We are going to determine the dependences of the optical gap on both the strength and the screening of the electron interaction in the intermediate region of strength and for any screening. Then it can be quantitatively shown how the electron interaction increases or decreases the optical gap.

Following the Feenberg-Jastrow Ansatz [20], the wave function of an interacting electron system is

$$\Psi (1, 2, \ldots N) = D(1, 2, \ldots N) \exp( \Sigma \mu \alpha \beta \phi \alpha \beta \), \quad (2.5)$$

where $D(1, 2, \ldots N)$ is the Slater determinant consisting of occupied states $\phi \alpha \beta$, $\Sigma \mu \alpha \beta \phi \alpha \beta$ is the correlation factor deter-
mined by the variational principle. As is known, the first part of the wave function (2.5) represents the individual particle mode and the second part the collective mode [21, 22]. Actually, in momentum space the second part can be written as

$$\tilde{\psi}_{ij} u_{ij} = \sum \tilde{C}_{k} \rho \cdot \rho \cdot \rho$$

where $C_{k}$ is the Fourier transform of $u(r)$ and $\rho$, the density fluctuation, which depicts the plasma in the case of Coulomb repulsion or the zero sound in the case of short-range repulsion. This many-body wave function (2.5) is a functional of electron's occupation configuration. In the half-filled case, the most interested states in our study are the ground state $\psi_{\downarrow}(1, 2, \ldots N)$ with full valence band and empty conduction band and the lowest single-particle excited state $\psi_{\uparrow}(1, 2, \ldots N)$ with one electron excited from the valence band to the conduction band. Since the collective mode is not excited in this excitation, $\psi_{\uparrow}$ and $\psi_{\downarrow}$ have the same $u_{ij}$ but different $D_{\downarrow}$ and $D_{\uparrow}$, which possess reversed
symmetries [23]. The optical transition is allowed between them and the optical gap \( E \) is

\[
E = E(\{\Psi,\}) - E(\{\Psi,\})
\]  
(2.6)

The energy \( E(\{\Psi\}) \) of the wave function \( \Psi \) (2.5) is

\[
E(\{\Psi\}) = \langle \Psi | H | \Psi \rangle / \langle \Psi | \Psi \rangle = \\
= \sum_{\text{occ.}} \varepsilon(k) + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}^2 |P(1,2) - n_0| v(1,2) |P(2) - n_0| \\
+ \frac{1}{2} \int d\mathbf{r} d\mathbf{r}^2 |P(1,2) - P(1)P(2)|v(1,2) + \frac{\hbar^2}{8m} \int d\mathbf{r} d\mathbf{r}^2 P(1,2) \left( \nabla_1 u_{13} \right)^2 \\
+ \frac{\hbar^2}{8m} \int d\mathbf{r} d\mathbf{r}^2 \int d\mathbf{r}^3 P(1,2,3) \left( \nabla_1 u_{13} \right) \cdot \left( \nabla_1 u_{13} \right),
\]  
(2.7)

where \( P(1,2,\ldots,n) \) is the \( n \)-particle distribution function, which satisfy the following integral equations [15]:

\[
P(1|\xi) = P(1|0) \exp\left[ \int_0^{\xi} d\xi' A(1|\xi') \right],
\]  
(2.8)

\[
P(1,2|\xi) = P(1,2|0) \exp\left[ \int_0^{\xi} d\xi' K(1,2|\xi') \right],
\]  
(2.9)
where
\[
A(1|\xi') = \int d^2u_{12} P(1,2|\xi')/P(1|\xi') \\
+ \frac{1}{2} \int d^2 \int d^3u_{23} [P(1,2,3|\xi')/P(1|\xi') - P(2,3|\xi')] ,
\]
(2.10)

\[
K(1,2|\xi') = u_{13} + \int d^3 (u_{13} + u_{23}) P(1,2,3|\xi')/P(1,2|\xi') \\
+ \frac{1}{2} \int d^3 \int d^4u_{34} [P(1,2,3,4|\xi') - P(1,2|\xi')P(3,4|\xi')] /P(1,2|\xi') .
\]
(2.11)

Since the density is not high, and there is no electron condensation under the screened repulsive interaction, the multi-body correlation is much less effective than the two-body correlation. Then, the three- and four-particle distribution functions can be expanded by the convolution approximation in terms of two-body correlation function \cite{22}, and the Eqs.(2.8) and (2.9) are closed. For the ground state \(\Psi_\infty\), its density \(P_\infty(1)\) and two-body distribution function \(P_\infty(1,2)\) can be get directly by solving the combined integral equations (2.8) and (2.9), which has been done in the previous work \cite{15}.

In order to get the optical gap from the Eq.(2.6), we
must first know the density $P_e(1)$ and two-particle distribution function $P_e(1,2)$ for the excited state. They can be obtained in the following way. Defining

\[ \widetilde{P}(1) = P_e(1) - P_e(1) \quad \text{and} \quad \widetilde{P}(1,2) = P_e(1,2) - P_e(1,2), \quad (2.12) \]

which are the differences of the densities and correlation functions between the excited state and ground state. Since the system contains $N$ electrons and $\psi_\omega$ is an extended single-particle excited state, both $\widetilde{P}(1)$ and $\widetilde{P}(1,2)$ are quantities with the order of $1/N$. Keeping this in mind and applying the Eqs.(2.8) and (2.9) to the excited state $\psi_\omega$, the equations for $\widetilde{P}(1)$ and $\widetilde{P}(1,2)$ can be derived:

\[
\frac{\dot{\widetilde{P}}(1|\xi)}{\widetilde{P}(1|\xi)} = \frac{\dot{P}(1|0)}{P(1|0)} + \int_0^t d\xi' \left\{ \int d2 \dot{P}(2|\xi') \frac{\delta A(1|\xi')}{\delta P(2|\xi')} ight\}, \quad (2.13)
\]

\[
\frac{\dot{\widetilde{P}}(1,2|\xi)}{\widetilde{P}(1,2|\xi)} = \frac{\dot{P}(1,2|0)}{P(1,2|0)} + \int_0^t d\xi' \left\{ \int d3 \dot{P}(3|\xi') \frac{\delta K(1,2|\xi')}{\delta P(3|\xi')} ight\}, \quad (2.14)
\]
From the Eq. (2.6), the optical gap can be expressed in terms of $\tilde{P}(1)$ and $\tilde{P}(1,2)$:

$$E_\varepsilon = 2\Delta + \frac{N}{n_0} \int d1 \int d2 \left[ \hat{P}(1,2)/2 - n_0 \hat{P}(1) \right] \psi(1,2)$$

$$+ \frac{\hbar^2}{8\pi n_0^2} \int d1 \int d2 \hat{P}(1,2)(\nabla_1 u_{11})^2$$

$$+ \int d1 \int d2 \int d3 (\nabla_1 u_{11}) \cdot (\nabla_1 u_{12}) \left[ \int dy \hat{P}(y) \frac{\delta P(1,2,3)}{\delta P(y)} \right]$$

$$+ \int dy \int dz \hat{P}(y,z) \frac{\delta P(1,2,3)}{\delta P(y,z)}, \quad (2.15)$$

where $\Omega$ is the volume of the cell and $n_0$ the average density. From this expression it can be seen that, although $\tilde{P}(1)$ and $\tilde{P}(1,2)$ are infinitesimal, their contribution to the optical gap is finite. On the right side of Eq. (2.15), the first term $2\Delta$ is the dimerization gap, the rest are the effect of the electron interaction on the excitation.

Thus, the core of our theory is to calculate $\tilde{P}(1)$ and $\tilde{P}(1,2)$ by solving the integral equations (2.13) and (2.14), from which the density and the correlation function of the excited state can be obtained. Substituting the obtained
III. Results and conclusion

Following the formulism established in the last section, the optical gap with the electron interaction can be calculated step by step.

First, deal with the band without the electron interaction. For the sake of simplicity, a periodical square well potential is taken as the electron-lattice interaction, which gives the band width $4t_0 = 12\text{eV}$ and the non-interacting dimerization gap $E_{\epsilon}^0 = 1.4\text{eV}$. With this band, the non-intera-
acting density $P(1|0)$ and $\tilde{P}(1|0)$ as well as the two-body distribution function $P(1,2|0)$ and $\tilde{P}(1,2|0)$ can be obtained straightforwardly.

Next, using the obtained $P(1|0)$ and $P(1,2|0)$ as the initial condition to solve the integral equations (2.8) and (2.9), we can get the density $P(1|\xi)$ and the two-body distribution function $P(1,2|\xi)$ under the electron interaction. Substituting $P(1|\xi)$ and $P(1,2|\xi)$ into the Eqs. (2.13) and (2.14), and using $\tilde{P}(1|0)$ and $\tilde{P}(1,2|0)$ as the initial condition, $\tilde{P}(1)$ and $\tilde{P}(1,2)$ can be obtained by numerically solving the Eqs. (2.13) and (2.14).

Finally, substituting all the obtained $P(1)$, $P(1,2)$ and $\tilde{P}(1)$, $\tilde{P}(1,2)$ into the Eq. (2.15), we get the optical gap.

In our theory, the essential quantity is $\tilde{P}(1,2)$, which reflects the electron correlation in the excited state. The feature of $\tilde{P}(1,2)$ is shown in the Fig. 1, apparently it appears the $2k$ oscillation, which is the characteristic of the correlation function. Notice that $\tilde{P}(1,2)$ itself is not
the two-body correlation function of the excited state, as is defined in Eq.(2.12); it is the difference of the two-particle distribution functions between the excited state and ground state. So, besides the behavior of $\langle \alpha^2 \rangle$, oscillation, the shape of the Fig. 1 looks quite different from the two-body correlation function of the ground state, which was shown in the Fig. 2 of Ref. 15.

The dependences of the optical gap $E_o$ (in the unit of $E_o = 1.4$ eV) on both the interaction strength $U_0$ (in the unit of $t_0 = 3$ eV) and the screening factor $\beta$ are shown in the Fig. 2. Each curve in the Fig. 2 has a fixed screening factor $\beta$. From these curves, it can be seen that the weak screening and the strong screening have qualitatively different effects of the electron interaction on the optical gap. In the case of the weak and usual screening, the electron interaction increases the optical gap; the weaker is the screening, the larger is the increase. But, if the screening is very strong ($\beta > 3$), the optical gap will be reduced by the electron interaction.
Based on these results, the dispute about the effect of the electron interaction on the optical gap can be clarified. As has been mentioned in the Introduction, the ratios between the interaction parameters $U$, $V$, $W$, $X$ cannot be assigned arbitrarily. The merit of using the screened Coulomb repulsion to describe the electron interaction is that it can correctly bring about the ratios. With our band and the Wannier function, it is straightforward to calculate the ratios of the off-diagonal elements to the diagonal ones. These ratios depend on the screening, their values are given in the following table, where it shows $X$ is negative. Painelli and Girlando predicted it and talked its meaning [24]. Combining this table and the Fig.2, the controversy is easy to be settled. From the table it is seen that, if the screening is small, the off-diagonal elements $W$ and $X$ are much smaller than the diagonal ones, and they can be neglected. So in this case, the electron interaction can be described by the extended Hubbard model. At the same time, the Fig.2 shows that the optical gap is increased by the electron interaction if the screening is weak. It is just the
result got by the former school. The table also shows when the screening increases, the ratios of $W/V$ and $|X|/V$ rapidly increase. If the screening becomes very strong, the off-diagonal terms can be big enough to compete with the diagonal ones; then the bond-charge repulsion $W$ and the other off-diagonal elements should be considered. Meanwhile, the Fig. 2 indicates that the electron interaction will suppress the optical gap if the screening is strong, just as the latter school claimed. Thus, our theory discloses the origin of the conflicting conclusions between these two schools. Now it becomes clear that each side of the dispute has their own limitation: the former is valid in the weak or usual screening, whereas the latter, valid in strong screening.

<table>
<thead>
<tr>
<th>\beta</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W/V$</td>
<td>0.02</td>
<td>0.10</td>
<td>0.26</td>
<td>0.43</td>
</tr>
<tr>
<td>$X/V$</td>
<td>-0.06</td>
<td>-0.18</td>
<td>-0.45</td>
<td>-0.77</td>
</tr>
</tbody>
</table>
Here, we see the behavior of the optical gap is quite similar to that of the dimerization of the polymer, although the optical gap is determined by the excitation of the polymer, whereas the dimerization is only related to the ground state. For the dimerization, it has been figured out that the electron interaction initially enhances the dimerization if the screening is weak; but if the screening is strong enough ($\beta > 1.5$), the electron interaction will suppress the dimerization [15]. For the optical gap, the difference is that it requires even stronger screening, $\beta > 3$, and only then the optical gap will be suppressed by the electron interaction.

In summary, our conclusion is that the effect of the electron interaction on the optical gap depends on the screening: for weak and usual screening, the electron interaction increases the optical gap, and the increase can be so large that the electron interaction becomes the main origin of the optical gap; conversely, for very strong screening, the electron interaction decreases the optical gap, and the optical
gap is mainly produced by the electron-phonon interaction.

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References

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19. In some cases, the excitonic effect (electron-hole correlation) can change the optical absorption at the band edge. See the papers: H. Hayashi and K. Nasu, Phys. Rev. B32, 5295 (1985); G. Sebastian and G. Weiser, Phys. Rev. Lett. 46, 1156 (1981). This effect is not involved in this paper, since the dispute is mainly on the effect of e-e interaction on the single-particle excitation.
23. It has been pointed out in Ref.[21] that the interaction between the individual mode and the collective mode can be neglected. In this approximation, the individual degrees of freedom and the collective degree span different spaces. The orthogonization between $D_\alpha$ and $D_\beta$ ensures that $\Psi_\alpha$ and $\Psi_\beta$ are orthogonal each other.

Caption of Figures

Fig. 1 The curve of $N\cdot\tilde{P}(1,2)$ with $X_1 = 0$.

Fig. 2 The dependence of the optical gap $E_\varepsilon$ on the interaction strength $U_\circ$ with different screening $\beta$. 