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AN INVESTIGATION OF AN ALTERNATIVE TO ACCEPTANCE SAMPLING THROUGH A MARKOV CHAIN ANALYSIS OF A MANUFACTURING PROCESS QUALITY CONTROL PROGRAM

by

Daniel F. Harrington

September, 1990

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In this thesis, we investigate the examination of a manufacturer's in-house quality program as an alternative to acceptance sampling. The manufacturing process addressed is one which consists of a production section, capable of producing items at one of two levels of fraction nonconforming, and a quality control section which consists of a single p-chart. The quality levels that result from this manufacturing process are represented using a Markov chain. A method of estimating the fraction of nonconforming items produced by the process is developed. Confidence intervals on this fraction nonconforming are obtained and these values considered for use in an alternative acceptance criteria for lots. When the upper confidence limit on the lot fraction nonconforming does not exceed the Acceptable Quality Level, there is considerable confidence that lots randomly selected from the manufacturing process will be acceptable without acceptance sampling.

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An Investigation of an Alternative to Acceptance Sampling through a Markov Chain Analysis of a Manufacturing Process Quality Control Program

by

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ABSTRACT

In this thesis, we investigate the examination of a manufacturer's in-house quality program as an alternative to acceptance sampling. The manufacturing process addressed is one which consists of a production section, capable of producing items at one of two levels of fraction nonconforming, and a quality control section which consists of a single p-chart. The quality levels that result from this manufacturing process are represented using a Markov chain. A method of estimating the fraction of nonconforming items produced by the process is developed. Confidence intervals on this fraction nonconforming are obtained and these values considered for use in an alternative acceptance criteria for lots. When the upper confidence limit on the lot fraction nonconforming does not exceed the Acceptable Quality Level, there is considerable confidence that lots randomly selected from the manufacturing process will be acceptable without acceptance sampling.

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I. INTRODUCTION

In March 1988, the Secretary of Defense issued a memorandum giving priority to the Department of Defense Total Quality Management (TQM) effort. This effort was to focus on "quality as the vehicle for achieving higher levels of performance" with the ultimate goal being a "quality-equipped, quality-supported soldier, sailor, airman, and Marine." [Ref. 1]

Rising costs, decreasing budgets and urgency of product delivery has caused the Department of Defense to look closely at its quality assurance methodology. Long-term direction stated in the Department of Defense Total Quality Management Master Plan [Ref. 1] calls for

> establishing meaningful contract terms and conditions...rewarding/reinforcing contractor quality/reliability/producibility...emphasizing quality in award-fee incentives...instituting a Department of Defense contractor quality excellence award...and emphasizing contractor's control and monitoring of subcontractors.

Determining and assuring the quality of items acquired from contractors and their vendors is an increasingly expensive endeavor. In light of this, less costly alternatives to acceptance sampling are being sought. In June 1990, Hammons [Ref. 2], seeking one such alternative, developed a Markov chain to estimate the performance of a manufacturing process. It was shown that, in some cases, acceptance sampling may be unnecessary if it can be verified that the vendor's quality assurance program is *satisfactory*. Satisfactory, in this context, means that the

production process is monitored and control is maintained through a quality program that supplies sufficient *numerical evidence* to support an estimate of product quality.

Not all manufacturing processes require extensive scrutiny when quality is the concern. Some processes operate in a fashion which produces high quality products with such regularity that it may not be worth the time and resources needed to assure item quality. When items have low monetary value, or minimal operational impact, the expenditure needed to verify the producer's quality program may outweigh the cost of accepting a low quality item. The aforementioned concerns are reasons to accept some items without quality assurance. *Field* evaluation for these items may be the most cost effective course of action, reserving as open the option to reinstate quality assurance practices if item performance degrades to an unacceptable level.

The use of Markov chain methods to analyze quality control techniques that was found in current literature concentrates on modeling acceptance sampling plans themselves, not control of the overall manufacturing process. Brugger [Ref. 3] while using a Markov chain to analyze the inspection sampling plans given in ANSI/ASQC Z1.4 reports that

> While this paper dealt with sampling plans, the methods described could of course be used in other suitable applications.

Throughout this thesis we will explore an alternative to acceptance sampling by using a Markov chain analysis to verify a producer's quality program. We will examine a quality control practice typical of many manufacturing processes. This practice could represent a small quality program or a portion of a larger quality

program in a manufacturing process. Production will be classified as either *in control*, meaning the fraction of items produced not conforming to the quality characteristic specifications is acceptable; or *out of control*, meaning the fraction of items nonconforming is too great. Our approach will be to determine the proportion of time the process is in control and the proportion of time the process is out of control through the use of a Markov chain, and calculate the fraction of items produced that do not meet the required quality specifications. In some cases, rather than using the current method of acceptance sampling, this calculated fraction of nonconforming items may provide information for alternative acceptance criteria.

A. CURRENT PROCEDURES

A longstanding method for quality assurance practiced by the Department of Defense is acceptance sampling. The Department of Defense primarily uses MIL-STD 105D as its directive for acceptance sampling when inspection is by attributes. (Acceptance sampling when an item is judged nonconforming by variables is directed by MIL-STD 414.) These directives establish the sampling plan used when accepting or rejecting lots from a vendor.

In MIL-STD 105D, an Acceptable Quality Level or AQL is used to determine the sampling plan. This level is the "poorest level of quality...that the consumer would consider acceptable" in a lot, and is usually given in lot proportion nonconforming

[Ref. 4:p. 170]. The Acceptable Quality Level allows for a certain percentage of nonconforming items be present in a lot. Acceptance sampling is used to determine if a lot meets, or does better than this Acceptable Quality Level.

Single-sample acceptance plans are easily understood and implemented. A sample of predetermined size is identified in a lot. Each item in the sample is inspected for a certain quality characteristic. This characteristic may be a physical measurement such as weight or size; it may be a time-oriented calculation such as reliability or availability; or it may be a sensory-related assessment such as comfort or taste. If the number of items found not conforming exceeds a predetermined number, set by the sampling plan, then the lot is rejected. Conversely, the lot is accepted if the number of nonconforming items does not exceed the predetermined number.

The purpose of sampling in this manner is to determine the fate of a particular lot. It is not intended to estimate the lot quality. If several lots of identical quality are inspected, sampling in the above manner may reject some and accept others.

Acceptance sampling provides the consumer with a number of advantages, the first of which is savings. Sampling is generally less expensive than a policy of 100% inspection or *screening*, primarily because there is less inspecting. (Clearly, if inspection is destructive, 100% inspection is not practical.) A second advantage of sampling is the message sent to the vendor. Following a screening inspection, only the nonconforming items would be returned to the vendor. If sampling is used, an

entire lot is returned in place of just the nonconforming items. This "...provides a stronger motivation to the vendor for quality improvements" [Ref. 5:p. 353].

Of course, acceptance sampling does have its disadvantages. Foremost is the fact that bad lots may be accepted and good lots may be rejected. Although sampling is usually better than screening, it is still costly in both time and manpower. Records must be maintained justifying acceptance or rejection which add to the consumer's administrative burden and cost. In an effort to reduce costs, the Department of Defense is seeking alternatives that offer similar protection to that enjoyed under the current methods of acceptance sampling.

B. THIS THESIS

This thesis will pursue, as an alternative to acceptance sampling, examination of the in-house quality program a vendor has established to maintain quality. The numerical evidence provided by a satisfactory in-house quality program can provide information consistent with acceptance sampling. In other words, if a manufacturer is ensuring product quality through in-house programs, and this can be verified in the form of a quality estimate, the Department of Defense could, in some instances, forgo acceptance sampling in favor of the vendor's quality program.

Our effort begins in Chapter II where the concept of Statistical Process Control (SPC) and the place it takes in a producer's quality program is discussed. Control chart theory is introduced and supported by an example.

Chapter III presents a simple manufacturing process example that has its production process monitored through the use of Statistical Process Control. A three state Markov chain is used to model the impact of the manufacturing process' quality control program. The proportion of time that process production is in control and out of control is found and the fraction nonconforming produced is calculated. Confidence intervals are obtained for a lot fraction nonconforming. An alternative to that of acceptance sampling is addressed. The concept is then extended to explore a more complex manufacturing process. The employment of the Markov model is introduced along with the particular numerical evidence that must be obtained from the manufacturing process which is necessary to complete the calculations.

Chapter IV shows an application of the model through a solved numerical example. Explored here are some alternatives to acceptance sampling when the manufacturing process presented in Chapter III is representative of the process under consideration. The Markov model sensitivity is addressed through further numerical example.

In Chapter V, we briefly summarize our findings. Recommendations are made and suggestions for further study in this area are presented.

II. STATISTICAL PROCESS CONTROL

Statistical Process Control is a methodology which may be used as part of a vendor's quality control program. Quality programs using this methodology "let the process do the talking" while statistical "listening tools" monitor product quality [Ref. 6:p. 10]. An understanding of Statistical Process Control is necessary if we are to judge the vendor's quality program instead of using of acceptance testing. One of the primary *tools* used to maintain control of a process, the control chart, is introduced in this chapter.

A. THE CONTROL CHART

Manufacturing processes typically yield nonconforming items, for one or more reasons, in a random manner. Although naturally occurring production variations yielding nonconforming items must be accepted, many factors which cause nonconforming items can be identified and corrected. Untrained production personnel, poor input material, or production machinery slipping out of calibration are among these identifiable causes. A useful *listening tool* for identifying if a correctable factor might be present is the control chart.

First introduced in the mid 1920's by Walter A. Shewhart, control charts apply statistical hypothesis testing to monitor a production process. A production process is said to be *in control* when the level of nonconforming items produced is due only

to natural production variations. Infrequently, the process will begin to produce a differing level of nonconforming items. If identifiable causes are present, and the level of nonconforming items is other than normal, the process is said to be operating *out of control*. Control charts are used to reveal if a process has *shifted* from being in control to being out of control.

The control chart is a progressive plot monitoring quality, and is employed at a specific location in the manufacturing process. The x-axis represents the ordering of samples taken from the production process. The y-axis represents some aspect of quality as measured by the sample, such as the sample's fraction of nonconforming items. (A fraction nonconforming chart is known as a *p-chart*.) A center line on the chart represents the average value of the quality characteristic when the process is in control. Control limits, one high and one low, depict the allowed control chart tolerance. Generally the control limits are set three standard deviations of the process average above and below the center line (process average).

Control charts are easily employed during production. Samples, taken at predetermined intervals of length τ, are inspected and the quality measure is recorded. The sampling interval length may be time, such as every hour, or it could be a previously determined number of items produced, such as every 10,000. The process is listened to by plotting the sample results on the control chart. If the sample results fall outside the control limits, or a non-random pattern of sample results is observed, the process is declared out of control. When an out of control

determination is made, the cause, or causes for the new level of nonconforming items are sought and corrected.

The probabilistic results of a control chart can be likened to that of a hypothesis test, with the *null* hypothesis being that the process is in control. When a sample result falls outside the control limits we reject the hypothesis that the process is in control. If a sample result falls inside the control limits, we can only say that we fail to reject the hypothesis. As in hypothesis testing, the possibility of making a type II error exists.

Control charts have associated operating characteristic curves (OC curves). The probability of concluding that the process is in control is given as a function of a process parameter, such as proportion nonconforming. Interpretation of the control chart OC curve is much the same as that of a sampling plan OC curve. The probability of the chart yielding a decision of in control, when the process is not operating at the process average (central line), has a similar interpretation to that of accepting a bad lot. This is the risk of making a type II error or "...the chance of *not* catching a shift in the process average on the first sample..." (in fact, any sample) "...taken after the shift has occurred" [Ref. 4:p. 426].

B. A CONTROL CHART EXAMPLE

It is useful to give an example of a control chart. The type of control chart we will use for our example is a p-chart. Suppose that the fraction of nonconforming items produced when the process is in control, $\mathbf{p_1}$, is 0.05. This serves as our center

line. If samples of size **n** = 50 are used, then when the process is in control, the standard deviation of the sample's fraction nonconforming is

$$\sigma_p = \sqrt{\frac{(p_1)(1-p_1)}{n}} = 0.03082 \quad .$$

The control limits for our p-chart are the standard three-sigma limits and thus the upper control limit is 0.1425. Figure 1 shows this particular p-chart with points plotted for 13 samples.

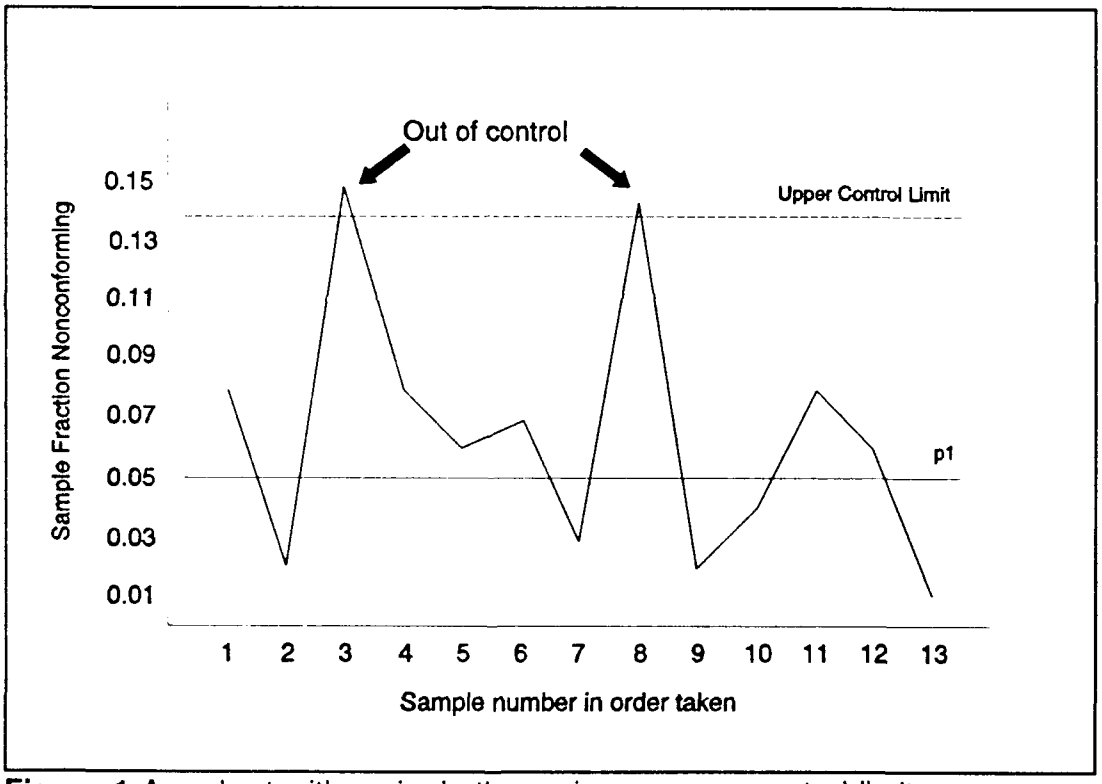

Figure 1 A p-chart with a single three-sigma upper control limit.

Random samples of 50 items, taken from the items produced in the previous sampling interval, are inspected for the desired quality characteristic. The fraction nonconforming from each sample is plotted against the time-ordered sample number.

The first two samples remained below the upper control limit and production is determined to be in control. The third sample plotted above the upper control limit revealing the process to be out of control. Typically, a shift to out of control production can be traced back to a cause factor in the production process. In this case, corrective action of some sort was taken and subsequent samples showed this action to bring the process in control. Sampling continued every interval and the determinations were made concerning the condition, in control or out of control, of the production process.

This type of listening tool is a basic part of Statistical Process Control. In the next chapter, we will examine a production process which uses Statistical Process Control as part of its quality program. A method of process quality control verification is explored which leads to calculations of alternative acceptance criteria. We will take a stochastic approach.

III. USING A MARKOV MODEL TO EXAMINE THE IMPACT OF A QUALITY CONTROL PROGRAM

In this chapter, the impact of a quality control program on a simple manufacturing process will be modeled as a Markov chain. Statistical Process Control, in the form of a control chart, will be the statistical foundation of the quality control program. Using this Markov model, we present a methodology one might use to find, as *satisfactory*, representative quality control programs. (Satisfactory was previously defined as meaning that the quality program supplies sufficient numerical evidence to support an estimate of product quality.) Our goal will be to determine the proportions of time the manufacturing process operates either in control, or out of control, which will permit us to calculate an estimate of the fraction of nonconforming items produced. In turn, we will use this estimate to form possible alternative acceptance criteria to that of acceptance sampling. After modeling a quality control program, we will expand this approach to encompass a more involved manufacturing process.

A. A SIMPLE MANUFACTURING PROCESS EXAMPLE

The manufacturing process we will consider consists of a set of production stages, producing items at a constant rate, and a process quality control section, responsible for implementing Statistical Process Control. In this process, the production stages, located prior to the process quality control section, yield one of

two fractions of nonconforming items. When production is of acceptable quality, the fraction of nonconforming items produced and arriving at the quality control section will be p_1, while when the process shifts to the second, unacceptable level, the fraction of nonconforming items produced will be p_2. The process quality control section will consist of a single p-chart. Figure 2 shows a representation of this example manufacturing process.

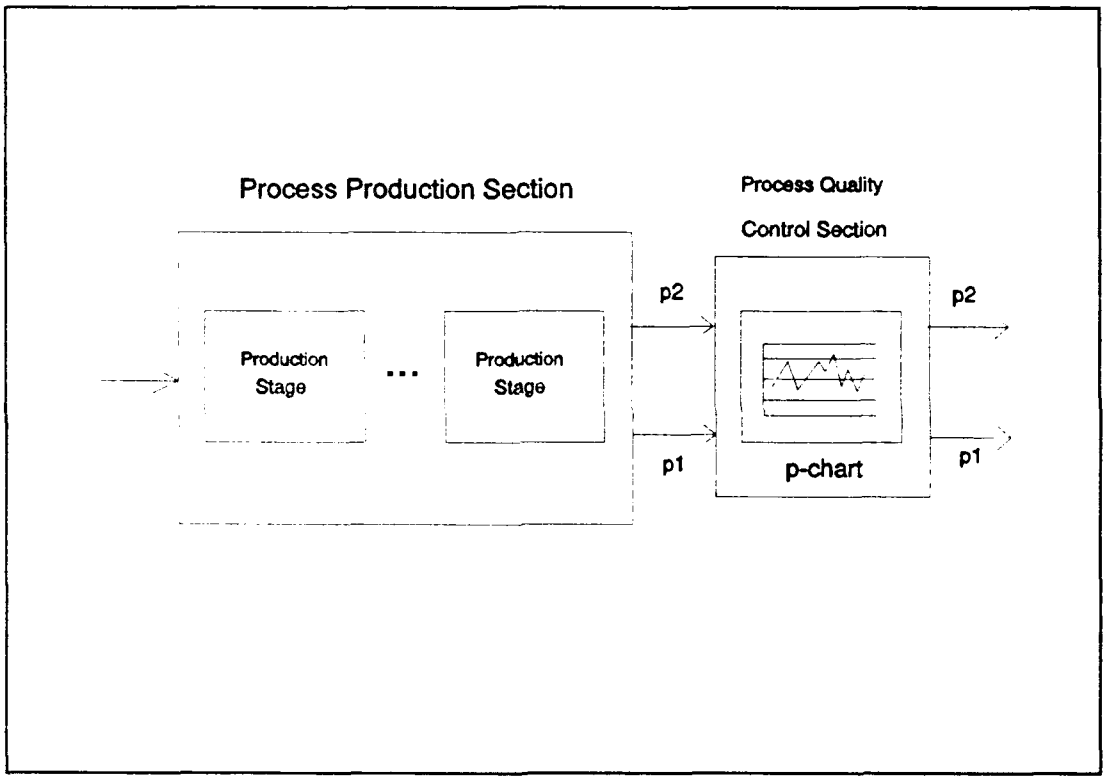

Figure 2 A manufacturing process composed of a Process Production Section producing one of two fractions nonconforming (p_1 or p_2), and a Process Quality Control Section employing a p-chart.

The process quality control section, the p-chart, is used to determine when the manufacturing process has shifted from in control to out of control. At the end of each sampling interval of length τ, a random sample of production items is drawn,

and inspected for the desired quality characteristics. The results of the sampling inspection are recorded on the p-chart, and a determination is made regarding whether the process is in control or out of control.

B. A MARKOV REPRESENTATION OF A MANUFACTURING PROCESS QUALITY CONTROL SECTION

A Markov chain exhibits the property that the one-step transitions from a Markov chain state to another depends only upon the current state and the one-step transition probability to the next state. This independence from the past is known as the Markov property. The quality levels resulting from a manufacturing process, consisting of a process production section and a process quality control section (a p-chart) can be represented using a Markov chain.

The Markov chain representation of the example manufacturing process introduced earlier is closely related to the sampling interval τ of the process quality control section. Over the course of a sampling interval, the process production section may or may not have experienced a shift from $\mathbf{p}_1$ to $\mathbf{p}_2$, and the p-chart determination of the process quality control section (in control or out of control) may or may not have correctly identified this shift. Combinations of these events form three Markov chain states. The states for our Markov chain will be defined as

State I - The process starts the sampling interval yielding $\mathbf{p}_1$, and remains yielding $\mathbf{p}_1$ throughout the sampling interval,

State S - The process starts the sampling interval yielding $\mathbf{p}_1$, shifts to yielding $\mathbf{p}_2$ during the sampling interval, and remains yielding $\mathbf{p}_2$ for the remainder of the interval, and

State O - The process starts the sampling interval yielding $\mathbf{p_2}$, and remains yielding $\mathbf{p_2}$ throughout the sampling interval.

The level of nonconforming items produced by the process production section, ($\mathbf{p_1}$ or $\mathbf{p_2}$) and the p-chart determination of the process quality control section (in control or out of control) can be seen in Figure 3 which shows how the state-to-state Markov transitions might occur.

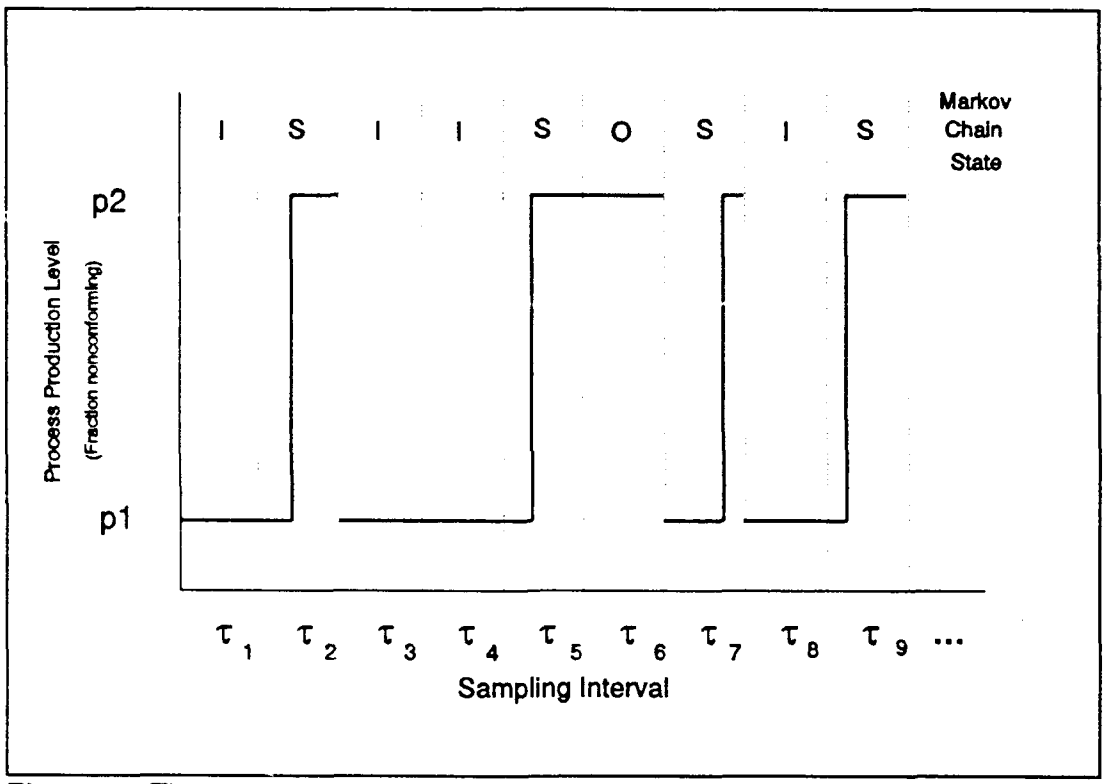

Figure 3 The manufacturing process Markov chain plot. This plot shows the relationship of the process level and the p-chart determination to the Markov chain state.

In the first sampling interval, denoted by Markov chain state **I**, production started at and remained at $\mathbf{p_1}$. The second sampling interval exhibits a production level shift from $\mathbf{p_1}$ to $\mathbf{p_2}$; thus the Markov chain state is denoted by **S**. In the sixth sampling interval the process was in state **O**, with production at a level of

nonconforming equal to $\mathbf{p_2}$ for the entire interval. This occurs because the p-chart failed to correctly identify an out of control process in the previous, the fifth, interval. Production will remain at a level equal to $\mathbf{p_2}$ until a correct p-chart determination is made.

Transitions among the three Markov chain states **I**, **S**, and **O** result from two probabilistic occurrences in the manufacturing process. The first is the p-chart determination of the process production level, $\mathbf{p_1}$ or $\mathbf{p_2}$. When we conclude the process average fraction nonconforming is $\mathbf{p_1}$, given that production is at $\mathbf{p_2}$, a type II error has occurred. We will define the probability of making this type II error as $\mathbf{P_a}$. The complimentary probability, $\mathbf{1\text{-}P_a}$, is the probability we conclude production is at process average fraction nonconforming, $\mathbf{p_2}$, given that production is, in fact, at that level. The second probabilistic occurrence, the probability that the production level shifts from $\mathbf{p_1}$ to $\mathbf{p_2}$, must be carefully defined to ensure the Markov property of independence from the past is observed.

The manufacturing process shift can be viewed as a *time to failure* of the process, an analogue of a machine in a maintenance model. We will consider production at $\mathbf{p_1}$ to be synonymous with that of a production process *success*, and production at $\mathbf{p_2}$ to be synonymous with that of a *failure*. The manufacturing process, yielding a fraction of nonconforming items equal to $\mathbf{p_1}$, may begin to yield a fraction of nonconforming items equal to $\mathbf{p_2}$ in a random amount of time. This process shift, a failure of the process production section, may be due to any number of causes on the production line. Perhaps a production unit slipped out of calibration and caused

the shift, or an employee, inattentive to his job, was the cause. The time elapsed from when the successful process (yielding $\mathbf{p_1}$) fails (shifts and begins to yield $\mathbf{p_2}$), may be exponentially distributed. This is not unlike a machine in a maintenance model.

We will not draw any further conclusions concerning this probability distribution. However, we will invoke the memoryless property at times τ, 2τ, 3τ... . This maintains the Markov property of independence of past states. The exponential probability that the manufacturing process shifts during a sampling interval is

$$\delta = 1 - e^{-\lambda\tau} .$$

However, if we express the exponential shift rate parameter λ, in units of shifts per τ, the resultant expression for δ is independent of the sampling interval length and can be expressed as

$$\delta = 1 - e^{-\lambda} .$$

The complimentary probability, $1-\delta$, is the probability that no shift occurs.

Recall that our goal was to determine the proportions of time the manufacturing process operates either in control, or out of control and calculate an estimate of the fraction of nonconforming items produced. Our approach is straightforward. To estimate the manufacturing process fraction nonconforming $\mathbf{p'}$, we will multiply the fraction nonconforming level the production process is operating at by the proportion of time the process spends producing that particular level, and sum over all states. The proportions of time the process operates in each state are commonly referred to as the *stationary probabilities*. Stationary probabilities are

symbolically represented with the notation π_{state}, where the subscript denotes the state of interest. This notation allows us to conveniently represent the equation for the manufacturing process fraction nonconforming as

$$p' = \sum_{i \in \phi} \pi_i \, p_i \quad ; \qquad \phi = \{I, S, O\} \quad .$$

The state S holds particular interest in that it is the state during which two differing levels of fraction nonconforming are produced, $\mathbf{p}_1$ and $\mathbf{p}_2$. To complete the calculation of $\mathbf{p}'$, we need to know the fraction nonconforming level produced while in this state. This fraction nonconforming, which we will refer to as $\mathbf{p}^*$, has a lower bound of $\mathbf{p}_1$ and an upper bound of $\mathbf{p}_2$. Given that the production process shifts from $\mathbf{p}_1$ to $\mathbf{p}_2$ over the sampling interval $(0, \tau]$, and that the time until the shift is exponentially distributed with rate parameter λ, we derive the expression

$$Q = \frac{1 - (1+\lambda)e^{-\lambda}}{\lambda(1-e^{-\lambda})} \quad , \tag{1}$$

which is the expected value of the proportion of time spent in state S attributable to the fraction nonconforming level of $\mathbf{p}_1$. The full derivation is given in Appendix A. With this result, the relationship for $\mathbf{p}^*$ is defined as

$$p^* = Q\,p_1 + (1-Q)\,p_2 \tag{2}$$

The process average $\mathbf{p}^*$ for state S has an associated p-chart type II error probability. This type II error occurs when we conclude the process average fraction nonconforming is $\mathbf{p}_1$, given that production for the sampling interval is actually $\mathbf{p}^*$.

We will define the probability of making this error as $\mathbf{P_a^*}$. The value for $\mathbf{P_a^*}$ can be determined from the p-chart OC curve as a function of $\mathbf{p^*}$.

The two probabilistic occurrences in the manufacturing process over a sampling interval: the p-chart determination when the process is at $\mathbf{p_2}$ or $\mathbf{p^*}$, and the probability of a process shift, permit calculation of the one-step transition probabilities among the Markov chain states **I**, **S**, and **O**. These one-step transition probabilities can be shown in the transition probability matrix,

$$
\begin{array}{cc}
 & \textit{NEXT STATE} \\
 & \begin{array}{ccc} I & S & O \end{array} \\
\begin{array}{c} \textit{PRESENT STATE} \\ \\ \end{array}
\begin{array}{c} I \\ S \\ O \end{array}
\begin{pmatrix}
(1-\delta) & \delta & 0 \\
(1-P_a^*)(1-\delta) & (1-P_a^*)\delta & P_a^* \\
(1-P_a)(1-\delta) & (1-P_a)\delta & P_a
\end{pmatrix}
\end{array}
$$

This matrix reflects the previously explained state-to-state Markov transitions. For example, a transition from state **O** to state **S** requires that the p-chart correctly determine production to be at the $\mathbf{p_2}$ level $(\mathbf{1-P_a})$, and that during the next sampling interval, a production process shift back to $\mathbf{p_2}$ occurs $(\boldsymbol{\delta})$. These transition probabilities can also be shown in the form of the Markov chain transition diagram as seen in Figure 4.

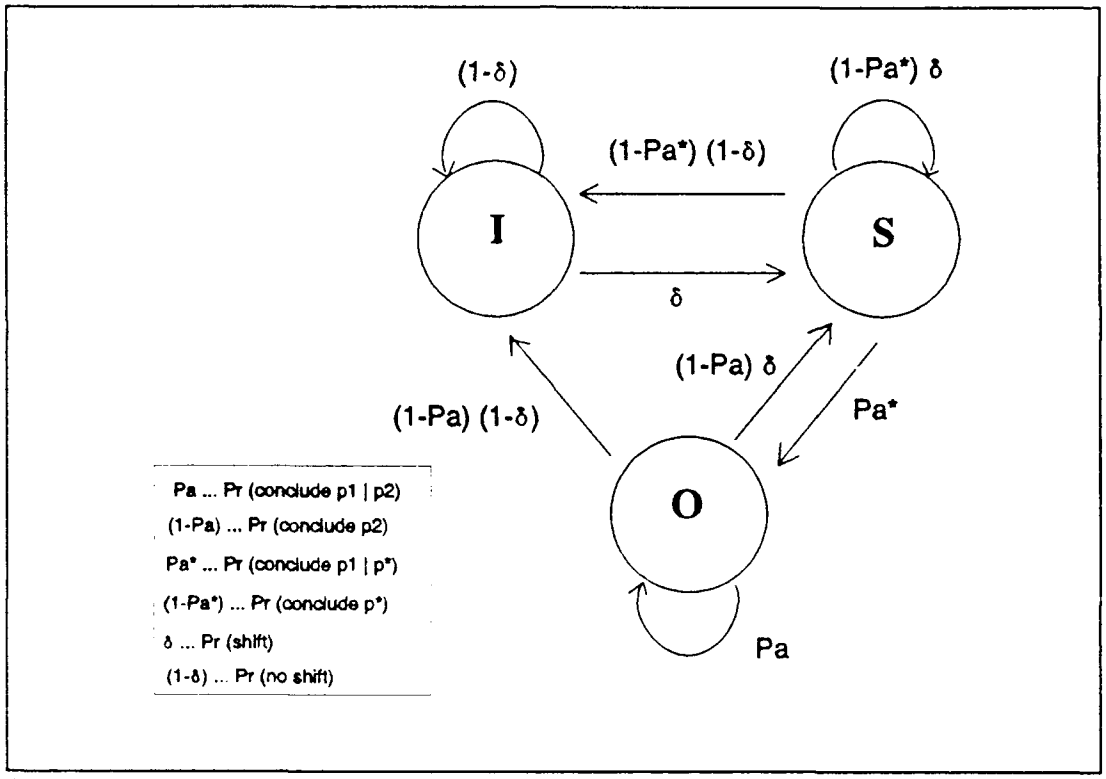

Figure 4 The probability transition diagram for the Markov chain states I, S, and O. The transition probabilities are the probabilistic outcomes of a p-chart's determination of a production process shift occurrence.

C. STATIONARY PROBABILITIES

The probability transition matrix,

$$
\begin{array}{cc}
 & \textit{NEXT STATE} \\
 & \begin{array}{ccc} I & S & O \end{array} \\
\begin{array}{c} \textit{PRESENT STATE} \\ \\ \\ \end{array}
\begin{array}{c} I \\ S \\ O \end{array}
& \begin{pmatrix} (1-\delta) & \delta & 0 \\ (1-P_a^*)(1-\delta) & (1-P_a^*)\delta & P_a^* \\ (1-P_a)(1-\delta) & (1-P_a)\delta & P_a \end{pmatrix}
\end{array}
$$

can be solved for the stationary probabilities by solving

$$\pi_j = \sum_{i,\, j \in \phi} \pi_i P_{ij} \quad ; \qquad \phi = \{I, S, O\} \quad ,$$

with the restriction that

$$\sum_{j \in \phi} \pi_j = 1 \quad ,$$

where the one-step transition probabilities $\mathbf{P}_{ij}$, are taken from the probability transition matrix [Ref. 7:p. 152].

To symbolically find the stationary probabilities we substitute the one-step transition probabilities into the above equations and solve

$$\pi_I = \pi_I(1-\delta) + \pi_S(1-P_a^*)(1-\delta) + \pi_O(1-P_a)(1-\delta) \quad ,$$

$$\pi_S = \pi_I \delta + \pi_S(1-P_a^*)\delta + \pi_O(1-P_a)\delta \quad ,$$

$$\pi_O = \pi_S P_a^* + \pi_O P_a \quad ,$$

and

$$\pi_I + \pi_S + \pi_O = 1 \quad .$$

The resulting stationary probabilities

$$\pi_I = \frac{(1-P_a)(1-\delta)}{1 - P_a + \delta P_a^*} \quad , \tag{3}$$

$$\pi_S = \frac{\delta(1 - P_a)}{1 - P_a + \delta P_a^*} \quad , \tag{4}$$

and

$$\pi_O = \frac{\delta P_a^*}{1 - P_a + \delta P_a^*} \quad , \tag{5}$$

represent the long run proportion of time each process state is experienced.

Most control charts will be designed so that the value of $\mathbf{P_a}$ is small, probably near 0.1, and certainly less than 0.5. Since the chance of a shift would be expected to also be small, and again certainly less than 0.5, the steady state equations show that the process will spend the greatest proportion of sampling intervals in control. The proportion of sampling intervals spent out of control will be smaller than those in which a shift occurs. In short, we should have $\pi_I > \pi_S > \pi_O$.

D. FRACTION NONCONFORMING CALCULATION

The stationary probabilities give us the necessary input to calculate the fraction nonconforming. Specifically, production is operating at a process average fraction nonconforming of $\mathbf{p_1}$ (in state **I**) the proportion of time equal to π_I. Likewise, the process is operating at a process average fraction nonconforming of $\mathbf{p_2}$ (in state **O**) the proportion of time equal to π_O and at process average fraction nonconforming $\mathbf{p^*}$ (in state **S**) the proportion of time equal to π_S.

Our approach, as described earlier, was to take the stationary probabilities, multiply them by their respective process fraction of nonconforming items yielded, and sum over all states. The resulting calculation,

$$p' = \pi_I p_1 + \pi_S p^* + \pi_O p_2 \quad , \tag{6}$$

is the long run process fraction of nonconforming items yielded by the manufacturing process.

Addressing the stationary probabilities in a somewhat different manner, an equivalent interpretation of π_I is the probability that the process is operating at $\mathbf{p_1}$. We will refer to this probability hereafter as $\mathbf{Pr(p_1)}$. The probability that the production process is operating at $\mathbf{p_2}$ is π_O, and will be referred to as $\mathbf{Pr(p_2)}$. Likewise, π_S will be referred to as $\mathbf{Pr(p^*)}$. Therefore, an equivalent representation of Equation (4) would be

$$p' = Pr(p_1)\ p_1 + Pr(p^*)\ p^* + Pr(p_2)\ p_2 \quad . \tag{7}$$

If control of the manufacturing process was maintained as described by the model, and lots are formed from a random sampling of the production items, then the expected number of nonconforming items in a lot of size **N** is determined to be

$$E[\#\ nonconforming\ items\ in\ a\ lot] = E[\#_{lot}] = N\,p' \quad ,$$

and the variance of the number of nonconforming items in a lot is

$$Var[\#\ nonconforming\ items\ in\ a\ lot] = \sigma^2_{lot} = Np'(1-p') \ .$$

A one-sided 95% upper confidence limit on the fraction of nonconforming items in a lot, $\mathbf{p}_{lot}$, is found to be

$$Pr\left(p_{lot} < p' + 1.645 \sqrt{\frac{p'(1-p')}{N}} \right) \doteq 0.95 \ . \tag{8}$$

This confidence limit is based on the normal approximation to the binomial distribution. Reasonable results are obtained when N and $\mathbf{p'}$ are large (N > 30 and p' > 0.10). (Another rule of thumb for using this approximation is that N(p') and N(1-p') be greater than or equal to 5.) When $\mathbf{p'}$ is small (p' < 0.01) and N is still large, a poisson approximation to the binomial distribution should be made. The confidence interval for small values of N in combination with a range of $\mathbf{p'}$ between 0.01 and 0.50 can be obtained in the National Bureau of Standards Tables. [Ref. 4:pp. 572-3]

A one-sided upper confidence limit on the fraction of nonconforming items in a lot is useful because it can be stated with a certain amount of confidence that the number of nonconforming items in the lot is expected to be no greater than that limit. When this value of fraction nonconforming is compared to the Acceptable Quality Level, valuable information is gained when considering alternate acceptance criteria.

E. VERIFICATION OF THE PRODUCER'S QUALITY PROGRAM

If alternative acceptance criteria are to be considered, and a fair comparison made to acceptance sampling, then we must establish a common ground for measure. We will consider using the Acceptable Quality Level as this measure. We can recall that when using acceptance sampling, the Acceptable Quality Level or AQL was the standard against which each lot is measured. The AQL is typically expressed as the "maximum fraction nonconforming for the supplier's process that the consumer would consider to be acceptable as a process average for the purposes of acceptance sampling" [Ref. 4:p. 170]. When acceptance sampling leads to the acceptance of a lot, the inference is made that the lot quality level is equal to, or lower than the AQL.

Verification of the producer's quality program through the results of the Markov model can also directly employ the AQL. When the upper confidence limit on the lot proportion nonconforming does not exceed the AQL, we should have considerable confidence that lots randomly selected from this process will be acceptable without acceptance sampling. In particular, if

$$p' + 1.645 \sqrt{\frac{p'(1-p')}{N}} \leq AQL \ , \tag{9}$$

we will have at least 95 percent confidence in the quality of an individual lot.

The manufacturing process we have examined in structuring this approach has been a simple one. In the next section, we will consider an expanded manufacturing process.

F. APPLICATIONS TO EXPANDED MANUFACTURING PROCESSES

The methods for estimating quality, which were provided in the previous section, can be extended to larger manufacturing processes. Many manufacturing processes have more than one production section and associated process quality control sections. One such process may consist of a production section, monitored by a p-chart, followed by another production section and its associated p-chart.

We will consider two variations of this expanded process and show how output quality estimates may be obtained. In the first case, we set the process shift probability of the follow-on production process equal to zero. In other words, the second p-chart serves as a back-up to the first. In the second case, we examine the situation when the follow-on process shift is greater than zero. The diagram for this expanded manufacturing process is shown in Figure 5.

For each of these variations, the initial production process and its quality control section will perform as was previously described. A double prime (″) notation will identify variables belonging to the second production and quality sections.

The calculations involved in the examination of these expanded processes can be more simply approached if we apportion the fraction nonconforming levels

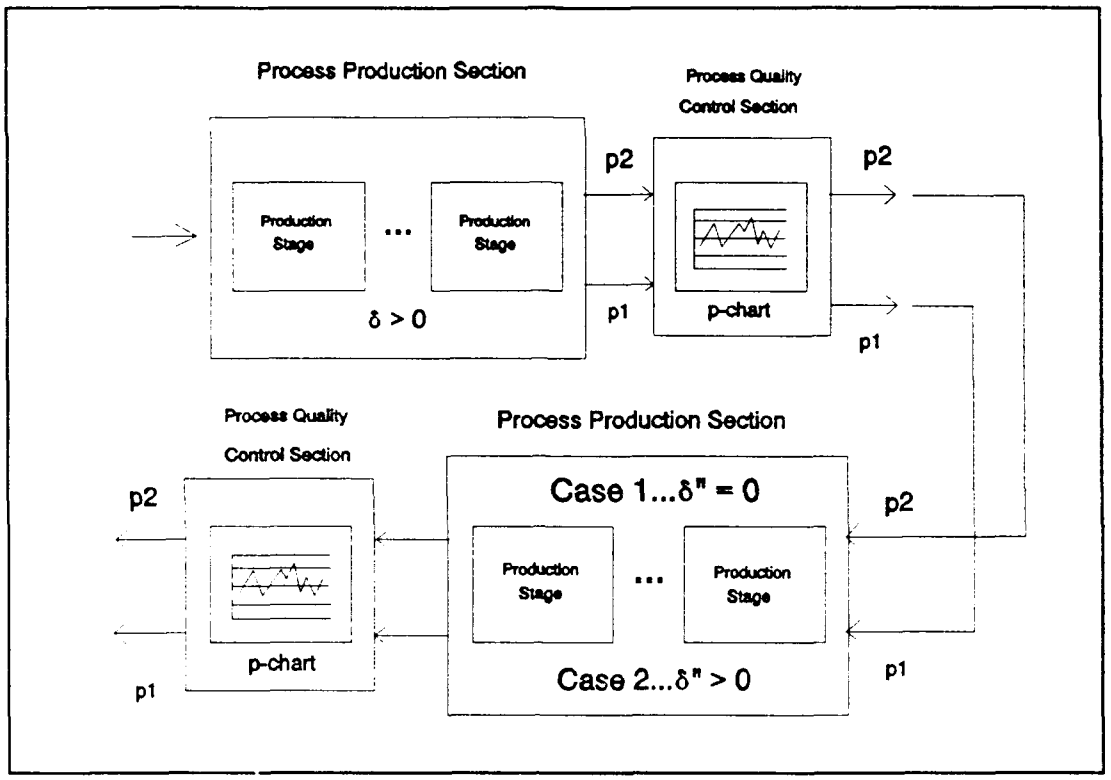

Figure 5 An expanded manufacturing process consisting of two Process Production Sections, each having a Process Quality Control Section.

produced while in state **S** directly to the states **I** and **O**. This means that the approximate proportion of time the first manufacturing process is operating at level $\mathbf{p_1}$ would be

$$Pr(p_1)' \doteq Pr(p_1) + Q(Pr(p^*)) \quad , \tag{10}$$

and an approximate value for proportion of time spent operating at level $\mathbf{p_2}$ is

$$Pr(p_2)' \doteq Pr(p_2) + (1-Q)(Pr(p^*)) \quad . \tag{11}$$

1. Case 1

The manufacturing process we will consider consists of two parts. The first part is a production section (producing one of two process averages, $\mathbf{p_1}$ and $\mathbf{p_2}$), and a quality control section (p-chart) similar to that which we have introduced. The second part is similar to the first except the process shift probability δ'' is taken to be zero. This corresponds to having two successive control charts ensuring item quality.

To find the fraction nonconforming produced by this manufacturing process we will use a familiar approach. We will determine the probabilities that the manufacturing process is yielding items at a process fraction nonconforming equal to $\mathbf{p_1}$, (this will be represented by $\mathbf{Pr(p_1)''}$), and at a process fraction nonconforming equal to $\mathbf{p_2}$, (represented by $\mathbf{Pr(p_2)''}$); multiply by the process fraction of nonconforming items ($\mathbf{p_1}$ and $\mathbf{p_2}$ respectively); and sum for the resulting expected value.

The probability $\mathbf{Pr(p_1)''}$ that the manufacturing process is yielding items with fraction nonconforming of $\mathbf{p_1}$ is equal to

$$Pr(p_1)'' = Pr(p_1)' + (Pr(p_2)')\ (1 - P_a'') \quad . \tag{12}$$

This represents that proportion of time the first production section is operating at $\mathbf{p_1}$ plus that proportion of time the second p-chart catches an error made by the first. The probability that the process is operating at $\mathbf{p_1}$, and the complimentary probability $\mathbf{1\text{-}Pr(p_1)''}$, also represented by

$$Pr(p_2)'' = Pr(p_2)' - (Pr(p_2)')\ (1 - P_a'')\ , \tag{13}$$

are substituted into

$$p'' = Pr(p_1)''\ p_1 + Pr(p_2)''\ p_2\ . \tag{14}$$

The fraction of nonconforming items exiting the manufacturing process after the second section control chart is then specified as

$$p'' = \left[\ Pr(p_1)' + (Pr(p_2)')\ (1 - P_a'')\ \right] p_1 + \left[\ (Pr(p_2)')\ P_a''\ \right] p_2\ . \tag{15}$$

2. Case 2

In Case 2, we will examine a manufacturing process similar to the one presented in Case 1, however the second production section's process shift probability δ'', will be greater than zero. This means that an in-control process may shift to out of control after the first quality control section.

In this second case, to specify the probability $\mathbf{Pr(p_1)''}$ that the manufacturing process is yielding items with a fraction nonconforming equal to $\mathbf{p_1}$, and the probability $\mathbf{Pr(p_2)''}$ for that of a process fraction nonconforming equal to $\mathbf{p_2}$, a close examination of the second production section and its associated control chart is needed. The events that can occur with respect to the process average fraction nonconforming after the first process production section and its quality control section (the first p-chart), are:

1. The second production section can be entered at a level of nonconforming items equal to $\mathbf{p_1}$, and continue to yield $\mathbf{p_1}$ for the remainder of the sampling interval,

2. The second production section can be entered at a level of nonconforming items equal to $\mathbf{p_1}$, shift to $\mathbf{p_2}$, and be correctly identified by the second p-chart at the end of the sampling interval,

3. The second production section can be entered at a level of nonconforming equal to $\mathbf{p_1}$, shift to $\mathbf{p_2}$, and *not* be correctly identified by the second p-chart at the end of the sampling interval,

4. The second production section can be entered at a level of nonconforming equal to $\mathbf{p_2}$, and be correctly identified by the second p-chart at the end of the sampling interval,

5. The second production section can be entered at a level of nonconforming equal to $\mathbf{p_2}$, and *not* be correctly identified by the second p-chart at the end of the sampling interval.

These events occur with probabilities of

Event 1 : $(\mathbf{Pr}(\mathbf{p_1})')(1-\delta'')$,

Event 2 : $(\mathbf{Pr}(\mathbf{p_1})')\ \delta''\ (1-\mathbf{P_a}'')$,

Event 3 : $(\mathbf{Pr}(\mathbf{p_1})')\ \delta''\ \mathbf{P_a}''$,

Event 4 : $(\mathbf{Pr}(\mathbf{p_2})')\ (1-\mathbf{P_a}'')$, and

Event 5 : $(\mathbf{Pr}(\mathbf{p_2})')\ \mathbf{P_a}''$.

The fraction of nonconforming items yielded by this process is found as before. The events that conclude the manufacturing process is operating at a process fraction of nonconforming items yielded equal to $\mathbf{p_1}$ are numbered 1, 2, and 4. Therefore, the probability $\mathbf{Pr}(\mathbf{p_1})''$, that the manufacturing process is yielding items with a fraction nonconforming equal to $\mathbf{p_1}$ is

$$Pr(p_1)'' = \left[(Pr(p_1)')(1-\delta'') + (Pr(p_1)')\delta''(1-P_a'') + (Pr(p_2)')(1-P_a'')\right]. \quad (16)$$

The remaining events, 3 and 5,

$$Pr(p_2)'' = \left[(Pr(p_1)') \, \delta'' \, P_a'' + (Pr(p_2)') \, P_a'' \right] , \tag{17}$$

represent the probability $\mathbf{Pr(p_2)''}$ that the manufacturing process is yielding items at a process fraction of nonconforming equal to $\mathbf{p_2}$. Substituting the probabilities of occurrence $\mathbf{Pr(p_1)''}$ and $\mathbf{Pr(p_2)''}$ into Equation (14), we find,

$$p'' = \left[(Pr(p_1)') \, (1 - \delta'') + (Pr(p_1)') \, \delta''(1 - P_a'') + (Pr(p_2)') \, (1 - P_a'') \right] p_1$$

$$+ \left[(Pr(p_1)') \, \delta'' \, P_a'' + (Pr(p_2)') \, P_a'' \right] p_2 , \tag{18}$$

is the fraction of nonconforming items exiting the manufacturing process after the second section control chart.

G. EMPLOYMENT OF THE MARKOV MODEL

The employment of this approach requires that certain parameters concerning the manufacturing process be known. In particular, we need to know the values for $\mathbf{p_1}$, $\mathbf{p^*}$, and $\mathbf{p_2}$ (the levels of process average fraction nonconforming); $\mathbf{P_a}$ and $\mathbf{P_a^*}$ (the probabilities that the control chart makes a type II error); and $\boldsymbol{\delta}$ (the probability that the production process shifts from $\mathbf{p_1}$ to $\mathbf{p_2}$). These parameter values may be calculated from the *numerical evidence* provided by a manufacturer's existing quality program. Specification of the control chart will provide us with the value for $\mathbf{P_a}$, while manufacturing process data will support calculation of the remaining values. The levels of process average fraction nonconforming, $\mathbf{p_1}$ and $\mathbf{p_2}$ could be determined

from sampling data. However, the value for the shift between the two levels δ, may be the most difficult to obtain.

A reasonable approximation for δ could be calculated by assuming that the time until the production process shift occurs is exponentially distributed. The shift probability δ would be represented by

$$\delta \doteq 1 - e^{-\hat{\lambda}} \quad . \tag{19}$$

The exponential shift rate parameter, the *arrival rate of a failure* of the production process $\hat{\lambda}$, could be estimated using maximum likelihood techniques.

In its simplest form, the information required to calculate $\hat{\lambda}$ need only be a record of the run lengths of in-control determinations as made by the quality control section's p-chart. The number of sampling intervals between out-of-control determinations could be recorded and expressed as r_i (i = 1, 2,..., n). The value of r_1 would be the length of the first run of in-control p-chart determinations. Likewise, the value of r_2 would be the length of the second run of in-control p-chart determinations, and so forth. If these values are available, the maximum likelihood function for λ,

$$L_R(\lambda) = \prod_{i=1}^{n} (\lambda\, e^{-\lambda r_i}) = \lambda^n e^{-\lambda \Sigma r_i}$$

can be solved by first taking the natural logarithm of both sides,

$$\ln L_R(\lambda) = n \ln \lambda - \lambda \Sigma r_i \ ;$$

then taking the derivative with respect to λ,

$$d \ln L_R(\lambda) = \frac{n}{\lambda} - \Sigma r_i \ ;$$

setting it equal to zero, and solving for $\hat{\lambda}$,

$$\hat{\lambda} = \frac{n}{\Sigma r_i} \ .$$

The maximum likelihood estimate of $\hat{\lambda}$ completes the information we needed to calculate values for δ, $\mathbf{p}^*$, and $\mathbf{P_a}^*$. [Ref. 8:p. 365]

The approach presented in this chapter examined the impact of an existing quality control program (a p-chart) on the production section of a simple manufacturing process. In the next chapter, a numerical example will be offered reviewing the calculations necessary for model employment.

IV. A SOLVED NUMERICAL EXAMPLE

In this chapter we will examine, with more detail, the impact of a quality control program on a simple manufacturing process through a solved numerical example. We will assess the manufacturing process' quality control program by calculating the 95 percent upper confidence limit on the lot proportion nonconforming and comparing it to the **AQL**. Finally, the sensitivity of the quality control section is addressed through alterations of the sampling interval and control chart parameters.

A. AN EXAMPLE MANUFACTURING PROCESS

Our example manufacturing process will be similar to the one introduced in Chapter III. It will consist of a continuously-operating process production section capable of producing items at either fraction nonconforming level $\mathbf{p_1}$ or $\mathbf{p_2}$, and a process quality control section, consisting of a single p-chart, capable of detecting a shift to the higher fraction nonconforming level $\mathbf{p_2}$ at least 90 percent of the time ($\mathbf{1-P_a} = 0.90$). The shift from $\mathbf{p_1}$ or $\mathbf{p_2}$ will be assumed to occur in an exponential manner. The production rate of manufacturing process will be 100,000 items per hour and for the purposes of quality control, samples will be taken every $\tau = 1$ hour. The *in control* process average fraction nonconforming $\mathbf{p_1}$, determined from production data, was found to be 0.07. In other words, on average, 7 percent or 7 items of every 100 produced are expected to be nonconforming. When the

production process *fails*, the process average fraction nonconforming produced $\mathbf{p_2}$ is equal to 0.12.

To successfully employ the three-state Markov approach introduced in Chapter III, we need to know the values for $\mathbf{p_1}$, $\mathbf{p^*}$, $\mathbf{p_2}$, $\mathbf{P_a}$, $\mathbf{P_a^*}$, and $\boldsymbol{\delta}$. Typically, as is the case for our example, the values for $\mathbf{p_1}$, $\mathbf{p_2}$, are known from sampling data and the value of $\mathbf{P_a}$ can be derived from the p-chart OC curve. The value for $\mathbf{P_a}$ is set at .10. The remaining values $\mathbf{p^*}$, $\mathbf{P_a^*}$, and $\boldsymbol{\delta}$ must be calculated from information derived from the manufacturing process itself.

First, we will approximate a value for the manufacturing process shift probability $\boldsymbol{\delta}$. Equation (20) represents this probability,

$$\delta \doteq 1 - e^{-\hat{\lambda}} \quad ,$$

provided that an estimate of the exponential shift rate parameter $\hat{\lambda}$ can be calculated. The maximum likelihood method suggested in Chapter III will be used to calculate $\hat{\lambda}$.

Secondly, the values for $\mathbf{p^*}$, the expected fraction nonconforming produced when the process is in state S (Equation (2)),

$$p^* = Q\,p_1 + (1-Q)\,p_2 \quad ,$$

and $\mathbf{P_a^*}$ will be calculated. We can recall that $\mathbf{P_a^*}$ is the associated type II error determined from the p-chart OC curve as a function of $\mathbf{p^*}$.

1. Calculation of the Manufacturing Process Shift Probability δ

Suppose the number of sampling intervals between out-of-control determinations, as made by the p-chart, has been recorded as suggested in Chapter III. The data might look like as is given in Table 1.

Table 1. EXAMPLE IN-CONTROL RUN LENGTH DATA

Run #	Run Length of in-control Determinations
1	14
2	35
3	18
4	9
5	27
...	...
50	17

Since the run length of in-control determinations, as made by the p-chart, is to be used to calculate the exponential shift rate parameter, it is useful to discuss possible p-chart errors. The type II error probability, the probability that the p-chart does not identify a process shift given that one has occurred over the sampling interval of interest, is equal to 0.10. If such an error had been made, it would, in most cases, be identified during the next sample test, because the probability that the p-chart identifies a process shift on at least the second sample, given that its type II error is equal to 0.10, is $1-(0.10)^2$ or 0.99. A misidentification of this type would only

amount to a *one* sampling interval difference, on the high side, between out-of-control determinations. The probability of a type I error, a false alarm initiated by the p-chart determining that production is at $\mathbf{p_2}$ given that it is actually at $\mathbf{p_1}$, is considerably small. Therefore, a maximum likelihood estimate is believed to be a satisfactory estimate of λ because the errors associated with the p-chart do not seriously effect the required data.

The maximum likelihood function for the 50 observations (runs), $\mathbf{L_R}$ (λ), is given by

$$L_R(\lambda) = \prod_{i=1}^{49} (\lambda\, e^{-\lambda r_i}) \prod_{i=50}^{50} (\lambda\, e^{-\lambda r_i})$$

and can be minimized, yielding

$$\hat{\lambda} = \frac{49}{\sum_{i=1}^{50} r_i} .$$

For the purposes of our example, let us assign $\hat{\lambda} = 0.04$ shifts per sampling interval τ. Therefore, the shift probability δ is represented by

$$\delta \doteq 1 - e^{-0.04} ,$$

or $\delta = 0.0392$.

2. Calculation of $\mathbf{p^*}$ and $\mathbf{P_a^*}$

To calculate $\mathbf{p^*}$, the expected fraction of nonconforming items produced by the manufacturing process when in the states identified as **S**, we need to first calculate the expected value of the proportion of time spent in state **S** attributable to the fraction nonconforming level $\mathbf{p_1}$. Using the maximum likelihood estimate for $\hat{\lambda}$ and Equation (1),

$$Q = \frac{1 - (1+\hat{\lambda})e^{-\hat{\lambda}}}{\hat{\lambda}(1-e^{-\hat{\lambda}})} ,$$

the value for **Q** is calculated to be 0.4967. Substituting this value into Equation (2)

$$p^* = Q\,p_1 + (1-Q)\,p_2$$

the value of $\mathbf{p}^*$ equal to 0.0952 is calculated.

In order to calculate $\mathbf{p}^*$'s associated probabilities $\mathbf{P_a}^*$, and $\mathbf{1\text{-}P_a}^*$, the OC curve for the p-chart must be defined. Duncan [Ref. 4:p. 448] offers a method for construction of OC curves for p-charts where the value of $\mathbf{P_a}$ is expressed as

$$P_a = Pr\left[\frac{LCL - p_2}{\sigma_{p_2}} \le z \le \frac{UCL - p_2}{\sigma_{p_2}}\right] ,$$

where **z** is a standard normal deviate. Since we are primarily concerned with identifying a shift to a higher fraction of nonconformities, we will concentrate on the upper control limit's discriminatory ability. The three-sigma upper control limit (UCL) is

$$UCL = p_1 + 3\sqrt{\frac{p_1(1-p_1)}{n}} ,$$

and the expression for σ_{p_2} is

$$\sigma_{p_2} = \sqrt{\frac{p_2(1-p_2)}{n}} .$$

Using the above equations, we define a relationship for the ordinate of the normal probability distribution

$$z = \frac{\left(p_1 + 3\sqrt{\frac{p_1(1-p_1)}{n}}\right) - p_2}{\sqrt{\frac{p_2(1-p_2)}{n}}}, \tag{20}$$

where **n** is the p-chart sample size.

For our example manufacturing process described above, we desire the p-chart to identify a sample fraction nonconforming of $\mathbf{p_2}$ at least 90 percent of the time or $\mathbf{1\text{-}P_a}$ = 0.90. Therefore, the value of the ordinate of the normal probability distribution must be less than -1.28, the value for the tenth normal percentile. This ordinate corresponds to the probability of a type II error or $\mathbf{P_a}$ = 0.10. Given that the values of $\mathbf{p_1}$ and $\mathbf{p_2}$ are 0.07 and 0.12 respectively, a sample size of 559 items yields an ordinate value of -1.282 or the tenth percentile. The p-chart OC curve would therefore have the ability to identify a fraction nonconforming equal to $\mathbf{p_2}$ at least 90 percent of the time, if a sample size of 559 items is inspected each sampling interval.

Using Equation (20), the relationship for the ordinate of the normal probability **z**, we can calculate the value for $\mathbf{P_a^*}$. Substituting $\mathbf{p^*}$ for $\mathbf{p_2}$ and maintaining the sample size **n** = 559, the calculated value for **z** is found to be 0.5779 and its associated probability $\mathbf{P_a^*}$ is equal to 0.7190.

To summarize, our example manufacturing process produces 100,000 items an hour. A sample size of **n** = 559 items will be examined every sampling interval, τ = 1 hour. The process shift, from a process average fraction nonconforming of $\mathbf{p_1}$ = 0.07 to a process average fraction nonconforming of $\mathbf{p_2}$ = 0.12, is exponentially distributed with a shift rate parameter of $\hat{\lambda}$ = 0.04 shifts per τ. If a shift occurs, it will be determined $\mathbf{1\text{-}P_a}$ = 90 percent of the time, therefore $\mathbf{P_a}$ = 0.10. Additionally,

the fraction of nonconforming items produced when in state **S** is $\mathbf{p}^* = 0.0952$, and the probability that the p-chart misidentifies this fraction nonconforming is $\mathbf{P_a}^* = 0.7190$.

B. AN EVALUATION OF THE QUALITY CONTROL PROGRAM

Combining the calculated values δ, and $\mathbf{P_a}^*$ with the known value $\mathbf{P_a}$, and substituting into Equations (3), (4), and (5) for the stationary probabilities,

$$\pi_I = \frac{(1-P_a)(1-\delta)}{1 - P_a + \delta P_a^*} ,$$

$$\pi_S = \frac{\delta(1-P_a)}{1 - P_a + \delta P_a^*} ,$$

and

$$\pi_O = \frac{\delta P_a^*}{1 - P_a + \delta P_a^*} ,$$

we calculate the stationary probabilities to be, $\pi_I = 0.9316$, $\pi_S = 0.0380$, and $\pi_O = 0.0304$. Following the notation introduced in Chapter III, the stationary probability π_I is represented by $\mathbf{Pr(p_1)} = 0.9316$, the stationary probability π_S is represented by $\mathbf{Pr(p^*)} = 0.0380$, and the remaining stationary probability π_O represented as $\mathbf{Pr(p_2)} = 0.0304$.

Using these stationary probabilities and their respective fractions nonconforming produced while in these states, we substitute into Equation (7),

$$p' = Pr(p_1)\ p_1 + Pr(p^*)\ p^* + Pr(p_2)\ p_2 \quad ,$$

and find the process average fraction nonconforming $\mathbf{p'}$ to be equal to 0.0723.

If items chosen at random from the manufacturing process are formed into lots of size N = 1000, then a one-sided 95 percent upper confidence limit on the fraction of nonconforming items $\mathbf{p}_{lo}$, determined from Equation (8), is equal to 0.0860. If this upper confidence limit is no greater than the Acceptable Quality Limit, then we will have at least 95 percent confidence in the quality of these lots.

C. FURTHER ASSESSMENT OF QUALITY PROGRAMS

A great deal of insight can be gained if we *situationally* apply the results of the three-state Markov model, but first we should use some basic common sense. For instance, we can set an upper bound or *best case* 95 percent upper confidence limit on the fraction of nonconforming items in a lot. If we assume that the production process never shifts to process average $\mathbf{p}_2$ = 0.12 but continues to operate at process average $\mathbf{p}_1$ = 0.07, then the 95 percent upper confidence limit on the fraction of items nonconforming in a lot of 1000 is found to be 0.0833. We will be more than 95 percent confident that a lot of size N = 1000 contains no more than 83 items that are *nonconforming*. If this fraction of nonconformities in a lot, when compared to the designated Acceptable Quality Level, is not acceptable, then the manufacturing process must be rejected because its best effort does not meet the specifications.

It should be noted that when acceptance sampling is used, the Acceptable Quality Level is not intended to be a producer's target value for the production process. However, when considering alternative acceptance criteria, the AQL could serve the producer as a target value for the 95 percent upper confidence limit for the manufacturing process average fraction nonconforming.

Once we have assessed the manufacturing process' capability to meet the Acceptable Quality Level, alteration of the process quality control parameters, or adjusting the sensitivity of the process quality control section, might prove interesting.

These adjustments would be made to the sampling interval τ or to the ability of the control chart to identify a process shift ($\mathbf{P_a}$).

1. Sampling Interval Sensitivity

Suppose the upper confidence limit of 86 nonconforming items per 1000, obtained in the example, is satisfactory. In fact, suppose we could tolerate as many as 89 nonconforming items as our upper bound. (This number would be calculated using the Acceptable Quality Level (AQL) for the item.) If the sampling interval during process control is doubled to two hours, the corresponding shift rate would be $\hat{\lambda} = 0.08$ per τ. Holding the values of $\mathbf{p_1}$, $\mathbf{p_2}$, and $\mathbf{P_a}$ equal to those presented in the example, the new 95 percent upper confidence limit for $\mathbf{p'}$ is calculated to be 0.0884. This confidence bound has an associated number of nonconforming items of 88. The difference is slight, only three items, however 88 nonconforming items is still acceptable, and the savings gained from halving the number of required samples to meet the AQL could be passed on to the consumer.

Following similar logic, if a two-fold increase in sampling interval caused little change to the output, what would that of a three-fold increase exhibit? If we triple the sampling interval during process control, the corresponding shift rate would be $\hat{\lambda} = 0.12$ per τ. The new one-sided 95 percent confidence limit for $\mathbf{p'}$ is calculated to be 0.0905. This confidence limit has an associated number of nonconforming items of 90 which is be above that required. So, whereas a doubling of the sampling interval proved helpful, too much of an adjustment became detrimental.

2. Control Chart Sensitivity

Suppose that upon examination of the manufacturing process, numerical evidence reveals the 95 percent upper bound on $\mathbf{p'}$ to be 0.0860, as was established earlier, and we are given information based on the Acceptable Quality Level that the

95 percent upper confidence limit is not to exceed 0.0880. We know that the sample size is equal to 559 items, and that, given this sample size, the probability that the p-chart will conclude the process is operating at level $\mathbf{p_1}$ when in fact it is operating at $\mathbf{p_2}$ is equal to 10 percent. Rather than increase the sampling interval, as was done previously, we will look at decreasing the p-charts' ability to identify a process shift on the first sample taken after the shift has occurred or increasing the probability of making a type II error.

If the sample size is decreased to that of 235 items per sampling interval then the corresponding value for $\mathbf{P_a}$ is equal to approximately 0.50. This adjustment decreases the sample size by 324 items and still meets the AQL with a 95 percent confidence limit on $\mathbf{p'}$ equal to 0.0878. In this case, the savings gained from the decreased sample size may be passed on to the consumer.

3. Nonconformities Occurring After Quality Control

Suppose the initial production process remains the same, but a follow-on process, for instance, packaging the manufactured items must take place. If a probabilistic estimate concerning the *damage* to an item during a packaging process was known, then a revised estimate for the number nonconforming can be easily made. For example, if we know that a *good* item is damaged $\mathbf{q} = 0.01$ percent of the time when packaged, a revised estimate of the fraction on nonconforming items is

$$E[\textit{fraction of nonconforming items}] = (1-p')q + p' .$$

Using the first numerical example $\mathbf{p'}$ value of 0.0725, the revised estimate on the fraction nonconforming is found to be 0.0818 with a 95 percent upper confidence limit of 0.0961. This estimate means that we would be more than 95 percent certain that a lot of 1000 items set for delivery would contain at least 903 items *conforming*.

The same approach could handle many nonconformities that occur after the process quality control section and could even be extended to encompass transportation and storage if reliable estimates are available regarding the probabilistic results of these transactions.

In this chapter we calculated the fraction of nonconforming items produced by a manufacturing process using the three-state Markov model introduced earlier. This calculation was possible because the relevant inputs, the values of $\mathbf{p}_1$, $\mathbf{p}^*$, $\mathbf{p}_2$, $\mathbf{P}_a$, $\mathbf{P}_a^*$, and $\hat{\lambda}$, were obtainable from the numerical evidence provided by the manufacturer's use of Statistical Process Control. Also shown was the model's ability to furnish *suggestions* as to the sampling interval length and the discriminatory power of the control chart, based on the Acceptable Quality Level.

V. SUMMARY AND SUGGESTIONS FOR FURTHER STUDY

In this thesis we examined the quality control practices typical of some manufacturing processes in an effort to gain information concerning product quality for the purpose of establishing an alternative to acceptance sampling plans. A three-state Markov model was used to represent a simple manufacturing process and a 95 percent upper confidence limit for the process average fraction nonconforming was calculated. The calculation of this confidence limit was contingent upon certain statistical evidence obtained from the manufacturing process itself.

A. AN ASSESSMENT OF STATISTICAL PROCESS CONTROL, VS. SAMPLING

Since this approach and others, based in Statistical Process Control, may be used as possible alternatives to acceptance sampling plans, they should, at least, maintain the characteristics enjoyed under acceptance sampling. While acceptance sampling does not attempt to control quality, it does provide

- Long run protection for the consumer,
- A level of protection for the consumer against accepting bad lots,
- Minimal sampling, inspection, and administrative burden as compared to 100 percent inspection, and
- Limited information concerning the quality of the product in the form of a sample mean, range, number or percentage nonconforming.

The additional advantages enjoyed under Statistical Process Control are that

- A certain confidence can be associated with the product quality information,
- The manufacturer is encouraged to keep his process *in control*,
- The manufacturer is protected against having lots rejected when his production process *in control*, and
- Comparisons between manufacturers based upon process capability is possible.

When alternative acceptance criteria is sought, close examination of the manufacturer's quality program is essential. Specifically, when a manufacturer implements Statistical Process Control, the manufacturing process itself becomes the source of the data needed implement this alternative. Provided that this data is available, the calculated estimate of the fraction nonconforming items and its associated confidence limit could afford decision makers a pre-delivery glance at the expected lot quality. If the 95 percent confidence limit conforms with that prescribed by the Acceptable Quality Level for lots, the decision maker may consider *qualifying* the manufacturing process' as a candidate for alternative acceptance criteria.

B. RECOMMENDATIONS AND SUGGESTIONS FOR FURTHER RESEARCH

While this thesis examined only one type of manufacturing process, we believe that manufactures who have quality control programs which use a well-structured and implemented statistical methodology could be targeted for alternative acceptance criteria. We recommend that the Department of Defense consider this Markov approach and other similar approaches founded in Statistical Process Control.

Although this particular approach proved worthwhile, it is not all encompassing, and further research into the examination of quality control programs for the purpose of developing alternative acceptance criteria is needed. While the manufacturing process examined in this thesis, one operating at only two differing process levels, certainly has its applications, a process which operated at more than two nonconforming levels, or one in which the process shift is other than exponential, could be examined.

If we view the consumer to be a manufacturing process further down a production line, then the notion of two or more production processes, each producing component parts for a larger product, comes to mind. Examination of alternative acceptance criteria for a process which manufactures an item from a network of input processes would prove useful.

It is sincerely hoped that the approach presented in this thesis will be beneficial to those members of the Department of Defense responsible for ensuring the quality of acquired items.

APPENDIX A

To be determined is **Q**, the expected value of the proportion of time spent in state **S** attributable to the fraction nonconforming level of $\mathbf{p_1}$ given that a shift, from $\mathbf{p_1}$ to $\mathbf{p_2}$, occurs in an interval of length $(0, \tau]$,

$$Q = \frac{E[\ shifttime \mid shiftoccurs\ (0,\tau]\ ]}{\tau} .$$

This expected value can be represented as

$$Q = \frac{\int_0^\tau x\ f(x)\,dx}{P[\ shiftoccurs\ (0,\tau]\ ]\ \tau} .$$

Assuming that a shift from $\mathbf{p_1}$ to $\mathbf{p_2}$ is exponentially distributed with a shift rate parameter of λ, the relationship for **Q** is rewritten as

$$Q = \frac{\int_0^\tau x\ \lambda e^{-\lambda x}\,dx}{\tau\ (1-e^{-\lambda\tau})} .$$

Evaluating the integral and simplifying we find

$$Q = \frac{\left[\frac{1}{\lambda} - \left(\frac{1}{\lambda} + \tau\right)e^{-\lambda\tau}\right]}{\tau\left[\ 1-e^{-\lambda\tau}\ \right]} = \frac{1-(1+\lambda\tau)e^{-\lambda\tau}}{\tau\ \lambda(1-e^{-\lambda\tau})} .$$

Recalling that the exponential shift rate parameter is expressed in units of shifts per τ, thus $\tau = 1$, and the expression for Q becomes

$$Q = \frac{1-(1+\lambda)e^{-\lambda}}{\lambda(1-e^{-\lambda})} .$$

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