MODIFIED TWO-FOCAL-LENGTH OPTICAL CORRELATOR

Kenneth H. Fielding, Capt, USAF; Dr. Joseph L. Horner; Charles K. Makekau, 1Lt, USAF
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APPROVED:

RICHARD PAYNE
Chief, Electro-Optic Device Tech Division
Directorate of Solid State Sciences

APPROVED:

HAROLD ROTH
Director of Solid State Sciences

FOR THE COMMANDER:

JAMES W. HYDE III
Directorate of Plans & Programs

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We present a modification to our previously reported 2f optical correlator. This modification removes a lens from the system and maintains a variable Fourier transform scale property while providing equal correlation performance.
Contents

1. INTRODUCTION 1
2. THEORY 2
3. IMPLEMENTATION 4
4. CONCLUSION 5
REFERENCES 7
1. 4f Vander Lugt Correlator

2. Evolution of the Optical Correlator From a 4f to a 2f System:
   (a) 4f System; (b) 3f System; (c) Modifying the 3f System;
   (d) 2f System

3. Modifying the 2f: (a) Moving the Working End of the Correlator to the Collimation Lens; (b) the Modified System
1. INTRODUCTION

We recently reported an optical correlator architecture whose physical length is two-lens-focal lengths (2f).\(^1\) This architecture is contrasted to the classical 4f correlator design of Vander Lugt.\(^2\) Other researchers have investigated 2f architectures\(^3,4,5\); however, our design used simple single thin lens elements and keeps the highly desirable variable scale feature. We now describe a modification to this system eliminating a lens element.
In reality, the 4f Vander Lugt design should be a 5f system when one takes into account the lens used to collimate the laser light. The total Vander Lugt system is shown in Figure 1.

![Collimation Lens Collimation Lens Input L_1 Filter L_2 Correlation](image)

**Figure 1. 4f Vander Lugt Correlator**

We assume here all lenses are the same but acknowledge they can be different. We quickly highlight the derivation of the 2f system for coherency to the reader.

The working end of the 4f correlator is shown in Figure 2a. The input is moved in back of the first Fourier transform (FT) lens \( L_1 \) providing the variable scaling to the FT, but also introducing the quadratic phase factor at the filter plane. The phase factor is proportional to the distance from the input to filter, \( d \), and the Fourier spectrum at the filter is given by

\[
A(x_2, y_2) \alpha \exp \left[ \frac{jk}{2d} \left( \frac{x_2^2 + y_2^2}{d} \right) \right] F(f_{x_2}, f_{y_2})
\]

where \( F() \) is the Fourier transform of the input signal and \( f_{x_2}, f_{y_2} \) are spatial frequencies equal to \( (x_2, y_2)/\lambda d \). A phase compensation lens \( L_3 \) with focal length equal to \( d \) is placed next to the filter to eliminate this term. The phase compensation term is

\[
\alpha \exp \left[ \frac{-jk}{2d} \left( \frac{x_2^2 + y_2^2}{d} \right) \right].
\]

This configuration is what we called the 3f system and is shown in Figure 2b. We move the second FT lens, \( L_2 \), to the phase compensation lens, \( L_3 \) in Figure 2c, and using the thin lens combination formula.
\[ l = l_1 + l_2 \]

(3)

combined these two elements into a single element \( L_4 \). In doing so, we introduce another quadratic phase factor at the correlation plane which is irrelevant since we detect the correlation with a camera in intensity. This system is what we called the \( 2f \) correlator and is shown in Figure 2d.

![Diagram](image)

**Figure 2.** Evolution of the Optical Correlator From a \( 4f \) to a \( 2f \) System: (a) \( 4f \) System; (b) \( 3f \) System; (c) Modifying the \( 3f \) System; (d) \( 2f \) System.
After several months of working with the 2f correlator, it became apparent we could use this idea of combining lenses on the front end of the system. Since the light is parallel after the collimation lens, we can move the entire 2f correlator until the first FT lens touches the collimation lens which is illustrated in Figure 3a.

We then use the lens combination formula to combine these lenses and replace them with a single lens element, \( L_5 \), eliminating one lens from the system. The final system is shown in Figure 3b. The working end of the 4f system can be backed up in a similar manner, but no lenses can be removed. Note that our correlator no longer uses collimated light, but we have a virtual collimation at lens \( L_5 \).

3. IMPLEMENTATION

In practice, one does not generally have the exact required lenses on hand so care must be taken to employ this system and we illustrate by a brief example. Suppose the collimation lens has a focal length of 20 cm and the FT lenses of the 4f system are 50 cm. Using the lens combination formula, the single lens, \( L_5 \), that is located in place of the collimation/first FT lens is 14.28 cm, not a typically stocked lens. Suppose the closest lens you have to 14.28 cm is 12 cm and you need to maintain the 50 cm input to filter distance (proper scale) because of the pixel size of the spatial light modulator you are using in the filter plane. We now have two variables, the collimation and FT lens focal length combination to give 12 cm. Let us pick the
FT lens length to be 55 cm giving a variable scale factor of +/- 10 percent on both sides of the exact required distance of 50 cm. Using the lens combination formula and solving for the collimation lens focal length we find its value is 15.35 cm. Lens L₄ should be placed 15.35 cm from the pinhole and the filter 55 cm from this lens. The input object is located 5 cm from the lens (50 cm from the filter) and can be adjusted for exact scale.

The same reasoning can be applied to the compensation/second FT lens. The thin lens combination of the required 50 cm compensation lens (50 cm from input to filter) and 50 cm FT lens is 25 cm, but you only have a 10 cm lens on hand for L₄. Knowing the compensation portion must stay at 50 cm, the calculated FT lens must be 12.5 cm. The correlation is found 12.5 cm behind L₄. Because correlators without filters are imaging systems and there is not a lens between the input and L₄, we can also use simple geometrical optics to calculate the position of the correlation plane. Using the Gaussian lens formula, \( 1/f = 1/s₁ + 1/s₂ \), and substituting 10 cm for \( f \) and 50 cm for \( s₂ \), \( s₁ \) is 12.5 cm as above (Figure 2d).

One can easily see an endless combination of possibilities for choosing the parameters to suit the problem and equipment on hand. The only pitfall we see is if the combined lens for the input stage is not large enough, the illumination spot size may not cover the entire input. However, increasing the objective strength in the lens-pinhole spatial filter will usually solve this problem.

4. CONCLUSION

We describe a 2-lens correlator that uses two-lens-focal lengths in the working end of the system. This system is typically one-half the physical length of the 4f Vander Lugt system, eliminates one lens element, has the variable Fourier transform scale property, and provides equal performance as a correlator, as we have shown experimentally.
References


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