DTIC FILE COPY

(3)

Conversion of Electrostatic and Electromagnetic Waves in a Plasma at the Peak of a Parabolic Density Profile

D.E. Hinkel-Lipsker, B.D. Fried, and G.J. Morales

PPG-1338 December 1990

AD-A230 560

DTIC
ELECTE
JAN 09 1991
S D
D

University of California at Los Angeles
Department of Physics
Los Angeles, CA 90024-1547

Conversion of Electrostatic and Electromagnetic Waves in a Plasma at the Peak of a Parabolic Density Profile

D.E. Hinkel-Lipsker, B.D. Fried, and G.J. Morales
Department of Physics, University of California, Los Angeles
Los Angeles, California 90024-1547

Abstract

Analytic expressions for the reflection, transmission, and mode conversion coefficients for electromagnetic and Langmuir waves in an unmagnetized plasma with a parabolic density profile are found for both the "direct" problem (incident electromagnetic wave) and the "inverse" problem (incident Langmuir wave). In contrast to the linear profile problem, the absorption depends explicitly on the value of the collision frequency (cold model) or temperature (warm plasma), but a transformation of parameters relates the results for these two limits.

PACS Numbers: 52.40Db, 52.35Lv, 94.20Bb

The conversion of electromagnetic waves into electrostatic modes in an inhomogeneous, unmagnetized plasma is a topic of broad interest which has important consequences for laboratory and space-plasma studies. Most of the analysis of this problem has concentrated on the simpler case of a plasma with a linear density profile[1-7]; in fact, only recently[6] has an analytic solution been obtained for this case by a scale-separation technique. The present study uses this technique to solve the more technically demanding parabolic density profile problem, a physically important situation of current interest to ionospheric heating at the peak of the F-layer[8], Langmuir turbulence studies in basic laboratory devices[9], beat wave acceleration schemes, and some aspects of laser plasma interactions.

Previous analytical studies of this problem have considered the structure of the electromagnetic wave, as done by Baños[10] using Langer's method, and by Lundborg and Thidé[11,12] through an analytic uniform-approximation method. However, these investigations have not yielded the structure of the electrostatic fields excited at plasma resonance, and have not predicted the experimentally important mode conversion coefficients. The present study yields relatively simple analytic expressions for these previously unknown quantities and simultaneously solves the inverse mode conversion problem, a situation of diagnostic interest since it permits the sampling of microscopic plasma phenomena at remote locations.

In the direct problem, an electromagnetic wave of frequency ω is obliquely

DTIC COPY INSPECTED

Statement "A" per telecon Dr. Charles Roberson. Office of Naval Research/code 1112AI.

A-1

VHG 1/8/91

incident on the plasma at an angle θ relative to the density gradient, with ω chosen equal to the electron plasma frequency ω_p at the peak of the parabolic density profile (Fig. 1). This wave encounters a cut-off at the layer $\omega_p(z) = \omega\cos\theta$, where some of the wave is reflected, while the remainder tunnels to the plasma resonance where $\omega = \omega_p(0)$. In this region, a portion of the incident wave energy is transformed into plasma oscillations for the cold, collisional problem, and, for a warm plasma, into Langmuir waves having cut-offs at $\omega_p(z) = \omega(1-\beta^2\sin^2\theta)^{1/2}$, where $\beta^2 \equiv 3T_e/m_e c^2$. The remaining energy tunnels out to the electromagnetic cut-off, where it becomes the transmitted fraction of the electromagnetic wave. Fig. 1 depicts this process for the warm, direct problem.

We also consider the warm, inverse problem, where an incident (leftgoing in Fig. 1) electrostatic wave in the plasma gives rise to two outgoing electrostatic waves [a rightgoing wave resulting from the reflection at the electrostatic cut-off and a leftgoing wave transmitted through the evanescent layer around $z = 0$] and two obliquely propagating, outgoing electromagnetic waves produced by mode conversion in the region around $z = 0$.

In general, the electrons are treated as a warm, collisional fluid with an unperturbed density $n_o(z) = \overline{n}(1 - z^2/L^2)$ and collision frequency ν. The ions are static, and quasineutrality is assumed. The fluid equations are linearized by setting $n(\vec{r},t) = n_o + \{n_1(z)\exp[ik_x x - \omega t] + \text{c.c.}\}$, etc. We assume that there are no zero order fields or flows ($\vec{E}_o = \vec{B}_o = \vec{u}_o = 0$). The fluid equations, together

with Maxwell's equations then yield a set of equations[6,7] which couple E_z and B_y.

This set of equations can be expressed in terms of scaled coordinates and wavenumbers appropriate for the parabolic profile problem: Z and N for the electromagnetic mode, and ζ and K for the electrostatic mode, where

$$\zeta \equiv k_o^{1/2} z/(\beta L)^{1/2} \equiv Z/\beta^{1/2}, \tag{1}$$

and

$$K \equiv \beta^{1/2}(k_x L)/(k_o L)^{1/2} \equiv \beta^{1/2} N \ , \tag{2}$$

with $k_o \equiv \omega/c$.

When the frequency matches the plasma frequency at the peak of the profile, the scaled set of equations have the form

$$E_z'' + (\zeta^2 + i\nu_o/\beta - K^2)E_z = bB(Z), \tag{3}$$

$$\rho'' + (\zeta^2 + i\nu_o/\beta - K^2)\rho = -2\zeta E_z/\alpha, \tag{4}$$

$$d^2B/dZ^2 - 2Z(Z^2 + i\nu_o - q)^{-1}dB/dZ + (Z^2 + i\nu_o - q)B = S, \tag{5}$$

where $k_o\rho \equiv \nabla \cdot \vec{E}$, $B \equiv iB_y$, $b \equiv iN\alpha(1-\beta^2)\beta^{-3/2}$, $\alpha \equiv (k_o L\beta^2)^{1/2}$, $\nu_o \equiv k_o L\nu/\omega$, $S \equiv S(Z) = -2iNZQ(Z^2+i\nu_o-q)$, with $Q \equiv (1-\beta^2)\rho - \alpha^{-1}E_z'$, $q \equiv N^2 = (k_x L)^2/(k_o L)$, and the prime denotes differentiation with respect to ζ.

Eqs. (3) – (5) describe wave propagation and conversion in a plasma with a parabolic profile. The electrostatic modes vary rapidly, i.e., on the short

scale (ζ), whereas the electromagnetic modes vary much more slowly, i.e., on the longer scale Z. The coupling of the two modes depends on the angle of incidence through N. The larger the angle of incidence, the stronger the coupling; however, as N increases, so does the tunnelling distance to the mode conversion point, and the competition between these two phenomena causes a peak in the mode conversion coefficient for a finite angle of incidence.

Cold, Collisional Problem. For this case, we let $\beta^2 = 0$ in the set of equations (3) – (5), but keep $\nu_o \neq 0$. In this limit, equations (3) and (4) yield algebraic relations for ρ and E_z in terms of B. When these expressions are substituted into the righthand side of (5), we obtain a differential equation for B whose homogeneous solutions are derivatives of parabolic cylinder functions. This equation is solved[7] using a Green's function technique, together with the approximation that on the righthand side of (5), which can be shown to be proportional to $B(Z)$, we replace $B(Z)$ by $B(0)$. We then self-consistently determine $B(0)$ as well as $B(Z)$ for $Z \gg 1$ and $Z \ll -1$, from which the reflection and transmission coefficients can be found. We also compute the mode conversion coefficient $|\eta|^2$ (the ratio of power collisionally dissipated in plasma oscillations to the incident power in the electromagnetic wave) which satisfies the energy conservation equation

$$|\eta|^2 = 1 - |R|^2 - |T|^2. \tag{6}$$

The final results can be most succinctly expressed in the form

$$|T|^2 = (1+e^{q\pi})^{-1}\frac{\nu_o|\Gamma|^4}{|\nu_o^{1/2}|\Gamma|^2+X|^2}, \tag{7}$$

$$|\eta|^2 = \frac{2\nu_o^{1/2}|\Gamma|^2}{|\nu_o^{1/2}|\Gamma|^2+X|^2}\frac{\mathrm{Im}[(\mathrm{i}+\mathrm{e}^{q\pi/2})\mathrm{X}]}{1+\mathrm{e}^{q\pi}}, \tag{8}$$

where $X \equiv 2q\pi^2\exp[q\pi/4+3\pi i/4]/[i+\exp(q\pi/2)]$, $|\Gamma| \equiv |\Gamma[(1+iq)/4]|$ is the gamma function[13], and $|R|^2$ is determined from (6). The converted energy from just one side of the mode conversion layer is half of the amount given by (8). Note that $|\eta|^2$ vanishes as $\nu_o \to 0$, in contrast to the linear profile problem where $|\eta|^2$ has a non-zero value in the cold, collisionless limit. In the limit $q \to 0$, $|\eta|^2 = 0$, and $|R|^2 = |T|^2 = 0.5$, in agreement with the earlier work of Baños.[10]

Fig. 2 depicts the dependence of the energy flux coefficients on the parameter $q \equiv k_oL\sin^2\theta$ for $\nu_o = .01$. For this case, the peak of $|\eta|^2$ occurs at $q \equiv q_o \approx 0.05$, which is smaller than for the linear profile problem[6,7], where $q_l \equiv (k_oL)^{2/3}\sin^2\theta \approx 0.5$. [It follows from (8) that for small q, $|\eta|^2$ is proportional to $q/|q+a\nu_o^{1/2}|^2$, where a is of order 10 and is independent of q, so $|\eta|^2$ peaks at a value of $q \approx 10\nu_o^{1/2}$.] For larger ν_o, the peak of $|\eta|^2$ moves to higher q, as does the minimum of $|R|^2$, and the tail on the mode conversion coefficient falls off more slowly. As q increases, the tunnelling distance for the electromagnetic wave becomes large. This thick evanescent layer, which causes most of the incident energy to be reflected, accounts for the fall-off of $|T|^2$ and $|\eta|^2$ and the large value of $|R|^2$ (approaching 1) seen in Fig. 2 for $q > 1$.

Warm, Direct Problem. In this case we set $\nu_o = 0$ and keep $\beta \neq 0$. In (3), the field E_z is on the short electrostatic scale (ζ) whereas its source on the righthand side, which is proportional to $B(Z)$, is on the longer electromagnetic scale. We utilize this disparity in scales to replace $B(Z)$ by $B(0)$ on the righthand side of (3). Equation (3) is then just a parabolic cylinder equation driven by a constant source. The solutions are determined by using the Green's function for outgoing waves.

Given this solution to E_z [in terms of $B(0)$] and the boundary conditions for the direct problem, it immediately follows that $\rho(\zeta) = \alpha^{-1} E_z'$. The variables E_z and ρ thus have functional forms expressible in terms of parabolic cylinder functions with argument ζ and their derivatives, with amplitudes proportional to the quantity $B(0)$.

In order to determine $B(0)$, we substitute $\rho(\zeta)$ and $E_z(\zeta)$ into the source term on the righthand side of (5). This equation has as homogeneous solutions the derivatives of parabolic cylinder functions, and these are used to form the Green's function for $B(Z)$ satisfying the boundary conditions that $B(Z)$ consist of leftgoing (incident) and rightgoing (reflected) waves for $Z \gg 1$, and a leftgoing (transmitted) wave for $Z \ll -1$. We then make the approximation that the parabolic cylinder functions with argument Z, which are multiplied by $E_z(\zeta)$ and $\rho(\zeta)$ in the Green's function expression for B, can be replaced by their $Z = 0$ values during integration on the rapidly varying ζ scale.

From the resulting expression for $B(Z)$, we self- consistently determine $B(0)$ [by evaluating $B(Z = 0)$] and then calculate $B(Z \gg 1)$ and $B(Z \ll -1)$, which give the reflection and transmission amplitudes, respectively. The results can most easily be expressed as follows. Each of the quantities $|T|^2$, $|R|^2$, and $|\eta|^2$ for the warm, direct problem is equal to the corresponding quantity for the cold, collisional problem with the substitutions $\nu_o \rightarrow \beta$ and $X \rightarrow i\pi M^2$, where $M^2 \equiv iq\gamma_{es} \exp[(1+\beta)q\pi/4][\int_{-\infty}^{+\infty} I(\zeta)d\zeta]/\{[i+\exp(q\pi/2)][i+\exp(\beta q\pi/2)]\}$, and $\gamma_{es} \equiv \Gamma[(1+iq\beta)/4]/\Gamma[(1-iq\beta)/4]$. The function $I(\zeta)$, which represents the solution to the driven parabolic cylinder equation with a constant source, is analogous to the function[7] $Gi + iAi$, which is an outgoing solution to the driven Airy equation, encountered in the linear density profile problem.

Fig. 3 exhibits the warm energy flux coefficients as a function of q for electron temperature $\beta^2 = 10^{-4}$. Since M^2 and X have essentially the same q dependence as long as $\beta q \ll 1$, it is not surprising that these curves are similar to those of Fig. 2 for the cold, collisional case. In both the cold and warm cases, the small q form of $|\eta|^2$ is $abq/|a+ibq|^2$, where a and b are independent of q, so the maximum value of $|\eta|^2$ is 0.5.

Warm, Inverse Problem. Solution of the inverse problem proceeds in a manner analogous to that of the direct problem, with one technical complication which is a consequence of the difference in boundary conditions for the direct and inverse problems. In the solution to the ρ equation, (4), the boundary conditions

for the inverse problem permit a homogeneous solution as well as the particular solution. The coefficient ρ_o of this homogeneous solution constitutes a second unknown constant, in addition to $B(0)$. We determine these by evaluating the Green's function expression for $B(Z)$ at $Z = 0$,as in the direct problem, and also using the z component of Ampère's Law evaluated at $Z = 0$. We find that the effect of ρ_o is simply a correction of order $\beta^{1/2}$ to the constant $B(0)$, so we can neglect the ρ_o term.

The energy flux coefficients for the inverse problem, shown in Fig. 3 as a function of q for $\beta^2 = 10^{-4}$, are determined in a manner identical to that for the direct problem. Only a single mode conversion coefficient is plotted, since we have shown by explicit calculation that $|\eta|_{\mathrm{I}}^2 = |\eta|_{\mathrm{D}}^2$ (where the I and D subscripts refer to the inverse and direct cases, respectively), a result which follows from time reversal symmetry. Fig. 3 also reveals an additional near symmetry: $|R|_{\mathrm{I}}^2 \approx |T|_{\mathrm{D}}^2$ and $|T|_{\mathrm{I}}^2 \approx |R|_{\mathrm{D}}^2$. We have analytically demonstrated that these equalities are valid to within terms of order $\beta^{1/2}$, but have not been able to show exact equality.

From the present results it is clear that the degeneracy between cold and warm plasma behavior found in the linear profile problem is not present for the parabolic profile. In the latter case, all mode conversion coefficients depend explicitly on ν_o and β. However, it is possible to relate the warm absorption coefficients to the collisional results through a transformation of parameters. For

non-linear studies, of relevance to ionospheric and laboratory experiments, it is worth emphasizing that for the same input power, the experimentally measurable ratio of peak electric fields is

$$\frac{|E_z|_{\mathrm{PAR}}}{|E_z|_{\mathrm{LIN}}} = \frac{18.6}{(\beta\pi)^{1/2}(13.1\beta + .05)^{1/2}} \frac{L_{\mathrm{PAR}}}{L_{\mathrm{LIN}}}, \tag{9}$$

where L_{LIN} is the scale length of the linear density profile and L_{PAR} is the curvature of the parabolic density profile.

The present study clearly illustrates the usefulness of the scale-separation technique. It allows an analytic solution of the direct and inverse conversion problems for both linear and parabolic density profiles from a unified point of view. We believe that the method may be of value in similar situations involving magnetized plasmas.

This work was supported by the Office of Naval Research, U.S.DOE, and the UCLA Dissertation Year Fellowship.

References

1. N.G. Denisov, Zh. Eksp. Teor. Fiz. **31**, 609 (1956) [Sov. Phys. JETP **4**, 544 (1951)].
2. A.D. Piliya, Zh. Tekh. Fiz. **36**, 818 (1966) [Sov. Phys. Tech. Phys. **11**, 609 (1966)].
3. D.W. Forslund, J.M. Kindel, K. Lee, E.L. Lindman, and R.L. Morse, Phys. Rev. A **11**, 679 (1975).
4. R.W. Means, L. Muschietti, M.Q. Tran, and J. Vaclavik, Phys. Fluids **24**, 2197 (1981).
5. K.G. Budden, *Radio Waves in the Ionosphere*, (Cambridge University Press, Cambridge, 1985), p. 446 ff.
6. D.E. Hinkel-Lipsker, B.D. Fried, and G.J. Morales, Phys. Rev. Lett. **62**, 2680 (1989).
7. D.E. Hinkel-Lipsker, "Resonant Absorption in an Inhomogeneous, Unmagnetized Plasma", Ph.D. Dissertation, UCLA, 1990.
8. A.Y. Wong, P.Y. Cheung, M.J. McCarrick, J. Stanley, R.F. Wuerker, R. Close, B.S. Bauer, E. Fremouw, W. Kruer, and B. Langdon, Phys. Rev. Lett. **63**, 271 (1989).
9. B.S. Bauer, A.Y. Wong, L. Scurry, and V.K. Decyk, Phys. Fluids B **2**, 1941 (1990).
10. A. Baños Jr., J. Math. Phys. **14**, 963 (1973).

11. B. Lundborg and B. Thidé, Radio Science **20**, 947 (1985).

12. B. Lundborg and B. Thidé, Radio Science **21**, 486 (1986).

13. P.J. Davis in *Handbook of Mathematical Functions*, edited by M. Abramowitz and I.E. Stegun (Dover, New York, 1972), 10th ed., pp 253ff.

Figure Captions

Figure 1. Schematic of the direct problem. An incident electromagnetic wave (EM_I) gives rise to a reflected wave (EM_R), a transmitted wave (EM_T), and outgoing Langmuir waves (ES_{OUT}). The inner and outer dashed lines represent the electrostatic and electromagnetic cut-offs, respectively.

Figure 2. Cold, collisional energy flux coefficients as a function of $q \equiv k_o L \sin^2\theta$ for $\nu_o = .01$.

Figure 3. Warm, energy flux coefficients as a function of $q \equiv k_o L \sin^2\theta$ for $\beta^2 = 10^{-4}$.

Figure 1

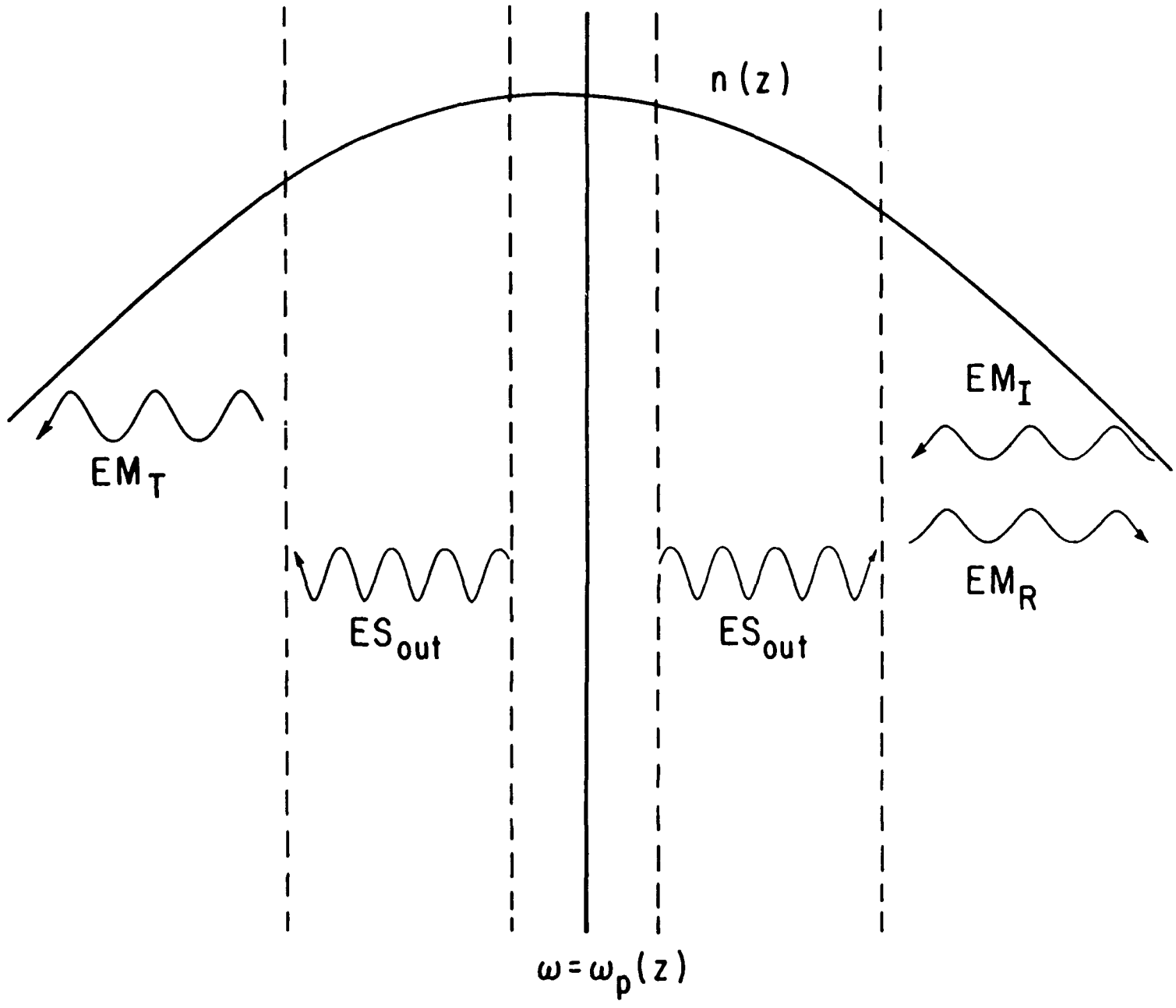


Figure 2

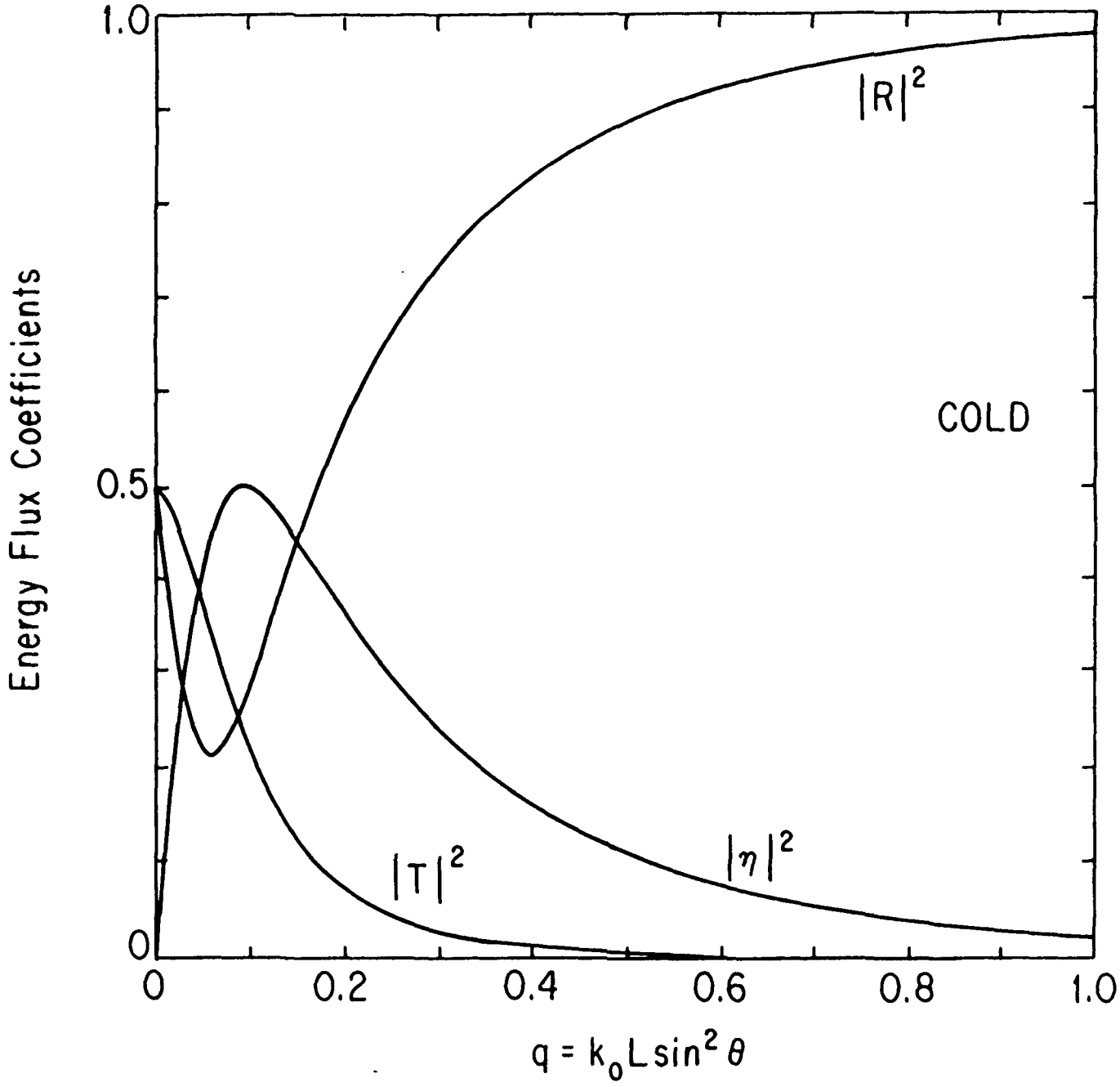
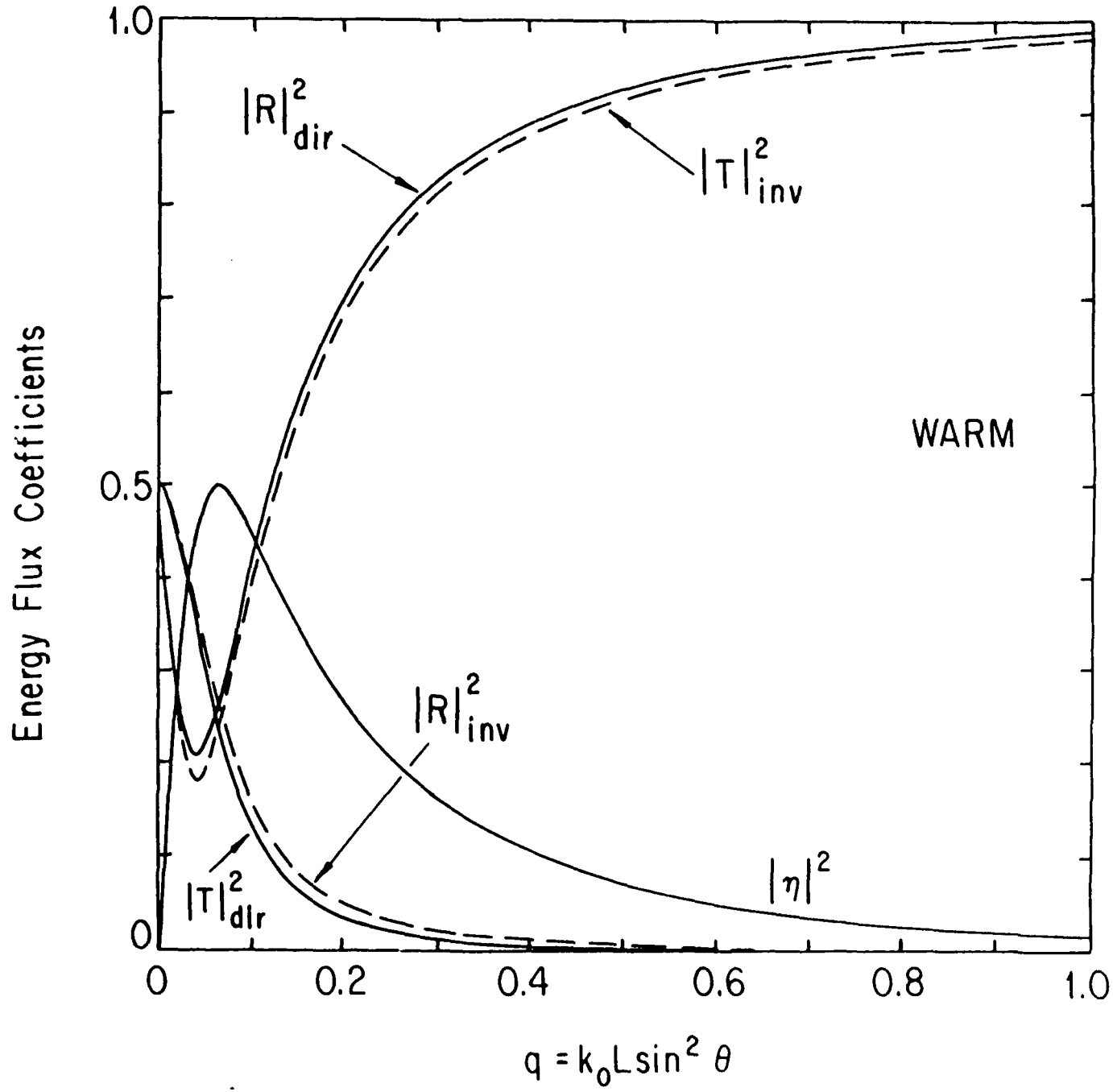

1.0
0.5
0
Energy Flux Coefficients
$|R|^2$
COLD
$|T|^2$
$|\eta|^2$
0
0.2
0.4
0.6
0.8
1.0
$q = k_0 L \sin^2\theta$

Figure 3


1.0
0.5
0
0
0.2
0.4
0.6
0.8
1.0
Energy Flux Coefficients
$q = k_0 L \sin^2 \theta$
$|R|^2_{dir}$
$|T|^2_{inv}$
$|R|^2_{inv}$
$|T|^2_{dir}$
$|\eta|^2$
WARM

PPG-1328 "Two-Groove Approach to the Kinetics of Particle Cluster Aggregation," M. Vicanek and N.M. Ghoniem, sub. to J. Comp. Phys., September 1990.

PPG-1329 "Spatial Coherence and Electromagnetic Wave Generation During Electron Beam Experiments in Space," P.L. Pritchett, sub. to Advances in Space Research, September 1990.

PPG-1330 "Electromagnetic Waves-Particle Interactions at the Doppler-Shifted Cyclotron Resonance Using Ducted Whistler Wave," P.R. Strauss, September 1990.

PPG-1331 "Unified Theory and Comparative Study of CARMs, ICLs and FELs," K.R. Chen, J.M. Dawson, A.T. Lin, and T. Katsouleas, sub. to Phys. Fluids B., September 1990.

PPG-1332 "Does Ion Tearing Exist?" R. Pellat, F.V. Coroniti, and P.L. Pritchett, sub. to Phys. Rev. Lett., September 1990.

PPG-1333 "Real Time Relativity," W. Gekelman, J. Maggs, and L. Xu, sub. to Computers in Physics, October 1990.

PPG-1334 "The Effects of Mobility Coalescence on the Evolution of Surface Atomic Clusters," M. Vicanek and N.M. Ghoniem, sub. to Thin Solid Films, October 1990.

PPG-1335 "On the Accuracy of the Folker-Planck Approximation to the Hierarchy to Master Equations," C. Stone, M. Vicanek, and N.M. Ghoniem, sub. to J. Comp. Phys., October 1990.

PPG-1336 "Technology of Theory for ISTP: Mission Oriented Theory", M. Abdalla, J. Berchem, F. V. Coroniti, J. Radar, D. Schriver and R.J. Walker, November 1990.

PPG-1337 "Absorption at the Ion-Ion Hybrid Resonance in the Presence of Incoherent Density Fluctuations," H. Romero and G.J. Morales, November 1990.

PPG-1338 "Conversion of Electrostatic and Electromagnetic Waves in a Plasma at the Peak of a Parabolic Density Profile," D.E. Hinkel-Lipsker, B.D. Fried, and G.J. Morales, sub. to Phys. Rev. Lett., December 1990.