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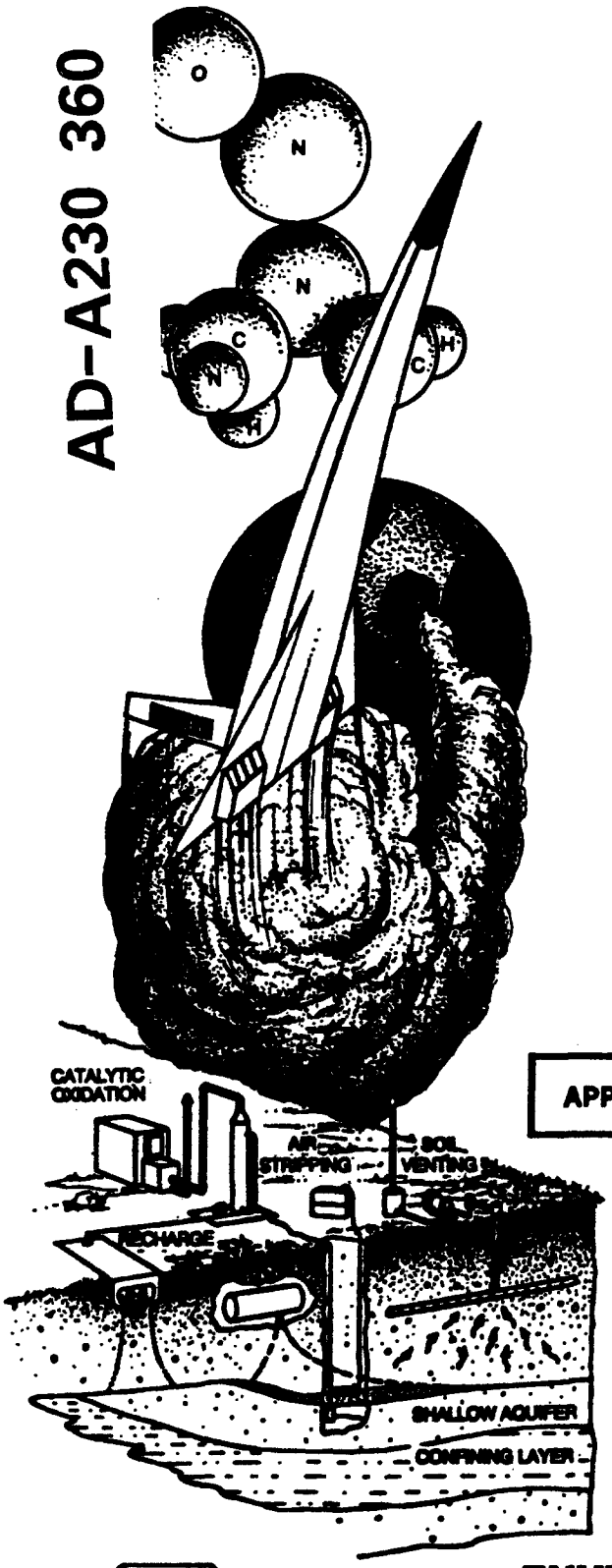

"EXEIS" EXPERT SCREENING AND OPTIMAL EXTRACTION/INJECTION PUMPING SYSTEMS FOR SHORT-TERM PLUME IMMOBILIZATION

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AIR FORCE ENGINEERING AND SERVICES CENTER
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This report presents the EXEIS family of micro-computer based programs for achieving short-term contaminant plume containment. EXEIS is applicable if contaminated water cannot be extracted and water cannot be imported to or exported from the site. There are two main purposes and types of users. For persons relatively unfamiliar with groundwater remedial actions, an expert screening system gives guidance concerning whether extraction/injection (E/I) pumping, slurry wall or sheet piling are most appropriate. For personnel more experienced in remedial actions, management models compute optimal E/I strategies for short-term containment. Via deterministic and stochastic multiobjective optimization models, uncertainty in both planning horizon and aquifer parameters is addressed.

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SUMMARY

Presented is a family of computer programs designed to aid achieving short-term immobilization of groundwater contaminant plumes. The programs are intended for situations in which: import or export of water from the site might be prohibited, and extracting and treating the contaminated water is unauthorized, impractical or cannot be initiated rapidly enough to prevent undesirable consequences.

Among these programs are an expert screener, deterministic and stochastic versions of a multi-objective optimization model, and a post-processor. For ease of use, programs run on an IBM-AT with 640K bytes of RAM, 30 MEG internal hard disk and math coprocessor, and are designed for two types of users and corresponding data availability. Programs are designed to run in under thirty minutes and to address uncertain knowledge of the aquifer system and the amount of time containment will be necessary.

The rule-based expert system is designed for use by persons only slightly familiar with groundwater hydraulics and management, for whom data collection might be difficult. It is a screening tool that can be utilized by base-level personnel or others when considering the practicality of remedial actions proposed by contractors. It conducts preliminary evaluation of whether slurry wall and sheet piling methods of plume containment are practical. It coarsely compares the relative costs of those methods with the cost of plume containment by extraction and injection (E/I) of water. The E/I method assumes an octagonal configuration of both pumping and observation wells.

An octagonal configuration is used because, when unable to import or export water, such an arrangement requires less pumping to halt a plume than does a configuration consisting of an equal number of extraction and injection wells arranged in two parallel lines. It also provides greater lateral control over down-gradient dispersion. An octagonal arrangement can be later converted to a pump and treat system in which contaminated water is extracted from the plume center, treated and injected via the octagonally arranged wells outside the plume boundary. This arrangement can also be more easily installed in the field than an arrangement that might more closely correspond to a plume's shape.

The expert system utilizes its own knowledge base and user-supplied information. Its inference engine uses forward-chaining for soil/site characterization. Its backward-chaining theorem-prover handles user interaction and checks the validity of input data. If queried, it explains why it requests certain information. It also adjusts its confidence in its assessment based on the user's confidence in his answers and how much assistance the user requires from the system.

E/I systems are generally less expensive to install and operate than alternative containment systems for short time periods. If screening shows an alternative method to be less costly for the expected planning horizon, that method should be strongly considered for implementation. This is so because the expert system helps the alternative methods be competitive by comparing them with a relatively intensive E/I arrangement having two pumping wells per side. (In tested short-term scenarios, non-E/I alternatives rarely competed favorably against E/I systems having only one well per side.) Coarse comparison of the feasibility and costs of using the different systems for periods of several years is performed.

Because expected users might not have access to detailed cost information, current unit prices are assumed.

Optionally, the expert system creates a simple data file for use with the optimization model. This permits easy preliminary computing of an optimal E/I strategy, and can be useful to individuals evaluating proposed containment actions. It uses field data, its own prior knowledge and Bayesian statistics to compute conditional probability distribution functions for aquifer parameters. This permits the plume-containment strategy to be calculated for a user-specified confidence level. This option provides a linkage between the two types of users. As an orientation tool it enables the less-technical user to see the E/I design process. Because the data file it develops can be easily modified to suit more rigorous design criteria, it can be a reconnaissance-level design tool for the more-technical user. It also aids the advanced user by developing statistical aquifer parameters needed for the probabilistic version of the management model.

The optimization model is designed for a more hydrogeologically oriented user. It computes the time-varying pumping (extraction and injection) rates that will best modify the potentiometric surface near the plume to contain the contaminant. Such optimal unsteady rates cause more rapid stagnation area evolution than any steady rates that can be computed. (For time periods less than that needed to reach steady-state, optimal unsteady strategies also require less pumping and cost to contain a plume than do steady strategies.)

Although pumping rates can vary with time, they are fixed to be the same in magnitude for all wells in any time step. This is done: to fit within specified computer RAM memory, to keep computer processing time to under a half hour, to permit utilizing more time steps in the optimization models than would otherwise be possible, and to facilitate avoiding the need for import or export of water. This assumption implicitly supports the use of a well-point system for containment, although other systems can be used.

Ideally, pump and treat action can commence at the end of the period of optimal pumping. If this is not possible, one can implement either: another unsteady pumping strategy or steady pumping rates that will maintain the achieved surface. In both cases there is eventual danger that contaminant will reach initially upgradient extraction wells.

A variety of well system configurations are possible. In regular octagonal systems the numbers of extraction and injection wells are equal. As a result, total injection equals total extraction and no import or export of water is needed, enhancing use of the approach for short-term action.

In the most curtailed configuration, there are three injection and three extraction wells, one in the middle of each of six sides of the octagon. In that design, there are no wells on the octagon sides parallel to the initial direction of groundwater flow. For regular octagonal systems, there can also be two or four pumping wells on each of the eight sides. In elongated plume systems there can be more wells on the sides parallel to the initial direction of flow (in such systems total extraction might not equal total injection).

There are both deterministic and stochastic versions of the model.

Each version has two major parts - a simulation component that develops hydraulic influence coefficients and a program that organizes and submits data to a formal optimization algorithm. In their simulation component, both versions incorporate the Theis well function for unsteady flow. They compute influence coefficients (potentiometric surface response to unit discharges or recharges at selected locations).

Using these coefficients, the deterministic version is most accurate for homogeneous isotropic aquifers, although it can approximately simulate anisotropic conditions. However, like most models, it cannot explicitly address uncertain knowledge of the aquifer. A standard approach to considering uncertainty is to perform exhaustive numbers of simulations via Monte Carlo techniques. To avoid that need and reduce processing time, a stochastic version is also presented.

The stochastic version computes modified, probabilistically-based influence coefficients and chance-constraints to consider the weakness of uncertain knowledge of the aquifer. This permits the user to directly compute strategies that have an acceptable, preselected, probability of achieving the stated objectives.

Changes in head predicted using these coefficients are accurate if transmissivity changes with time are insignificant (less than ten percent). If this criterion is violated, new transmissivities should be computed and the optimization model run again. This process can be repeated until the desired accuracy is attained, enhancing use of the models for confined or unconfined situations.

This use of analytically-based influence coefficients for simulation can be preferred over finite difference or finite element simulation in some slightly heterogeneous systems. This occurs when there is insufficient data, time or money to justify calibration and use of spatially distributed approaches. If heterogeneity is not well defined, the model's ability to compute probabilistically based pumping strategies is desirable. This model also has an advantage over finite difference models by being able to compute head response at predetermined points that are not necessarily at the centers of cells. Finally, the computation of influence coefficients using the analytic Theis equation is more rapid than computation using alternative techniques, speeding microcomputer processing.

The second part of each optimization model uses commercial solution algorithms to compute the optimal unsteady pumping strategy needed (for a particular well configuration) to contain the plume. The model uses a weighting technique to permit the user to discriminate between the two components of the bi-objective function. These objectives include minimizing operating and maintenance costs (without discounting) and minimizing final head differences resulting across the plume.

The ability to select a compromise strategy or a strategy purely reflecting one of those objectives is important because the plume might need to be contained beyond the length of the period of optimal unsteady pumping. If one is confident that a more permanent action can commence at the end of the period of optimal unsteady pumping, one will prefer to use the objective of minimizing cost for that period. The longer beyond that time that one might need to contain the plume, the more one will prefer the hydraulic smoothness objective. As surface smoothness increases, the pumping and funding needed to maintain the surface decreases. The

previously mentioned post-processor computes the steady pumping needed to maintain the achieved potentiometric surface.

After developing optimal pumping strategies, the contaminant migration that would result from strategy implementation was simulated. With proper well placement, each strategy assured that the contaminant did not reach the encircling observation wells. Such tests were conducted for a variety of hypothetical isotropic and anisotropic situations. The deterministic model was also applied to a boron plume at Otis Air Base, Massachusetts. The resulting strategy almost entirely halted the plume during an 8 week planning period. Without pumping, significant contaminant movement would have occurred.

The stochastic version was examined by varying both the standard error of the aquifer parameters and the required reliability of the solution. As uncertainty of aquifer parameters increases or as the confidence required in the result increases, pumping in each time period decreases and the final gradient worsens.

In summary, appropriate technology is utilized in preparing a system of micro-computer based programs for achieving short-term contaminant plume containment. There are two main purposes and types of users.

- For persons relatively unfamiliar with groundwater remedial actions, an expert screening system gives guidance concerning whether E/I pumping, slurry wall or sheet piling are most appropriate.
- For personnel more experienced in remedial actions, management models compute optimal E/I strategies for short-term containment. Via deterministic and stochastic multiobjective optimization models, uncertainty in both planning horizon and aquifer parameters is addressed.

PREFACE

This report was prepared by the Biological and Agricultural Engineering Department, University of Arkansas, Fayetteville AK 72701, and the Agricultural and Irrigation Engineering Department, Utah State University, Logan UT 84322-4105, under Intergovernmental Personnel Act agreements for the Air Force Engineering and Services Center (HQ AFESC/RDVW), Tyndall Air Force Base FL 32403-6001.

This report summarized work done between 23 June 1986 and 30 December 1988. HQ AFESC/RDVW project officers were Capt Ed Heyse, Capt Mike Elliott, and Mr Bruce Nielsen. The greater portion of this report was in fulfillment of a dissertation requirement. Therefore, there are some deviations from ESL format. It is being published because of its interest to the worldwide scientific and engineering community.

This report has been reviewed by the Public Affairs Office (PA) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nationals.

This technical report has been reviewed and is approved for publication.

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TABLE OF CONTENTS

Section	Title	Page
I	INTRODUCTION	1
	A. OBJECTIVES	1
	B. BACKGROUND	1
	1. Expert System	2
	2. Optimization Model	2
	C. SCOPE	5
II	REVIEW OF LITERATURE	7
	A. EXPERT SYSTEM	7
	B. OPTIMIZATION MODEL	8
III	METHODOLOGY	14
	A. EXPERT SYSTEM	14
	A. OPTIMIZATION MODEL	15
	1. Contaminant Plume Identification	16
	2. Well System Configuration	17
	3. Model Theory	19
	4. Final Potentiometric Surface and Steady-State Pumping Determination	39
IV	APPLICATION, RESULTS, AND DISCUSSION	41
	A. DETERMINISTIC MODEL APPLICATION TO HYPOTHETICAL SITUATION	42
	1. Analysis of the Weight Factor	42
	2. Analysis of Varying the Objective or Requiring Steady Pumping	48
	3. Analysis of vAriation in Physical Properties for the Hypothetical Problem	50
	4. Effects of Long-Term Steady Pumping	64
	5. Evaluation of Safety Factor for Equation	64
	B. DETERMINISTIC MODEL APPLICATION TO THE HAZARDOUS WASTE SITE, OTIS AIR BASE, MA	66
	1. Input to the Deterministic Model	68
	2. Results	73

TABLE OF CONTENTS
(CONTINUED)

Section	Title	Page
	C. STOCHASTIC MODEL	75
V	USE OF THE EXPERT SYSTEM	86
	A. LOGIC FLOW	86
	1. Soil Characterization	90
	2. Site Characterization	94
	3. Contaminant Characterization	95
	4. Economic Analysis	96
	B. DEVELOPMENT OF INPUT FILE, SMODEL.DAT	97
	C. LOADING THE SYSTEM	97
	D. SYSTEM EXAMPLE	98
VI	USE OF THE OPTIMIZATION MODEL	99
	A. PARTS FO THE MODEL	99
	B. SETTING UP THE MODEL	102
	C. RUNNING THE MODEL	104
	1. Data Input File	104
	2. GAMS-MINOS Output File	104
	3. Procedure to Run the Deterministic and Stochastic Evolutionary and Terminal Steady-State Version	108
VII	CONCLUSIONS AND RECOMMENDATIONS	111
	A. EXPERT SYSTEM	111
	B. OPTIMIZATION MODEL AND POST-PROCESSOR	112
	1. Deterministic Version	113
	2. Stochastic Version	116
	RECOMMENDATIONS FOR FUTURE RESEARCH	117
REFERENCES		118
APPENDIX		
	A. DERIVATION OF COEFFICIENTS FOR EQUATION 18	123
	B. ANALYSIS OF UNCERTAINTY IN DRAWDOWN	126

TABLE OF CONTENTS
(Concluded)

Section	Title	Page
	C. DERIVATION OF EQUATION 30	131
	D. EXAMPLE OF EXPERT SYSTEM ANALYSIS	133
	E. DATA INPUT FORMAT	146
	F. OUTPUT FILE MODEL2LST FROM GAMS-MINOS USING MODEL2.DAT	148
	G. OUTPUT FILE MODEL2.CAL FROM HEAD2.FOR USING MODEL2.DAT	162

LIST OF TABLES

Table	Title	Page
1	Effect of Weight Factor on Optimal Unsteady Pumping Strategies for Hypothetical Problem (model run 1d)	45
2	Effect of Anisotropy on Optimal Unsteady Pumping Strategy for Hypothetical Problem	52
3	Effect of Anisotropy on Steady Pumping Strategy for Hypothetical Problem	54
4	Effect of Duration of Evolutionary Era on Optimal Unsteady and Steady Pumping Strategies for Hypothetical Problem (Tmax = 1255 m^2/d and Tmin/Tmax = 0.67)	59
5	Effect of Well Configuration on Optimal Unsteady and Steady Pumping for Hypothetical Problem (Kmax = 82 m/d w/varying saturated thickness, Kmin/Kmax = 0.30)	61
6	Optimal Unsteady and Steady Pumping Strategies for Boron Plume at Otis Air Force Base	74
7	Effect of Aquifer Parameter Uncertainty on 95% Reliable Optimal Unsteady Pumping Strategy for Hypothetical Problem (run 1d)	79
8	Effect of Aquifer Parameter Uncertainty on 80% Reliable Optimal Unsteady Pumping Strategy for Hypothetical Problem (run 1d)	80
9	Standard Normal Deviate $F(\rho)$ Corresponding to the Reliability Function	82
10	Summary of Trends Produced by Stochastic Analysis	84

LIST OF FIGURES

Figure	Title	Page
1	Definition of Anisotropic Hydraulic Conductivity	26
2	Flow Chart for Solving Linearized Chance Constrained Groundwater Mangement Model	33
3	Cross-section Demonstrating Sample Stochastic Constraints on Final Water Levels	36
4	Contaminant Plume in the Hypothetical Area	44
	a. Initial Assumed Concentrations b. Resulting Concentrations After 38 Days of No Pumping (run 1d)	
5	Resulting Potentiometric Surface Elevations and Hydraulic Gradients After 8 Days of Optimal Unsteady Pumping (run 1d)	46
6	Resulting Potentiometric Surface Elevations and Hydraulic Gradients Resulting from 30 Days of Steady Pumping Immediately Following 8 Day Optimal Pumping (run 1d)	47
7	Run 1d Simulated Using Solute-Transport Model (T_{max} = 1255 m^2/d, T_{min}/T_{max} = 1.0)	49
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	
8	Run 2d Simulated Using Solute-Transport Model (T_{max} = 1255 m^2/d, T_{min}/T_{max} = 0.67)	55
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	

LIST OF FIGURES
(CONTINUED)

Figure	Title	Page
9	Run 3d Simulated Using Solute-Transport Model (T_{max} = 1255 m^2/d, T_{min}/T_{max} = 0.30)	56
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	
10	Run 4d Simulated Using Solute-Transport Model (T_{max} = 370 m^2/d, T_{min}/T_{max} = 0.25)	57
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	
11	Run 6d Simulated Using Solute-Transport Model (K_{max} = 82 m/d, K_{min}/K_{max} = 0.30, Regular Octagon shifted 1 grid to the east)	62
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	
12	Run 7d Simulated Using Solute-Transport Model (K_{max} = 82 m/d, K_{min}/K_{max} = 0.30, Irregular Octagon shifted 1 grid to the east)	63
	a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping	
13	Resulting Concentrations After 12 Additional Weeks of Steady Pumping Using Solute-Transport Model	65
	a. Run 2d b. Run 6d	
14	Study Area for Otis AFB Contamination Problem	67
15	Steady-State Groundwater Table Elevations	69

LIST OF FIGURES
(CONCLUDED)

Figure	Title	Page
16	Contaminant Plume Extent and Location of Proposed Well System	70
17	Resulting Concentrations After 40 Weeks if No Pumping Strategy is Implemented for the Otis AFB Plume	76
18	Resulting Concentrations After 8 Weeks of Optimal Unsteady Pumping for the Otis AFB Plume	77
19	Resulting Concentrations After Week 40 Following 32 Weeks of Steady Pumping for the Otis AFB Plume	78
20	Flow Chart of User Options	87
21	Flow Chart of Programs Used and Files Developed	88
22	Flow Chart of Expert System	89
23	Soil Type Selection Table	91
24	Soil Fact Database	91
25	Drainage Selection Table	92

LIST OF PROGRAMS

Program	Title	Page
1.	BW.BAT	164
2.	FORT2.BAT	164
3.	GM.BAT	164
4.	GAMS.BAT	164
5.	FORT.BAT	164
6.	FORT1.BAT	164
7.	EXP.BAT	164
8.	MODEL2.FOR	165
9.	SMODEL.FOR	180
10.	BOB2.GMS	197
11.	HEAD2.FOR.	199
12.	MODEL2.DAT	204
13.	SMODEL.DAT	206
14.	XCON.FOR	208

NOMENCLATURE

a intersection of contaminant plume ellipse and x-axis, (L).

$\bar{A}$ stochastic coefficient produced by taking the partial derivative of drawdown with respect to transmissivity, (T/L^2).

b intersection of contaminant plume ellipse and y-axis, (L).

$\bar{B}_{i,j,k}$ the unit response function for a stimulus at well i on an observation point j at time period k; calculated using the mean values of transmissivity and effective porosity, (T/L^2).

$\hat{\delta}_{i,j,t-k+1}$ the incremental drawdown at a well j in time period t caused by a unit volume of pumping at well i in time k, $(1/L^2)$.

$[B]$ an I x TT matrix of the sum of influence coefficients describing the effect on the head at each pumping well i caused by unit pumping at all other pumping wells in each time step t, $(1/L^2)$.

(B_o) a 1 x TT row vector of the sum of influence coefficients describing the effect on the head at the contamination source by a unit of pumping at all pumping wells in each time step t, $(1/L^2)$.

$[B_{jd}]$ a jd x TT matrix of the sum of influence coefficients describing the effect on the head at each observation well j (that is down-gradient of the contaminant source) caused by unit pumping at all pumping wells in each time step t. The jd value is the number of observation wells downstream of the contamination source, $(1/L^2)$.

c'_t present value cost of pumping a unit volume of water a unit vertical distance in time period t, $(\$/L^4)$.

c''_t present value maintenance cost of pumping a unit volume in time period t, $(\$/L^3)$.

$[C_e]$ an I x TT array containing total present value cost per unit volume of total pump maintenance costs plus energy costs associated with raising water a distance equal to the initial static lift at each pumping well, ($\$/L^3$).

$[C_h]$ an J x TT array containing the weighted unit contributions (linear) to the final difference in head between the contaminant source and the J observation wells, caused by the initial difference and pumping at each of the I pumping wells, ($\$/L^3$).

$E(s_{j,t})$ mean of drawdown at observation well j at the end of time period t, (L).

$E_{i,j,k}$ stochastic unit response function for stimulus at well i and response at point j for time period k, (T/L^2).

E(T) mean of transmissivity, (L^2/T).

$E(\phi)$ mean of effective porosity.

ϕ effective porosity.

$\{f(Q)\}$ a J x 1 column vector. The vector contains the weighted squared contributions to the final difference in head between the contaminant source and the j observation wells caused by pumping at each of the I pumping wells in all time steps, ($). Each term is the squared product of a row of the $[K_h]$ matrix and the {Q} vector.

f(Q) standard deviation of drawdown, (L).

$F^{-1}[\rho]$ standard normal deviate corresponding to a normal cumulative distribution function.

$\{g(Q)\}$ an I x 1 column vector. It is the product of $[K_e]$ matrix and the {Q} vector.

$h_{i,g}$ ground elevation at pump i, (L).

$h_{i,0}$ head at pump well i at time 0, (L).

$h_{i,t}$ head at pump well i at time t, (L).

h_i^L lower limit on head at pump i, (L).

h_i^U upper limit on head at pump i, (L).

$(h_{j,TT})_d$ head at observation well j which is down-gradient of the contamination source at the end of the modeling period TT, (L).

$h_{o,TT}$ head at contaminant source at end of modeling period TT, (L).

$\{H^L\}$ an I x 1 column vector of lower limits on hydraulic head in pumping wells, (L).

$\{H^U\}$ an I x 1 column vector of upper limits on hydraulic head, (L).

I total number of pumping wells.

i_x hydraulic gradient in the x direction, (L/L).

i_y hydraulic gradient in the y direction, (L/L).

J total number of observation wells.

K_x hydraulic conductivity in the x direction, (L/T).

K_y hydraulic conductivity in the y direction, (L/T).

K_{θ_k} hydraulic conductivity in the direction θ_k degrees counter-clockwise from the x-axis (L/T).

$[K_e]$ an I x TT array containing present value energy costs associated with raising a unit volume of water a distance equal to the dynamic drawdown (+ or -) at I pumping wells, caused by pumping at all wells, ($\$/L^6$).

$[K_h]$ a J x TT matrix. Each element is a weighted response of the final difference in head between the contaminant source and the J observation wells caused by unit pumping at each of the I pumping wells in a particular time step, (L^{-2}).

η porosity.

$\overline{P}$ stochastic coefficient produced by taking the partial derivative of drawdown with respect to effective porosity.

q Darcy's velocity, (L/T).

q^L lower limit on pumping at all wells, (L^3/T).

q^H upper limit on pumping at all wells, (L^3/T).

q_t pumping at all wells at time t, (L^3/T).

Q_o initial estimate of optimal pumping for stochastic model, (L^3/T).

$\{Q\}$ a TT x 1 column vector of unknown pumping values, (L^3/T), (these values vary in time, but for a time step are equal in absolute value for all wells).

(Q) a 1 x TT row vector of unknown pumping values, (L^3/T).

$\{Q^L\}$ a TT x 1 column vector of lower limits on pumping, (L^3/T).

$\{Q^U\}$ a TT x 1 column vector of upper limits on pumping, (L^3/T).

Q_p the steady-state pumping at well p to maintain existing potentiometric surface at observation well o, (L^3/T).

r distance from stimulus i to observation point j, (L).

r_e effective radius of the pump well, (L).

r_p radius of the pump well, (L).

sdt standard deviation of transmissivity.

sds standard deviation of effective porosity.

s.f. safety factor.

$s_{i,t}$ calculated drawdown at pump i at time t, (L).

$s^*_{i,t}$ specified upper limit on drawdown, (L).

SL side length of a regular octagon, (L).

S2 length of sides parallel to gradient for irregular octagon, (L).

s_p the drawdown at pump well p that is to be maintained, (L).

s_o the drawdown at observation well o that is to be maintained, (L)

TT total time for optimal pumping strategy, (T).

T transmissivity, (L^2/T).

T_{avg} average transmissivity between pump well p and observation well o, (L^2/T).

t time period within time T,(T).

θ angle formed by the x-axis counter-clockwise to Kmax

θ_k angle formed by the x-axis counter-clockwise to the line connecting the pumping well and any other well on the octagon.

u Boltzman variable.

μ reliability.

v seepage velocity, (L/T).

$var(s_{j,t})$ variance of drawdown at observation well j at the end of time period t.

var(T) variance of transmissivity.

$var(\phi)$ variance of effective porosity.

(V) variance of field data.

(Vo) variance of prior probability density function.

W_f weight factor to convert the square of hydraulic head differences to dollars, $(\$/L^2)$.

$(\bar{X})$ mean of field data.

$(\bar{X}o)$ mean of prior probability density function.

Y a constant made up of initial head terms times the weight factor, (\$).

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SECTION I

INTRODUCTION

A. OBJECTIVES

The objectives of this study are:

1. To present an expert system that performs preliminary screening and recommends an appropriate method for short-term groundwater contaminant plume containment. The system queries the user for input of aquifer parameters, contaminant information, time parameters and confidence in this input. The system outputs a decision that describes the type of solution it feels is best and its confidence in this decision. Designed to be used primarily by persons inexperienced in groundwater hydrology, the system answers questions concerning the assumptions it is making and its decision-making process.

2. To present a procedure for determining the optimal time-varying sequence of extraction and injection of water needed for short-term containment of a groundwater contaminant plume. Procedure assumes that pumping of contaminated water is not permitted and that exporting or importing water is generally not desireable. Included is guidance on well siting, development and use of a deterministic simulation-optimization model, and guidance on interpreting model results. The model is intended for use by persons somewhat experienced in groundwater hydrology. Influence coefficients within the model are computed using the Theis well function for unsteady-state flow in a confined aquifer. As an approximation, application of the influence coefficients is extended to a hypothetical heterogeneous anisotropic aquifer. Saturated thickness and transmissivity may differ at each well and hydraulic conductivity can vary with direction. Assumed are a miscible contaminant plume, 2-D flow, and the absence of vertical density gradients. The safety factor used to determine plume extent includes consideration of hydrodynamic dispersion. Both advective and dispersive transport were simulated when testing the computed optimal pumping strategies and verifying that the hypothetical plumes would be contained.

3. To present a multiperiod stochastic groundwater contaminant management model that also develops optimal pumping strategies using the Theis equation. This model is intended for use by an experienced hydrogeologist. It considers the random characteristics of temporally constant transmissivity and effective porosity. The stochastic management model is formulated by transforming the objective function and constraint equations containing random aquifer properties into chance-constrained expressions that specify system performance reliability requirements. The model is applied to the same hypothetical system mentioned above.

EXEIS (Expert Screening and Optimal extraction/Injection Pumping Systems) is a family of computer programs developed to reach these objectives. EXEIS is an aid in developing optimal strategies for short-term containment of groundwater contaminant plumes in situations when extraction and treatment of contaminated groundwater is impractical or unfeasible.

B. BACKGROUND

1. Expert System

Pressure to protect groundwater has increased as the public has realized the serious threat posed by groundwater contamination. Remediation or prevention of groundwater contamination is increasingly important for all water users. Inadequate response to contaminant situations may result in unnecessary damage. Excessive response may be unnecessarily expensive. Timely decisions must be made to develop corrective strategies for each particular contamination situation. Systematic development of tools or methodologies is needed for optimizing remedial actions. The tool presented in this report integrates an expert system with an optimization algorithm to compute an optimal strategy for containing a contaminant plume.

Expert systems are computer programs designed to emulate the logic and reasoning processes humans would use to solve problems in their fields of expertise. Interest in expert systems has grown rapidly with the emerging availability of artificial intelligence-based techniques and tools. By emulating human reasoning to combine objective and subjective knowledge, expert systems expand the availability of specialized expertise.

Many solutions exist for contamination problems. Solution selection must be situation-specific and be based on the expertise of the decision-maker(s). The presented expert system accomplishes systematic and efficient evaluation of alternatives and intelligent strategy selection.

2. Optimization Model

Individuals, industries and government agencies face many situations requiring remediation or prevention of groundwater contamination. There is a clear need for techniques for optimizing, to the extent possible, response to groundwater contamination problems. The purpose of this study is to present one of those methodologies.

The presented technique is applicable for groundwater contaminant situations best solved by modifying the potentiometric surface in the vicinity of the contaminant source. Example contaminant sources include spilled hazardous chemicals and toxics leaked from waste facilities as well as petroleum spills or leaks from underground storage facilities.

Appropriate potentiometric surface modification can:

- prevent groundwater from contacting the source of contamination and becoming contaminated, and
- prevent contaminated groundwater from spreading beyond the immediate site.

Methods of modification include construction of artificial barriers to groundwater flow and/or extraction/injection of water from/to the aquifer. Cost of installing and maintaining the different types of artificial barriers varies greatly, as does their reliability. This

study describes models for optimizing extraction/injection (E/I). This approach has comparatively low installation expense and good reliability, but is commonly used as a transitional element of remedial actions. It is less often used as a long-term solution.

An overview of numerical computer models and, specifically, the computer model developed in this study is appropriate. A groundwater model is a numerical representation of a natural system. To make the model an acceptable representation (though it can never be exact) one must simulate the natural system as closely as possible using available aquifer information and the basic laws governing flow in porous media.

Incorporation of these laws into the optimization model is achieved using the "response-matrix" approach. An external groundwater simulation component is used to develop unit response functions for input into the optimization component. Decision variables often include pumping and drawdown in the objective function.

This study incorporates these two components into the optimization model; a simulation component to develop the unit response matrix and an optimization or "management" component.

a. Simulation Component

The simulation component incorporates equations describing the relationships between the physical properties of and the processes in a system. Simulation models are used to investigate the behavior of the system when it is subjected to specified levels and/or patterns of stimuli. In a groundwater simulation model pumping is most often the stimulus and groundwater potentiometric levels (or drawdowns) are the responses being investigated.

As with other resource management, groundwater management is generally performed in an uncertain environment. This uncertainty is ascribed mainly to lack of perfect knowledge about an aquifer system, inherent variability of aquifer parameters and flow characteristics and other factors such as system cost and revenue, engineering design and system operation. This uncertainty affects our capability to predict system response to management decisions.

To consider uncertainty in aquifer parameters and subsurface flow, groundwater flow can be treated as a stochastic process and aquifer parameters can be considered random variables. Therefore, this model provides two versions of the simulation component that interface with the optimization component:(1) a deterministic version and(2) a stochastic version.

Two basic laws governing steady groundwater flow are Darcy's law and the Law of Continuity. The simulation component (both versions) uses these laws, as well as the Theis well function, to predict plume movement and generate the unit-response function matrix. This study looks at three separate phases of contaminant plume containment. The first phase uses Darcy's law to predict the steady-state movement of

the plume (accomplished in the expert system). The second phase uses the Theis well function for unsteady-state flow to predict the response of the potentiometric surface to a unit stimuli (performed in the simulation component). The third phase uses the Theim equation for steady radial flow to a well to predict what stimulus is required to maintain the new potentiometric surface (computed in the post-processor). The Theis well function and the steady radial flow equation are derived from the Law of Continuity. The derivation of these equations is found in texts. (References 1, 2, and 3).

Because of uncertain knowledge of aquifer parameters and to the neccessity of making approximations and assumptions, models provide only rough estimates of real world processes. All these attributes of the modeling process are sources of error. Because these errors introduce uncertainty into groundwater modeling, future projections cannot be made with absolute certainty. The validity of these mathematical equations and the errors introduced by numerical methods are discussed in many texts. The stochastic version addresses only those errors in hydraulic head estimation caused by uncertain knowledge of aquifer parameters. The stochastic version also establishes tolerances within which the parameters of the physical system may vary without appreciably affecting the model results. These tolerances are measured by the 'reliability' that the user demands from his model.

The guidelines for when to use the deterministic version and when to use the stochastic version are situation-dependent. In most cases it is advantageous to compare the results from both versions.

b. Optimization Component

A simulation model per se cannot generally predict the physically feasible limits of a response. As a result, it may predict potentiometric surfaces below the base of the aquifer, or it may estimate pumping in excess of that which is possible. This is due to the fact that the solution space for a simulation model is not constrained. Responses prescribed by a simulation model will often not be physically feasible in the field if input data to the model are poorly related to the actual properties of the system. Therefore, an optimization model is used in conjunction with the simulation model.

The optimization component consists of:(1) an objective function, (2) constraints and(3) bounds. The objective function is an equation, the value of which is maximized or minimized. This objective function is a mathematical description of a specific policy goal. Values of variables in the model are systematically changed by the algorithm until an optimal objective value is obtained. Both the objective function and the constraints are mathematical expressions in terms of system properties (model parameters) and conditions (state and decision variables). In addition to functional constraints, limits (bounds) may be imposed on the system variables so that the variables cannot assume undesirable values.

The optimization component seeks to identify the best possible

solution; i.e. the solution providing the optimal value of the objective function. The final optimal solution consists of the optimal objective value and a value for each system variable. In most cases, the specific combination of variable values at the optimum is as important to the investigator as the optimum objective value.

Whenever there is more than one objective to be achieved, multi-objective optimization is required. The dual objective function in the optimization model of this study uses a weighting factor to simultaneously consider the dual objectives of minimizing the total cost of pumping and maximizing the degree to which a horizontal potentiometric surface is attained across a groundwater contaminant plume.

C. SCOPE

Accomplishments of this study include:

1. An expert system was developed for analyzing various methods of groundwater contaminant containment. This includes practical validation of the system by testing with several hypothetical situations. The optimization model was run with the suggested input from the expert system.

2. An appropriate weighting factor was found for the bi-objective function of minimizing pumping costs while assuring stabilization of the contaminant plume. This was obtained by sensitivity analysis using a hypothetical situation. Comparisons were made of optimal solutions developed when emphasizing only the hydraulic objective, when emphasizing solely the economic objective, and when merely minimizing the volume of water pumped.

3. Verification was made that contaminant is contained by implementing the optimal E/I strategy computed by the deterministic management model. This was accomplished by simulating contaminant movement using a 2-D method of characteristics (MOC) solute transport model (Reference 4).

4. Analysis was made as to how changes in uncertainty of the aquifer parameters and required reliability of the results affect the final objective function and variable values computed by the stochastic model.

5. Determination of steady pumping values required to maintain the potentiometric surface needed to control the contaminant plume is made. (This potentiometric surface was attained by unsteady pumping).

6. The methodology was applied to a contaminant plume at Otis Air Force Base in Massachusetts is made.

Each of the preceding actions is supported by presenting optimal strategies and results of implementing those strategies in summary tables and graphic contour maps. The tables allow the comparison of different weighting factors in the deterministic model, different aquifer parameters or well configurations in the deterministic model, and different aquifer uncertainties and reliabilities in the stochastic model. Contour maps show the movement of the plumes predicted by the

solute transport model as a result of the pumping strategies recommended by the deterministic model. They are used to demonstrate the acceptability of the plumes resulting from strategy implementations.

SECTION II

LITERATURE REVIEW

A. EXPERT SYSTEM

Reference 5 provides a good overall review of artificial intelligence and expert systems-a rapidly developing field. It describes HYDRO (2) as the most successful application of expert systems to a water resource problem. HYDRO was developed to aid in the calibration of a large hydrologic watershed model. It uses watershed characteristics to calculate initial parameter values and calculates the "most likely" values and certainty factors. A unique feature permits the user to specify how the certainty factors associated with the parameter estimates are used.

Another example of the application of an expert system to water resources is given by Reference 6. Cuena reports the development of an expert system designed to operate flood control dams during emergencies and to plan for best handling of flooding in flood prone areas. The system includes a series of simulation models that predict the hydrologic condition of a watershed. These models permit the expert system to provide guidance on operation based upon updated, predicted conditions. The system is driven by a set of physical rules (that describes relations between rainfall, inflow, and flood level) and a set of operational rules (for civil defense and dam operation).

Reference 7 presents an expert system for aiding the operation of an activated sludge wastewater treatment facility. Production rules, typically of the "if-then" structure, are used for knowledge representation. Production rules define the paths by which an input into the system can reach a goal state (terminal conclusion). The program requests additional information to resolve inconsistencies. Control strategies are produced and directions for future efforts are presented.

Reference 8 describe a comprehensive expert system to control city-wide flooding and pollution. The system incorporates the experiences of several experts in model verification, sensitivity analysis, calibration and validation. It provides information on storm intensity, sewer system flows, pollutant concentrations, and status of diversions and storage. It directs excess flows through diversion structures and indicates when to bypass the sewage treatment plant.

Expert system use in agriculture has been proposed and documented by several authors. Reference 9 suggests application in decision support (in diagnosing plant and animal disease and developing marketing strategies, and machine intelligence; developing new sensors and manipulators). Reference 10 developed a skeletal expert system called ADAM (Adaptive Assembler for Models) that allows a user to easily custom build models involving conventional equations and

human expertise. In a related paper, Reference 11 describes several methods of representation and reasoning useful for specific types of problems. They discuss two widely used rule paradigms-pattern-matching and parameter-driven systems. They describe how forward- and backward-chaining are implemented in each system.

Specific applications of systems in agriculture have been shown. Reference 12 developed an expert system from an off-the-shelf software shell to control a greenhouse misting system that allows dynamic implementation of a grower's perceived optimal misting strategy. Reference 13 developed an expert system for sizing and selecting machinery for whole-farm cropping systems which integrates a whole-farm management linear program (LP) with the knowledge-based expert system.

A system to aid in identifying and assessing groundwater pollution sources has been presented by Reference 14. The paper presents an approach for identifying and locating a finite number of groundwater pollution sources. A pattern recognition algorithm is used as a secondary knowledge base. The finite sequential recognition algorithm is accessed from within the knowledge base. The expected risk in the pattern classification decision and a heuristic confidence threshold is compared to determine the acceptability of the source identification.

Reference 15 developed a system to demonstrate the utility of applied artifical intelligence to aid in the assessment of the potential for groundwater contamination. The system incorporates expert knowledge coupled with a chemical transport/degradation model and supporting data bases. An evaluation of 12 polynuclear aromatic compounds contained within a wood preserving waste that has been applied to a soil system is presented.

To date, there are no published expert systems designed for aiding the management of existing groundwater contaminant plumes. The system presented in this report partially fills that void. It determines whether extaction/injection is the best containment approach for a particular contamination situation. The system also facilitates using this information as input to a previously described optimization program (Reference 16) that develops extraction/injection strategies.

B. OPTIMIZATION MODEL

Reference 17 reviews many applications of optimizing groundwater management. In this section we mention only those relevant to groundwater quality management and/or potentiometric surface evolution.

Some early efforts to identify strategies for managing groundwater quantity and quality resources focused on simulation of groundwater flow and mass transport. Discharge and contaminant input rates were known or assumed (References 18, 19, 20, 21, 22, and 23).

Later, groundwater hydraulic management models were developed to

systematically relate the hydraulic behavior of the flow system to the cost of utilizing scarce aquifer supplies. This was accomplished by coupling the physical principles of groundwater flow and optimization theory (Reference 17).

Aquifer management research has also treated the problem of groundwater pollution control. Groundwater management models can be classified according to objective or formulation. Concerning objective, models belong to one of two categories (Reference 17). One type develops management strategies optimal with respect to groundwater hydraulics. The second category develops strategies that optimize economic and other consequences of water policies.

Relatively few studies have used stochastic concepts at the macroscopic scale in subsurface flow models. The work done in this area can be categorized into the three major causes of uncertainty in model solutions. Such models have considered uncertaintycaused by: (1)measurement errors in the input parameters, (2) spatial averaging of the input parameters, and (3) the inherent stochastic nature of heterogeneous porous media.

Reference 24 studied error propagation. They investigated the influence of errors in initial head, transmissivity and effective porosity on the drawdown patterns predicted by the Theis equation for pumpage from a homogeneous isotropic confined aquifer. They utilized uniform frequency distributions for the input parameters, noting this is the usual Bayesian "know-nothing" prior distribution. They produced plots that show the growth through time of the per cent error in hydraulic head at various radial distances from a pumping well with various input errors. They also concluded that a far more general and better (yet mathematically complicated) method of investigating error would be to consider the parameters as stochastic processes.

Reference 25 looked at the sensitivity of groundwater models with respect to variations in transmissivity and effective porosity. The sensitivity formalism is applied to the Theis equation by taking the partial derivative with respect to a particular parameter. They describe a first-order formulation for evaluating the effect of hydraulic head resulting from small changes in aquifer parameters. They obtained sensitivity coefficients with respect to each of these parameters. In general, they discovered that a 20 percent deviation in transmissivity or effective porosity can be handled adequately (computed drawdown error of less than 5 percent) by a first-order approximation. Their formulation is used in this study.

The work of Reference 26 combines aspects of approaches 1 and 2. They used a numerical simulation model of transient flow to a well in a confined aquifer. They utilized Monte Carlo simulation to investigate the effect on the solutions of normally distributed measurement errors in initial head, boundary heads, pumping rate, aquifer thickness, hydraulic conductivity and storage coefficient. In addition they analyzed the uncertainties introduced into the solutions by choosing spatially averaged parameter values at each grid point in

the nodal mesh used in the numerical method. They assumed that within each nodal block, each input parameter (for example, hydraulic conductivity) can be represented by a general linear function that fully describes the spatial trends within the block. The uncertainties in the values of the coefficients of this general linear function (which are related to the number of measurements available) lead to uncertainty in the spatially averaged value used at each node. The result is a normal distribution for the hydraulic conductivity values. This normal distribution identifies the approach as having more in common with the analysis of measurement errors (category 1) than with stochastically defined media (category 3) where hydraulic conductivity is usually recognized as being log normally distributed.

Reference 27 falls into category 3. He concluded that the most realistic representation of a nonuniform homogeneous porous media is a stochastic set of macroscopic elements in which the two basic hydrogeologic parameters (hydraulic conductivity and porosity) are represented by log-normal and normal frequency distributions, respectively.

The groundwater flow equation is an integral part of any numerical groundwater model. Incorporation of this equation into a management model is achieved via either "embedding" or "response matrix" methods (Reference 17). In the embedding method, numerical approximations of the governing flow equation are directly included as constraints in an optimization model. In such cases drawdowns and pumpings often are decision variables.

The embedding method was first presented in Reference 28. Using one- and two-dimensional examples, they showed that the physical behavior of a groundwater system could be included as an integral part of an optimization model. They used finite-difference approximations to simulate both steady and unsteady flow.

Reference 29 applied the embedding method to a hypothetical case involving steady-state control of hydraulic gradients to insure stationarity of a fluid stored in an aquifer.

Another application of the embedding approach to control hydraulic gradients was reported in Reference 23. Their objective was to minimize pumping while containing a plume of contaminated groundwater, dewatering two excavation areas and obtaining water for export from the system. They used cells to represent the wells and steady-state pumping was used. The solution included nodal locations where either pumping or injection wells should be located. The solution also included optimum pumping rates and steady-state hydraulic head distribution over the 99 active nodes.

Reference 30 developed an influence coefficient method for optimally modifying a steady-state surface to satisfy a groundwater contaminant concentration criteria. They used the embedding method for a 25 cell subsystem of a larger study area.

In the response matrix method an external groundwater simulation model is used to develop unit responses. Each unit response describes the influence of a unit stimulus (e.g., pumping) upon hydraulic heads at points of interest throughout a system. These coefficients, Dirac delta functions, (References 31 and 32) are also termed discrete kernels (References 33 and 34) or response values (References 35 and 36). An assemblage of the unit responses, a response matrix, is included in the management model. Decision variables in the objective function often include pumping and drawdown values.

Reference 37 is perhaps the first that considered the response matrix method for use in groundwater management modeling. He considered two objectives, maximization of water production and minimization of the production costs for a well field. Linear and quadratic objective functions were proposed for the respective objectives. The Theis unsteady-state formula (Reference 38) was used to calculate drawdown responses. Constraints were formulated so that drawdowns were controlled according to pump and well facility limitations. The second objective function was quadratic because water production costs were assumed to be directly proportional to the products of variable lifts and discharge rates. However, no solutions were presented.

Reference 1 presented a hypothetical example of managing a 25-cell aquifer system. Developed strategies were to maintain groundwater elevations above specified minimum levels at specific locations in order to prevent poor quality lake water from entering the aquifer. The model determined pumping locations needed to minimize cost of delivering water at a specific location. A computer simulation model was used to generate response coefficients.

Reference 39 maximized the degree to which spatially distributed target potentiometric surface elevations are attained by the end of a planning period. They used linear programming and the response matrix approach.

Reference 40 used a response matrix comprised of velocity responses to determine the optimal pumping to prevent a contaminant plume from reaching production wells.

Reference 41 also used the velocity response matrix approach. Their model minimized the cost of extracting a contaminant plume subject to achieving desired groundwater velocities within a specified time period. Their model determined well location and timing and rates of pumping for a 4-year period of aquifer restoration. Extraction wells were located within the plume boundaries in the presented hypothetical situation. In applying their model it was assumed that the extracted contaminated water can be appropriately treated and utilized or disposed of.

Reference 42 developed a multiperiod stochastic groundwater management model utilizing the Cooper-Jacob equation and the concept of unit response functions. He concluded that effort should be expended to better evaluate transmissivity and its variability. Variation in

effective porosity was shown to have little effect on drawdown at all reliability levels tested (.90 and greater) and can be treated as deterministic. When the uncertainty of transmissivity is large the normality assumption for random drawdown may not be appropriate. He also concluded that first order analysis may not be appropriate for assessing the statistical properties of drawdown. He reiterated that there have been other investigations regarding the appropriatenesss of first order analysis applied to situations where variation of system components is large.

Reference 43 developed a methodology for estimating the elements of parameter matrices in the governing equation of flow in a confined aquifer. The estimation techniques for the distributed parameters inverse problem pertain to linear least squares and generalized least squares methods. Secondly, a nonlinear maximum likelihood estimation approach to the inverse problem is presented. The statistical properties of maximum likelihood estimators are derived, and a procedure to construct confidence intervals and do hypothesis testing is presented.

Numerical modeling techniques for groundwater investigation and management purposes are well established. Coupling of groundwater simulation methods with linear and quadratic programming techniques will become common management practice. However, application of these techniques to real-world problems concerning water quality are still relatively uncommon in the literature.

Reported applications of optimization to groundwater contamination problems mainly address the extraction and treatment of contaminated groundwater. A different procedure, proposed in this study, is needed if the contaminated groundwater cannot be readily treated. In that case, the groundwater should be immobilized in the aquifer until appropriate treatment equipment is available. For short periods this can be accomplished most economically by siting extraction and injection wells outside the plume, rather than inside it. These can be used in an attempt to create a zero gradient across the plume.

Over a short period one cannot be certain to achieve a horizontal water surface. Therefore, the proposed model's objective function includes a goal programming approach to the hydraulic portion. This goal programming attempts to achieve a target relationship between hydraulic heads. In addition, there is an economic component for minimizing the cost of pumping to obtain these target hydraulic heads. There is a weighting factor which allows the user to determine whether the model should emphasize economics or hydraulics.

In the presented model all constraints describing water level response to pumping utilize the response matrix approach. Both deterministic and stochastic versions use influence coefficients developed for the Theis unsteady-state flow equation. Stochastic constraints differ from those used by Reference 42 in that both the hydraulic portion of the objective function and the drawdown constraints are affected by uncertainty in aquifer parameters.

For example, in our model the user may wish to be 95 percent sure that the model-predicted head change at observation wells is equaled or exceeded in the field, and simultaneously that the predicted drawdown at pumping wells are not exceeded. This is accomplished in the presented model by incorporating a 95 percent confidence level for the drawdown constraints and a 5 percent confidence level for the hydraulic portion of the objective function.

SECTION III

METHODOLOGY

A. EXPERT SYSTEM

Most commercially available expert system shells are based on a single computational model (i.e. production rules, deductive retrieval, etc.). A system that would combine approaches and would be able to link with an optimization model was needed. At least part of what constitutes expertise in a particular domain is the ability to select a problem solving strategy which works, but is somehow better than the alternatives.

Therefore, a rule-based expert system was developed specifically for this project using the FORTRAN-77 language. All rule-based systems have three elements: facts, rules and a reasoning strategy. Facts consist of knowledge about the states or values of objects that describe the problem. Facts are dynamic because they change as the system executes. Rules contain knowledge about relationships between these facts. They are static. The part of the knowledge system that uses the rules to reason out the problem is contained in a group of inference and control strategies collectively referred to as the inference engine.

Specifically, the presented contamination containment expert system uses production rules (if-then rules) to control the data acquisition phase, uses a forward-chaining system for soil/site characterization and uses a backward-chaining theorem-prover to handle user interaction.

The core of the expert system is in the inference engine where the determination of the best method of containing a groundwater contaminant plume (so there is no forward movement of the plume) is made. Factors considered are type of contaminant, soil and aquifer characteristics, site characteristics and cost.

When building an expert system one must first decide what knowledge the system will contain and how the system will be used. In the presented system the knowledge domain was purposely kept narrow. It focuses on just one aspect of groundwater contamination. Assuming groundwater is already contaminated, the system only needs knowledge for deciding how best to prevent contaminant movement or increased contamination. The system does not try to perform a human risk assessment nor does it try to determine the best way to clean up the aquifer. However these are forseeable additions to an enhanced system.

The system can determine if particular input is needed, thus permitting information exchange. Domain information is used by the system in three ways: (1) To aid the user in organizing all needed information to analyze a contamination problem. (2) To use model results to propose the best possible containment strategy for a

particular problem, and (3) To evaluate the overall confidence in the solution based on subjective and statistical confidence of input parameter estimations and of the user's understanding of model assumptions.

An expert system should avoid alienating the user by treating him as if he knows nothing about the topic. The general purpose of an expert system is to make decisions, but the degree of decision-making should depend on user expertise. This system was designed assuming its user is familiar with the basic terminology and underlying principles of soil charaterization, groundwater flow, and the basic parameters needed to solve the problem.

The user may ask the system "why" in response to any question. The system will respond with a brief and sometimes general explanation of why certain input is important. In some cases the system indicates how data may be used by the model.

To evaluate a contamination problem, human experts systematically characterize existing soil, site, and pollutant conditions. Modular design allows the expert system to use the same approach. Separate modules perform soil, site, and pollutant characterizations. Each of these three modules contains submodules which check major assumptions, estimate input parameters, access small databases, issue warnings, and offer explanations and advice.

To avoid redundancy, documentation and use of the expert system are described in Section V. A listing of a sample session using the expert system is contained in Appendix IV.

B. OPTIMIZATION MODEL

Before discussing the optimization model, some terms should be defined. "Aquifer" refers to a single-layered saturated geological formation in which the velocity of groundwater movement is not dependent upon vertical position. Above and below this saturated formation the velocity of groundwater movement is negligible compared with the velocity of groundwater in the formation itself. "Pumping" is either extraction or injection of water from/to the aquifer. Extraction and injection are respectively, positive and negative in sign. Only nonpressurized injection is permitted by the management models. Reference 41 considers pressurized injection as occurring if water in the injection wells rises above the ground surface. "Potentiometric surface",in this study, is either the water table in an unconfined aquifer or the hydrostatic pressure level of the water in a confined aquifer. The water level in a well (or piezometer) penetrating a confined aquifer defines the hydrostatic pressure level at that point. A change in potentiometric surface elevation is referred to as "drawdown." Drawdown is considered positive if it produces a reduction in elevation of the potentiometric surface. The configuration of wells used in this study to contain a contaminant plume is either a regular or irregular octagon. A "regular octagon" is an eight-sided figure in which all sides are equal in length. Sides are not equal in length in

an "irregular octagon." All interior anles are 135 degrees in either configuration.

1. Contamination Plume Identification

Using the model requires being able to estimate the size of the contaminant plume at the future time of extraction/injection (E/I) strategy implementation. The initial task is to assess the nature and magnitude of the contaminant plume and its velocity of travel. Knowing that the proposed E/I system should be functioning at a future time t, one can predict the size of the plume at that time.

One can describe the contaminant plume using the standard equation for an ellipse:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

a = point of intersection of ellipse and x - axis, (L);
b = point of intersection of ellipse and y - axis, (L).

Assume the contaminant plume source is at the origin of the X-Y coordinate system and X increases in positive value down-gradient from the pollutant source. To compute the future x,y coordinates of the farthest downgradient extent of the contaminant plume ('a' and 'b' respectively in the ellipse equation), begin with the Darcy velocity,q.

$$q = -Ki \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

where:
K = hydraulic conductivity, (L/T);
i = hydraulic gradient, (L/L).

The seepage velocity is computed by: $v = q/\eta = Ki/\eta$ .....(3)

where:
η = porosity

Therefore the down-gradient limits of the plume are predicted as:

$$a = a' + \frac{K_x i_x t}{\eta} \text{ (s.f.)} \qquad b = b' + \frac{K_y i_y t}{\eta} \text{ (s.f.)} \quad \ldots\ldots\ldots\ldots (4)$$

where:

a' = initial extent of contaminant plume in X direction at time 0;
b' = initial extent of contaminant plume in Y direction at time 0;
K_x, K_y = hydraulic conductivity in X and Y direction, (L/T);
i_x, i_y = hydraulic gradient in X and Y direction, (L/T);
t = time from initial contaminant discharge (t = 0) to activation of

pumping containment system, (T);

s.f. = Appropriate safety factor based on the uncertainty of the geologic and aquifer data, the relative amount of infiltration into the aquifer, and an average dispersivity value.

≈ 1.0 + coefficient of variation for transmissivity + infiltration factor (reference: Section IV)

2. Well System Configuration

The containment well-point system is arranged in an octagonal shape completely encircling the assumed elliptically shaped contaminant plume. An octagonal (regular or irregular) shape is selected because it can be configured to closely encircle an elliptical plume. Its straight sides and 45 degree deflection angles promote easy calculation of well locations and simplifies well installation in the field. The length (SL) of each side of a regular octagon is a function of 'a'.

$$SL = \frac{a}{0.5+\cos(45^\circ)} \quad \text{(5a)}$$

If an irregular octagon is used side lengths are determined individually. All sides except the two parallel to the hydraulic gradient are calculated using Equation (5a) with 'b' distance in place of 'a'. The two sides parallel to the hydraulic gradient (called S2) are calculated as:

$$S2 = 1.2[a-SL\cos(45^\circ)] \quad \text{(5b)}$$

Sides S2 (parallel to the initial direction of the hydraulic gradient) will be longer than the other sides of the octagon. The octagon should be positioned so that it is symmetrical with respect to the x-axis (a line in the direction of the hydraulic gradient and through the contaminant source). Sides of length S2 should have approximately 83 percent of their length down-gradient of the source. These equations are only approximations. If the user has a good idea of the limits of the plume a drawing should be made of the plume and the octagon situated using the drawing.

Spacing of the wells is also determined by the user. The only requirement is that the spacing be an even multiple of the side length.

The first step in computing maximum well spacing is to determine the "effective radius of influence" of available well pumps. This radius is a function of time (it increases as time increases). Using the planned pumping period (TT) it can be calculated using (Reference 3):

$$r_e = \sqrt{\frac{4Tu}{\phi}} \bullet \sqrt{TT} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \quad (6)$$

where:

T = average transmissivity, (L^2/T);

$u = W^{-1}\left\{\frac{4\pi Ts}{q^u}\right\}$, inverse of the Theis well function which is explained later in this section;

s = drawdown; in this case it is one-half the drawdown required at the most down-gradient point on the octagon to achieve the initial potentiometric surface elevation of the source (L);

q^u = upper limit on pumping (L^3/T);

ϕ = effective porosity;

TT = total planned pumping period.

Therefore, the radius of influence is actually the maximum spacing that should be used in the model for the pumping wells. Any larger spacing would require a longer pumping period by the wells to achieve the drawdown (at the lowest potentiometric surface elevation of the octagon) needed to stabilize the plume. The required drawdown is that needed to reach the potentiometric surface elevation at the contaminant source. This assumes the pumping rate is at the maximum value and the pumping wells on each side of an observation well will have an equal effect on that observation well. Because the upper limit on pumping and the total time period are used the actual spacing should be something less than r_e. However, the required drawdown, s, assumes the potentiometric surface elevation of the source does not change during the pumping period. This is only true when using a regular octagon. An irregular octagon, in general, produces positive drawdown at the source making the drawdown assumption a conservative one. One-half the required drawdown is used assuming the pumping wells on each side of the observation point equally influence the drawdown. Spacing can be varied with consecutive model runs to determine the best spacing. Observation wells (where achieved potentiometric surface elevations are monitored) are located midway between pumping wells. From the theory of superposition these midpoint potentiometric surface elevations are least affected by an extraction and injection scheme. Therefore, one attains as nearly level a potentiometric surface as possible within a specified time frame by minimizing the absolute difference between the heads at the observation wells and the head at a selected point within the system (normally the contaminant source) at the end of the pumping period.

The presented model assumes pumping values (q) at all well points are equal in a particular time step. This assumption is made because the normal use of the model is for emergency action where a well point system with a common pump would be used. In addition, due to memory and speed limitations of working with a PC, it allows larger well systems to be analyzed.

3. Model Theory

The management objective is to contain the plume by producing a horizontal hydraulic gradient (i.e. as nearly horizontal as possible) at a specific time for a minimal cost. Ideally, a horizontal potentiometric surface would be attained precisely when it is most convenient for planning and management purposes. Physically, depending on the situation, there may be no conceivable sequence of pumping that can cause complete convergence to a horizontal surface within the desired time (Reference 39). It may be that the best that can be achieved is to minimize the difference between horizontal target elevations and those actually attained by the end of the specified period.

Specifically, model objectives include minimizing operating and maintenance (O&M) costs of pumping and minimizing the difference between potentiometric surface levels achieved at observation wells and the potentiometric surface elevation at the plume source. Simultaneous consideration of both goals makes this a multiobjective optimization. To be able to compare the economic portion of the objective function with the hydraulic portion, a weighting factor is introduced in the hydraulic portion of the objective function. The purpose of this weighting factor is to: (1) provide common units for otherwise noncommensurate objectives, (2) provide a way of emphasizing achievement of one objective at the expense of the other and developing a pareto optimum). The weighting factor is discussed in greater detail in Section IV, "Application, Results, and Discussion." The groundwater management model is theoretically appropriate for a uniform system and practically applicable for a heterogeneous and nonisotropic aquifer with the following assumptions: (1) aquifer is nonleaky and infinite in horizontal extent; (2) pumps produce a radial flow pattern; (3) wells fully penetrate the entire thickness of aquifer; and (4) potentiometric surface gradient prior to pumping is uniform throughout the entire aquifer. Approximations are also made to apply the model to a heterogeneous nonisotropic system.

a. Deterministic Version

The objective function used in the model minimizes, for a predetermined time period, the present value of cost of groundwater extraction/injection plus the squares of final head deviations of the observation wells from the final head at the source:

$$\min: \sum_{t=1}^{TT} \sum_{i=1}^{I} \left[c_t'(h_{i,g} - h_{i,0} + s_{i,t})q_t + c_t''q_t \right]$$

$$+W_f \sum_{j=1}^{J} \left[(h_{o,TT} - h_{j,TT})^2 \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

Based on the following constraints:

$$q^L \le q_t \le q^U \quad \ldots \quad (8)$$

$$h_i^L \le h_{i,t} \le h_i^U \quad \ldots \quad (9)$$

$$h_{o,TT} \le (h_{j,TT})_d \quad \ldots \quad (10)$$

where:

I = total number of pumping wells;

$s_{i,t}$ = drawdown at pump i at time t, (L)

$$= \sum_{k=1}^{t} \sum_{j=1}^{I} \hat{\delta}_{i,j,t-k+1} \, q_k \quad \ldots \quad (11)$$

$h_{i,g}$ = ground elevation at pump i, (L);

$h_{i,0}$ = head at pumping well i at time 0, (L);

$h_{o,TT}$ = head at contaminant source at end of modeling period TT, (L);

$h_{j,TT}$ = head at observation well j at the end of the modeling period TT, (L);

$\hat{\delta}_{i,j,t-k+1}$ = the incremental drawdown at a well i in time period t caused by a unit volume of pumping at well j in time period k. The subscript t-k+1 ensures the correct coefficient is multiplied by the correct pumping value, (T/L^2);

q^L = lower limit on pumping at all wells, (L^3/T);

q_t = pumping at each well at time t, (L^3/T);

q^U = upper limit on pumping at all wells, (L^3/T);

h_i^L = lower limit on head at pump i, (L);

$h_{i,t}$ = head at pump i at time period t, (L);

$= h_{i,0} - s_{i,t}$

h_i^U = upper limit on head at pump i, (L);

$(h_{j,TT})_d$ = head at each observation well j which is down-gradient of contamination source at the end of the modeling period TT, (L);

c'_t = present value of the cost of pumping a unit volume of water a

unit vertical distance in time t, ($/L^4);

c''_t = present value of the maintenance cost of pumping a unit volume in time period t, ($/L^3);

W_f = weight factor to convert the square of hydraulic head differences to dollars. This varies depending on economic factors and physical parameters, ($/L^2).

The last head term in equation (7) is not summed over time because we are concerned solely with the final potentiometric surface. In addition to the upper and lower limits on pumping, equation (8), total injection cannot exceed total extraction during any time step and pumping is the same at each well for a particular time period. This eliminates need for disposal or acquisition of water.

(a) Unit response functions

The first step in developing an optimal strategy is to calculate unit response functions (also known as influence coefficients) in the simulation component of the optimization model using an analytic expression. Unit response functions describe relationships between state variables of an aquifer system such as drawdown and management decision variables such as pumping.

The continuous form of convolution relations between aquifer drawdown and discharge for a linear flow system can be expressed as:

$$s_{i,t} = \sum_{i=1}^{I} \int_{0}^{t} \left[\hat{\delta}_{i,j,TT-t+1}\, q_t\right] dt \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

The time-dependent drawdown response function, $\hat{\delta}_{i,j,t}$ represents incremental drawdown of each observation point i at time t resulting from a unit impulse of pumping at each discharging well j applied at time t = 0. When the time scale is discretized, equation (12) can be expressed in an equivalent form as equation (11):

$$s_{i,t} = \sum_{i=1}^{I} \sum_{k=1}^{t} \hat{\delta}_{i,j,t-k+1}\, q_k \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

In groundwater management practices, the entire planning horizon is generally divided into operational intervals. An operation policy or management decision may vary from one operational interval to another but generally remains the same within each operational interval. As a result, for groundwater management, the discrete form of the convolution relation, equation (11), is more practical than the continuous form.

(b) Theis well function

For the deterministic version and the stochastic version of the optimization model the Theis well function is used to compute influence coefficients. It is based on unsteady flow in a confined aquifer. The Theis equation can also be applied to an unconfined aquifer if the change in aquifer saturated thickness with time is small compared to the saturated thickness itself.

Influence coefficients are a function of transmissivity, effective porosity, time and distance between wells. They are used to calculate heads which in turn affect operating costs and final hydraulic gradient. The influence coefficients ($\hat{\delta}_{i,j,t}$) are calculated using equation (13) (Reference 33). They are positive for extraction wells and negative for injection wells.

$$\hat{\delta}_{i,j,k} = \begin{cases} \psi_{i,j,k}; & \text{for } k = 1 \\ \psi_{i,j,k} - \psi_{i,j,k-1}; & \text{for } k > 1 \end{cases} \quad \ldots\ldots\ldots\ldots (13)$$

where:

$$\psi_{i,j,k} = \left(\frac{1}{4\pi T}\right) W(u_{i,j,k})$$

and

$$u_{i,j,k} = \frac{r^2 \phi}{4Tk} \quad \ldots\ldots\ldots\ldots (14)$$

$u_{i,j,k}$ = Boltzman variable at time k (dimensionless)

$W(u_{i,j,k})$ = Theis well function at time k (dimensionless)

T = transmissivity, (L^2/T);
ϕ = effective porosity (dimensionless)
r = distance from stimulus j to point of observation i,(L)

The well function for the Theis equation can be written:

$$W(u_{i,j,k}) = \int_{u_{i,j,k}}^{\infty} \left[\frac{e^{-v}}{v}\right] dv \quad \ldots\ldots\ldots\ldots (15)$$

The well function is a form of the so-called exponential integral (Reference 44). These integrals cannot be evaluated in terms of elementary functions. Therefore, an alternative expression in the form of a series expansion is used (Reference 44, p.43):

$$W(u) = -.5772157 - \ln(u) - \sum_{n=1}^{\infty} \frac{(-1)^n u^n}{nn!} \quad \ldots\ldots\ldots\ldots\ldots (16)$$

where .5772157 is Euler's constant. The series converges very rapidly for small u. However, for large u, much computer time is consumed before the equation converges. Reference 44 (p.44) developed an expansion specifically for large u to complement equation 16. If the series is to converge more rapidly with increasing u it will have to proceed in inverse powers of u (for example in proportion to u^{-n}). With this in mind, a series expansion for large u is:

$$W(u) = \left[\frac{e^{-u}}{u} \right] \sum_{n=0}^{\infty} \frac{(-1)^n n!}{u^n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (17)$$

Equation (17) is called an asymptotic series. That is, there is an optimal n that gives the best accuracy for any given x. This type of series must be cut off at a finite n (the optimal n). Therefore, the absolute value of each term is compared with the one immediately preceeding it. When terms begin to increase in magnitude the calculation is stopped. In this study it was seen that if u is greater than 5, equation (17) is as accurate as equation (16) when compared to values tabulated by Wenzel (Reference 1). In addition, for u > 50, equation (17) required only one-tenth as many terms as equation 16 to obtain the final value.

(c) Matrix notation for objective function and constraints

In matrix notation the objective function can be described as shown below (derivation of the expression and all coefficients is in Appendix I):

$$\text{min.:} \quad Z = [C_e]\{Q\} + [C_h]\{Q\} + \{g(Q)\} + \{f(Q)\} + Y \quad \ldots\ldots\ldots (18)$$

I = total number of pumping wells;
J = total number of observation wells (I always equals J);
TT = total number of time steps;
$[C_e]$ = the linear economic portion of the objective function. It is an I x TT array containing total present value cost per unit volume of total pump maintenance costs plus energy costs associated with raising water a distance equal to the initial static lift at each pumping well, ($\$/L^3$);
$[C_h]$ = the linear hydraulic portion of the objective function. It is an J x TT array containing the weighted unit contributions (linear) to the final difference in head between the contaminant source and the J observation wells, caused by the initial difference and pumping at each of the I

pumping wells, ($/$L^3$);

{Q} = a TT x 1 column vector of unknown pumping values, (L^3/T), (these values vary in time, but for a time step are equal in absolute value for all wells);

(Q) = a 1 x TT row vector of unknown pumping values, (L^3/T);

{g(Q)} = the quadrtic economic portion of the objective function. It is an I x 1 column vector. It is the product of $[K_e]$ matrix and the {Q} vector. It is quadratic in q since each element equals:

$$= \sum_{t=1}^{TT} c'_t \left[\sum_{k=1}^{t} \sum_{i=1}^{I} (\hat{\delta}_{i,j,t-k+1}\, q_k) \right] q_t$$

$[K_e]$ = an I x TT array containing present value energy costs associated with raising a unit volume of water a distance equal to the dynamic drawdown (+ or -) at I pumping wells, caused by pumping at all wells, ($/$L^6$);

{f(Q)}= the quadratic hydraulic portion of the objective function. It is a J x 1 column vector. The vector contains the weighted squared contributions to the final difference in head between the contaminant source and the j observation wells caused by pumping at each of the I pumping wells in all time steps. Each term is the squared product of a row of the $[K_h]$ matrix and the {Q} vector.

$$= W_f \left\{ \sum_{t=1}^{TT} \left[\sum_{i=1}^{I} (\hat{\delta}_{j,i,TT-t+1} - \hat{\delta}_{0,i,TT-t+1}) \right] q_t \right\}^2$$

$[K_h]$ = a J x TT matrix. Each element is the final difference in head between the contaminant source and the J observation wells caused by unit pumping at each of the I pumping wells in a particular time step, (L^{-2});

Y = a constant made up of initial head terms squared times the weight factor, ($).

$$= W_f \sum_{j=1}^{J} (h_{o,0} - h_{j,0})^2$$

The matrices produced as a result of the matrix multiplication for each term of the objective function are not all the same size but this is unimportant. Summing all elements of the product matrices yields a resultant single value for the objective function.

The objective function is subject to the following constraints (in matrix form):

$$\{Q^L\} \leq \{Q\} \leq \{Q^U\} \quad (19)$$

$$\{H^L\} \leq [\mathbb{B}]\{Q\} \leq \{H^U\} \quad (20)$$

$$h_{o,0}-(\mathbb{B}_o)\{Q\} \leq \{h_{jd,0}\}-[\mathbb{B}_{jd}]\{Q\} \quad (21)$$

where:

$\{Q^L\}$ = a TT x 1 column vector of lower limits on pumping;

$\{Q^U\}$ = a TT x 1 column vector of upper limits on pumping;

$\{Q\}$ = a TT x 1 column vector of unknown pumping values;

$\{H^L\}$ = an I x 1 column vector of lower limits on hydraulic head in pumping wells;

$[\mathbb{B}]$ = an I x TT matrix of the sum of influence coefficients describing the effect on the head at each pumping well i caused by unit pumping at all other pumping wells in each time step t;

$\{H^U\}$= an I x 1 column vector of upper limits on hydraulic head;

$(\mathbb{B}_o)$ = a 1 x TT row vector of the sum of influence coefficients describing the effect on the head at the contamination source by a unit of pumping at all pumping wells in each time step t;

$\{h_{jd,0}\}$ = a jd x 1 column vector of initial heads at each observation well down-gradient of the source;

$[\mathbb{B}_{jd}]$= a jd x TT matrix of the sum of influence coefficients describing the effect on the head at each observation well j (that is down-gradient of the contaminant source) caused by unit pumping at all pumping wells in each time step t. jd is the number of observation wells downstream of the contamination source.

(d) Anisotropic conditions

To accomodate anisotropic conditions within the aquifer a method is used that is similar to the method used in SUTRA, a finite-element simulation model for fluid-density-dependent groundwater flow (Reference 45). The anisotropic permeability field in two dimensions can completely be described by Kmax, Kmin and θ ; where Kmax is the maximum hydraulic conductivity, Kmin is the minimum hydraulic conductivity assumed to be at 90 degrees to Kmax and θ is the counter clockwise angle from the x-axis (which is in the direction of the hydraulic gradient) to Kmax (Figure 1).

Reference 2 shows that if the anisotropic conditions can be described by a maximum hydraulic conductivity and a minimum hydraulic conductivity at 90 degrees to the maximum then the hydraulic conductivity in any direction is described by an ellipse with major axis equal to $\sqrt{Kmax}$ and minor axis equal to $\sqrt{Kmin}$.

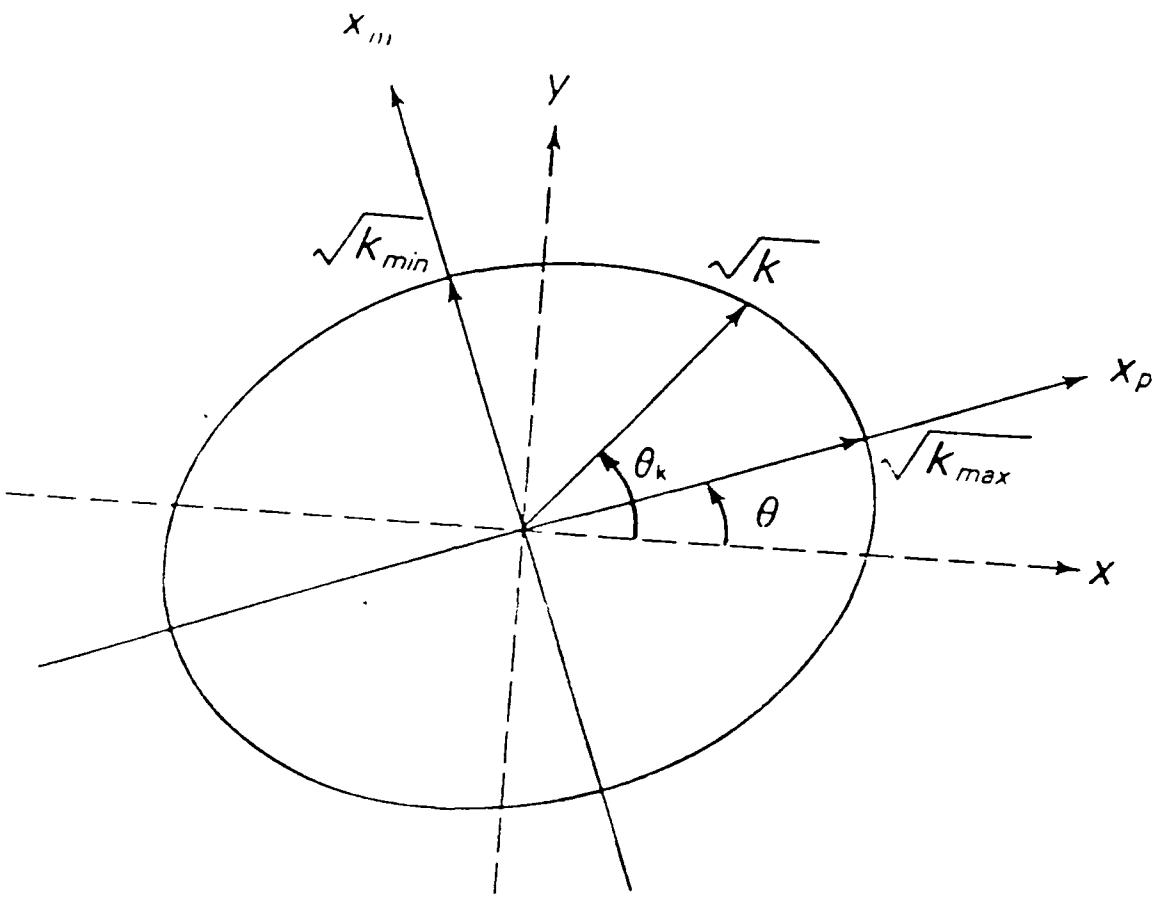


Figure 1. Definition of Anisotropic Hydraulic Conductivity

For simplicity, assume Kmax coincides with the x-axis, θ is the counter-clockwise angle from Kmax (x-axis) to any direction, d. The relationship between velocity, v, and hydraulic conductivity, K, in any direction is given by:

$$v_d = -K_d \bullet \frac{\partial h}{\partial d}$$

and the components of velocity in the x and y directions are:

$$v_x = -Kmax \bullet \frac{\partial h}{\partial x} = v_d \cos\theta \qquad v_y = -Kmin \bullet \frac{\partial h}{\partial y} = v_d \sin\theta$$

Now, since h = h(x,y),

$$\frac{\partial h}{\partial d} = \frac{\partial h}{\partial x} \bullet \frac{\partial x}{\partial d} + \frac{\partial h}{\partial y} \bullet \frac{\partial y}{\partial d}$$

Geometrically, $\partial x/\partial d = \cos\theta$ and $\partial y/\partial d = \sin\theta$. Substituting these relationships and the first three equations (solved for the partial derivatives) into the equation for $\partial h/\partial d$ and simplifying gives:

$$\frac{1}{K_d} = \frac{\cos^2\theta}{Kmax} + \frac{\sin^2\theta}{Kmin}$$

Solving this equation for K_d (now K_{θ_k} from Figure 2) and assuming that Kmax can be at any angle from the x-axis (Figure 2) gives equation (22):

$$K_{\theta_k} = \frac{Kmin * Kmax}{[Kmin * \cos^2(\theta_k - \theta)] + [Kmax * \sin^2(\theta_k - \theta)]} \quad \ldots\ldots\ldots (22)$$

where:

K_{θ_k} = the hydraulic conductivity in the direction θ_k degrees counter-clockwise from the x-axis;

θ_k = the angle formed by the x-axis counter-clockwise to the line connecting the pumping well and another well on the octagon;

θ = the angle formed by the x-axis counter-clockwise to the direction of Kmax.

Knowing the rectangular coordinates of each pumping well and observation well as related to the x-y axis system of the plume ellipse we can calculate the hydraulic conductivity.

Saturated thicknesses of the aquifer are given as individual values for each pumping well and observation well. The saturated thicknesses at the pumping well and corresponding observation well are averaged and multiplied by K_{θ_k} to obtain an average transmissivity. The average transmissivity value is used in the calculation of the unit response functions and steady-state pumping values.

(e) Optimization component

GAMS/MINOS (Reference 44) is the code used to solve the optimization problem. It determines the optimal pumping (extraction and

injection) values to contain the contaminant plume for a minimum value of objective function. GAMS(General Algebraic Modeling System) is a preprocessor that converts input data into standard MPS format for the optimization program MINOS (Modular In/Core Nonlinear Optimization System)(Reference 47).

b. Stochastic Version

The Theis well function is again the basic groundwater flow equation used by the simulation component. The deterministic version of the groundwater contaminant plume management model is used as the starting point for development of the stochastic management model.

Once again the goal is to determine the optimal pumping rates for a specified planning horizon such that undesirable consequences do not occur. The stochastic approach allows the incorporation of uncertainty of aquifer parameters within the model. The model can use a probability distribution for each aquifer parameter. The model then will generate optimal pumping values that will produce no undesirable results for a specified reliability (confidence limit).

(1) Stochastic unit response function

The deterministic unit response function, $\hat{\delta}$, can be obtained from a distributed parameter groundwater simulation model. However, when hydrogeologic information of an aquifer system is lacking or unavailable, a closed form analytical solution to an idealized condition can be utilized to derive a stochastic unit response function.

Since the unit response function characterizes an aquifer pumping-drawdown relationship, a groundwater management model can be very easily formulated once the response functions are defined. The deterministic management model detailed previously in section III does not consider the random nature of aquifer parameters. The stochastic model presented below has the same objectives, but incorporates probability in all equations that use response functions. Probability is considered via information on the probability density function (pdf) of transmissivity (T) and effective porosity (ϕ).

Values for transmissivity and effective porosity are normally derived from a pump well test. Such a test provides in situ values of aquifer parameters averaged over a large and representative aquifer volume. Therefore, T and ϕ should be treated as random variables. Because the response function $\hat{\delta}$ is computed using the random variables T and ϕ, it too is random in nature.

The deterministic objective function equation (equation 7), drawdown constraint equation 9 and the observation well potentiometric head constraint equation 10 are all functions of the probabilistic response function. Therefore, it is more appropriate and realistic to examine both objective function and constraints probabilistically; particularly when aquifer information is scarce.

In a stochastic environment, one wishes to specify limitations on allowable risk or required reliability of constraint performance. The necessary reliability for attaining the objective and satisfying the constraints can be represented by a confidence limit. This reliability states the model's confidence in the resulting potentiometric surface. The reliability can be determined based on the confidence of the model user in his estimates of aquifer parameters.

The following development is based on the procedure proposed by Tung (Reference 42) for the drawdown constraint. The restriction that drawdown at any point j at the end of the period t resulting from pumping over the entire well field cannot exceed (or has to exceed) a specified value is the basis for the analysis. In this case the specified value is that which is calculated by the model. The drawdown is based on a specified reliability, ρ.

For the drawdown constraint at pumping wells, there is a ρ confidence that the actual drawdown at a pumping well will not exceed the $s_{j,t}$ drawdown value calculated by the stochastic model. Representing the actual drawdown using equation (11) yields equation (23a) below. Rigorous testing of the validity of this constraint would be accomplished by (1) using a random number generator to create a large set of possible combinations of transmissivities and porosities, (2) creating one set of $\hat{\delta}$ for each combination developed in the previous step, (3) using equation (11) to compute the drawdowns that would result from using the optimal pumping strategy developed by the stochastic model. If the sampling is large enough, ρ percent of the drawdowns computed in this step should be less than the $s_{j,t}$ computed by the stochastic model.

$$\Pr\left\{\sum_{i=1}^{I}\sum_{k=1}^{t}\hat{\delta}_{i,j,k}q_{t-k+1} \le s_{j,t}\right\} \ge \rho \text{ ; for all j and t . . . . .(23a)}$$

The calculated value, $s_{j,t}$, is limited by the drawdown constraint, equation (9); all such calculated drawdowns at pumping wells will be less than that specified by the drawdown constraint except for the drawdowns at the tightly constrained pumping wells. At such wells the stochastic drawdown will equal the constraint value. At the tightly constrained wells there is a ρ probability that an actual drawdown is less than the stochastically created drawdown. At all other pumping wells the probability will be greater than ρ.

Heads at observation wells affect the objective function and constraint equation (10). There must be ρ confidence that the actual drawdown at an observation well will be greater than the value, $s_{j,t}$, calculated by the stochastic model. However, to express this in the same

form as equation (23a), it is stated that there is a $1-\rho$ confidence that the actual drawdown will be less than that calculated by the stochastic model. This is expressed as:

$$\Pr\left\{\sum_{i=1}^{I}\sum_{k=1}^{t}\hat{\delta}_{i,j,k}q_{t-k+1} \le s_{j,t}\right\} \le 1-\rho \text{ ; for all j and t} \quad . . . (23b)$$

In equations (23a) and (23b) the sequence of summation and notation for the increments t and t-k+1 has been reversed from that in equation (11). This provides a more clear derivation of the stochastic coefficients. This reversal has no effect on the final results.

A probabilistic statement of the drawdown constraint (or any statement where drawdown is used, such as the objective function) like equation (23), is not mathematically operational, so further modification is needed. To make equation (23) operational, it is necessary to assess statistical properties to random terms in this chance-constrained expression.

There have been a number of field investigations and laboratory experiments assessing the probability distribution of aquifer transmissivity and hydraulic conductivity. Most findings indicate that the hydraulic conductivity has a log normal distribution. Because the response function, $\hat{\delta}$, computed by the Theis equation, is a nonlinear function of transmissivity and effective porosity, the probability function of $\hat{\delta}$ as well as drawdown at any observation point cannot easily be determined. Therefore, a first-order analysis is used to estimate the statistical properties of the unit response function and drawdown at each observation point.

First-order analysis is useful in estimating statistical characteristics such as the mean and variance of a function involving random variables. In first-order analysis, the function containing random variables is expanded in Taylor series about the mean values of random variables, i.e.

$$f(x) = f(u) + f'(u)[x-x(u)] + \frac{f''(u)}{2!}[x-x(u)]^2 \; . . .$$

$$. . + \frac{f^n(u)}{n!}[x-x(u)]^n \quad . . . . . (24)$$

in which f(x) is a function involving a random variable x, f(u) is the mean value of f(x) and x(u) is the value of the random variable at the mean, f(u).

Derivations of statistical properties of drawdown at each observation point, assuming independence of transmissivity and effective porosity, are given in Appendix II. Results are as follow:

$$E(s_{j,t}) = \sum_{i=1}^{I} \sum_{k=1}^{t} \bar{B}_{i,j,k}\, q_{t-k+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

where $\bar{B}$ is the same as $\hat{\delta}$ in the deterministic model;

$$var(s_{j,t}) = \left[\sum_{i=1}^{I} \sum_{k=1}^{t} \bar{A}_{i,j,k}\, q_{t-k+1}\, sdt \right]^2$$

$$+ \left[\sum_{i=1}^{I} \sum_{k=1}^{t} \bar{P}_{i,j,k}\, q_{t-k+1}\, sds \right]^2 \quad \ldots\ldots\ldots\ldots \quad (26)$$

in which $E(s_{j,t})$ and $var(s_{j,t})$ are the mean and variance respectively of drawdown at observation point j at the end of the t period; sdt and sds are the standard deviations of the transmissivity and effective porosity, respectfully and $\bar{B}$, $\bar{A}$ and $\bar{P}$ are coefficients that are functions of the mean transmissivity and effective porosity. As can be seen in equation (25), the mean drawdown is a linear function of pumping and represents the deterministic solution (50 percent reliability) but the variance (equation 26) is a quadratic function of pumping. Derivation of equations (25) and (26) enables the development of a deterministic equivalent for equations (23a) and (23b). As shown in the next section, the equivalent is mathematically operational and permits explicit incorporation of random characteristics of the aquifer properties in the management model.

The total drawdown at any control point is the sum of the drawdown created by many individual pumps. Since drawdown is a random variable the central limit theorem applies. That theorem states that, if n is large, a set of random variables has approximately a standard normal distribution. Therefore, the total drawdown at each observation point can be assumed to have a normal distribution with a mean and variance given by equations (25) and (26), respectively. Under the normality assumption the original chance constrained equations (23a) and (23b) can be expressed as:

$$Pr\left\{ Z \le \frac{s_{j,n} - E(s_{j,n})}{\sqrt{var(s_{j,n})}} \right\} \ge \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (27a)$$

for the drawdown constraint equation (9) and

$$\Pr\left\{Z \le \frac{s_{j,n} - E(s_{j,n})}{\sqrt{\mathrm{var}(s_{j,n})}}\right\} \le 1-\rho \quad \ldots\ldots\ldots\ldots\ldots\ldots (27b)$$

for the objective function and constraint equation (10). Z is a standard normal random variate with mean zero and unit variance. By substituting equation (25) into (27a) and (27b), and since $F^{-1}[\rho] = -F^{-1}[1-\rho]$, an equivalent expression can be written as:

$$\sum_{i=1}^{I} \sum_{k=1}^{t} \bar{B}_{i,j,k}\, q_{t-k+1} \pm \sqrt{\mathrm{var}(s_{j,t})}\; F^{-1}[\rho] \le s_{j,t} \;;$$

$$\text{for all } j \text{ and } t \quad \ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which $F^{-1}[\rho]$= a standard normal deviate corresponding to the normal cumulative distribution function of ρ. The plus sign on the left side of the equation produces the equation stating that there is a ρ probability that the actual drawdowns at pumping wells are less than the calculated value, $s_{j,t}$. The minus sign produces the equation stating that there is a $1-\rho$ probability that the actual drawdowns at observation wells are less than the calculated value.

Note that the second term in equation (28) involves a square root of the variance of drawdown at each observation point which, in turn, is a quadratic function of unknown decision variables q. The deterministic equivalent of a chance-constrained equation is nonlinear. Standard linear programming codes cannot solve problems with nonlinear constraint equations. However, as suggested by Tung (Reference 42), quasi-linearization can be employed to linearize the nonlinear term in equation (28).

This linearization is actually a trial and error method using an "estimate" of the optimal pumping to determine the stochastic coefficients. The iterative process is shown in a flow chart as Figure 2. In the process of linearization, the nonlinear term in equation (28) is expanded as a Taylor series, equation (23), about this estimate of optimal pumping, Qo_{t-k+1}.

$$f(q) = \sqrt{\mathrm{var}(s_{j,t})} \approx f(Qo) + \sum_{i=1}^{I} \sum_{k=1}^{t} \left\{ \left[\frac{\partial f(q)}{\partial q_{t-k+1}}\right]_{Qo_{t-k+1}} \right.$$

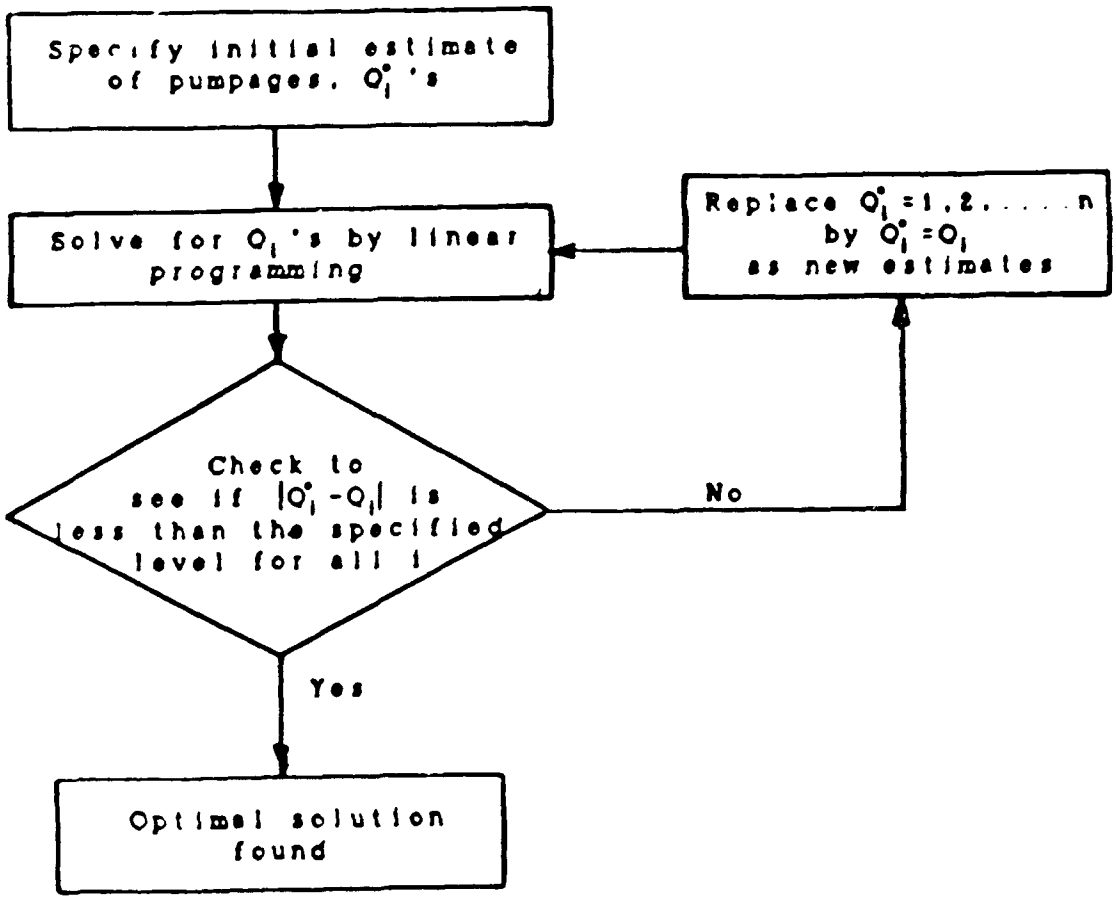


Figure [illegible] Flow Chart for Solving Linearized Charge Constrained Groundwater Management Model

$$[q_{t-k+1} - Qo_{t-k+1}]\Big\} + HOT \quad \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

in which HOT are the higher-order terms. After neglecting the higher order terms and some algebraic manipulations, the first-order linear approximation of the nonlinear terms (derived in Appendix III) can be expressed as:

$$f(q) = \sqrt{var(s_{j,t})} \approx \sum_{i=1}^{I} \sum_{k=1}^{t} \bar{D}_{i,j,k}\, q_{t-k+1} \quad \ldots\ldots\ldots\ldots (30)$$

where:

$$\bar{D}_{i,j,k} = \frac{1}{f(Qo)} [ft(Qo)\bar{A}_{i,j,k}sdt + fs(Qo)\bar{P}_{i,j,k}sds] \quad \ldots\ldots\ldots (31)$$

$$f(Qo) = \sqrt{ft(Qo)^2 + fs(Qo)^2}$$

$$ft(Qo) = \sum_{i=1}^{I} \sum_{k=1}^{t} \left[\bar{A}_{i,j,k} Qo_{t-k+1} \right] sdt$$

$$fs(Qo) = \sum_{i=1}^{I} \sum_{k=1}^{t} \left[\bar{P}_{i,j,k} Qo_{t-k+1} \right] sds$$

sdt is the standard deviation of transmissivity
sds is the standard deviation of effective porosity
$\bar{A}$ and $\bar{P}$ are defined by equations (48) and (50), respectively.

Finally, substituting equation (30) into equations (28a) and (28b) results in a linear approximation for the stochastic equivalent to the original deterministic constraint on drawdown:

$$\sum_{i=1}^{I} \sum_{k=1}^{t} E_{i,j,k}\, q_{t-k+1} \leq s_{j,t} \quad \ldots\ldots\ldots\ldots\ldots\ldots (32)$$

where:

$E_{i,j,k} = \bar{B}_{i,j,k} + F^{-1}[\rho]\bar{D}_{i,j,k}$ for drawdown constraint equation 9 and

$E_{i,j,k} = \bar{B}_{i,j,k} - F^{-1}[\rho]\bar{D}_{i,j,k}$ for the objective function and constraint equation (10).

Checking the signs for the $\bar{B}$ and $\bar{D}$ coefficients reveals that the stochastic unit influence coefficient, E, responds the same whether showing the influence of an injection or extraction well. At injection wells both $\bar{B}$ and $\bar{D}$ are negative values. Therefore, E is larger in absolute magnitude than the deterministic unit influence coefficient for the drawdown constraint. E is smaller than the deterministic coefficient for the objective function and constraint equation (10). At extraction wells both $\bar{B}$ and $\bar{D}$ are positive; producing a larger absolute value for E in the drawdown constraint and a smaller value for the objective function and constraint equation (10).

To convert the original deterministic model into a stochastic model replace the drawdown constraint equation (9) with equation (32) and use $E_{i,j,k}$ for $\hat{\delta}_{i,j,k}$ in the objective function. Clearly, $E_{i,j,k}$ can be considered as a stochastic unit response function derived from the Theis equation. And it should be noted that the deterministic model actually represents a reliability of .50 (when $F^{-1}[.50] = 0$).

(2) Reliabilty determination

There are drawdown terms (for observation wells) in the objective function and constraint equation (10) as well as in drawdown constraint equation (9) (for pumping wells). Reliability is treated differently in the two cases. Refer to Figure 3 during the following discussion.

Let's assume a reliability level of 0.95. In a drawdown constraint one wishes to be 95 percent sure that the change in water level does not exceed the prespecified maximum change (i.e. does not violate predetermined bounds on head). One uses the standard normal deviate ($F^{-1}[\rho]$) corresponding to a reliability of 0.95 for the drawdown constraint (i.e. $F^{-1}[.95] = 1.64$). The procedure described previously computes a stochastic unit response coefficient for the 95 percent confidence level. The coefficient is larger than a deterministic coefficient (which corresponds to a 50 percent confidence level). Since a unit pumping causes a greater change in head using the 95 percent probability influence coefficient less pumping is feasible before drawdown constraints become tight.

When considering the objective of raising water levels to prevent contaminant movement one wishes to be 95 percent confident that head changes equal or exceed calculated values. Therefore, with the objective function and constraint equation (10) one uses the standard

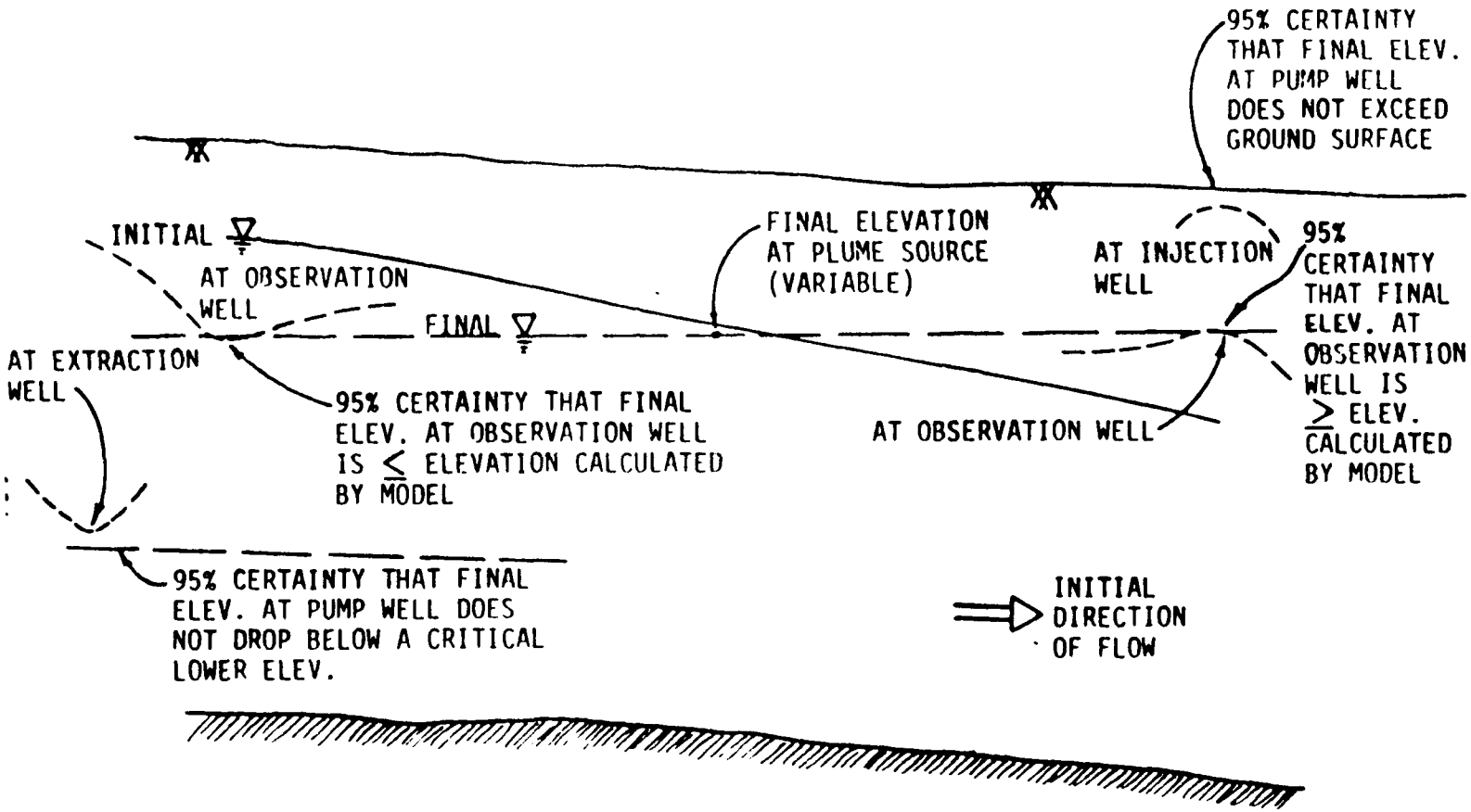


FIGURE 3. CROSS-SECTION DEMONSTRATING SAMPLE STOCHASTIC CONSTRAINTS ON FINAL WATER LEVELS

deviate corresponding to a reliability of .05. This produces stochastic influence coefficients that are numerically smaller than 95 percent of all deterministic influence coefficients. For identical pumping values the 95 percent probability change in water levels needed to achieve a horizontal gradient is much greater than that needed using deterministic coefficients. This guarantees that pumping values calculated by the model are equal to or greater than those required by the deterministic model to produce a horizontal gradient.

However, this guarantee also allows constraint equation (10) (which specifies that final heads at down-gradient observation wells are greater than final head at source) to cause the objective function value to be larger than an objective function value resulting from only trying to minimize the head differences between the contaminant source and all observation wells. Greater pumping values may actually cause the heads at the down-gradient observation wells to 'overshoot' the head at the source and produce a reverse gradient. This is demonstrated in Chapter 4 where the objective function and reverse gradient increase as aquifer parameter uncertainty increases. The "tight" down-gradient observation well is the one whose final head is equal to the final head at the source. All other down-gradient observation well heads are higher than the source head, therefore, producing a larger objective function value.

(3) Determination of aquifer parameters

Estimation of transmissivity and effective porosity has received much attention in the literature in recent years and was discussed in Section II, "Review of Literature". Equations (25) and (26) show that the mean and variance of transmissivity and effective porosity are needed in the stochastic version of the optimization model. Many methods for determining these statistics are described in the literature. Here a Bayesian approach is used to derive the mean and variance for transmissivity and effective porosity.

The Bayesian approach uses a prior (also called unconditional) probability distribution function (pdf) and a likelihood pdf to determine the mean and variance for the aquifer parameters. The mean and variance describe the posterior or conditional pdf used within the stochastic model. The prior pdf is based on knowledge of the aquifer obtained from past experience. This study suggests using aquifer material (soil type) as the basis for the prior pdf. The likelihood pdf is developed from current information (field or lab data) about the aquifer in question.

In the stochastic analysis portion of this study the standard deviation of transmissivity and effective porosity is varied to determine how these changes affect the objective function value. However, in a real situation, one would estimate a mean and variance for these aquifer parameters from a prior pdf and a "likelihood" pdf. The user would select a description of the soil type from a given list. Based on a range of values of transmissivity and effective porosity associated with each soil type (derived from numerous references), a

prior pdf mean ($\bar{X}o$) and variance (Vo) are determined. This determination is made by assuming that the range of values spans three standard deviations each side of the mean (99 percent confidence interval). With this assumption and assuming a log-normal pdf for transmissivity and a normal pdf for effective porosity one can compute the mean and standard deviation. If there are no field data values for the problem the prior pdf becomes the posterior pdf.

If one has field data values, the mean (X) and variance (V) are determined using standard equations for mean and variance of a data population. This mean and variance for the field data values define the likelihood pdf. The mean and variance for transmissivity are calculated using the natural log of all transmissivity values because these values are known to be normally distributed. The posterior pdf is related to the prior pdf and likelihood pdf as shown:

posterior distribution prior distribution x likelihood distribution

The mathematics of multiplying a normally distributed likelihood pdf by a normally distributed prior pdf have been previously derived (Lindley, 1970). Assuming the natural log data values for transmissivity and the data values for effective porosity are normally distributed, the posterior mean, E( ), and posterior variance, var( ) for either parameter are calculated from:

$$E(\ ) = \frac{1}{Vo^{-2} + V^{-2}} (Vo^{-2}\bar{X}o + V^{-2}\bar{X}] \quad \ldots\ldots (33a)$$

$$var(\ ) = (Vo^{-2} + V^{-2})^{-1} \quad \ldots\ldots (33b)$$

The expected value, E, and the variance, var, for effective porosity are used as the posterior mean and variance. However, because natural log values are used to determine the expected value and variance for transmissivity, these values must be converted back to represent the mean and variance of the actual transmissivity values. Standard equations for the mean and variance of a population which has a log normal pdf and the expected value and variance of its natural log values are known are used (Johnson and Kotz, 1970). These are:

$$mean = \exp\left\{E + \frac{(var)}{2}\right\} \quad \ldots\ldots (34a)$$

$$variance = \{\exp[(var) + 2E]\}\{\exp[(var)] - 1\} \quad \ldots\ldots (34b)$$

These two equations are used assuming the entire population of values is available. Since the prior pdf uses the knowledge of a large amount of data for each soil type this assumption is sound.

c. Iterative Procedure and Global Optimality

An iterative procedure is required to insure the convergence of the approximated solution to the true optimal solution in the stochastic model. In addition, the global optimum to the problem cannot generally be guaranteed in either model because of the nonlinear nature of the problem. Therefore, a few runs with new starting positions are suggested to increase the likelihood that the overall optimum is obtained.

4. Final Potentiometric Surface and Steady-State Pumping Determination

Both versions of the optimzation model (deterministic and stochastic) determine the optimal pumping strategy to stabilize, within a specified time frame, a contaminant plume. The potentiometric surface at the hypothetical observation wells resulting from this pumping strategy and the steady state pumping needed to maintain stability of the plume (by maintaining the achieved heads) for a finite period of time are then determined.

These values are computed by a post processor. Heads at the observation wells are calculated by subtracting optimal drawdown (eq. 11) from the original potentiometric surface elevations. To compute heads the post-processor uses deterministic or stochastic unit response functions($\hat{\delta}$ or E) as appropriate.

The steady-state pumping values are those that will maintain the potentiometric surface existing at the end of time step TT at each well. If dispersion effects are insignificant, this will result in a perpetually stable contaminant plume. In this computation it is assumed that only the two nearest pumping wells affect the potentiometric surface at an observation well. It is also assumed that the pumping wells on each side of an observation well have equal affect on the potentiometric surface at that observation well. This is reasonable since the pumping wells are equidistant from the observation wells. The result of these assumptions is a pumping strategy that may be greater than absolutely necessary. Knowing the drawdown at the pump well and the drawdown that is to be maintained at a specific distance from the pump well, steady-state pumping may be computed using the Thiem equation (Reference 3):

$$Q_p = 2\pi T_{avg}\left[\frac{s_p - s_o}{\ln(r_o/r_p)}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (35)$$

where:

Q_p = the steady-state pumping at well p needed to maintain existing potentiometric surface at observation well o, (L^3/T).

T_{avg} = average transmissivity between pump well p and observation

well o, (L^2/T).

s_p = the drawdown that is to be maintained at pump well p, (L).

s_o = one-half the average drawdown that is to be maintained at the observation wells on each side of the pumping well, (L).

r_o = the distance between the pump well and the observation well, (L).

r_p = the radius of the pump well, (L).

SECTION IV

APPLICATION, RESULTS AND DISCUSSION

The optimization model was tested in three ways. First, the deterministic version was used to develop optimal strategies for a hypothetical groundwater contamination problem. The physical properties of the aquifer, the time frame and the well configuration were varied. The contaminant transport that would result from the optimal pumping schemes was then computed using a two-dimensional (2-D) solute transport model. A strategy is considered successful if no contaminant reaches the observation or pumping wells that surround the plume. Second, the stochastic optimization model was applied to the same hypothetical groundwater contamination problem. Aquifer parameters' (transmissivity and effective porosity) coefficient of variation (ratio of standard deviation to mean) and required solution reliability were varied in consecutive runs. Again, the results of strategy implementation were computed using the solute transport model. Finally, an actual contamination problem at Otis Air Force Base, Massachusetts was simulated using the deterministic model. This testing provided a systematic analysis of the effect of varying aquifer parameters, time frame and physical assumptions on the optimization model and resulting pumping strategies.

The simulation component and optimization component were run on an IBM AT with 640K bytes of RAM, a 30 MEG internal hard disk with at least one floppy disk drive, and math coprocessor. This is the minimum system needed. The 2-D solute transport model (Reference 4) used to demonstrate the results of implementing the computed pumping strategies was run on an IBM 4381 mainframe computer using CMS (conversational monitor system). The mainframe was used because it is faster than the microcomputer. This allowed the AT to be used solely for optimization runs. However, the 2-D solute-transport model can be run on an AT.

Theoretically, the Theis equation (which is the basis for the deterministic and stochastic unit influence coefficients) is applicable only for confined aquifers. The rule of thumb has been that the Theis equation is also applicable for unconfined aquifers if the change in saturated thickness during pumping does not exceed 10 percent of the original thickness. The model allows drawdowns of 50 percent of the saturated thickness which presumably would make the Theis equation not applicable. This is a limitation of this model. The 2-D solute transport model uses the same transmissivities for all time periods. Therefore, it cannot accurately predict plume movements for large drawdowns. However, as is done in some subsequent examples, transmissivities resulting from the final heads at the end of the pumping period can be used as "the worst case" in the solute transport model to estimate the greatest transport that may result. When the worst-case transmissivities are used in the MOC model to test model-pumping strategies it is specifically mentioned. Otherwise, initial transmissivities are used. It appears that the safety factor used in plume movement calculation inherently provides some safety factor to overcome this limitation.

All model runs are designated with a number and either a "d" (for deterministic model run) or an "s" (for a stochastic model run).

A. DETERMINISTIC MODEL APPLICATION TO HYPOTHETICAL SITUATION

In the hypothetical situation a spill of toxic liquid occurred at a sandy soil location. The water table was located 5.8 meters (19 feet) below the ground surface. The aquifer saturated thickness is 15 meters (50 feet). Prior to well installation it was predicted that the spill would contaminate a surface area of 247,000 m^2 (910,000 ft^2). Prompt prevention of contaminant movement was important because of a domestic well located 24 meters (78 feet) from the downgradient edge of the plume. Use of equation (4) indicated the plume could reach the well within 8 days. A safety factor of 2 was used in the calculation of plume movement to account for dispersion and nonhomogeniety. The emergency response decision was to attempt to stablize the plume by the end of day 8.

Utilized physical parameters for model run 1d include a transmissivity of 1255 m^2/d (13,500 ft^2/d), and an effective porosity of 0.3. The original hydraulic gradient was 0.54 percent. Maximum and minimum acceptable pumping rates, based on available equipment, are 135 L/s and 0 L/s. This was based on the performance curve for a pump that can discharge 150 L/s against 6 meters of head at 80 percent efficiency. The upper limit on head at all injection wells was the ground surface (5.8 m above the initial water table). This should prevent pressurized injection (Reference 41). The lower limit on head at extraction wells corresponded to the elevation that would leave at least one half the saturated thickness of the aquifer (7.5 meters). A common rule-of-thumb is to leave at least one-third of the original saturated thickness. This is based on the fact that normally a well is screened for only the lower one-third of the aquifer. Leaving one-third of the original saturated thickness is also a common criteria based on energy-needed versus discharge-obtained relationships. In attempting to minimize violating the assumption of horizontal flow in the aquifer, one-half the initial saturated thickness was chosen as a lower limit on acceptable final saturated thickness. This value, however, depends on the situation.

1. Analysis of the Weight Factor

From the size of plume predicted using equation (4), a regular octagon with sides 274 meters long was selected. Unless the plume is extremely elongated in shape, a regular octagon produces the most economical pumping values and best hydraulic gradient (closest to horizontal). This is discussed in more detail later in this section. Figure 4a shows the initial plume concentrations and octagon location for run 1d (note that the octagon is centered on the plume origin). Figure 4b shows the resulting plume after 38 days if no pumping strategy is implemented. Economic coefficients (assumed constant in time because of a short pumping period) were: c'= $0.44/ha-m/m

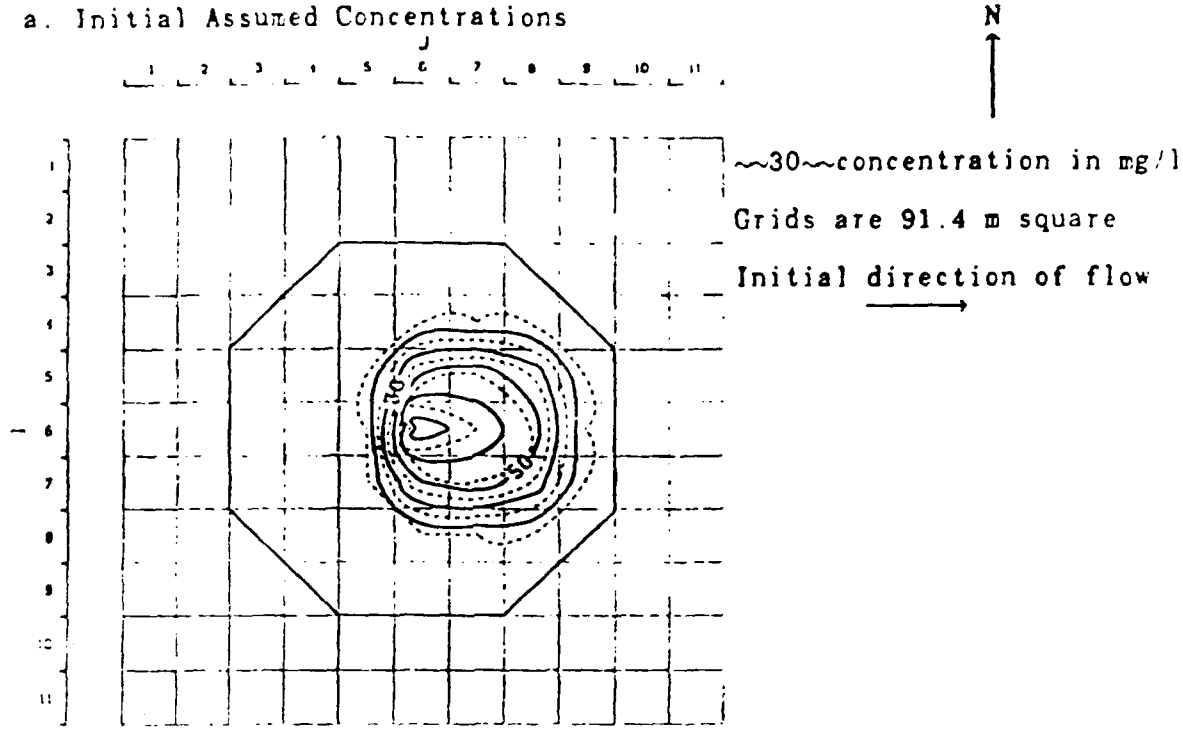
($0.18/ac-ft/ft and $4.13 x 10^{-6}ft^3/ft) and c"= $1.24/ha-m ($1.65/ac-ft and $3.78 x 10^{-5}/ft^3). The initially assumed well spacing was one half the side length (137 meters, corresponding to two pumping wells per side located at the one-fourth and three-fourth points). Varying the weight factor (W_f) in consecutive optimizations for model run 1d yielded the results of Table 1. The resulting observation well heads and final gradients (they are the same for all weight factors) for the 8 day optimal pumping strategy are shown in Figure 5. The resulting heads and gradients for the 30 days of steady pumping are shown in Figure 6. The average terminal gradient between contamination center and observation wells achieved for all these trials was 0.07 - 0.08 percent. The standard deviation (SD) of the final gradients for each run is shown to provide a measure of the "spread" of the final gradients. The constraint requiring the final heads at the observation wells initially down-gradient of the source to be equal to or greater than the final head at the source produces a gradient in the reverse direction of the original gradient. All final gradients referred to in the text and tables are in the reverse direction of the original gradient.

A tight constraint is one which, during the course of the optimization iterations of the model, reaches one of its bounds. For all the runs the upper limits on head at some injection wells were the tight constraints at optimality. These upper limits were tight for all weight factors at the same two wells: well 3 at days 1 and 2 and well 2 at days 3,4 and 5. The upper limit on head was also reached at wells 12 and 13 (these are symmetrical to wells 2 and 3). However, the optimization program did not declare these to be tight constraints (i.e. they were given no sensitivity values). The optimization program identifies tight constraints for the optimal pumping values by specifying a nonzero value for each tight constraint. A sensitivity value indicates the approximate improvement in the objective function that results from a unit relaxation of that particular constraint. For example, a sensitivity value of 11.3 for the tight constraint on head at injection well 2 indicates that the objective function would improve by 11.3 units if the upper bound on head at pumping wells was increased by 1 unit.

Optimization using weight factors of 0.1 and 0.01 result in final gradients that are almost 3 times the final gradients for those runs with weight factors of 1 or greater. Such gradients are unacceptable because the contaminant plume would extend outside the octagon by the end of the planning period.

It became obvious from the values for the four matrix components of the objective function produced by the optimization program (eq. 18 and the S1 through S4 values in the output file, Appendix V) that for weight factors of 1.0 or larger the two hydraulic head components (S1 and S3) have a much larger effect on the objective function than do the two economic components (S2 and S4). This assures plume stabilization. Weight factors of less than 1.0 produce better economic solutions but

a. Initial Assumed Concentrations



b. Resulting Concentrations After 38 Days of No Pumping (run 1d)

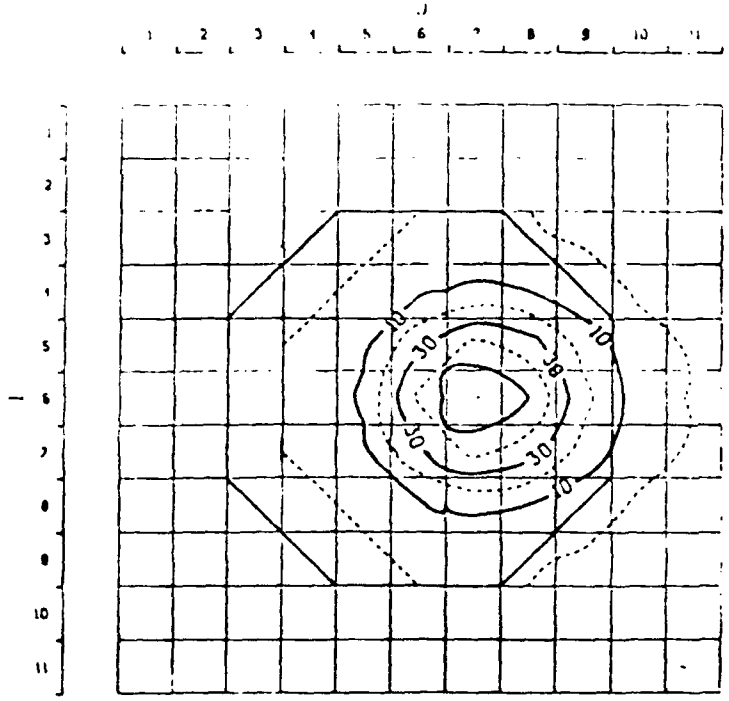


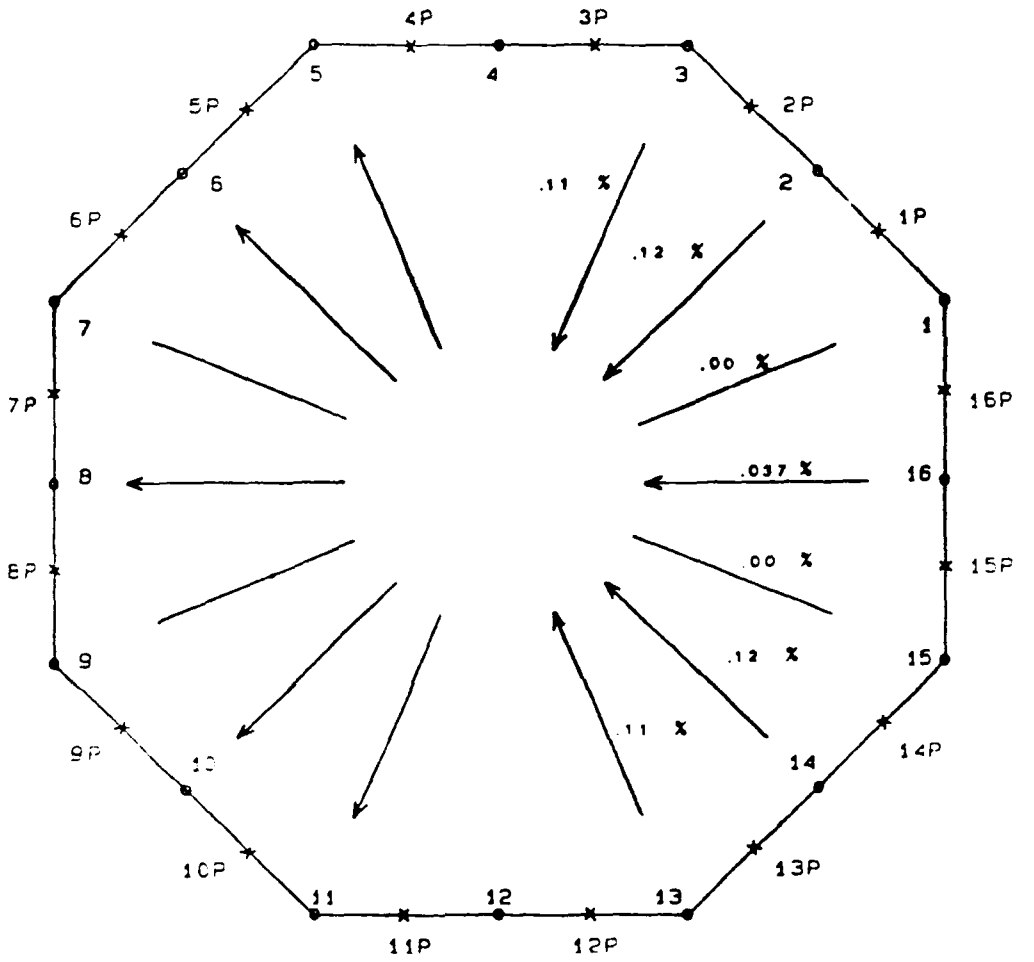
Figure 4. Contaminant Plume in the Hypothetical Area

TABLE 1. EFFECT OF WEIGHT FACTOR ON OPTIMAL UNSTEADY PUMPING STRATEGIES FOR HYPOTHETICAL CONTAMINATION PROBLEM (model run 1d)

Weight factor	1.0	10.0	100.0	1000.0
		Pumping(L/s)		
Day 1	96.1(3.35)*	96.1	96.1	96.1
2	90.1(3.15)	90.1	90.1	90.1
3	84.9(2.95)	84.9	84.9	84.9
4	80.2(2.80)	80.2	80.2	80.2
5	76.9(2.70)	76.9	77.3	77.6
6	36.9(1.28)	37.1	37.5	37.9
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
Avg gradient(%)	0.08	0.07	0.07	0.07
S.Dev of Gradient	0.058	0.056	0.056	0.056
Sum of head differences squared (m^2)	1.24(13.3)	1.24	1.22	1.19
Obj. function	15.6	135.3	1332.	12766.
O & M costs ($\$ \times 10^3$)	2.3	2.3	2.3	2.3

* values in parentheses are corresponding English units in ft^3/s or ft^2.

approximate scale: 1 in. = 450 ft.

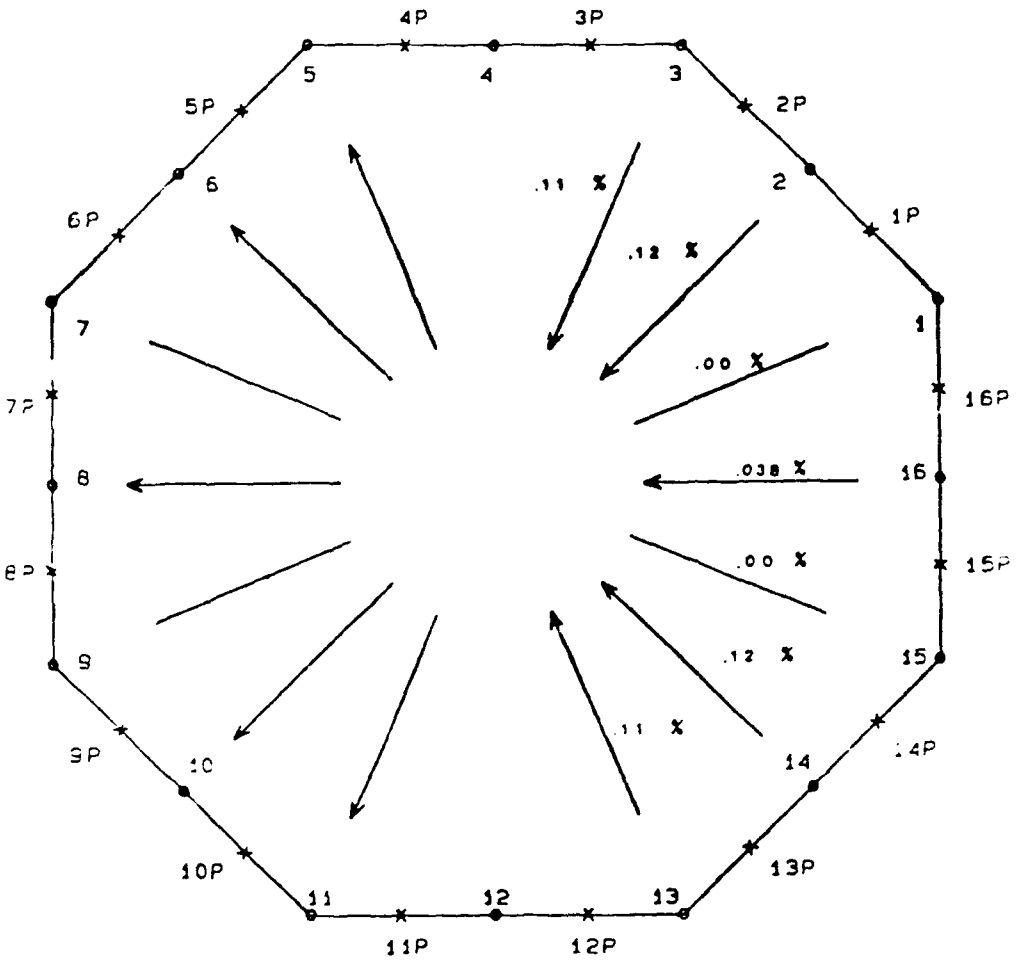


16 PUMP. WELLS
INITIAL GRADIENT = .54%

LEGEND:
1P = PUMP WELLS x
1 = OBSERVATION WELLS o

Figure 5. Resulting Potentiometric Surface Elevations and Hydraulic Gradients After 6 Days of Optimal Unsteady Pumping (Case 1a)

approximate scale: 1 in. = 450 ft.



16 PUMP. WELLS
INITIAL GRADIENT = .07%

LEGEND:
1P - PUMP WELLS x
1 - OBSERVATION WELLS o

Figure 6. Resulting Potentiometric Surface Elevations and Hydraulic Gradients Resulting from 30 Days of Steady Pumping Immediately Following 8 Day Optimal Pumping (run 1d).

should not be used because they cannot guarantee plume stabilization.

The ideal weight factor depends on many factors and may be problem-specific. A major consideration is the acceptable increase in potentiometric surface elevation at an injection site. This constraint is based on the desire to avoid pressurized injection. In a contamination problem with an initial water table at greater depth than that used in the assumed situation (providing for a larger upper limit on head), weight factors of 10, 100, and 1000 would probably produce increasingly smaller gradients.

To demonstrate the acceptability of the concentrations resulting from implementing the pumping strategy for the total 38-day planning period a 2-D solute transport simulation model was used (Reference 41). The model couples the ground-water flow equation with the conservative contaminant advection-dispersion solute-transport equation. The program uses an alternating-direction implicit procedure to solve a finite-difference approximation of the groundwater flow equation. It uses the method of characteristics (MOC) to estimate solute transport. The model assumes the contribution of molecular and ionic diffusion to hydrodynamic dispersion is negligible.

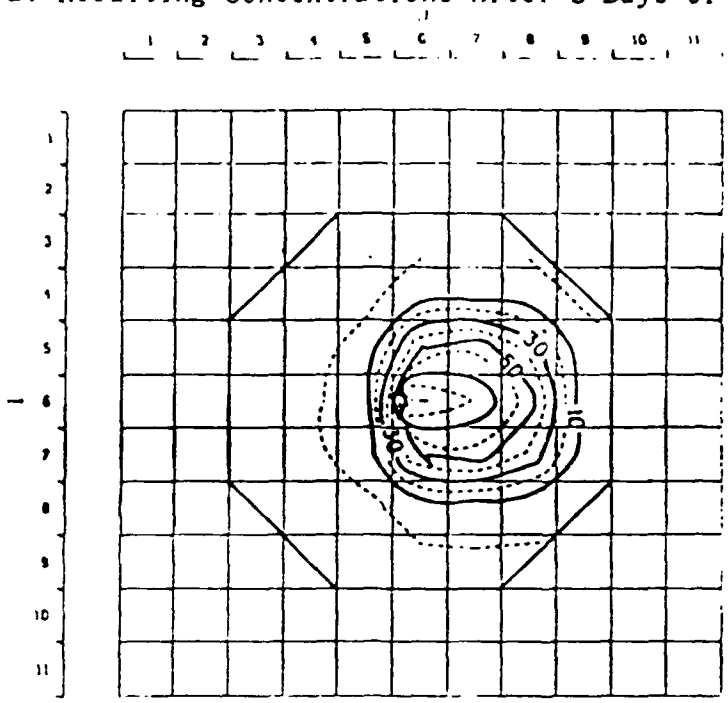
The initial concentrations of the contaminant plume (for all deterministic runs) when the pumping strategy is implemented are shown in Figure 4a. Subsequent contour maps for the hypothetical problem illustrate concentrations simulated by the MOC model resulting from the proposed unsteady and steady pumping strategies. The octagonal pumping well configuration is superimposed. It is assumed that the source of contamination has been eliminated and that vertical variation in concentration is negligible. As with the optimization model it is assumed that the contaminated area is a part of a larger aquifer extending in all directions. Because wells can only be located at nodes, not all wells could be located at the exact locations specified in the optimization program. However, using grids of 91.44 meters (300 feet) square allowed all but four wells to be located exactly. Transmissivities were developed from the final heads shown by the model's post-processor. Thus, the worst-case plume movements are determined.

Initially, the MOC model was run using longitudinal and transverse dispersivity values of 30.5 meters (100 feet). They were set equal to simulate similar soil pore structure in all directions. Figure 4b shows the plume concentration results at the end of 38 days if no pumping strategy is implemented. Figure 7a shows the plume concentrations after 8 days if the optimal strategy is implemented. Figure 7b shows the plume concentrations after an additional 30 days of the steady pumping rates computed by the post-processor. As can be seen, there is no appreciable plume movement during this time; indicating plume stability.

2. Analysis of Varying the Objectives or Requiring Steady Pumping

Runs were also made to compare (1) a purely hydraulic objective,

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

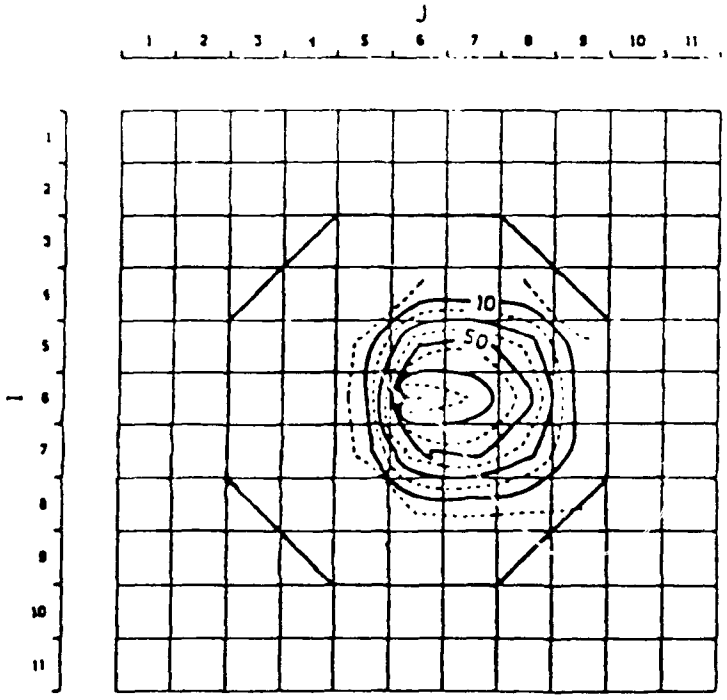


Figure 7. Run 1d Simulated Using Solute-Transport Model
(T_{max} = 1255 m^2/d. T_{min}/T_{max} = 1.0)

(2) a purely economic objective and (3) a minimization of the volume-of water-pumped objective. The first run of model 1d was made with W_f=1.0 and c′=c″=0.0 to emphasize hydraulics goal attainment. The results were the same as the original model 1d run. The second run set W_f=0.0 and c′ and c″ equal to their original values to emphasize only economics. The resulting pumping was during the final 5 days of the planning period only. Total pumping was less and the resulting final gradient was 0.134 percent. The third run minimized the total volume of water pumped by setting W_f=0.0, c′=0.0 and c″=1.0. The resulting pumping was during the final 3 days of the planning period only. Total pumping was reduced even further and the final gradient was 0.137 percent.

An additional constraint was added. It specified pumping for all time periods to be equal (steady pumping). The results were compared to the optimal unsteady results with W_f=1.0 and c′ and c″ equal to the original values. The total volume of water pumped with the steady pumping was over 350,000 ft^3 less than the unsteady strategy but the final gradient was .117 percent as compared to .07 percent with almost double the standard deviation.

3. Analysis of Variation in Physical Properties for the Hypothetical Problem

a. Varying Water Table Elevation

A slight variation of the hypothetical situation described in Section IV (run 1d with a weight factor of 1.0 in the objective function) was simulated. The initial water table was 2.5 meters higher than previously (3.3 meters below the ground surface rather than the original 5.8 meters). This reduced the upper limit on head at the injection wells from 5.8 meters to 3.3 meters. The final average gradient for this situation was a less satisfactory .10 percent (as compared to .08 percent). The tight constraint was again the upper limit on head at injection wells 2 and 3.

b. Varying Well Spacing

The sensitivity of optimal solutions to initially assumed well spacing was also tested. When well spacing of 274 meters (900 feet) (one pump per side at the mid-point and the observation wells at the corners) was used the resulting final gradient was unacceptable. This spacing is twice the initial spacing. This gradient was ten times that achieved using the 137 meters spacing. The solution was constrained by the upper limit on water table elevation at wells 2 and 3. A spacing of 68.5 meters (225 feet) was then used. This is one half the initial spacing (4 wells per side at the 1/8, 3/8, 5/8 and 7/8 points with the observation wells at the corners and at the mid-point between each pumping well). It produced a gradient equivalent to that of the initial spacing, and with only one-fourth of the O&M cost. However, the

capital cost could be twice that of the system using the initial well spacing of 137 meters.

In both of these cases there is an odd number of pumping wells per side. When this is the case two wells (the middle well on each of the sides of the octagon parallel to the hydraulic gradient) would be located on the y-axis. The model automatically deletes these two wells to maintain an equal number of injection and extraction wells and symmetry about the y-axis.

Once the end of the initial planning period has been reached and a hydraulic gradient has been achieved which will stabilize the plume one may wish to maintain the final hydraulic gradient. There is only one pumping value for each well which can maintain this new gradient. These are referred to as the steady pumping values. Final potentiometric surface elevations at the observation wells as a result of optimal pumping and the steady pumping required to maintain these elevations are determined using the post-processor. Figure 5 shows the observation well heads and final hydraulic gradients after the 8-day optimal pumping strategy. The results of run 1d are reiterated in Table 2 and Table 3 shows the steady pumping required for the next 30 day period to maintain plume stability for run 1d. Figure 6 shows the resulting observation well heads and final gradients. The most any observation well head changes during the steady pumping period is 0.01 meter.

c. Anisotropic Situations

Optimal strategies were developed for a variety of anisotropic situations and tested with the 2-D solute transport model. The original well configuration was used (well spacing of 137 meters). Original economic factors and a weight factor of 1.0 were used. Table 2 compares the isotropic aquifer run (run 1d) with the runs using Tmin/Tmax ratios of 0.67 (run 2d) and 0.30 (run 3d). In all three cases the maximum transmissivity was 1255 m^2/d.

The general trend is that as the minimum transmissivity decreases (and therefore the average transmissivity decreases) more pumping is needed for each day, but fewer days are used. The result is less total pumping required to achieve a nearly horizontal potentiometric surface. The lower transmissivity, being a measure of a slower flow of fluid through the aquifer, causes the model to require more pumping during each day. Greater pumping is needed to achieve approximately the same heads at the observation wells. At the end of pumping the gradient has been reversed more than is necessary and the final days (when there is no pumping) allow the potentiometric surface to rebound towards its initial gradient. The smaller transmissivity causes the potentiometric surface to rebound at a slower rate. As a result, as Tmin decreases, there are more days without pumping. The improvement in the final gradient as Tmin decreases is caused by the slower rebounding of the potentiometric surface. The slower rebounding actually permits more control, on a day to day basis, of the final surface. The tight

TABLE 2. EFFECT OF ANISOTROPY ON OPTIMAL UNSTEADY PUMPING STRATEGY FOR HYPOTHETICAL CONTAMINANT PROBLEM

	Tmax = 1255 m^2/d			Tmax = 370
Tmin/Tmax	1.0	0.67	0.30	0.25
Run Number	1d	2d	3d	4d
	Pumping(L/s)			
Day 1	96.1(3.35)*	96.4	96.7	32.1
2	90.1(3.15)	90.3	90.6	29.9
3	84.9(2.95)	85.2	86.2	28.8
4	80.2(2.80)	80.4	31.9	28.1
5	76.9(2.70)	58.2	0.0	27.5
6	36.9(1.28)	0.0	0.0	1.9
7	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0
Aver. Pumping (L/s)	58.1	51.2	38.1	18.5
Avg gradient(%)	0.08	0.06	0.05	0.10
S.Dev. of Gradient	0.058	0.040	0.039	0.076
Sum of head differences squared (m^2)	1.24(13.3)	0.78	0.58	2.34
Obj. function	15.6	10.6	7.73	25.9
O & M costs ($\$ \times 10^3$)	2.3	2.24	1.49	.72

* values in parentheses are corresponding English units in ft^3/s or ft^2.

constraints for all anisotropic runs were the upper limits on the potentiometric surface at pump wells 2,3,12 and 13. (Just as they were for the original hypothetical problem).

Model 4d in Table 2 contains the results of an anisotropic run using very low transmissivity values. A Tmax of 370 m^2/d (3,980 ft^2/d) and a Tmin/Tmax of 0.25 were used. These low transmissivities so restrict flow in the aquifer that there is a quick buildup of water in the injection wells. This causes the upper bound on head at the pumping wells to become tight at very small pumping values. Thus, permitted daily pumping is much less than previously. The tight constraints continue to be the upper water table at injection wells 2,3,12 and 13. However, the sensitivity values for these constraints are 10 times as large as the sensitivity values for the same constraints in the other anisotropic runs. As was explained previously, this greater sensitivity adversely affects the objective function. Physically, this large sensitivity indicates that water builds up around the injection wells instead of moving to the observation wells. The upper bound on head is reached quickly at the injection wells and the gradient in the contaminated area has changed very little. Therefore, a greater number of days is needed to achieve the nearly horizontal potentiometric surface. The resulting final average gradient is still much worse than the other anisotropic runs.

Optimal steady pumping strategies demonstrating the effect of anisotropy are shown in Table 3. As Tmin decreases, less steady pumping is required. A small transmissivity, which causes large head changes at pumping wells for a unit of pumping and at the same time restricts flow to the observation wells, requires less steady pumping to maintain the heads at the observation wells once they have been achieved. This restriction in flow causes slower natural changes in head at the observation wells, thus requiring less pumping to offset the attempt by the potentiometric surface to return to its steady-state gradient.

The contaminant movement resulting from implementing the strategies shown in Tables 2 and 3 were computed using the MOC model on the mainframe computer. Initial concentrations, the same as those of the original hypothetical problem, are seen in Figure 4a. Figure 4b shows the concentrations resulting after 38 days if no pumping scheme is instituted. The contour maps of plume concentrations resulting from the optimal pumping schemes during the first 8 days (Figures 8a,9a and 10a) indicate a slight movement of the plume to the west. This is caused by the reversal of the original gradient produced by the optimal unsteady pumping (the gradient now slopes to the west). Run 3d used the worst-case transmissivities as a check on the plume movement versus using the initial transmissivities. Significant difference in plume movement was found.

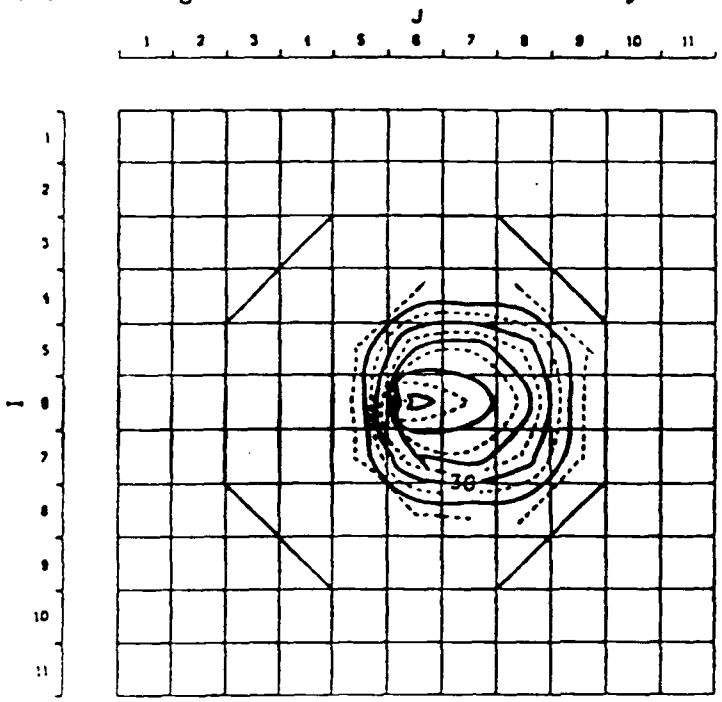
The concentrations resulting after day 38 from 30 days of steady pumping are shown as Figures 8b, 9b and 10b. The major difference when comparing the plumes after 8 days and after 38 days is that the highly concentrated plume source (90 mg/L) has dissipated by the end of 38

TABLE 3. EFFECT OF ANISOTROPY ON STEADY PUMPING STRATEGIES FOR HYPOTHETICAL PROBLEM

	Tmax = 1255 m^2/d			Tmax = 370
Tmin/Tmax	1.0	0.67	0.30	0.25
Run Number	1d	2d	3d	4d
	Pumping(L/s)			
Well 1	-18.9(0.66)*	-12.2	-4.46	-0.24
2	-15.6(0.55)	-9.20	-2.92	-0.70
3	-7.40(0.26)	-5.00	-2.64	-1.84
4	7.40(0.26)	5.00	2.64	1.84
5	15.6(0.55)	9.22	2.92	0.70
6	18.9(0.66)	12.2	4.46	0.24
7	19.4(0.68)	10.5	2.64	0.21
8	19.4(0.68)	10.5	2.64	0.21
9	18.9(0.66)	12.2	4.46	0.24
10	15.6(0.55)	9.20	2.92	0.70
11	7.40(0.26)	5.00	2.64	1.84
12	-7.40(0.26)	-5.00	-2.64	-1.84
13	-15.6(0.55)	-9.22	-2.92	-0.70
14	-18.9(0.66)	-12.2	-4.46	-0.24
15	-19.4(0.68)	-10.5	-2.64	-0.21
16	-19.4(0.68)	-10.5	-2.64	-0.21
Aver. Pumping (absolute L/s)	15.3	9.23	3.16	0.75

* values in parenthesis are corresponding English units in ft^3/s.

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

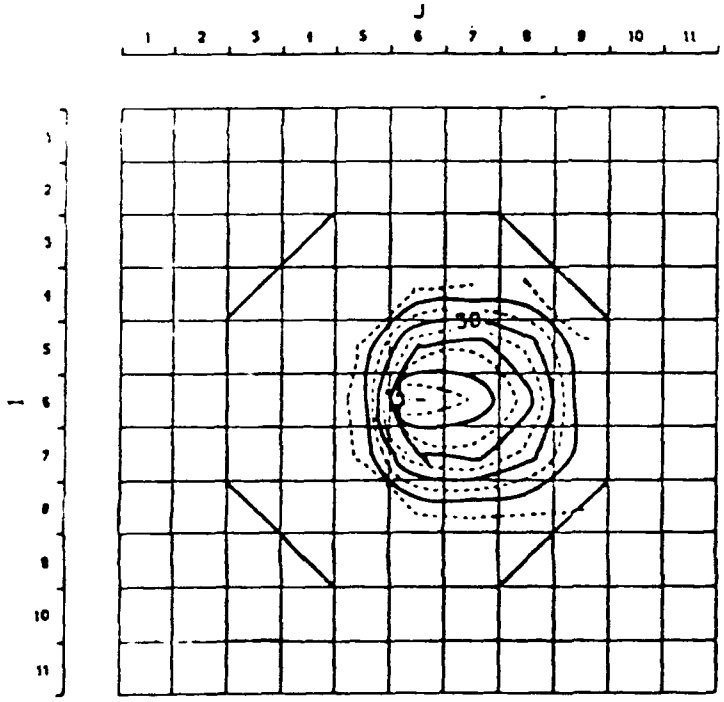
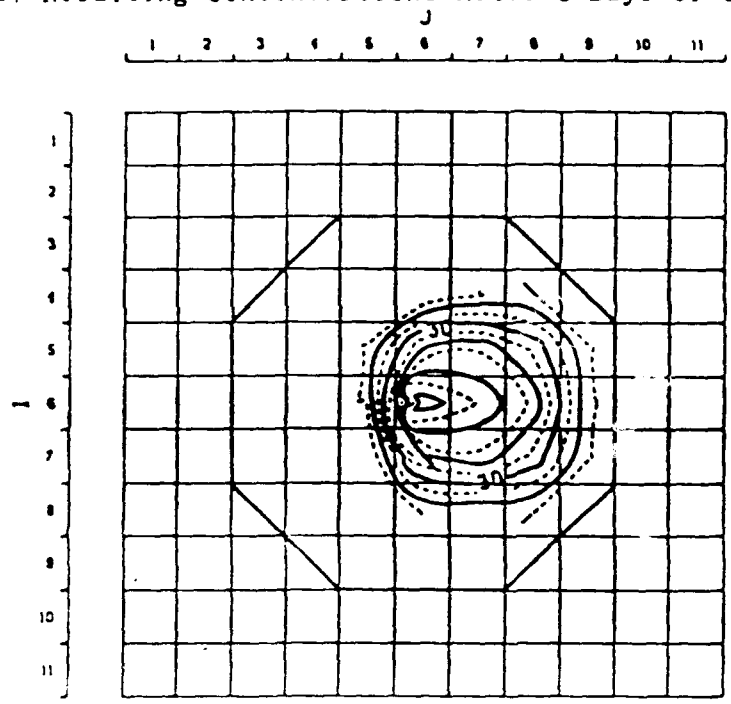


Figure 8. Run 2d Simulated Using Solute-Transport Model
(Tmax = 1255 m^2/d, Tmin/Tmax = 0.67)

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

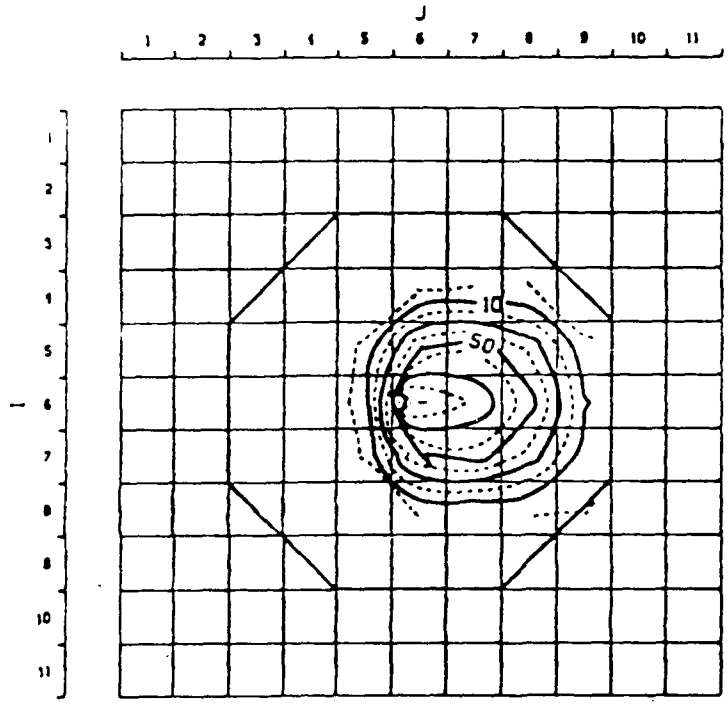
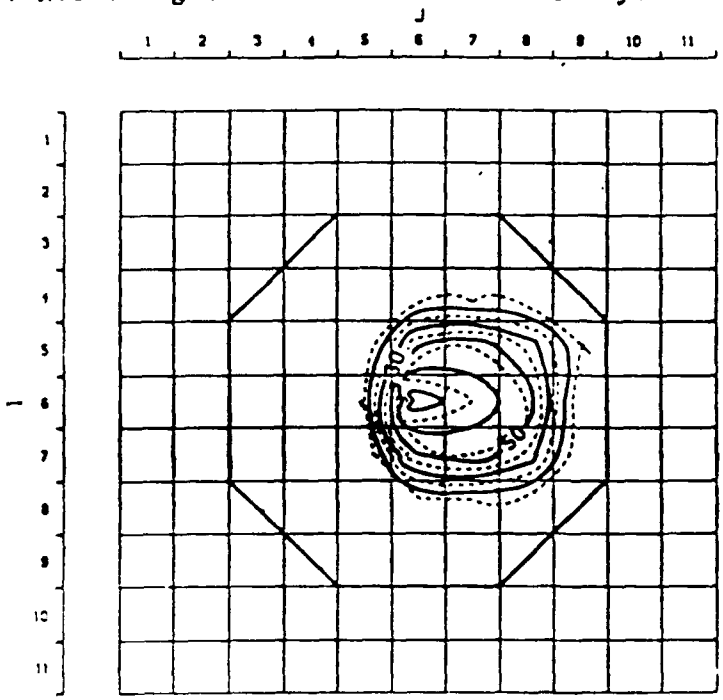


Figure 9. Run 3d Simulated Using Solute-Transport Model
(Tmax = 1255 m²/d, Tmin/Tmax = 0.30)

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

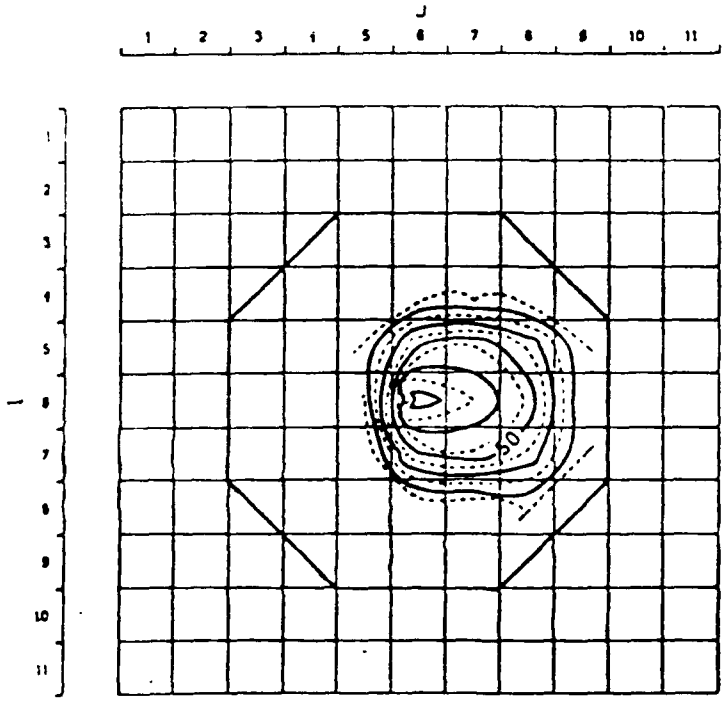


Figure 10. Run 4d Simulated Using Solute-Transport Model
(T_{max} = 370m^2/d, T_{min}/T_{max} = 0.25)

days. All other concentrations, including those along the outer boundary of the plume, remained relatively stable. Figure 10b shows that if transmissivity is low enough (Tmax = 370 m/d) even the 90 mg/L isoline remains stable during the steady pumping phase.

d. Varying the Total Time Period

The optimal strategy developed for an 8-day period for run 2d (Tmax = 1255 m^2/d and Tmin/Tmax = 0.67) was compared with a strategy developed for a 5-day time period. Table 2 illustrates that pumping is needed in only the first 5-days of the 8-day time period. One may surmise that 5 days is enough time to stabilize the plume. To test this hypothesis, an additional optimization was made using a 5-day time period. Table 4 permits easy comparison of both optimal strategies. The optimal pumping required for the 5-day period is less than that needed for the 8-day period. Therefore, the operation and maintenance costs (O&M) are less. However, the resulting gradient is steeper for run 5d and the steady pumping required to maintain this steeper reverse gradient is 3 to 7 times larger than that for run 2d.

Since the 5-day optimization (run 5d) showed no pumping in day 5, a 4-day optimization was made. This showed pumping in only the first 3 days, the O&M costs were reduced $200 and again the resulting final gradient was steeper. This steeper gradient caused the steady pumping values to more than double those required by the 5-day run. A 3-day run showed the same trend; slightly lower O&M costs but a steady pumping almost 3 times as large as that for the 5-day run. A 2-day run was unfeasible because the requirement that heads at observation wells initially down-gradient from the source be higher than the head at the source after two days could not be satisfied without violating bounds or constraints.

e. Varying Well Configuration

Finally, two optimizations (runs 6d and 7d) were performed to evaluate how changing the octagonal placement and shape in an area where the transmissivity varies spatially. This differs from the anisotropic transmissivity of previous runs. For model run 6d, a regular octagon of 274 meter side length with two pumping wells per side was used. Because of the varying saturated thickness (making it difficult to calculate the estimated plume movement) the octagon was shifted 91.4 meters (300 feet) to the east. Model run 7d used a modified version of the same pump arrangement. The two sides parallel to the gradient were only 183 meters long and had only a single pumping well. All other sides were as in the previous system. Initial concentrations (Figure 4a) and gradient were the same as for previous examples. For both runs, the maximum hydraulic conductivity was 82 m/d parallel to the hydraulic gradient and the minimum hydraulic conductivity was 25 m/d perpendicular to the gradient. The saturated thickness of the

TABLE 4. EFFECT OF DURATION OF EVOLUTIONARY ERA ON OPTIMAL STEADY AND UNSTEADY PUMPING STRATEGIES FOR HYPOTHETICAL PROBLEM

(T_{max} = 1255 m^2/d and T_{min}/T_{max} = 0.67)

	Optimal unsteady pumping			Steady pumping	
Time(days)	8	5		8	5
Run Number	2d	5d		2d	5d
	Pumping(L/s)			Pumping(L/s)	
Day 1	96.4(3.40)	96.4	Well 1	-12.2	-35.3
2	90.3(3.18)	90.3	2	-9.2	-34.4
3	85.2(3.00)	85.2	3	-5.0	-35.8
4	80.4(2.83)	27.0	4	5.0	35.8
5	58.2(2.05)	0.0	5	9.2	34.4
6	0.0	----	6	12.2	35.3
7	0.0	----	7	10.5	28.8
8	0.0	----	8	10.5	28.8
Avg gradient(%)	0.06	0.08	9	12.2	35.3
S.Dev. of gradient	0.040	0.063			
Sum of head differences			10	9.2	34.4
squared (m^2)	0.78(8.36)	1.74	11	5.0	35.8
Obj. function	10.6	20.3	12	-5.0	-35.8
O & M costs ($\$ \times 10^3$)	2.24	1.61	13	-9.2	-34.4
			14	-12.2	-35.3
			15	-10.5	-28.8
			16	-10.5	-28.8
Aver. Pumping (absolute L/s)	51.3	37.36		18.5	33.6

* values in parentheses are corresponding English units in ft^3/s or ft^2.

aquifer varied linearly from a high of 15 meters at the west end of the octagon to a low of 12 meters at the east end of the octagon. The resulting transmissivities range from 1230 m^2/d to 300 m^2/d.

Table 5 compares the unsteady and steady pumping strategies of runs 6d and 7d. Pumping cost is greater for the optimal unsteady strategy of the regular octagonal system (6d) than for the irregular system (7d). However, the final gradient is significantly better. The irregular system has difficulty achieving a horizontal potentiometric surface because the extraction wells are closer to the source than the injection wells. This causes the potentiometric surface at the source to drop rather than remain constant as with the regular octagon. A larger reverse gradient from the injection wells back to the source results. It must be kept in mind that two additional wells are required with the regular octagon strategy, thereby increasing the initial capital cost. The steady pumping values are not exactly symmetrical about the x-axis (for either the regular or irregular configuration) as they are for all other anisotropic cases. This may be caused by the shifting of the octagon to the east. This causes the injection wells down-gradient of the source to be farther from the source than the extraction wells up-gradient of the source. Therefore the final gradient is not constant from a down-gradient observation well through the source to an up-gradient observation well and different steady pumpings are required to maintain these gradients.

Strategies for runs 6d and 7d were very effective for the 8 day optimal pumping period but the 30-day steady pumping strategy did not immobilize the plume as well as previous runs. The optimal unsteady pumping strategies show very little movement of the contaminant plume (Figures 11a and 12a). However, the plume movements resulting from the steady pumping strategies are disappointing (Figures 11b and 12b). The dense portion of the plume has moved approximately 45-m east during this 30-day period even though the outer isoline remains fairly stable. All other scenarios (including the Otis Air Base problem mentioned later) show very little plume movement.

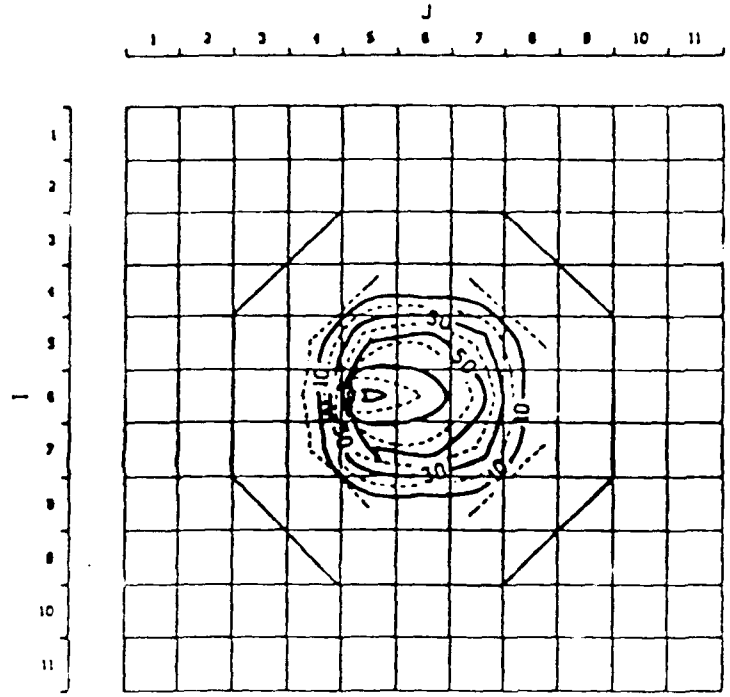
The deterministic version of the optimization model cannot guarantee global optimality because of the quadratic form of the objective function. A standard procedure to attempt to gain some assurance that global optimality has been found is to make a number of different optimizations, each using a different initial solution. For a problem requiring computation of 8 daily pumping values, at least 16 optimizations should be performed. The initial solutions for these optimizations are obtained by systematically employing initial pumping values at their upper or lower bounds. This procedure was performed with run 6d (16 different optimzations were performed) and it was found that all runs gave the same optimal unsteady pumping values. Thus, empirically at least, global optimality was attained for this hypothetical situation.

TABLE 5. EFFECT OF WELL CONFIGURATION ON OPTIMAL UNSTEADY AND STEADY PUMPING FOR HYPOTHETICAL CONTAMINANT PROBLEM (Kmax = 82 m/d w/varying saturated thickness, Kmin/Kmax = 0.3)

	Optimal unsteady pumping			Steady pumping	
Well Config.	Regular	Irregular		Regular	Irregular
Run Number	6d	7d		6d	7d
	Pumping(L/s)			Pumping(L/s)	
Day 1	84.4(2.97)*	84.4	Well 1	-3.58	-3.12
2	79.0(2.78)	79.3	2	-2.64	-3.12
3	73.1(2.57)	29.1	3	-2.58	-4.40
4	27.9(0.98)	0.0	4	2.64	0.31
5	0.0	0.0	5	2.90	3.12
6	0.0	53.3	6	4.34	1.99
7	0.0	0.0	7	2.58	1.99
8	0.0	0.0	8	2.58	3.12
Avg gradient(%)	0.04	0.13	9	4.26	0.31
S.Dev. of gradient	0.038	0.079			
Sum of head differences squared (m^2)	0.52(5.60)	2.99	10	2.84	-4.29
			11	2.56	-2.84
Obj. function	6.86	33.14	12	-2.50	-3.12
O & M costs ($\$ \times 10^3$)	1.24	.93	13	-2.58	-1.42
			14	-3.49	-1.42
			15	-1.84	
			16	-1.84	
Aver. Pumping (absolute L/s)	33.0	24.1		2.86	2.47

* values in parentheses are corresponding English units in ft^3/s or ft^2.

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

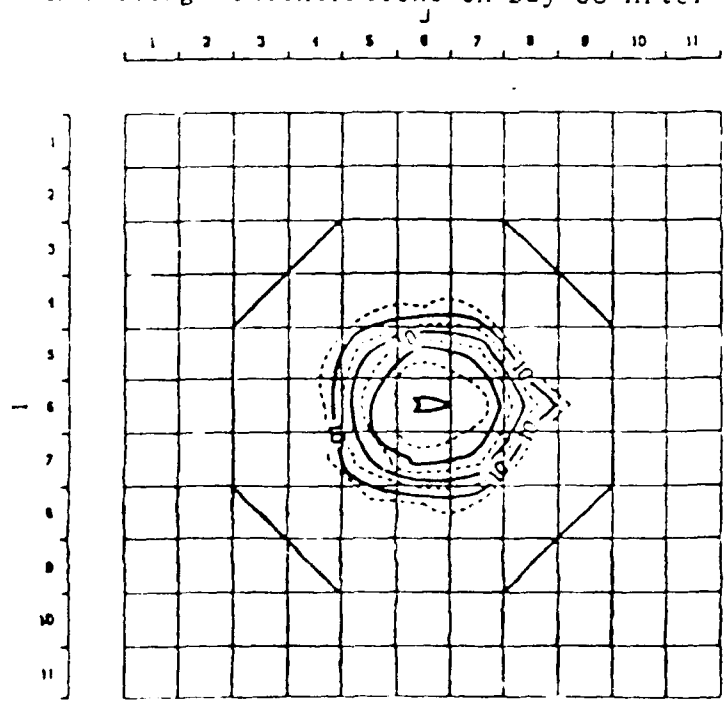
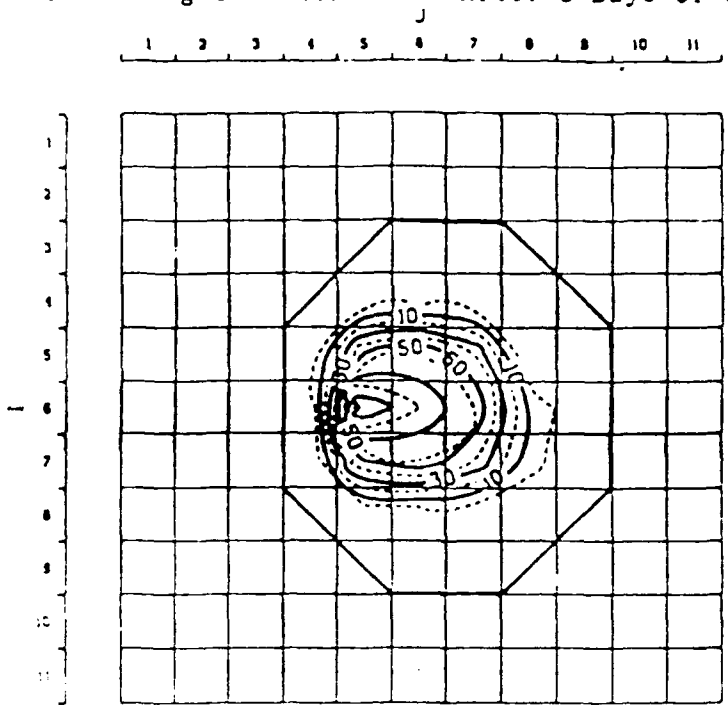


Figure 11. Run 6d Simulated Plume Solute-Transport Model
(Kmax = 82 m/d, Kmin/Kmax = 0.30,
Regular Octagon shifted 1 grid to the east)

a. Resulting Concentrations After 8 Days of Optimal Unsteady Pumping



b. Resulting Concentrations on Day 38 After 30 Days of Steady Pumping

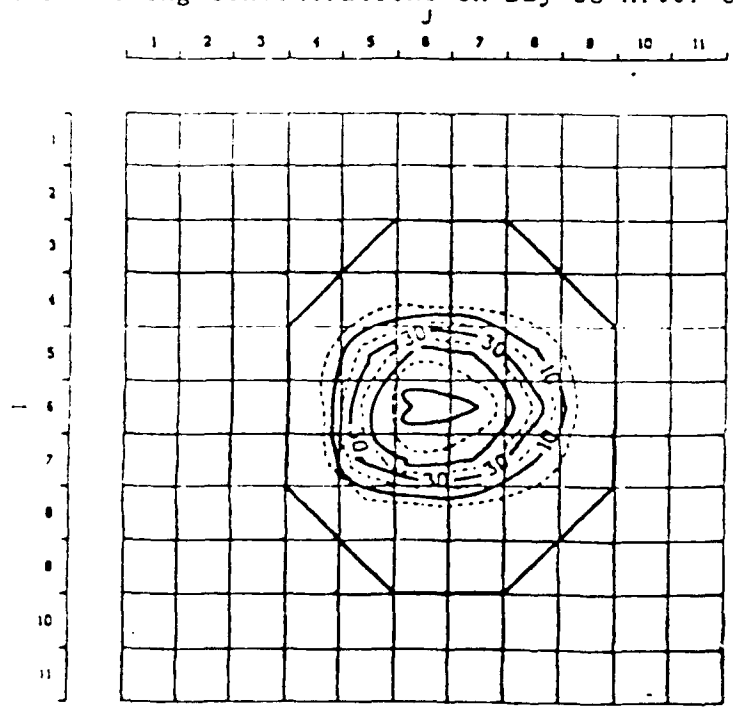


Figure 12. Run 7c Simulated Using Solute-Transport Model
(Kmax = 82 m/d, Kmin/Kmax = 0.30,
Regular Octagon shifted 1 grid to the east)

4. Effects of Long-Term Steady Pumping

Because of the reverse gradient (downward slope from east to west in the hypothetical problem) produced by the optimal unsteady pumping, the length of time the steady pumping can be used to maintain the reverse gradient is limited. Eventually, unless a new optimal unsteady strategy is implemented, the new gradient will cause contaminated water to reach the extraction wells. The results of 22 additional weeks of steady pumping were simulated using the MOC model for two of the anisotropic cases. Figure 13a shows the plume location for run 6d (regular octagon) and Figure 13b shows the plume location for run 7d (irregular octagon).

In both cases, the outer limit of the plume has reached the extraction wells. This contaminated water cannot be used to supply the injection wells. Ideally, this would be a good time to begin the actual withdrawal and treatment of the contaminated water. Since each contaminant problem is site-specific, there is no way to predict when the plume would reach the extraction wells. Careful monitoring of these wells is needed to guarantee contaminant-free water being used in the injection wells.

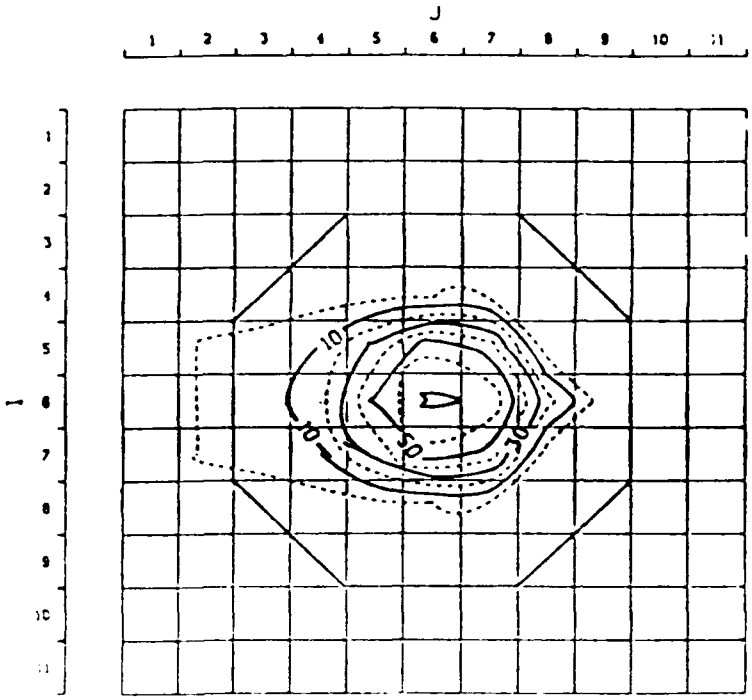
The model can be used to develop a new strategy to address the reverse gradient produced by the first optimal pumping strategy. The model accepts spatially variable heads at each pumping and observation well. Therefore, the model can be run again; only this time the down-gradient wells are those to the left of the contaminant source. The previous extraction wells become injection wells and a new optimal pumping strategy and steady pumping strategy are developed for another finite time period.

5. Evaluation of Safety Factor for Equation 4

An empirical equation was developed to guide the model user in determining an appropriate safety factor for equation 4. The original hypothetical problem (run 1d) was used in conjunction with the 2-D solute transport (MOC) model. Successive runs were made to determine a relationship between the safety factor and the uncertainty of transmissivity and the relationship between the safety factor and the relative infiltration of water. A dispersivity of 100 feet was used for all runs. This value for dispersivity is thought of as an "average" value (Reference 4), and, because it is greater than one-fourth of the cell size (300 feet) used in the MOC model, it is considered a conservative value for estimating plume movement (it predicts greater movement than normally associated with a particular set of conditions). Therefore, this safety factor incorporates dispersion as a source of plume movement and adjusts the size of the well octagon accordingly.

Except for the transmissivity values, all input to the solute transport model, including the optimal unsteady pumping values from run 1d, remained constant. The statistical software package, SAS, was used to generate random transmissivity values for a log-normal probability distribution. The SAS program used a mean transmissivity, E(T), of

a. Run 6d



b. Run 7d

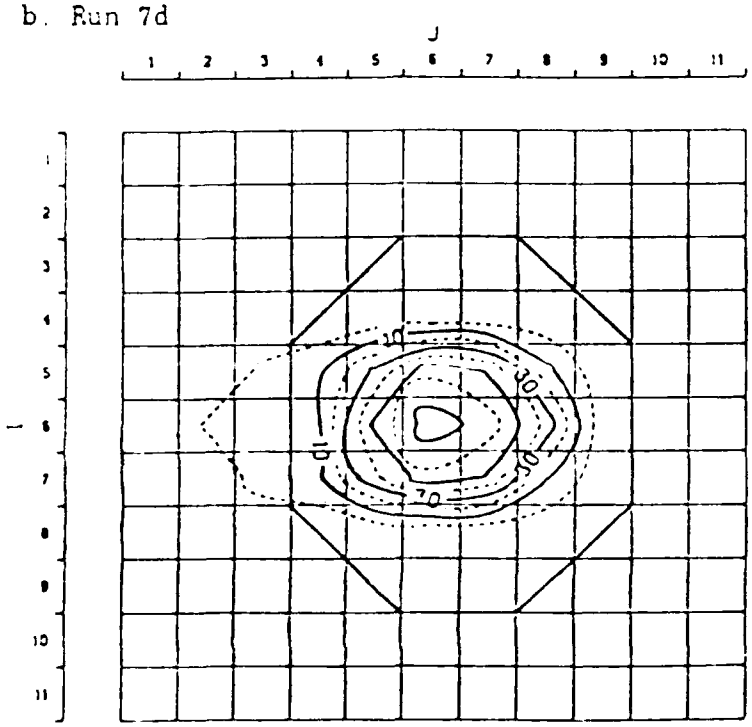


Figure 13. Resulting Concentrations After 12 Additional Weeks of Steady Pumping Using Solute-Transport Model

1255 m^2/d and coefficients of variation (CV = standard error/mean) for transmissivity of 0.2, 0.4 and 0.8. The program used a SAS function called RANNOR to generate 121 (1 for each grid of the model) random numbers which were normally distributed with a mean of 0.0 and standard error of 1.0. Therefore, the transmissivity value corresponding to each random number (rn) for a log normal distribution with a mean of $\exp\left\{E(T) + \frac{[CV * E(T)]^2}{2}\right\}$ and variance of $\{\exp[2 * E(T) + (CV * E(T))^2]\}\{\exp[(CV * E(T))^2] - 1\}$ was calculated using the following relationship:

$$T = \exp[E(T) + \{COV * E(T)\} * rn]$$

These transmissivity values were output in a format which could be added directly to the MOC input file and read into the 11 by 11 grid.

Twenty runs of the solute transport model were made for each of the three coefficient of variation values. Each run, for a constant CV, required a new "seed" value to begin its iterative calculation of the 121 random numbers. This insured a new set of values for each run. None of the 60 runs allowed the plume to leave the octagon of wells (which was sized using an arbitrarily chosen safety factor of 2). However, observing solute movement permitted developing an approximate relationship to help the model user:

$$s.f. \approx 1.0 + COV(\text{of transmissivity})$$

In addition, the same solute transport model was used to determine the relationship between infiltration rate and plume movement assuming constant transmissivity. Numerous runs were made, varying precipitation rate and infiltration rate. It was discovered that there is a slight increase in plume movement for increasing precipitation and/or infiltration. This information is used in a related expert system program and is discussed in detail in Section VI. The resulting equation is:

$$s.f. \approx 1.0 + COV + \text{infiltration factor}$$

B. DETERMINISTIC MODEL APPLICATION TO HAZARDOUS WASTE SITE, OTIS AIR BASE, MASSACHUSETTS

Data and description of this contaminated groundwater site are obtained from a preliminary report by Denis R. LeBlanc, (Reference 48).

Since 1936 disposal of treated sewage through infiltration beds has been allowed at Otis Air Base, Cape Cod, Massachusetts (Figure 14). The resulting plume of contaminated groundwater is in an underlying sand and gravel aquifer 2,000 feet wide, 75 feet thick and more than 8,000 feet long. Water in the plume contains elevated concentrations of chloride, sodium, boron, nitrogen, TCE, detergents and other constituents of the treated sewage. The plume was previously mapped and described in a study (Reference 48) by the U. S. Geological Survey in

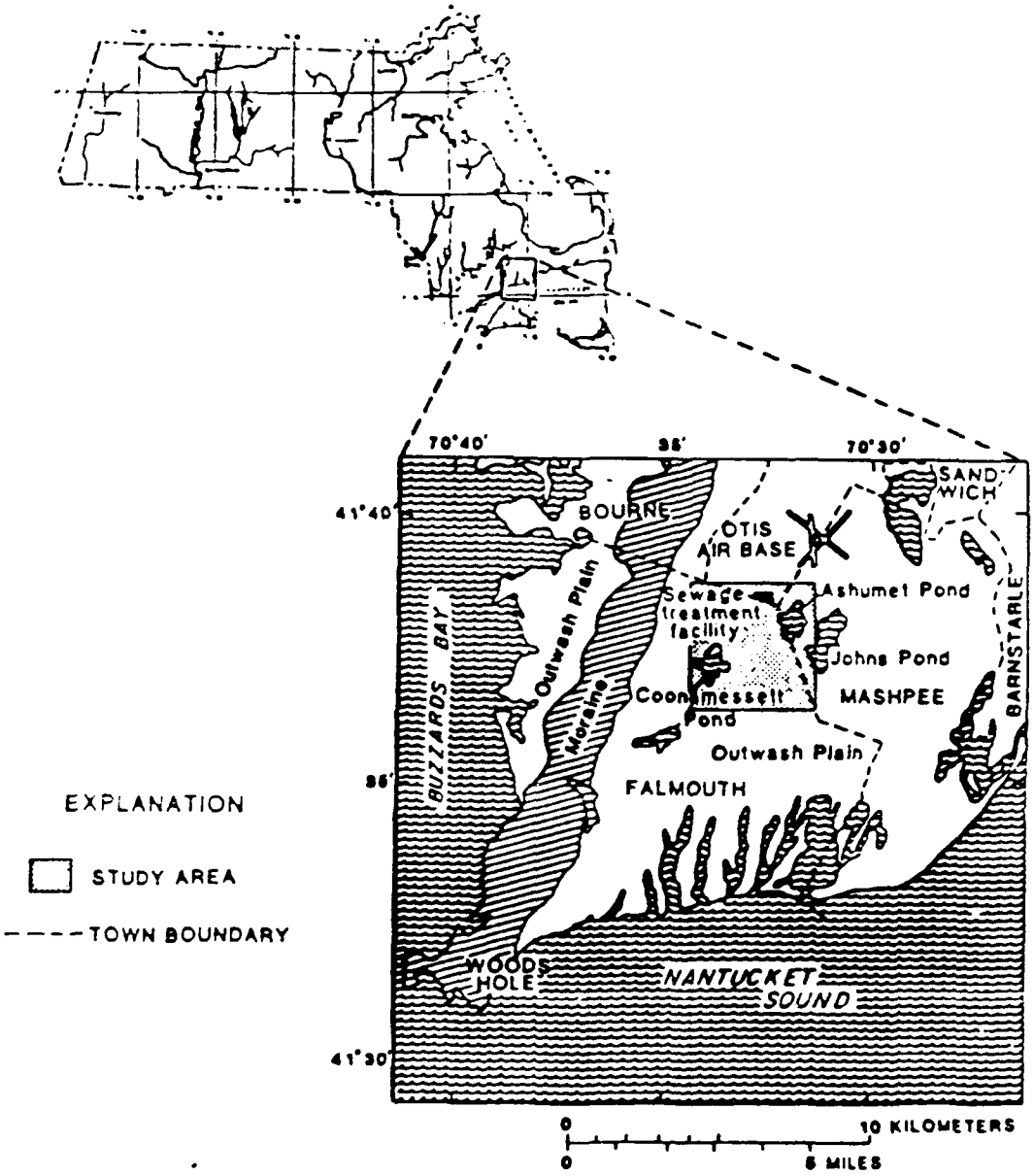


Figure 14. Study Area for Otis AFB Contamination Problem

cooperation with the DWPC (Mass. Dept. of Enviromental Quality Engineering, Division of Water Pollution Control).

More than 8 billion gallons of secondarily treated sewage have been discharged to the aquifer at the Otis Air Base sewage plant since 1936. Disposal is by rapid infiltration through sand beds. The aquifer that receives the treated sewage is composed of 90 to 100 feet of stratified sand and gravel outwash underlain by silty sand and till. Groundwater in the outwash is unconfined and moves southward toward Nantucket Sound at a rate of about 1 foot per day. The study area south of Otis Air Base is mostly rural, although many homes have been built since the plume was first mapped in 1978-79.

Groundwater in the aquifer is unconfined and the water table slopes uniformly to the south at an average rate of .17% except where it is distorted by ponds. The water table contour map (Figure 15) was prepared from water levels measured in November 1979. Water table levels in November 1979 were near average values for the period 1963-76 at ten long term monitoring sites on Cape Cod.

The only natural source of water to the aquifer is recharge from precipitation. The estimated average annual recharge rate is 21 in/yr. Recharge occurs over most of the study area. Direct surface runoff is negligible because the sandy soils are very permeable. Groundwater flow is nearly horizontal except near the ponds and presumably near the infiltration beds.

Most groundwater flowing through the study area discharges to Nantucket Sound and to streams, ponds and wetlands in southern Falmouth. The net discharge from the aquifer by pumping wells is small because most water is returned to the aquifer through irrigation and septic systems. Water also flows between the aquifer and the three large kettle-hole ponds. Ashumet Pond, which is located 1,700 feet southeast of the infiltration beds has no surface inlet or outlet. Johns Pond and Coonamessett Pond are drained by streams. Groundwater levels south of the Otis treatment plant are controlled, in part, by the relatively constant water levels along Johns and Coonamesett Ponds.

1. Input to the Deterministic Model

All data used by the optimization model has been verified (Reference 48) by simulating the history of the existing contaminant plume with the 2-D solute transport model. The predicted limits of the plume from the 2-D model corresponded very closely to actual limits of the plume.

Boron is a good indicator of the contaminated zone. Boron concentrations in the treated sewage between 1974 and 1980 were 10 to 50 times greater than boron concentrations in the uncontaminated groundwater. The major sources of boron in the sewage are cleaning agents and detergents. The plume delineated by the elevated boron concentrations is 2,000 feet wide and over 8,000 feet long (Figure 16). Contaminants from the disposal site may have moved farther than 8,000

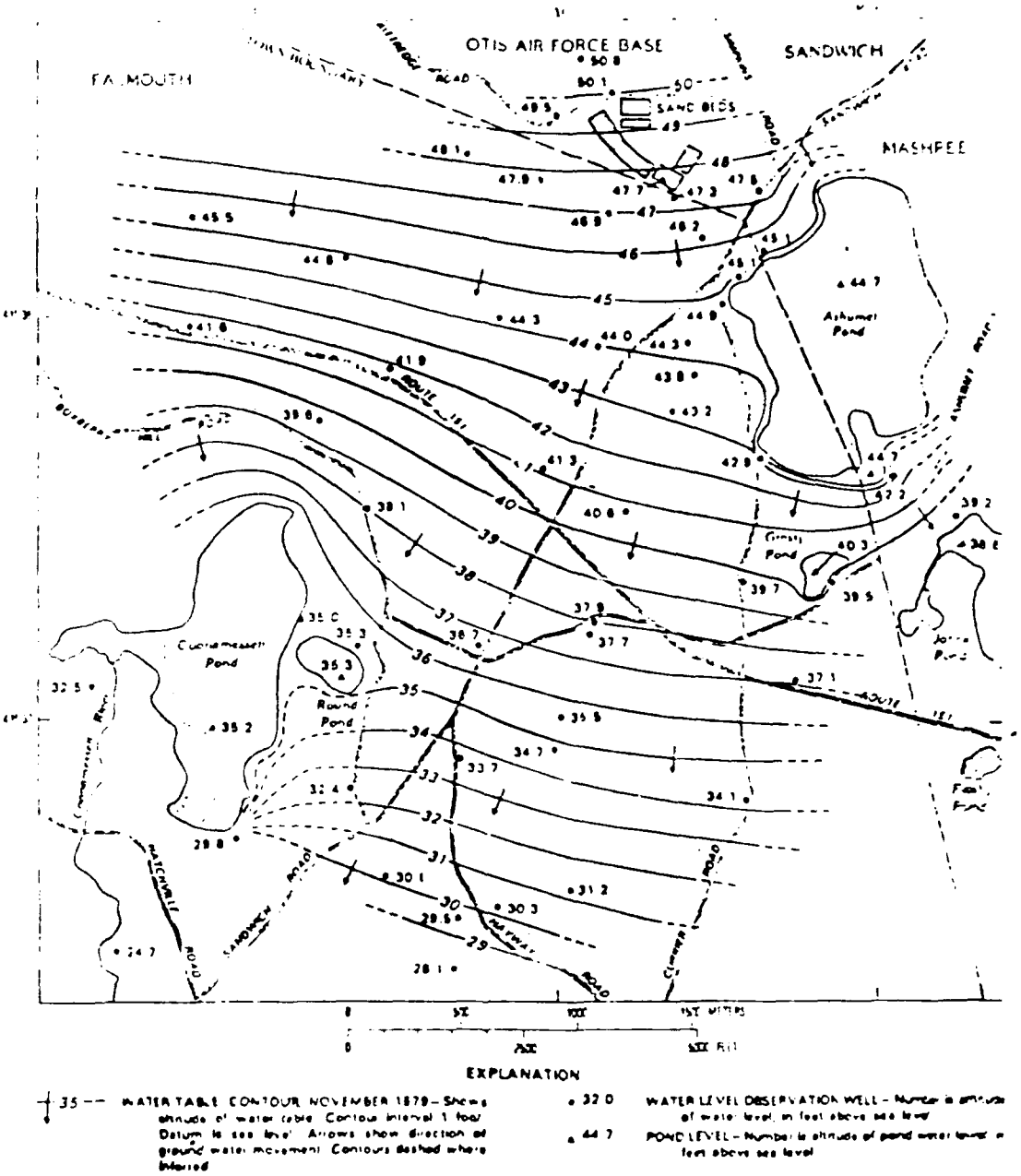


Figure 15. Steady-State Groundwater Table Elevations

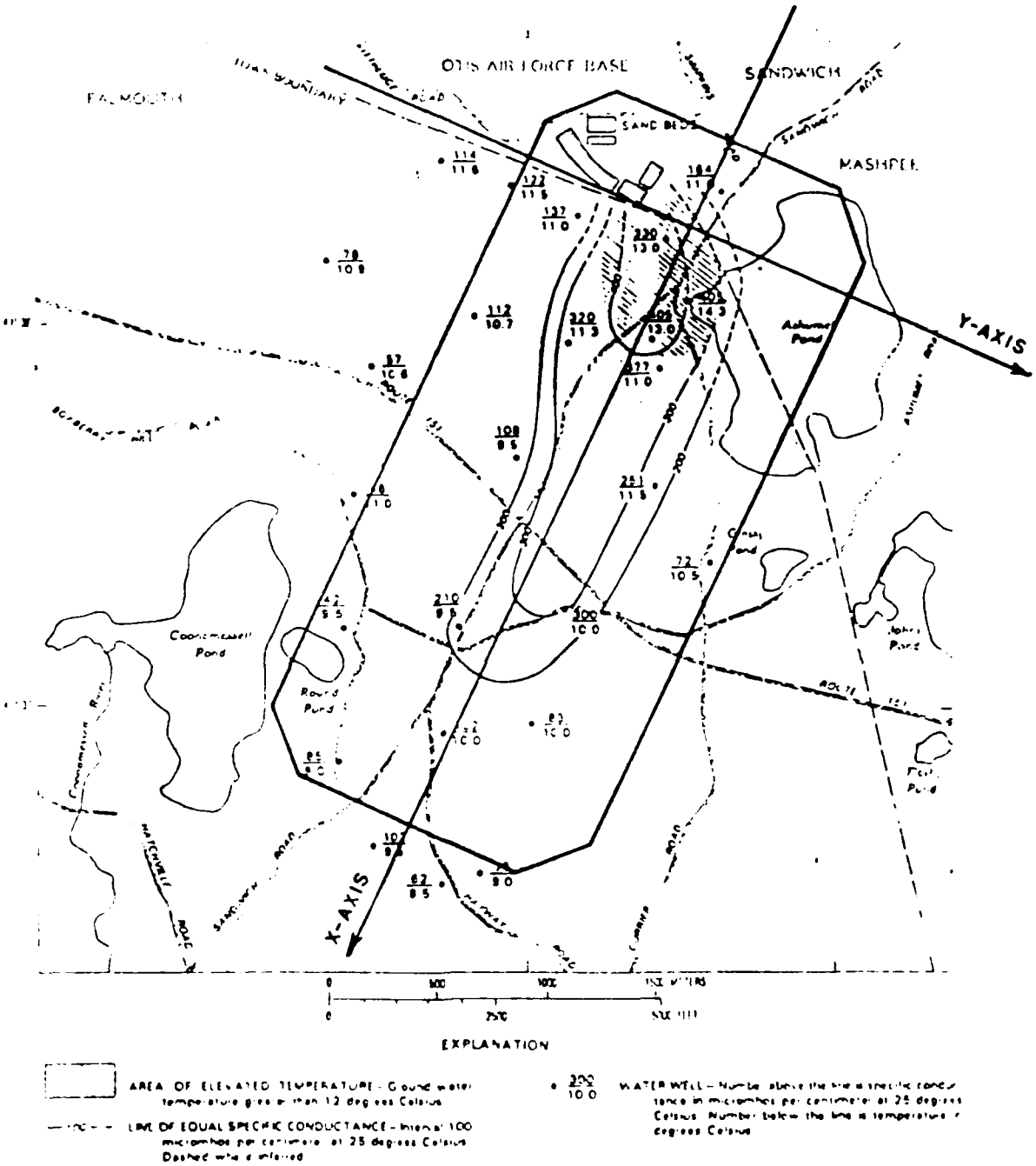


Figure 10 Contaminant Plume Extent and Location of Proposed Well System

feet down-gradient of the infiltration beds, but water samples were not collected beyond this distance in 1978-79. The longitudinal axis (x-axis in Figure 16) of the plume is oriented in the direction of groundwater flow shown in Figure 15. Spreading and dilution by hydrodynamic dispersion was evident along the toe and sides of the plume, but the contaminant concentrations in the center remain high as far as 6,000 feet down-gradient of the sand beds. The amount of spreading could not be determined precisely because the observation wells were spaced several thousand feet apart.

Although the plume is extensive, it is only about 75 feet thick and is contained almost entirely in the sand and gravel outwash. Its bottom boundary generally coincides with the contact between the permeable sand and gravel and the less permeable silty sand and till. A zone of uncontaminated groundwater that is 20 to 50 feet thick overlies the plume.

Application of the model requires simplification of the real system. Assumptions made in the modeling procedure must be considered when interpreting model results. Four major assumptions are (Reference 48):

1. The aquifer is formed only by the sand and gravel outwash. The underlying silty sand and till are at least 10 to 20 times less permeable than the outwash and the vertical hydraulic head gradients across the interface between outwash and fine-grained sediments are small. Therefore, it is reasonable to assume that the silty sand and till approximate an impermeable bottom boundary to the aquifer.

2. The aquifer can be represented by a single, two-dimensional layer in which the vertical variations in hydraulic head and solute concentration are negligible. The assumption of two-dimentional flow is reasonable because groundwater flow in the outwash is nearly horizontal.

3. The density and viscosity of the contaminated and uncontaminated groundwater are essentially identical; so only hydraulic head gradients affect the velocity distribution. The difference in total dissolved solids concentration between the treated sewage (155 to 178 mg/l) and the uncontaminated groundwater (39 mg/l) is relatively small and groundwater temperatures vary only slightly. Therefore, density differences due to solute concentration and temperature variations are negligible.

4. Groundwater levels and the velocity distribution do not change with time and represent a steady-state system before the pumping strategy is implemented. Although water levels fluctuate 1 to 3 feet seasonally, no long-term rise or decline of water levels has been observed since observations began in 1975. The short-term fluctuations are relatively uniform throughout the area and have little effect on the hydraulic gradient.

All aquifer parameters used were developed by Dennis LeBlanc

(Reference 48). The average hydraulic conductivity of the sediments was estimated from:(1) flow net analysis of the regional water table map,(2) measured hydraulic conductivity at four aquifer test sites near the study area and at three sites in similar sediment elsewhere on Cape Cod,(3) aquifer parameters used in a digital model of regional groundwater flow on Cape Cod and 4) an empirical equation relating grain size distribution to permeability. The estimates of hydraulic conductivity of the sand and gravel, obtained by the above methods, ranged from 140 to 220 ft/day. The isotropic hydraulic conductivity used in the model was 186 ft/day (Hmax and Hmin were assumed equal).

Porosity of the sand and gravel was estimated from:(1) measured porosity of the outwash near the sewage treatment plant and(2) measured porosity of similar outwash on Long Island, New York. The average porosity of samples near the sewage treatment plant was 0.32. The porosities of two core samples of outwash on Long Island were 0.34 and 0.38. From this data, the average porosity of the sand and gravel was estimated to be 0.35 for the model. Although the total pore space may not be available for flow due to dead-end pores and adhesion of water to the sediment grains, the effective porosity available for flow is essentially equal to total porosity in coarse-grained unconsolidated media.

The saturated thickness of the aquifer varies linearly from 115 feet at the north end of the plume near the infiltration beds to 90 feet at the south end of the plume.

An irregular octagon was situated as near as possible to the outline of the plume (Figure 16). The southern end of the octagon was located 2000 feet down-gradient from the extreme edge of the plume as a safety precaution to account for the uncertainty of the actual plume extent and uncertainty as to how much the plume will move before it can be stabilized. The width of the octagon is 1500-2000 feet away from the plume on each side for the same reasons. The two sides perpendicular to the hydraulic gradient (north and south ends) are 4000 feet in length with 4 pumping wells per side. The two elongated sides parallel to the hydraulic gradient are 9500 feet in length with nine pumping wells per side. The remaining four sides are 1000 feet in length with one pump per side. Wells are shown to be placed in Ashumet Pond when in reality they would be placed between the plume and the pond as regularly spaced as possible. Because of the extreme elongation of the plume the contaminant source (origin of the x-y axis in Figure 16) is located near one end of the octagon. All wells located down-gradient of the designated source (any on the positive x-axis side of the y-axis) are injection wells. Since there are 24 injection wells and only 6 extraction wells total injection exceeds total extraction.

The following parameters are not needed for the deterministic optimization model but are required by the solute transport model.

Hydrodynamic dispersion causes the plume to spread and mix with uncontaminated groundwater in the direction of flow and, to a lesser extent, perpendicular to flow. It is a function of groundwater velocity

and dispersivity, a property of the aquifer. Dispersivity of the outwash material was estimated based on values determined for similar aquifers. These values were computed by matching observed plumes with mathematical models by trial-and-error adjustment of dispersivity and other parameters. For the outwash at Otis Air Base assumed longitudinal dispersivity is 40 feet and transverse dispersivity is 13 feet.

Coonamessett and Ashumet Ponds act as drains to the groundwater flow system along which water levels are relatively constant. These ponds were specified as constant head boundaries in the solute transport model. This is accomplished by representing the boundaries as leakage nodes at which leakage is set to a high value (1.0 ft/s/ft). Leakage is the vertical hydraulic conductivity of the pond bottom divided by bed thickness.

The rate of areal recharge from precipitation was estimated by application of the Thornthwaite and Mather method to climatic data for Falmouth (Reference 48). The original recharge estimate, 21 in/yr, was adjusted downward to 19.8 in/yr during model calibration.

It must be kept in mind that this long-term type of contaminant problem is not best suited for the optimization model. The model is designed to predict optimal pumping strategies for smaller, emergency type groundwater contamination problems. In this particular problem the physical feasibility of having only 6 extraction wells to supply water to 24 injection wells would have to be addressed. In a more conventional emergency type problem the plume would not have extended so far down-gradient. An octagon more regular in shape could then be used.

2. Results

Table 6 summarizes the results of the deterministic model run. Eight weeks were needed to stabilize the plume. Pumping is needed during each week. Tight constraints are the upper water table limits at injection wells. The previous optimal strategies for the hypothetical contaminant problems allowed non-pumping days for "rebounding" of the hydraulic gradient; indicating that a shorter time period could be used in the optimal pumping strategy. This was demonstrated by varying the time from 8 days to 5 days with the Tmin/Tmax = 0.67 problem (runs 2d and 5d). All 8 weeks were used to pump in the Otis Air Base problem and reducing the time would not produce an optimal solution. However, a user could attempt a shorter time period by simply editing the input file.

The consequences of implementing these optimal pumping values and steady pumping values were then tested with the 2-D solute transport model. The input values to this model corresponded to those of the optimization model and additional parameters described in Section IV. The transmissivities used were those calculated using the resulting heads at the end of the pumping period. This produces the worst possible plume movement.

TABLE 6. OPTIMAL UNSTEADY AND STEADY PUMPING STRATEGIES FOR BORON PLUME AT OTIS AIR FORCE BASE

	Optimal pumping	Steady pumping			
	Pumping(L/s)	Pumping(L/s)			
Week 1	135.42(4.77)*	Well 1	-109.	Well 16	87.
2	118.36(4.17)	2	-109.	17	-87.
3	107.32(3.78)	3	-107.	18	-80.
4	99.21(3.50)	4	-106.	19	-97.
5	92.81(3.27)	5	-106.	20	-102.
6	87.56(3.09)	6	-106.	21	-104.
7	71.56(2.52)	7	-106.	22	-105.
8	23.83 0.84)	8	-104.	23	-105.
Avg gradient(%)	0.012	9	-99.	24	-106.
S.Dev. of gradient	0.008				
Sum of head differences squared (m^2)	99.6(1071)	10	-82.	25	-107.
		11	87.	26	-109.
Obj. function	1117.	12	101.	27	-110.
O & M costs ($\$ \times 10^3$)	46.00	13	106.	28	-111.
		14	106.	29	-110.
		15	101.	30	-111.
Aver. Pumping (absolute L/s)	92.0		102.2		

* values in parenthesis are corresponding English units in ft^3/s or ft^2. pumping is in cu.ft./s)

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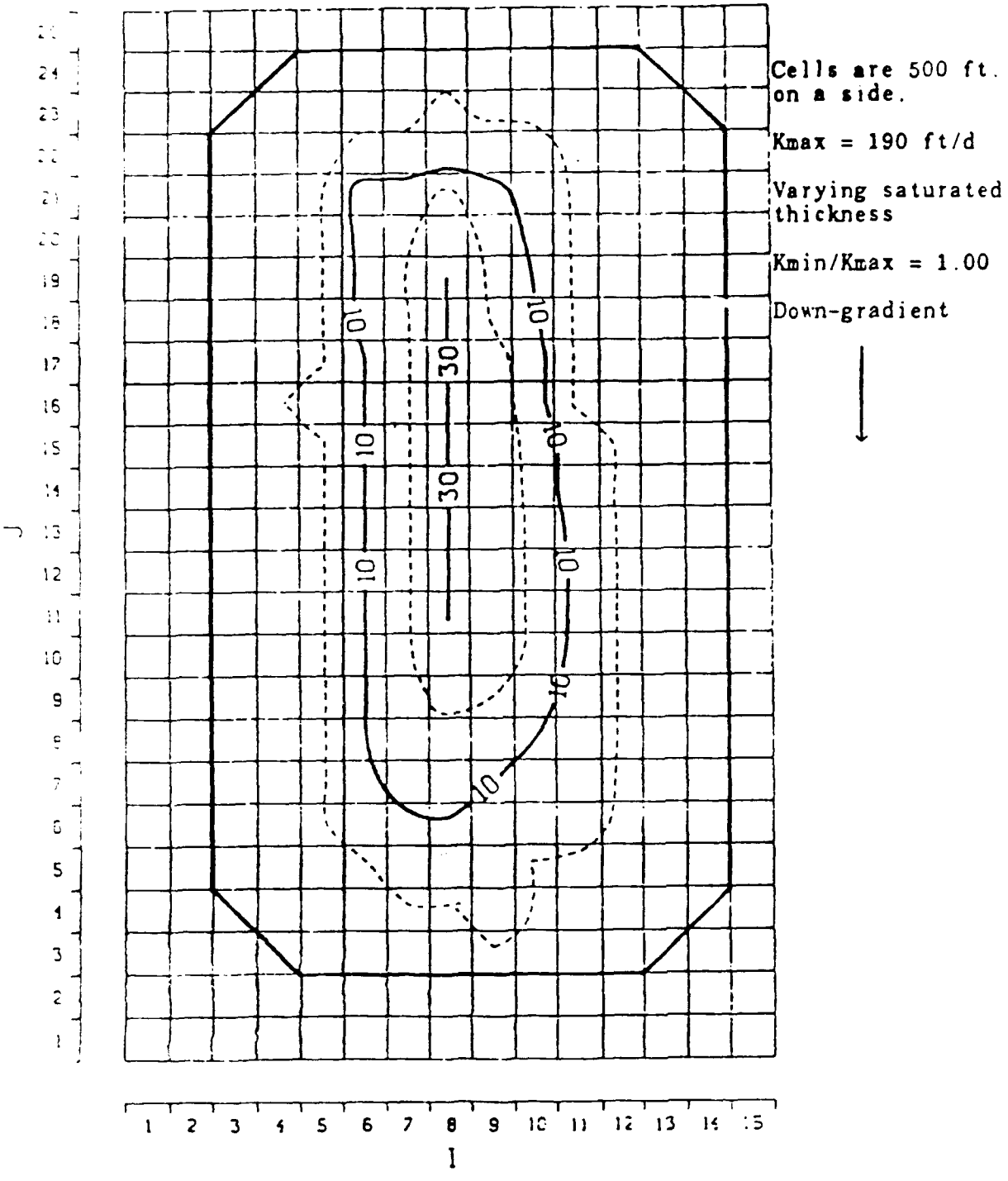


Figure 17. Resulting Concentrations After 40 Weeks if No Pumping Strategy is Implemented for the Otis AFB Plume

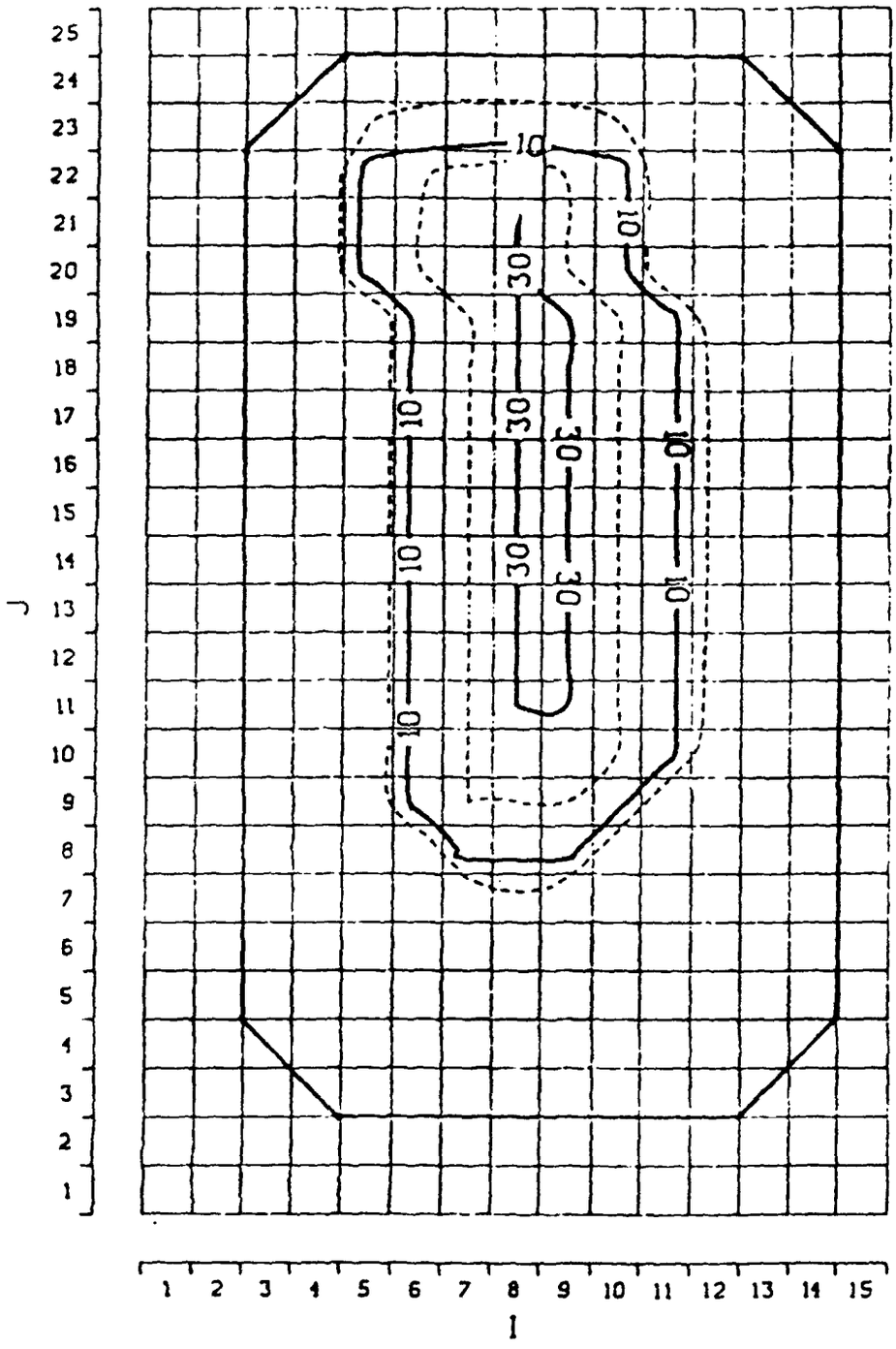


Figure 18. Resulting Concentrations After 8 Weeks of Optimal Unsteady Pumping for the Otis AFB Plume

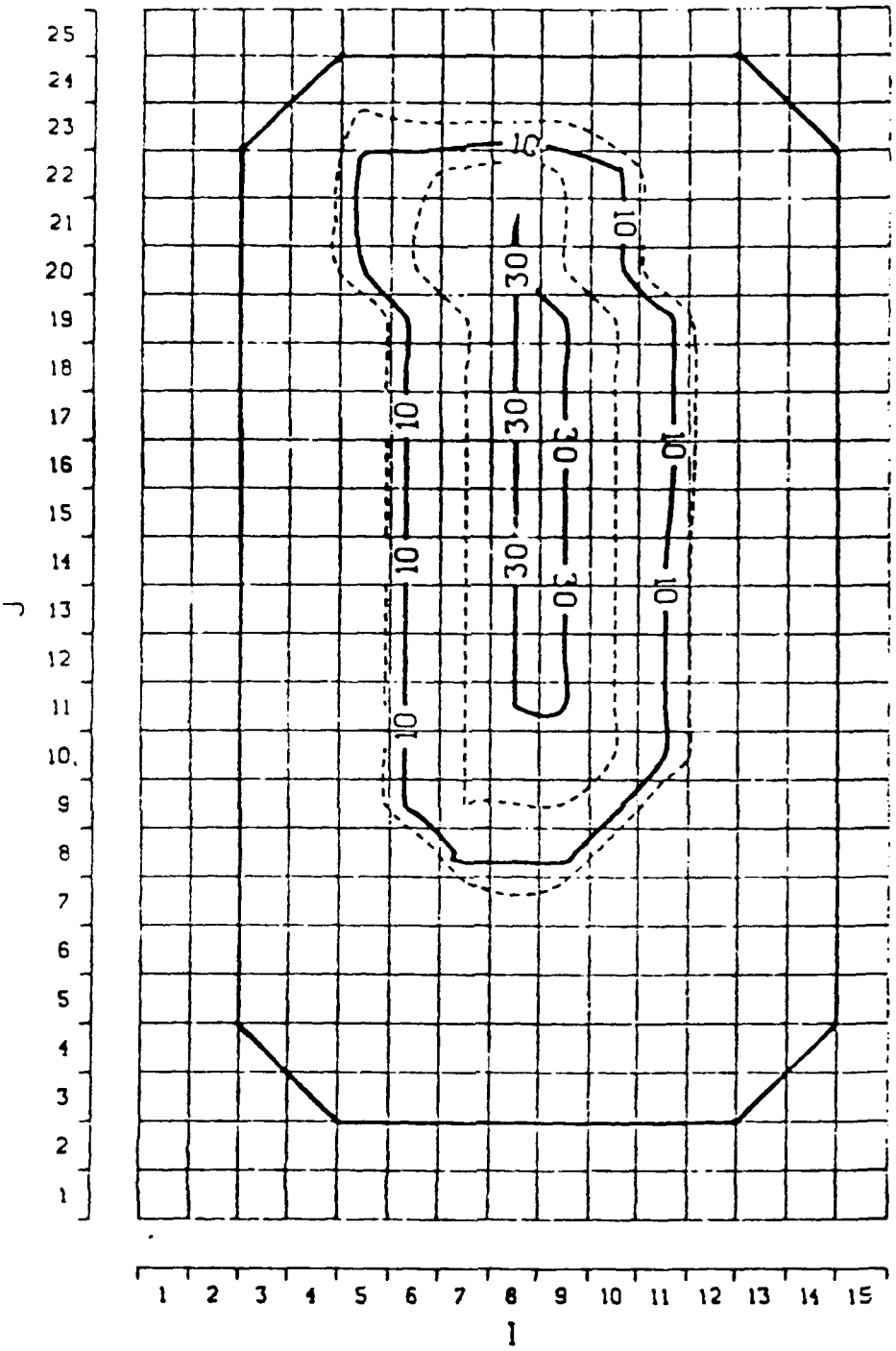


Figure 19. Resulting Concentrations After Week 40 Following 32 Weeks of Steady Pumping for the Otis AFB Plume.

TABLE 7. EFFECT OF AQUIFER PARAMETER UNCERTAINTY ON 95% RELIABLE OPTIMAL UNSTEADY PUMPING STRATEGY FOR HYPOTHETICAL PROBLEM (run 1d)

Run	1d	1s	2s	3s	4s	5s
	Pumping(L/s)					
Day 1	96.1	85.8	70.2	51.4	85.3	83.3
2	90.1	76.4	63.4	47.1	74.8	70.9
3	84.9	70.4	59.3	44.7	68.3	63.7
4	80.2	66.3	56.4	43.0	64.0	59.2
5	76.9	63.2	54.2	41.7	60.9	56.2
6	36.9	57.3	52.5	40.7	58.7	54.2
7	0.0	0.0	28.7	40.0	0.0	52.8
8	0.0	0.0	0.0	25.6	23.3	0.0
Aver. Pumping	58.14	52.42	48.1	41.77	54.4	55.0
Avg. gradient(%)	0.08	0.079	0.085	0.095	0.098	0.14
gradient SD	0.058	0.043	0.057	0.062	0.061	0.084
Sum of sqd. head diff.(m^2)	1.24	1.08	1.30	1.72	1.79	4.99
Obj. func.	15.63	13.54	15.66	19.82	21.18	55.53
O & M costs ($\$ \times 10^3$)	2.31	1.93	1.65	1.32	1.93	1.84

Model Run:
1d. Deterministic model

	Transmissivity CV	Effective porosity CV
1s.	0.2	0.2
2s.	0.4	0.2
3s.	0.8	0.2
4s.	0.2	0.4
5s.	0.2	0.8

TABLE 8.EFFECT OF AQUIFER PARAMETER UNCERTAINTY ON 80% RELIABLE OPTIMAL UNSTEADY PUMPING STRATEGY FOR HYPOTHETICAL PROBLEM (run 1d)

Run	1d	1s	2s	3s	4s	5s
	Pumping(L/s)					
Day 1	96.1	94.6	85.7	69.8	93.2	90.6
2	90.1	86.0	76.7	63.2	85.1	82.0
3	84.9	78.8	71.1	59.2	77.6	74.7
4	80.2	73.9	67.1	56.4	72.4	69.3
5	76.9	70.2	64.1	54.3	68.7	65.6
6	36.9	21.5	44.9	52.7	36.2	63.0
7	0.0	0.0	0.0	20.1	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	4.9
Aver. Pumping	58.1	53.1	51.2	47.0	54.1	56.3
Avg. gradient(%)	0.08	0.067	0.070	0.076	0.076	0.097
gradient SD	0.058	0.047	0.048	0.050	0.049	0.060
Sum of sqd. head diff.(m^2)	1.24	.77	.85	1.04	1.01	1.70
Obj. func.	15.63	10.37	11.03	12.80	12.89	20.36
O & M costs ($\$ \times 10^3$)	2.31	2.04	1.89	1.62	2.04	2.06

To analyze the predictability of these results consider the equation for the stochastic influence coefficient E (eq. 32) and reference Figure 3. Table 9 shows that, as reliability (ρ = F(z)) increases, z (which equals $F^{-1}[\rho]$) increases. Therefore, from equation (32) we see that, as reliability increases, E for the objective function and constraint 10 decreases and E for the drawdown constraint increases. In addition, as uncertainty of aquifer parameters increases (increasing CV), the standard deviation of the parameters increases ; thereby increasing the value of $\bar{D}$ (eq.31). In summary, an increase in uncertainty of aquifer parameters produces the same result as an increase in reliability; smaller E for the objective function and constraint 10 and larger E for the drawdown constraint.

As stated, for the drawdown constraints, increasing reliability or uncertainty of parameters produces a larger influence coefficient. This causes a greater reaction of the potentiometric surface to a unit of pumping. Therefore, this increase allows for less pumping during a unit of time because the upper bound on drawdown is reached more quickly. In the case of a reliability of .95 the $F^{-1}[.95]$ value (1.64) is equal to or larger than 95 percent of all $F^{-1}[\rho]$ values; thus the E value for a reliability of .95 is equal to or greater than 95 percent of E values for the same aquifer parameters. This confirms the stochastic constraint that the upper bound on drawdown will not be exceeded 95 percent of the time. Tables 7 and 8 reflect the trend of increasing reliability or increasing uncertainty of parameters and the resulting decrease in allowable pumping.

Why, then, does the pumping increase for the last time period or are there more time periods of pumping as reliability or CV increases? While the large coefficients are causing large head increases at the injection wells (thus restricting the amount of pumping) the small stochastic influence coefficients for the objective function and constraint 10 cause much smaller reaction of the potentiometric surface at the observation wells. Thus, lower pumping values caused by increasing the reliability or uncertainty have even a smaller effect on drawdown at the observation wells. Yet the goal is still to minimize the objective function. To do this, additional pumping periods are needed or more pumping is required during the last time period as reliability or uncertainty increases. This trend is shown in Tables 7 and 8. The objective function uses the large drawdowns at the pumping wells to calculate pumping costs; thus producing the highest costs. The objective function uses the small drawdowns at the observation wells to determine the differences in head; thus producing a large sum of head differences. Thus the objective function value is the largest possible for the input given and it should be the value calculated or less.

However, constraint 10, because it uses the smaller E values for the observation well head calculations, actually causes the hydraulic gradient to "overshoot" horizontal. The smaller E values produced at the .05 reliability level for observation well head calculations give

TABLE 9. **Standard Normal Deviate F(μ) Corresponding to the Reliability Function**

$$F(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{1}{2}t^2} dt$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.0429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.0981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.0987	0.9987	0.0987	0.9988	0.0988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

us a 95 percent confidence that the heads are those calculated (using these E values) or greater; thus causing the reverse gradient. Remembering that the final gradients are always reverse gradients, Tables 7 and 8 show that as reliability or uncertainty increase the final gradient is larger in the reverse direction. The confidence in the final gradient is further complicated by the fact that the target elevation (normally the head at the contaminant source) is itself stochastic. Therefore, the actual reliability of the final gradient would be something less than the specified value; but that reliability cannot be determined with precision.

Table 10 summarizes the trends that developed as uncertainty of aquifer parameters and reliability were systematically varied. As the coefficient of variation (CV) for transmissivity increases (runs 1s, 2s and 3s) the influence coefficients for the drawdown constraint increase and those for the objective function decrease. The expected result is decreased pumping for each time period (but larger total pumping), increased final average gradient and objective function value.

Runs 1s, 4s and 5s show the results of increasing the CV for the effective porosity while holding the transmissivity CV constant. The general trend for these runs is the same as those for runs 1s, 2s and 3s. The resulting gradient and objective function for runs 4s and 5s show a sharp increase from run 1s. The increased CV produces larger influence coefficients for the drawdown constraint and smaller coefficients for the objective function just as the increased CV for transmissivity does. However, the changes in these coefficients are small as compared to those produced by comparable increases in transmissivity CV; and cause only small differences in pumping between runs 1s, 4s and 5s. In comparison, the resulting gradient and objective function are much worse than those resulting from comparable tranmissivity changes in runs 2s and 3s.

To explain this difference (i.e. small increases in pumping, yet large increases in objective function and final gradient, for effective porosity CV increases as compared to large pumping decreases, yet small objective function and final gradient increases for comparable transmissivity CV increases) we look at the difference in sign between the $\bar{A}$ coefficients (equation 48) which are affected by changes in transmissivity CV and the $\bar{P}$ coefficients (equation 50) which are affected by changes in effective porosity COV. The negative sign with the $\bar{P}$ coefficient indicates it will affect the optimal strategy in an opposite manner than that of the $\bar{A}$ coefficient. As the CV of transmissivity is increased, there is a large change in pumping and a small change in gradient and objective function. For the same CV increase in effective porosity there is a small change in pumping and a large change in gradient and objective function. The two parameters (transmissivity and effective porosity) cause an opposite relationship between pumping and its effect on the objective function and the constraints.

TABLE 10. Summary of Trends Produced by Stochastic Analysis (hydraulic objective function)

Value affected	Increased reliability	Increased uncertainty in trans.	Increased uncertainty in eff. por.
1. Influence coef. used with:			
objec. func.	decrease	large decr.	small decr.
DD constraint	increase	large incr.	small incr.
2. Daily pumping	decreases	large decr.	small decr.
3. Total pumping	decreases	large decr.	small incr.
4. Gradient(reverse)	steeper & less smooth	steeper & less smooth	
5. Obj. func. value	increase	small incr.	large incr.

Table 8 displays results of the same variation in the CV of the two parameters, but at a reliability level of 0.80. As expected, the reduction in reliability increases the optimal pumping values and improves the final gradient and objective function. The smaller reliability produces smaller stochastic unit response coefficients. Resulting strategies and water levels are more similar to those from the deterministic model (reliability = 0.50) than are those developed using a 0.95 reliability.

Strategies for runs 5 and 6A have no pumping on day 7 and yet require pumping on day 8. This is a definite change in the overall pattern of the stochastically optimal pumping strategies. However, a look at the sensitivity values for the pumping during days 7 and 8 gives an indication that it is not a major change. The sensitivity value (amount the objective function would change with a unit increase in pumping during that day) associated with each pumping value for days 7 and 8 for those two runs are very small. For example, these sensitivities are in the range of 10^{-4} to 10^{-15} as compared to a sensitivity of 0.7 to 1.3 for the tight pumping value in most other runs. This indicates that the pumping for day 8 could also be 0 without any significant change in the objective function. Therefore, the 0 pumping for day 7 and a pumping value for day 8 of runs 5 and 6A could be 0 pumping for both days 7 and 8 without a dramatic change in the overall pattern of the results.

Comparisons to Tung's (Reference 42) analysis are difficult to make because his objective function was to maximize pumping which is not effected by the stochastic influence coefficient. The only constraint was on drawdown. In addition, the Cooper-Jacob equation (which is only appropriate for small values of the Boltzman variable; $u \leq 0.01$) used to derive the stochastic unit influence coefficient shows P to be equal to 0 except for the first time period. However, the general trends Tung speaks of concerning transmissivity apply to this analysis: (1) Increased pumping as reliability or CV decreases and (2) Uncertainty of transmissivity causes a larger change in pumping than does a comparable change in effective porosity. However, this study indicates effective porosity has an effect on the drawdown at the observation wells (something Tung considers negligible) and hence has an effect on the objective function value. In addition, the daily pumping increases with decreasing effective porosity CV but, at the same time, the total pumping decreases.

Table 10 summarizes the trends shown in this analysis.

SECTION V

USE OF THE EXPERT SYSTEM

An expert system has been developed which can be used as a preprocessor for a groundwater management model. The management model optimizes pumping to provide hydrodynamic control of a contaminant plume. With this expert system three methods of groundwater contaminant plume containment can be analyzed. They are bentonite slurry wall, steel sheet piling and extraction/injection pumping. One other method that is becoming more popular is a grout curtain. That method is not considered in this system because its costs are approximately the same as a slurry wall but it requires specialized equipment not usually available.

Before a model is run to determine the optimal pumping values for hydrodynamic control, one should determine whether a pumping well strategy is the most economical alternative. This expert system systematically analyzes a contamination problem by querying the user. The expert system asks the user for pertinent information about the contamination site, the aquifer, and the contaminant. Based on capital costs, the most economical containment method is determined. If the pumping strategy is selected, the system estimates operation and maintenance costs and determines how long the pumping strategy will remain the most economical. Sometimes the expert system will make an assumption if the user lacks knowledge about an input. However, for most questions a definite response is required. Therefore, it is recommended that before this program is run, the user compile as much information as possible about the aquifer (soil type, hydraulic gradient, depth and saturated thickness), the site (ground slope, precipitation, drainage) and the chemical makeup of the contaminant. Also needed is information on the available pumping plants (head vs pumping capacity curves) and time frame (how long before a pumping system can be in place, how much time exists before the plume must be stabilized, and how long the plume will need to remain stabilized).

The following discussion describes the procedure. Directions for loading the expert system and linking to the simulation model are found later in this section. Figure 20 is a flowchart of the user's options as he progresses through the expert system and/or the simulation model. Figure 21 shows the use of the provided programs and the files that are produced by running these programs. Figure 22 provides a flow chart of the questions and logic of the expert system. Appendix IV provides a simulated run of the expert system. This should be referred to when reading subsequent discussion.

A. LOGIC FLOW

The expert system first explains that it is analyzing three possible containment methods; slurry trench, sheet piling and pumping. It assumes that the physical system for each method would be octagonal in shape and would be centered on the assumed point source of the contaminant. An octagonal system of pumping wells would completely encircle the plume. The other two containment methods would only require

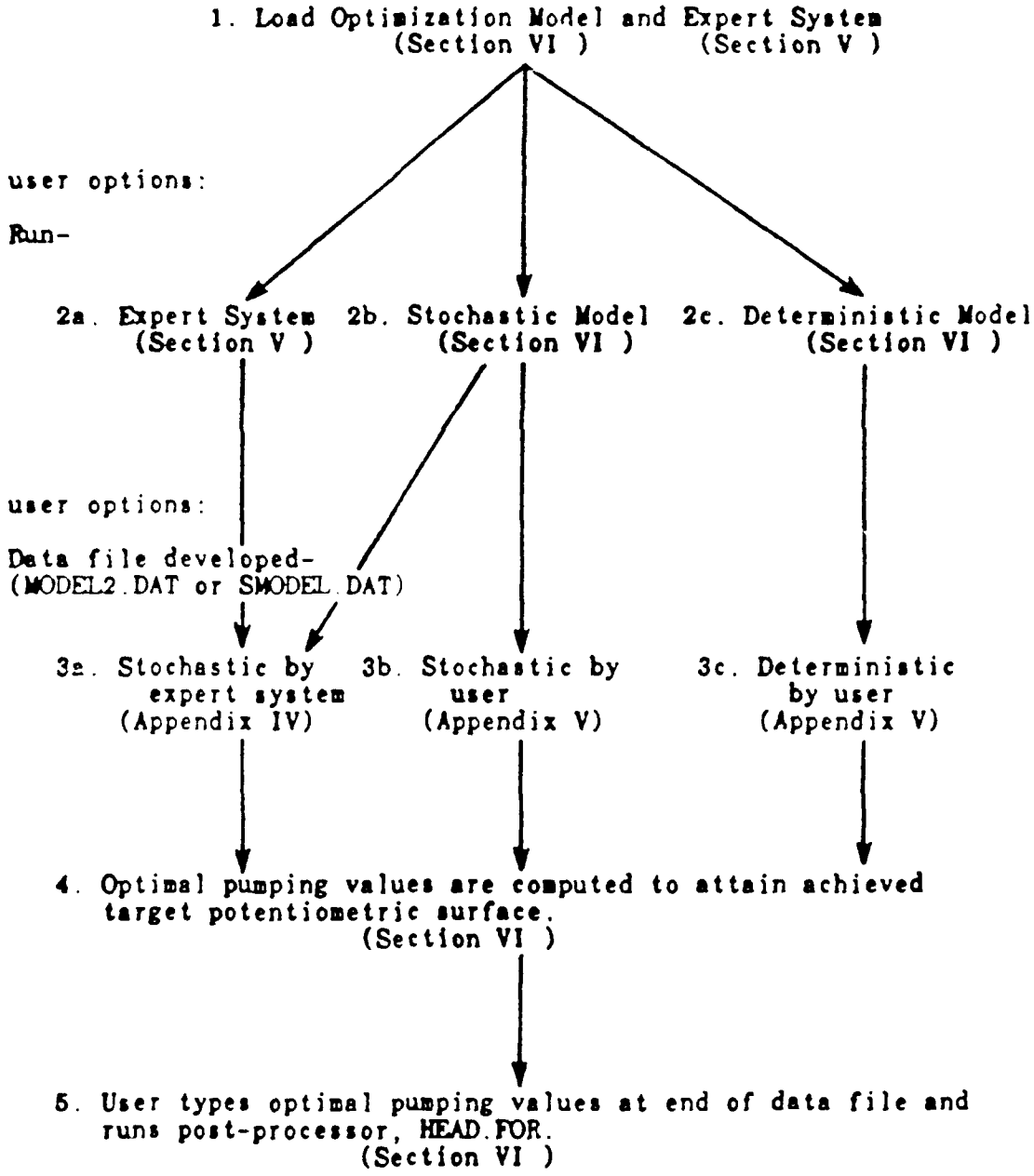
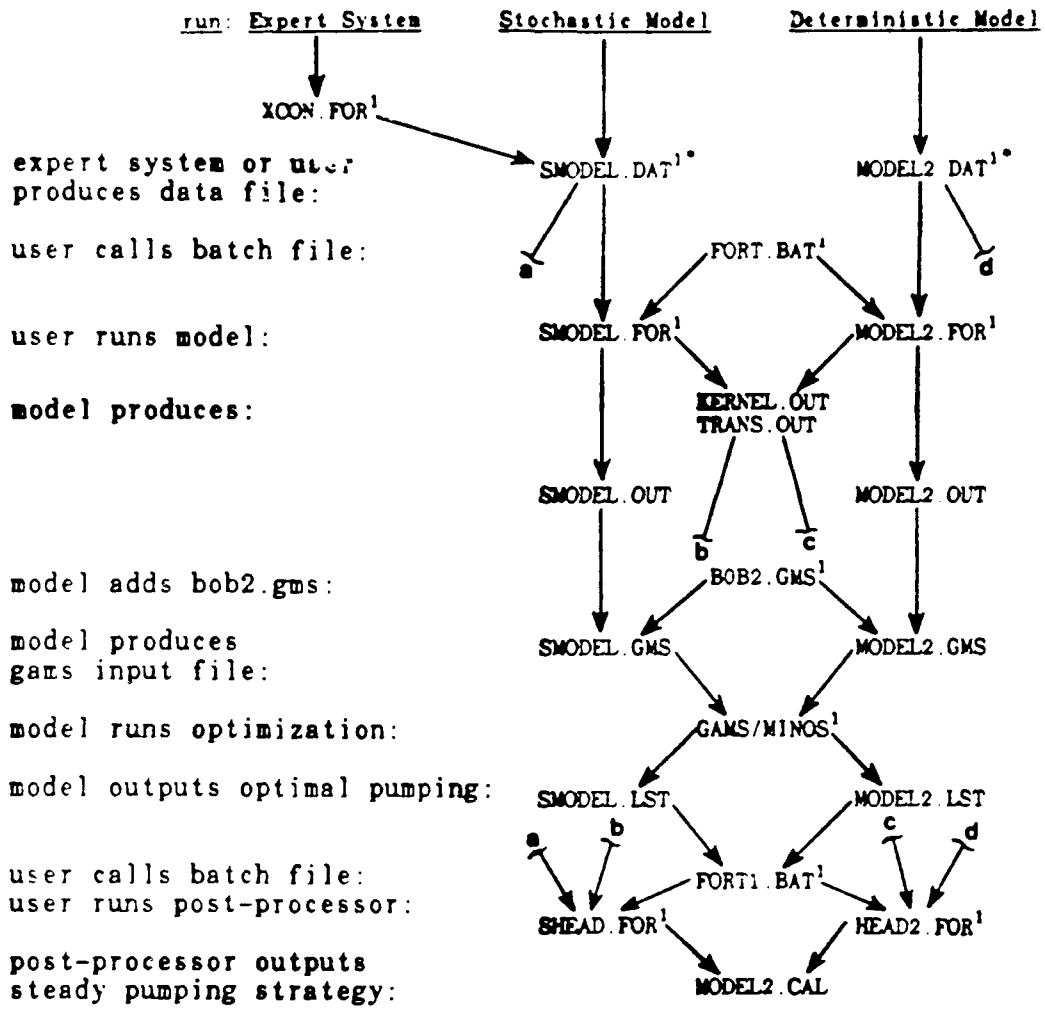


Figure 20. Flow Chart of User Options



[1]Programs provided on diskettes. All others are files developed during the running of programs.

*Data files are provided for demonstration only. User will need to use problem specific files for each contamination site.

Figure 21. Flow Chart of Programs Used and Files Developed

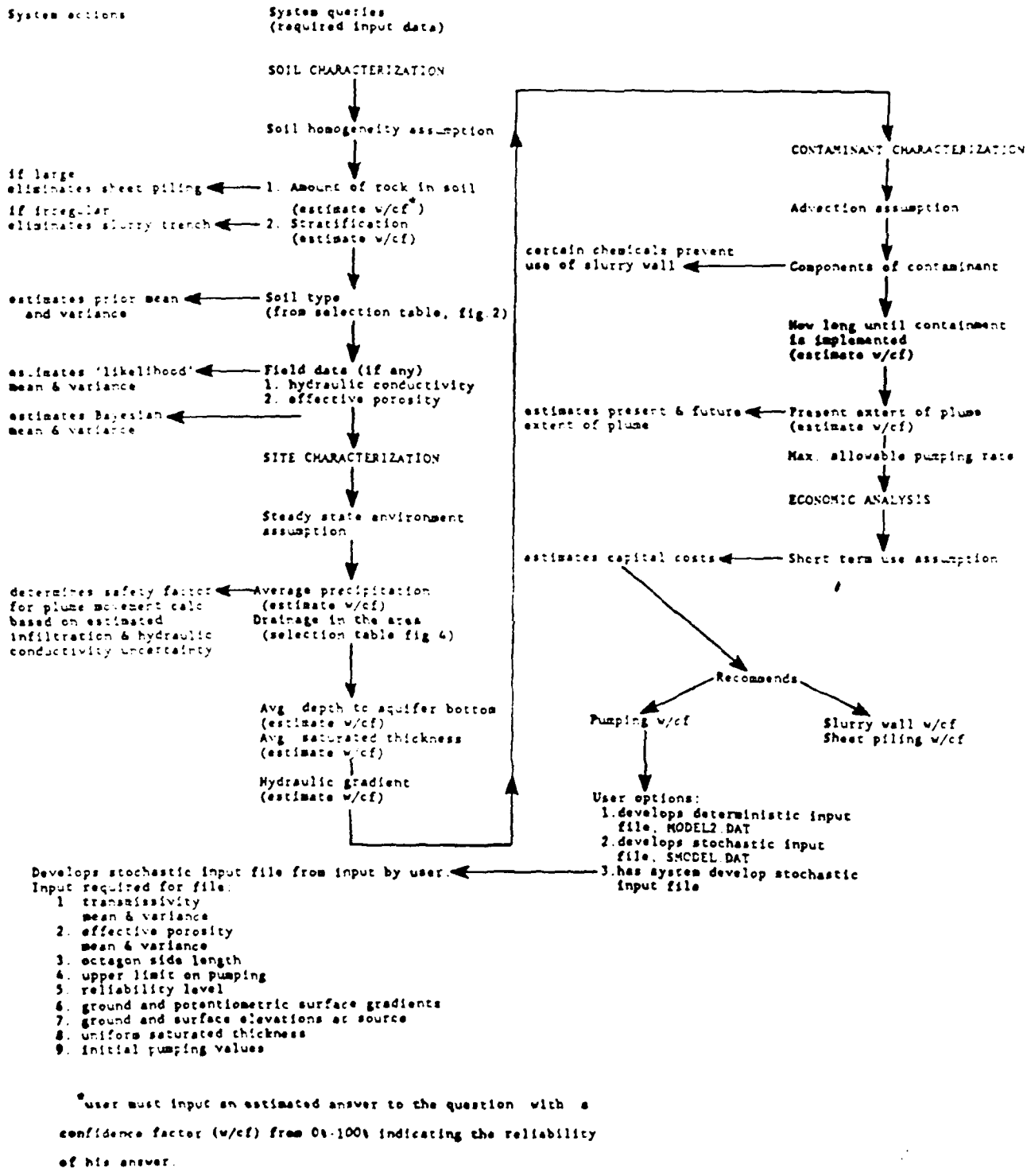


Figure 22. Flow Chart of Expert System

installation on the 5 sides of the octagon down-gradient from the contaminant source.

1. Soil Characterization

The first step in completing a comprehensive site evaluation is to characterize existing soil conditions. The system asks if the user understands the transport model assumption of soil homogeneity. If the user answers "no", "why", or "unknown", the system responds with a brief explanation and will ask the user if the assumption has been learned. If the user still does not understand, the system will repeat the same explanation. It makes no effort to clarify its explanation.

Without letting the user know, the expert system will lower its overall confidence in the consultation at appropriate times. These include each time the user: 1) does not understand a basic model assumption after the first time he is asked (reduction of 0.01 or 1% in confidence level), 2) needs aid in estimating input parameters (reduction of 0.01), and 3) the user has no field data for either hydraulic conductivity or effective porosity (reduction of .03 for each). Similarly, a human expert would most likely lower confidence in a consultation if a client did not demonstrate a basic understanding or provide exact information. The system starts with the smallest individual confidence factor given by the user as he enters data requested by the system. This approach is followed because the system can be no more confident in its recommendation than the user is in his least reliable data. The system then adjusts this confidence based on user responses as described previously. In short, the less a user knows about a given situation, the less confidence the system has in its recommendation for containing a contaminant plume.

Once the user understands the homogeneity assumption, the system asks the user for soil parameters. The first questions concern the amount of rock in the soil and the condition (stratification) of the interface between the aquifer material and the bedrock. The answers to these questions determine whether sheet piling or a slurry wall are viable alternatives for plume containment. If "unknown" is given as the answer to either of these questions the system assumes that particular method is a viable alternative (but simultaneously lowers the overall confidence). The user is then asked to select a soil type that best describes the soil of the aquifer from a selection table (Figure 23). Using this soil type, the system estimates ranges of effective porosity and hydraulic conductivity from a soil fact database (Figure 24).

The stochastic version of the optimization model requires a mean and variance for both transmissivity and effective porosity. The expert system computes these from a posterior probability distribution function (pdf). The pdf is computed using Bayesian theory, prior knowledge of what the pdf should be and, if current information is available, a "likelihood" distribution based on this current information. Bayes theorem states:

Soil Type	% clay	% sand	% silt
1. sand	<10%	>90%	>90%
2. sandy-loam	<20%	>85%	50-70%
3. sandy-clay	35-55%	60-85%	50-65%
4. silty-clay	40-60%	20-40%	40-60%
5. clay	>40%	30-75%	<60%
6. loam	5-25%	40-60%	75-95%

Figure 23. Soil Type Selection Table

Soil Type	Hydraulic Conductivity(ft/d)	Effective Porosity
sand	.26-1873	.13-.40
sandy-loam	.160-820	.16-.46
sandy-clay	.003-3.28	.01-.39
silty-clay	$(2.5\text{-}1970)10^{-3}$	.01-.28
clay	$(3.3\text{-}1300)10^{-6}$	.01-.46
loam	.066-52.5	.01-.46

Figure 24. Soil Fact Database

Drainage Class	Observable action
1.Very poorly drained	Water remains at or on the surface most of the year
2.Poorly drained	Water remains at or on the surface much of the year
3.Somewhat poorly drained	Soils are wet for significant portions of the year
4.Moderately well drained	Soils are seasonably wet (high spring water table)
5.Well drained	Water readily removed from the soil
6.Somewhat excessively	Water is rapidly removed from the soil (e.g. uniform drained sands)
7.Excessively drained	Very rapid removal of water, little or no retention

Figure 25. Drainage Selection Table

posterior pdf $\propto$ prior pdf x likelihood pdf

The expert system can manage three different situations: 1) the complete lack of field or lab data, 2) three or fewer field or lab values for each parameter, and 3) four or more field or lab values for each parameter. The upper limit of 4 field or lab values is purely arbitrary.

If no field or lab data is available the posterior pdf used by the optimization program is the prior derived pdf from the data of Fig. 24. The expert system bases its prior mean and standard deviation on the range of values in the soil fact database. This range is assumed to equal the mean ± 3 standard deviations. With this assumption the system calculates a mean (Xo) and standard deviation (Vo) based on the natural log values for the extremes of log normally distributed hydraulic conductivity and on the actual values for normally distributed effective porosity.

Field data values for hydraulic conductivity and effective porosity are then requested. If there are 4 or more field data values for these aquifer parameters, the "likelihood" pdf of Bayes theorem is developed using the mean (X) and standard deviation (V) of the field data values. Again, the natural log values are used for hydraulic conductivity and the actual values are used for effective porosity. Subsequently, this is the posterior pdf given to the optimization program. The prior pdf developed from the soil type is ignored.

If the "likelihood" mean for hydraulic conductivity (developed from the field data) is more than 3 "prior" standard deviations from the "prior" mean hydraulic conductivity (developed from the soil type) the user is warned that this seems to be contradictory information. The user is then given the option to change the soil type, change the field data values or simply continue with the program. Stochastic model simulations have shown that uncertainty of effective porosity does not have as large an effect on the optimal pumping strategy as does hydraulic conductivity. Therefore, the mean effective porosity from the field values is not compared to the mean effective porosity of the soil type. It is used as input by the user.

If there are less than 4 field values for these parameters, the likelihood pdf and prior pdf are multiplied together. (If only one value is given for a particular parameter the likelihood standard deviation is assumed to equal the prior standard deviation.) Multiplication of a normally distributed likelihood pdf by a normally distributed prior pdf has been previously demonstrated (Reference 49). The resulting formulas for computing the mean and variance of effective porosity and the mean and variance of the natural log values of transmissivity for the optimization program are:

Posterior mean

$$E(\ ) = \frac{1}{Vo^{-2}+V^{-2}}\left[Vo^{-2}Xo + V^{-2}X\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (33a)$$

Posterior variance

$$VAR(\ ) = \left[Vo^{-2} + V^{-2}\right]^{-1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (33b)$$

The expected value, E, and variance, VAR, for effective porosity are used directly in the stochastic optimization model. Because the natural log values have been used to determine E and VAR for transmissivity these values are actually the expected value and variance for the natural log values and not the actual values. Therefore, standard equations to determine the mean and variance of a log normally distributed parameter are used. E and VAR are the expected value and variance of its normally distributed natural log values (Reference 50). The equations used are:

$$\text{mean} = \exp\left[E + \frac{(VAR)^{-2}}{2}\right] \quad \ldots\ldots\ldots\ldots\ldots\ldots (34a)$$

$$\text{variance} = \left\{\exp\left[(VAR)^2 + 2E\right]\right\}\left\{\exp\left[(VAR)^2\right] - 1\right\} \quad \ldots\ldots\ldots (34b)$$

The equations are based on the assumption that the expected value and variance are for the entire population of transmissivity. This assumption is valid since the data used as the prior knowledge for each soil type is obtained from a very large set of information.

The user is required to specify a soil type. However, he might be much more confident in his field data (even though he has less than 4 values) than he is in the specific soil type. By repeating some of the field data values so that at least 4 values are input the program will ignore the soil type and will recommend, to the optimization program, the mean and standard error of the field data values. It should be understood that, if the posterior mean for the hydraulic conductivity is less than 0.002 ft/d, the pumping strategy is not considered a viable solution and therefore no economic analysis is performed for pumping.

2. Site Characterization

Once soil characterization is accomplished, the system asks questions to characterize the site environment. The system establishes whether the user understands the simplifying assumption of a steady state environment (that all conditions such as precipitation are assumed constant over the entire planning period) and that no other remedial action (such as a clay cap) has been attempted. If he does not, a brief explanation is given.

The system requests the average monthly precipitation in the contaminated area during the planning period. The user must input a value for this parameter since it will not be estimated by the expert system. The user is then asked to describe the study area drainage from a list of drainage classes (Figure 25). Precipitation and drainage input, along with the coefficient of variation (CV) for hydraulic conductivity, are used to estimate a safety factor. This safety factor is used in the calcualtion of the farthest extent of the plume at the present time. It is also used to estimate the additional distance the plume will travel before a containment strategy is implemented.

Extensive model simulations of hypothetical contamination problems have been performed to determine the effect of precipitation infiltration and the effect of uncertainty of hydraulic conductivity on plume movement. During all simulations a dispersivity value of 100 ft. was used. A safety factor has been developed incorporating infiltration and hydraulic conductivity uncertainty in its determination. This safety factor is used by the program to estimate future plume extent. This insures that the containment octagon is outside the limits of the plume at the time of containment strategy implementation. Model simulations have demonstrated that precipitation, drainage and hydraulic conductivity uncertainty (measured by its coefficient of variation--which equals standard error/ln[mean]) are the best indicators of the need for a larger safety factor in calculating plume movement.

The coefficient of variation is used directly as an addition to the nominal safety factor of 1.00 (i.e. if CV equals 0.43 the safety factor is 1.43). The precipitation range and drainage class selected by the user determines any additional increase in the safety factor. Increases range from 0.0 to 0.04 in increments of 0.02 for increasing precipitation ranges and from 0.0 to 0.03 in increments of 0.005 for the drainage classes ("very poorly drained" increases the safety factor by .03). It was found, however, that a safety factor should never be greater than 2.00 because safety factors greater than 2 produced octagons much larger than needed, no matter how large the coefficient of variation for hydraulic conductivity.

The system then asks for the average depth to the base of the aquifer, the average saturated thickness of the aquifer and the average hydraulic gradient (all three must have a confidence factor associated with them). These values are used to estimate plume movement and make economic comparisons between strategies.

3. Contaminant Characterization

The third and final knowledge base module characterizes the contaminant. The system queries whether the user understands the assumption that water is the contaminant carrier and that advection is the major mechanism of contaminant movement. The system asks what the pollutant is. If certain chemical compounds are specified (alcohol, hydrochloric acid, certain hydroxides, etc.) a bentonite slurry wall

is eliminated as a possible containment strategy. The permeability of a slurry wall may increase by a factor of 10 if any of these chemicals come in contact with it.

The user is asked to estimate the number of days until a containment strategy will be implemented (with a confidence factor). The farthest extent of the plume at the current time is then requested (assuming a point contaminant source). Next, the system estimates what the extent of the plume will be at the specified future time. It uses the current extent of the plume, hydraulic gradient and conductivity, the time until the containment strategy will be implemented and the safety factor.

The expert system assumes that contaminant spillage ceased prior to the current time. Future versions of the system may assume that contaminant is still entering the aquifer. In such case additional information will assist evaluating possible remediation strategies. Pertinent questions might include:

1. What total volume of contaminant has entered the aquifer?
2. Is contaminant still entering the aquifer?
3. At what rate is contaminant entering the aquifer?

4. Economic Analysis

By this point the system has eliminated containment methods that are inappropriate (because of irregular aquifer-bedrock interface, large volume of rock in the soil, too low of a hydraulic conductivity). It is conceiveable that none of the three containment methods are viable because of a particular sequence of user input. If this happens, the system informs the user, explains why none of the strategies are practical and terminates the program. Otherwise, the system informs the user it is assuming the possible use of suitable containment methods for only a short time period. Therefore, only capital costs are considered in this preliminary analysis.

Capital costs are based primarily on the extent of the plume (in 2 horizontal dimensions) and the depth to the bottom of the aquifer. Unit costs used in the analysis are based on federal estimates (Reference 51). Before the economic analysis is performed the user is told that the unit costs are based on 1986 prices (already updated from the reference). He must input a coefficient to convert these costs to whatever year is applicable. Comparisons are made between slurry walls, sheet piling and pumping (if all three are still acceptable approaches). These cost estimates are cursory estimates and include simple assumptions of pump spacing and size. If pumping is determined to be the most economical remedy based on capital costs the expert system calculates the length of time the pumping strategy can continue before the operation and maintenance costs exceed the additional cost of the next least costly strategy. These computations are based on the following assumptions:

1. Pumping is at the upper limit specified by the user for entire period,

2. Pumping lift is at the maximum allowable and corresponds to that which will leave only 1/2 the initial saturated thickness,
3. Pumps are replaced every 10 years,
4. Operating costs are $\$4.13(10^{-6})/ft^3/ft$ and increase by 1.5 times every 10 years and
5. Maintenance costs are $\$3.79(10^{-5})/ft^3$ and triple from beginning to end of each 10 year pump life-span.

B. DEVELOPMENT OF INPUT FILE, SMODEL.DAT

At this point, the user has the option to run the deterministic model or the stochastic model.

He may develop the input file for the deterministic model (MODEL2.DAT) or the stochastic model (SMODEL.DAT). The expert system suggests data that can be used for SMODEL.DAT (much of which can also be used with MODEL2.DAT). Alternately, upon request, the expert system will develop an SMODEL.DAT data file directly based on the following crude assumptions:

1. The previously calculated mean and coefficient of variation for transmissivity and effective porosity.
2. A configuration of 1 foot radius wells shaped into a regular octagon which is centered on the contaminant source. The wells are located at the one-fourth and three-fourths points of the sides of the octagon.
3. A previously input constant saturated thickness.
4. Ground elevations for each pumping well calculated from user input of a slope, the angle the direction of this slope makes with the x-axis (which is determined by the direction of the hydraulic gradient) and the ground elevation at the contaminant source.
5. Potentiometric surface elevations at all wells calculated from user input of the potentiometric surface elevation at the source and the previously input hydraulic gradient.
6. User input of estimated initial pumping values for the stochastic model to use in its iteration process. From our testing experience, the magnitude of these pumping values is not important as long as they are greater than zero and less than the upper limit on pumping.

C. LOADING THE SYSTEM

Before the following steps are performed the optimization model should be set up (Section VI). The expert system program should be located in the same subdirectory as MODEL2.FOR and SMODEL.FOR (i.e. BW). It should be run before either version of the optimization program is run.

The expert system program is begun by batch file EXP.BAT by typing EXP XCON NO (or YES). The first time this program is run on a particular computer it needs to be compiled and linked. Therefore, the

last word typed should be YES (to signify; yes the program needs to be compiled and linked). However, after the first run of the program, unless the listing file (XCON.FOR) is changed in some way, NO should be the last word typed so the program will immediately begin to run.

STEP	TYPED COMMAND(in all caps)
1. Load the optimization model (Section VI)	
2. If in root directory C: put yourself in subdirectory.	BW
3. The prompt should now read C:>BW	
4. *Copy the expert system, XCON.FOR, and the batch file, EXP.BAT, into the subdirectory (the floppy disk with the programs on it should be in a: drive).	COPY A:XCON.FOR COPY A:EXP.BAT
5. Run the expert system	EXP XCON NO (or YES)

* This instruction is repeated in Section VI under loading the optimization model.

NOTE: The batch file running the expert system erases any previous SMODEL.DAT file. If the user wishes to save any previous stochastic data file named SMODEL.DAT it needs to have its name changed by typing:

REN SMODEL.DAT NEWNAME

D. SYSTEM EXAMPLE

A complete validation process is the most important step in building a viable expert system. Unfortunately, it is the most difficult. Ideally, one uses documented field contamination problems to compare what the expert system recommends with what was done in the field or with what an "expert" recommended.

To date, the expert system has been tested on a hypothetical situation previously used to test the optimization program (Reference 16). The expert system run for the hypothetical situation run 2s (as described in Chapter 4 and Table 7) is shown in Appendix IV.

The final portion of Appendix IV is the program listing of the expert system developing the input file for the hypothetical contamination problem. The input file developed by the simulation in Appendix IV is very similar to Program 12. The potentiometric surface elevations will not agree exactly because the hydraulic gradient for the original problem was not constant as it has to be with the expert system problem. This causes the optimal pumpings to be slightly different from those shown in run 2s of Table 7. With the given input values the optimization program will determine the most economical pumping scheme to attain as nearly a horizontal gradient as possible within the 8 day time period specified by the user.

SECTION VI

USE OF THE OPTIMIZATION MODEL

The optimization model is used to determine optimal pumping values needed to produce hydrodynamic stabilization of a groundwater contaminant plume. This model can be used without using the expert system. In that case, the user will develop the input file himself (either SMODEL.DAT or MODEL2.DAT). The expert system can be used as a pre-processor for the stochastic version of the simulation model. The expert system will make suggestions of data to input into SMODEL.DAT or, if requested by the user, the expert system will develop a stochastic input file based on responses given the system by the user. This input file developed by the expert system is only for a simplified problem assuming a regular octagon with two 1 ft. radius wells per side, constant saturated thickness, uniform ground slope and uniform potentiometric surface slope.

This model requires a set of aquifer parameters and a pumping well layout. The model objective is to determine the optimal pumping required to stabilize a contaminant plume within a specified time frame. The model stabilizes the plume by reversing the hydraulic gradient and forming, as nearly as possible, a horizontal potentiometric surface around the contaminant source. 'Optimal' pumping can be defined as the most economic pumping value required or it can be the pumping that produces the best gradient or it can be the smallest volume of pumping needed to stabilize the plume. This depends on what part of the objective function is emphasized. The model simulates the reaction of the potentiometric surface to point stimulus (pumping). It uses unit response functions derived from the Theis well function for unsteady flow in a confined aquifer. Depending on the knowledge base of the aquifer parameters, a deterministic version and/or a stochastic version may be run.

A. PARTS OF THE MODEL

The three major model components, and their functions, are as follow:

1a. MODEL2.FOR (Program 8)

This FORTRAN program is used to prepare for running the deterministic version of the optimization program. To provide all data necessary for the optimization program it does the following, in order:

a. Reads input data from file MODEL2.DAT
b. Calculates x and y coordinates of all pumping wells and observation wells.
c. Calculates all transmissivities and stores them in file TRANS.OUT for use by the post-processor.
d. Calculates unit influence coefficients.
e. Sums the influence coefficients describing the effect of pumping at all pumping wells on head at each well. These are stored in file KERNEL.OUT for use by the post-processor.

f. Calculates the matrix coefficients (derived in Chapter 3 and Appendix I) needed for the objective function and the constraints in the optimization program.
g. Develops a file called MODEL2.OUT, containing, in GAMS format, all data required by the optimization program.

1b. SMODEL.FOR (Program 9)

This FORTRAN program is used to prepare for running the stochastic version of the optimization program. This program performs the same 7 functions as the deterministic preprocessor described above, however, it reads data from SMODEL.DAT. Instead of calculating transmissivities in step 3 it uses the mean and standard error of transmissivity and effective porosity to calculate coefficients $\bar{A}$ and $\bar{P}$ (equations 48 and 50 respectively). The coefficient derivations are presented in Appendices II and III. In step 4 this program calculates stochastic unit influence coefficients (E, equation 32) based on the uncertainty coefficients from step 3. The output file generated is called SMODEL.OUT.

2. GAMS-MINOS (ver. 2.04)

This program contains MINOS, a nonlinear optimization algorithm developed at Stanford (1983). It is linked with GAMS, a processor developed by the World Bank (1986) to facilitate use of MINOS and other optimization algorithms. GAMS reads the data in the format prepared by MODEL2.FOR or SMODEL.FOR. Data is converted by GAMS into standard MPS (mathematical programming system) format as required by MINOS. MINOS iteratively computes the optimal pumping values. MODEL2.LST or SMODEL.LST is the output file from MINOS corresponding to the deterministic and stochastic versions respectively. This file contains error messages if the program did not run to completion or it contains optimal pumping values and constraint values if a feasible solution is found. Tight constraints are those that have a value in the "marginal" column. These are the sensitivity values for the tight constraints. Tight constraints and sensitivity values are discussed in Section 3 of Volume I.

3. HEAD2.FOR or SHEAD.FOR (Program 11)

This FORTRAN program is the post-processor for the optimization program. HEAD2.FOR is used with the deterministic model and SHEAD.FOR is used with the stochastic model. The only difference between the listing of the two programs is that HEAD2.FOR reads data from MODEL2.DAT and SHEAD.FOR reads data from SMODEL.DAT. It uses optimal pumping values determined by MINOS, along with data from files KERNEL.OUT and TRANS.OUT to perform the following:

a. Reads input data from file MODEL2.DAT (or SMODEL.DAT) which now also includes the optimal pumping values entered by the user.
b. Reads the calculated transmissivities from file TRANS.OUT if the deterministic model is used or the mean transmissivity directly from the input file SMODEL.DAT if the stochastic model is used. It then reads the calculated influence coefficients

from KERNEL.OUT (these are either stochastic coefficients or deterministic depending on which model is run).

c. Calculates potentiometric surface elevations resulting at the observation wells from optimal pumping at the extraction/injection wells.

d. Calculates the steady pumping required to maintain the potentiometric surface produced by the optimal pumping. The method of computation is described in Section 3.4 of Volume 1.

e. Outputs optimal pumping values, steady pumping values and resulting potentiometric surface elevations into file MODEL2.CAL.

There are other files essential for easy running of the model. These are described briefly below:

4. BW.BAT (Program 1)

A batch file in the root directory that transfers the user to his subdirectory BW.

5. FORT2.BAT (Program 2)

A batch file in the subdirectory GAMSLIB that, when activated, directs the model to perform GAMS.BAT.

6. GM.BAT (Program 3)

A batch file in the root directory that, when activated, transfers the user to subdirectory GAMSLIB.

7. GAMS.BAT (Program 4)

A batch file in the subdirectory GAMS2.04 that runs the GAMS-MINOS optimization program.

8. FORT.BAT (Program 5)

A batch file in the user's subdirectory BW that runs model MODEL2.FOR or SMODEL.FOR.

9. FORT1.BAT (Program 6)

A batch file in the user's subdirectory that runs program HEAD2.FOR (or SHEAD.FOR).

10. BOB2.GMS (Program 10)

The portion of the MODEL2.GMS or SMODEL.GMS file that never changes. It is merged with MODEL2.OUT or SMODEL.OUT to form the .GMS file. The Q.L(T,J) value is the starting value for pumping used by the optimization program in the iteration process. It must be a value between the upper and lower bounds on pumping.

11. MODEL2.DAT (Program 11)

Input data file needed by the deterministic model.

12. SMODEL.DAT (Program 12)

Input data file needed by the stochastic model.

13. AUTOEXEC.BAT

Should include any DOS commands the user wants the computer to perform each time a user DOS command is given. This would include all paths and subdirectories the user wants the computer to search every time a command is given. An explanation of this file is found in any IBM DOS manual under batch file.

B. SETTING UP THE MODEL

Model set-up requires an IBM AT with internal hard disk, at least one floppy disk drive (drive A), 640K bytes of RAM and a math co-processor. PROFESSIONAL FORTRAN (an IBM product) is needed in directory, C:\BW, set-up below. The file PROFORT.LIB should be in the root directory C:. The EXEIS system diskette, two GAMS diskette and a MINOS diskette are provided. The following procedure is used:

STEP	TYPED COMMAND
While in the root directory C:	
1. Create subdirectory GAMS2.04.	MD GAMS2.04
2. Create subdirectory GAMSLIB.	MD GAMSLIB
3. Create your subdirectory for the models (BW is used as an example)	MD BW
4. Create a new path in the AUTOEXEC .BAT file to find the subdirectory GAMS2.04. This requires editing AUTOEXEC. BAT which should be found on all micros. To do this add the line:	C:\GAMS2.04
5. If necessary, edit the CONFIG .SYS file to reflect these minimums (values can be larger).	BUFFERS=10 FILES=16
Insert EXEIS system diskette in drive a:	
6. Create a batch file similar to BW .BAT (if the subdirectory is called something other than BW change BW to the new name in all other programs).	COPY A:BW.BAT

7. Create a batch file similar to GM .BAT (Copy the file from diskette)	COPY A:GM.BAT
Move to subdirectory GAMS2.04:	CD\GAMS2.04
Insert GAMS diskettes, one at a time into drive a:	
8. Copy GAMS files from GAMS system disk I & II into subdirectory.	COPY A:GAMS*.*
Insert MINOS diskette into drive a:	
9. Copy all files from MINOS5 diskette into subdirectory.	COPY A:*.*
Insert EXEIS system diskette into drive a:	
10. Copy batch file GAMS.BAT from diskette	COPY A:GAMS.BAT
Move to subdirectory GAMSLIB:	CD\GAMSLIB
Insert GAMS II diskette into drive a:	
11. Copy GAMS examples from GAMS system II diskette into GAMSLIB.	COPY A:*.GMS COPY A:*.LST COPY A:*.IDX
Insert EXEIS system diskette into drive a:	
12. Copy batch file GAMS.BAT from diskette	COPY A:GAMS.BAT
13. Copy batch file FORT2.BAT from diskette	COPY A:FORT2.BAT
Move to subdirectory BW:	CD\BW
14. Copy remaining programs to run model.	COPY A:MODEL2.FOR COPY A:SMODEL.FOR COPY A:HEAD2.FOR COPY A:SHEAD.FOR COPY A:FORT.BAT COPY A:FORT1.BAT COPY A:BOB2.GMS COPY A:MODEL2.DAT COPY A:SMODEL.DAT
15. Copy expert system programs from diskette. This instruction is repeated in Section V.	COPY A:XCON.FOR COPY A:EXP.BAT

You are now ready to create the data file, MODEL2.DAT or SMODEL.DAT, in the user's subdirectory (or use the sample data files MODEL2.DAT or SMODEL.DAT) or have the expert system develop the stochastic input file, and run the model. (NOTE: There can only be one file by these names at a time on the hard disk. As described below, if you wish to save a data file rename it before you or the expert system prepare a new data input file.)

C. RUNNING THE MODEL

First decide whether to run the deterministic (MODEL2.FOR) or the stochastic (SMODEL.FOR) model. The decision is affected by the knowledge of aquifer parameters (the field data and confidence in it). If the aquifer data set for the contaminated site is large, use MODEL2.FOR; if not, use SMODEL.FOR. The expert system is available to assist the user before SMODEL.FOR is run.

Second, prepare a sketch of the area showing the plume extent, proposed well configuration, potentiometric surface and ground elevations and saturated thicknesses (example: Figures 15 & 16). This data needs to be fairly accurate for the deterministic version. The stochastic version requires estimates of the same data. However, it is a simple matter to convert a deterministic data file to a stochastic data file and it is recommended that in most cases both versions of the model be run for comparison.

To reiterate, the mean and variance for both transmissivity and effective porosity are determined from available field data and equations 33 and 34 as explained in section III. Then the future plume extent and, therefore, the size of the octagon, are determined from the standard error ($\sqrt{\text{variance}}$) of transmissivity as explained in Section IV and Section III (equations (4),(5) and (6)).

Optimal well spacing is determined by successive model runs in which only the well spacing is changed. Spacing of the wells should be varied based on two criteria: (1) Spacing has to be an even multiple of side length and (2) Spacing should never exceed the radius of influence (equation 6).

The contaminant "source" can be located anywhere inside the octagon. In either model, the source is actually the point about which the potentiometric surface will rotate in an effort to achieve a horizontal gradient. This will also tend to be the point at which the highest concentrations of contaminant will be located after all pumping is complete. As an example, in an Otis Air Base problem (described in Section IV) the "source" could have been specified at a point down-gradient of the actual contaminant source. In that case, the optimal pumping would cause the plume to move toward the designated source point rather than remain at the original source.

1. Data Input File

An input data file is now generated using the editor available

with your computer or by having the expert system generate a stochastic input file, SMODEL.DAT. The expert system will erase any existing SMODEL.DAT file before it begins to run. If the user wants to save any old SMODEL.DAT file (i.e. the sample data file) it should be renamed before running the expert system. For example, renaming to a new name of SMODELBW.DAT can be done by typing REN SMODEL.DAT SMODELBW.DAT while in subdirectory BW.

The input data file can be given any name. If MODEL2.DAT is not used for the deterministic version or SMODEL.DAT is not used for the stochastic version the statements in MODEL2.FOR or SMODEL.FOR that "open" these files must be revised with the editor to reflect the new .DAT file name.

Appendix V explains the data input format for both versions of the model. Program 12 is a data input file for the deterministic version and Program 13 is a data input file for the stochastic version. Program 12 and Program 13 differ only in card 2. The deterministic data file (Program 12) specifies a maximum hydraulic conductivity of 270 ft/day and a minimum hydraulic conductivity of 180 ft/day. The stochastic data file (Program 13) specifies a coefficient of variation for the effective porosity of 0.80, a covariance for the transmissivity of 0.70, a reliability of 0.95 with a corresponding standard normal deviate of 1.64 (from Table 9) and a mean transmissivity of 13,500 ft^2/day. The stochastic version requires the same information as the deterministic model regarding ground and potentiometric surface elevations and saturated thicknesses.

It has been assumed that the saturated thickness (and therefore the transmissivity) does not change with time during the optimal unsteady pumping scenario. This is true for a confined aquifer for which the Theis equation is appropriate. However, with an unconfined aquifer the saturated thickness will vary with time. If the user wishes to try to increase the realism in modeling an unconfined aquifer, a second run of the same contaminant problem can be performed. This second run may use a time-average saturated thickness for each well. The new saturated thickness is found by averaging the original saturated thickness used in the problem with the final saturated thickness resulting from the optimal pumping. This final saturated thickness would be obtained from the final heads at the observation wells as calculated by the post-processor, HEAD2.FOR (or SHEAD.FOR), and written into file MODEL2.CAL. Alternatively, the user may wish to test the worst case situation by using the final saturated thickness rather than time-average saturated thickness.

If field data is limited, estimates for ground and potentiometric surface elevations and saturated thickness may be very crude. Uncertain knowledge of these parameters should be represented in the model by a larger value for coefficient of variation (CV) for effective porosity and transmissivity (>0.20) than was originally calculated. A small reliability should also be used (a small reliability results in large pumping values thus guaranteeing a better chance of containing the plume; ref: Section IV).

The coefficient of variation (CV = standard error/mean) is a measure of the "spread" of the field data. It should be calculated from the mean and variance of the aquifer parameters as described by equations 33 and 34. Reliability is a measure of how confident you want to be in pumping values containing the plume. As seen in Section IV, less pumping is allowed to be 95% confident that the bounds on head are not exceeded than is allowed to be 80% confident. To reiterate what was explained in Section V, if a 50% reliability is used the model is actually solving a deterministic optimization using the average values for hydraulic conductivity and effective porosity

In the stochastic version initial pumping values for each time period are required at the end of the data file. These initial values are used by the stochastic model as a starting point for the iteration process. These initial pumping values can never have a value of zero for the program to work properly. When using the pumping values from a previous run as the initial values for the next run replace any 0.0 value with a small positive value (i.e. <1.0).

2. GAMS-MINOS Output File

Appendix VI is an example of the output file, MODEL2.LST from a deterministic optimization. If it were the output from a stochastic optimization, the file would be named SMODEL.LST. The deterministic output contains HCMIN and HCMAX values on lines 20 and 21. The stochastic output contains mean transmissivity, covariance of transmissivity, covariance of effective porosity, reliability and corresponding standard normal deviate and the beginning value of pumping for each time period used in the stochastic iteration process (Figure 2).

Output file, MODEL2.LST, consists of two parts. First (numbered lines 2 through 433) is a reproduction of the input file, MODEL2.GMS, that GAMS reads and inputs to MINOS. This input file consists of MODEL2.OUT (data generated by program MODEL2.FOR) in lines 2-358 and BOB2.GMS (data that normally remains constant in lines 359-433. The batch file, FORT.BAT, merges files MODEL2.OUT and BOB2.GMS, calls the new file MODEL2.GMS and copies it into the GAMSLIB subdirectory.

The lines of the input file, MODEL2.GMS or SMODEL.GMS, that are preceeded by an * are comment statements. These lines are either input data not normally shown in a GAMS input file or statements clarifying the file. Most of the input data is labeled and explained in Chapter 3. The time vector, TT(T), and table, IND(L,M), are needed to multiply the correct pumping value, q_k, by the correct coefficient (i.e. $\hat{\delta}_{t-k+1}$ or E_{t-k+1}) and to indicate the correct number of terms (t) corresponding to the time period being examined. If day 3 is being examined, only 3 terms,

$\hat{\delta}_3 * q_1 + \hat{\delta}_2 * q_2 + \hat{\delta}_1 * q_3$, are included in the drawdown calculation.

The results of the optimization run are the unnumbered lines in the output file, MODEL2.LST. This output has been purposely edited to reduce its length. If the solution is unfeasible MINOS will print EXIT-UNFEASIBLE SOLUTION instead of EXIT-OPTIMAL SOLUTION FOUND. The marginal column will indicate the unfeasibility by printing "INFES" next to the constraint that is not satisfied. An unfeasible solution usually means that more time is needed to reach the objective, however, each situation is different. Some knowledge of optimization theory is needed to diagnose the problem and select corrective action.

If the solution is feasible, the output file contains the value for all constraints during all time periods. Tight constraints are designated by a value in the "marginal" column. Some of the lower water table and upper water table constraint values in the example output file (Appendix VI) have been purposely deleted to shorten the length of the file. Output labe'ed EQU WTH shows the tight constraints in this example are the upper limit on drawdown at injection well 3 for days 1 and 2 and at injection well 2 for days 3 and 4. A pecularity with our runs is that the upper limit on the water table is shown as a negative lower bound rather than a positive upper bound. However, it also shows the actual water table level as increasing in the negative direction at the injection wells. Therefore, the resulting values and tight constraints are correct even though the signs of all values of output in EQU WTH are opposite of what one might expect.

On the last day of pumping, observation wells down-gradient of the source must have potentiometric surface elevations no lower than that at the source. This is spelled out in constraint equation GRAD. Output line EQU GRAD shows that head at observation well 1 is the tight constraint for the final time period. This is confirmed in Appendix VII which shows the elevation of the potentiometric surface at observation well 1 to be the same as that at the source. Optimal pumping values are labeled VAR Q in the output file. If there is no pumping during a particular time period the marginal value indicates by how much the objective function value would increase if a unit of pumping was provided during that particular time period. The output file, SMODEL.LST, for the stochastic version would look exactly like MODEL2.LST except the output values (optimal pumping, objective function, tight constraints, etc.) would be similar to the values shown in Tables 7 and 8. *NOTE: All influence coefficients are the computed responses to 1000 units of pumping, not to merely a single unit. This is reflected in Appendix VI where the optimal pumping values are shown multiplied by 1000.

When the hydraulic portion of the objective function dominates (W_f is 1.0 or greater) the optimal strategy involves pumping in the early time periods followed by a rebounding of the water table toward its steady-state level. As is the case in Appendix V, there is no pumping in days 6,7 and 8. Therefore, the run could be repeated simply by changing the total time in the data input file to 5 days. Experience has shown that this does reduce operation and maintenance (O & M) cost for the

optimal pumping. However, the resulting final gradient is steeper in the reverse direction (from the original steady-state gradient) than the gradient resulting from the 8 day optimal strategy. Therefore, the steady pumping values needed to maintain this steeper reverse gradient are much larger than those needed to maintain the gradients developed after 8 days by the original model; sometimes over twice as large.

VAR S1 through S4 are values of portions OB1 through OB4 of the objective function. These values correspond to the 4 terms of the matrix objective function, equation 18. S1 corresponds to the $[C_h]\{Q\}$ term, S2 to the $[C_e]\{Q\}$ term, S3 to the $\{f(Q)\}$ term and S4 to the $\{g(Q)\}$ term. Y in equation 18 is represented by 'CON' under the scalar heading at the beginning of the output file. Summing VAR S2 and VAR S4 and multiplying by 1000 provides an estimate of the O & M cost for the optimal pumping. Line VARIABLE MIN.L contains the objective function value. As a check, subtracting S2 and S4 from the objective function value should yield the "sum of elevation differences sqd" value from MODEL2.CAL (Appendix VI).

3. Procedure to Run the Deterministic and Stochastic Evolutionary and Terminal Steady-State Models

This procedure assumes that the model (both versions) has been set up on a hard disk as described in Section V. MODEL2.FOR is used for deterministic optimization. For stochastic optimization substitute SMODEL.FOR in the following steps. If the user's subdirectory is other than BW, make appropriate substitutions in the following:

a. When the C: prompt appears type BW.

   This transfers the user to his subdirectory.

b. Create a data input file with an editor as described in Section VI.

   The listing of MODEL2.FOR or SMODEL.FOR must be edited to reflect the name that has been given to the data file. Change the "OPEN" statement for the .DAT file in either MOD EL2.FOR or SMODEL.FOR and HEAD.FOR to reflect this new name. These "OPEN" statements are near the beginning of each program.

   The listing of MODEL2.FOR or SMODEL.FOR may be edited to achieve only an economic objective function or only a hydraulic objective funtion. For strictly an economic objective function the economic coefficients, c_k and c_{kk}, must have a value greater than zero assigned to them and the weight factor, W_f, must be zero as shown on the lines near the bottom of pages 166 and 167 of Program 8 listing for MODEL2.FOR or as shown in the middle of page 182 and near the top of page 183 of Program 9 listing for SMODEL.FOR. If only a hydraulic objective function is desired then the economic coefficients must be zero and the weight factor given a value of one.(See 7 statements in each file marked with ►).

c. Set the beginning pumping value, Q.L(T,J), in BOB2.GMS for the optimization iteration process.

This value must be between the upper and lower bounds on pumping. To check global optimality the user may input values for pumping in each time period and then making successive runs as described in Section V. For example, he may type Q.L('1',J) = 150.0; for time period 1, Q.L('2',J) = 0.0; for time period 2, etc.

d. Run the model by typing (in all capital letters):
FORT MODEL2 (or SMODEL) BOB2 YES (or NO)

This begins the FORT.BAT program, in which %1 corresponds to MODEL2 and %2 corresponds to BOB2. YES or NO designates whether to compile the program MODEL2.FOR before it is run. The program must be compiled the first time it is run on a particular computer. Until changes are made in the program listing, there is no subsequent need to recompile and relink the program. This creates an object (.OBJ) file and an executable (.EXE) file. Typing NO skips the compilation and link steps and immediately begins running the program; thereby saving computer time.

Compilation and linking takes about 5 minutes. During this time the screen will echo the commands of file FORT.BAT until C:\BW>MODEL2 appears on the screen. At this time the preprocessor begins calculating the influence coefficients and preparing the input for GAMS. The preprocessor takes about 10 minutes for a problem the size of MODEL2.DAT.

When the preprocessor is finished the FORT.BAT program combines the output from MODEL2.FOR, called MODEL2.OUT, with BOB2.GMS and copies it into the GAMSLIB subdirectory as file MODEL2.GMS. These batch file commands are echoed to the screen. The GAMS-MINOS program is then run. As this program is run (it takes 6-8 minutes for a problem the size of MODEL2.DAT) the screen shows if any errors have been detected. If there are no errors, the screen shows a summary line for each iteration of the program. When the GAMS-MINOS program is completed the screen shows EXIT, specifies whether an optimal or unfeasible solution has been obtained, and lists a summary of program results. The user is then transferred to his subdirectory.

When running the stochastic version an unfeasible solution is likely to result if uncertainty of either aquifer parameters is large (CV larger than approximately 0.30). This large CV may not allow the heads at some of the observation wells down-gradient of the source to rise above the head at the source within the time period the user specifies (constraint 10). The output file, SMODEL.LST, will show which wells do not meet the constraint. Under the heading EQU GRAD the marginal column will show INFES (unfeasible). However, the pumping values computed by the model will probably be the best pumping strategy possible for that

particular situation and the majority of the gradients will be reversed from the original gradient.

The output file, MODEL2.LST (or SMODEL.LST), is placed in the user's subdirectory, BW. This listing file is similar to Appendix V if an optimal solution is found. If not optimal, the listing file will either indicate what part of the problem is unfeasible or it will indicate where and what the user errors are.

e. Transfer the optimal pumping values from MODEL2.LST (or SMODEL.LST) to the input file.

With the editor, examine the output file, MODEL2.LST (or SMODEL.LST), delete what is not needed, and obtain a printout. Transfer the optimal pumping values to the bottom of the input file (either MODEL2.DAT or SMODEL.DAT) in the format described in Appendix V. These should be in the correct order and should include days with no pumping. However, the stochastic version must have a nonzero value for pumping. In lieu of 0 pumping put a small value such as 0.1. The pumping values should be located immediately following the last saturated thickness by either adding on to the end of the file or by inserting them in place of the pumping values used as initial estimates for the stochastic model.

f. Run the post-processor, HEAD2.FOR (or SHEAD.FOR), by typing: FORT1 HEAD2 (or SHEAD.FOR)

This begins the FORT1.BAT program which runs HEAD2.FOR (or SHEAD.FOR). The output file, MODEL2.CAL, will contain the final potentiometric surface elevations at all observation wells. It also contains the steady pumping values at all pumping wells (minus signs indicate injection pumping) needed to keep the observation well heads constant.

SECTION VII

CONCLUSIONS AND RECOMMENDATIONS

The three major components to the presented EXEIS expert/optimizer system are the expert system, the optimization model, and the post-processor. First, the expert system is used to determine whether pumping is the most economical method of containing a specific groundwater contaminant plume. If requested, the system also develops an input data file for the optimization program. The dual-objective optimization program determines the unsteady pumping that will most optimally contain the plume. A deterministic version of this program is used if the user is confident in his information concerning the physical system. A stochastic version is used if his knowledge is less certain. That version considers the effect of aquifer parameter uncertainty on the optimal unsteady pumping values and predicted hydraulic heads. Finally, a postprocessor uses the optimal unsteady pumping values to determine the hydraulic heads at all pumping and observation wells and what steady pumping is required at each well to maintain those heads.

A. EXPERT SYSTEM

An expert system is developed to provide assistance in assessing how best to contain a plume of contaminated groundwater. The system requests, from the user, pertinent information about soil and site characteristics, and the contaminant plume. Based on this information, the system analyzes three containment methods; slurry wall, sheet piling and pumping. The system recommends a containment method and (if pumping is the chosen method) suggests the data that should be used in the optimization program.

The expert system compares the three containment methods based on the physical characteristics of the contamination problem and approximate capital costs of each method. Initially, operating costs for the pumping strategy are not included in the analysis of which strategy is most economically desirable. Operation and maintenance (O&M) costs cannot be accurately estimated until pumping values are obtained from the optimization model. However, if pumping is initially computed to be the least expensive method, its O&M costs are estimated based on a worst-case scenario. In that case, the expert system states how long the pumping strategy could be used before it's O&M costs are such that another method of containment would be less expensive.

The system uses Bayesian statistics to determine aquifer parameter values that should be used to incorporate uncertain knowledge of aquifer parameters into the stochastic version of the optimization model. In addition, this system can create an input file for that model. This option is applicable only for physically simple contamination problems. However,it is beneficial because it speeds user familiarization with the optimization process. It also adds understanding the difference between stochastic and deterministic pumping strategies. By selecting a reliability of 50% in this option, the user, in effect, causes the computation of a deterministic optimal strategy. This can then be

compared with stochastic strategies developed using other reliabilities.

This system provides a well-structured method of analyzing a contamination problem. In so doing, it develops analytical values for transmissivity and effective porosity. It also recommends a design for an octagonal well system to be used in the optimization program.

B. OPTIMIZATION MODEL AND POSTPROCESSOR

An efficient method for optimizing extraction/injection pumping strategies for contaminant plume containment within an aquifer is presented. Optimal extraction/injection strategies are computed using specialized groundwater management models. There are two versions of the optimization model. The deterministic version accepts input for a nonhomogeneous anisotropic aquifer and should be used if the user has a good set of data he is confident in. The stochastic version uses average values for the aquifer parameters and incorporates uncertainty in these parameters by using the standard deviation of each parameter and a required reliability in the model solution. It is suggested that both versions be run to see the effect of aquifer parameter uncertainty on the model solution. Strategies are developed for a predetermined well or well-point system surrounding a hypothetical contaminant plume.

The groundwater management model uses simulation based on analytical expressions. These are most perfectly applicable for a confined, homogenous aquifer with the following assumptions: (1) aquifer is nonleaky and infinite in horizontal extent, (2) pumps produce a radial flow pattern, (3) wells fully penetrate the entire thickness of aquifer, and (4) potentiometric head prior to pumping is at steady-state conditions. As is common practice, use of these analytical expressions is extended to more complex and realistic physical settings.

The objective function of the management model uses a weighting factor to provide a common basis for simultaneous evaluation of both economic and hydraulic criteria. A range of weight factors (W_f) was tested with this multiobjective model. Sensitivity of strategies to W_f was tested using the deterministic version. Weight factors equal to or greater than one produced a gradient of less than 0.1 per cent. Named run 1d, a run using a W_f of 1.0 is used as a base comparison in the discussion below. This strategy included pumping in the first 5 days of an 8-day planning period.

Additional testing of the deterministic version compares the effects of varying W_f and cost coefficients (c' and c'' in $\$/L^4$ and $\$/L^3$). First, a pumping strategy developed for a purely economic objective (W_f=0.0) is compared with a strategy developed using only a hydraulic objective ($c'=c''=0.0$). The unsteady pumping strategy developed with the hydraulic objective is almost exactly that produced by the original model run 1d. The strategy developed using only economics emphasized pumping late in the planning period (the opposite of the hydraulic

objective strategy). The total volume pumped is less than for the hydraulic objective run but the resulting final gradient is much steeper than when emphasizing hydraulics (0.134 per cent vs. 0.07 per cent).

Finally, the original run of model 1d (W_f=1.0 and c' and c" equal to their original values) is made with the additional constraint that the pumping during all time periods be equal. This is done to compare the results of unsteady pumping and steady pumping. The steady pumping strategy did require a smaller volume of water to be pumped but the resulting final gradient is 0.117 per cent as compared to 0.07 per cent for the original unsteady strategy. An unsteady pumping strategy is superior to a steady pumping strategy during the period of potentiometric surface evolution because it produces a better (closer to horizontal) and smoother final gradient.

The ideal weight factor is dependent on many factors and may be problem-specific. A major factor is the maximum acceptable increase in water table elevation at an injection site. This constraint is based on the desire to avoid pressurized injection. However, because the greatest concern is to keep the plume contained, using a weight factor of 1.0 and ignoring the economics (using cost coefficients of 0.0) produced the most satisfactory gradients.

It has been decided that for short term planning periods, where contaminant cleanup is planned immediately after stabilization of the plume, the economic objective need be the only consideration (use a weight factor of 0). If the plume needs to be held stable for a long period of time the hydraulic objective should be considered to produce as near a zero hydraulic gradient as possible. For long stabilization periods the plume tends to drift towards the extraction wells and contaminated water might be extracted before desired. Using only the economic objective produces the least cost strategy to stop the plume movement down-gradient but it also accelerates the drifting toward the extraction wells.

1. Deterministic version

The deterministic version is tested by running a variety of hypothetical contaminant situations. These situations are developed by systematically varying the aquifer parameters for the original hypothetical problem (run 1d). The optimal deterministic pumping strategies developed for all hypothetical situations has greater pumping at the beginning of the modeling period than at any other time. Initial changes in head at the observation wells caused by these large pumping values are greater than needed. Therefore, the aquifer "rebounds" (i.e. the potentiometric surface moves toward its original steady-state elevations) during the zero-pumping days to achieve a nearly horizontal gradient.

To subsequently maintain as nearly a horizontal surface as possible steady pumping values are calculated in a postprocessor. The steady pumping holds the potentiometric surface at the same elevations as those

achieved when the rebounding is completed. In the tested cases, these pumping values varied slightly from well to well. In practice, one may wish to use the average steady pumping value for all wells. The smoother the potentiometric surface is by the end of the period of unsteady pumping, the more appropriate this approach is.

The results of implementing a proposed optimal strategy are simulated using a 2-D solute transport model (the model uses the method of characteristics, MOC). This is done to demonstrate that the optimal strategies are effective. Without implementation the plume migrates beyond acceptable limits. Implementing the proposed optimal unsteady pumping strategy and steady pumping strategy contains most of the contaminant within the boundaries of the original plume.

Sensitivity of optimal deterministic strategies is evaluated with respect to anisotropy, planning period duration and well configurations. In all cases, the resulting pumping schemes were tested with the 2-D solute transport (MOC) model. In general, the results showed very little movement of the contaminant plume. However, in some cases it had large movement in the densely contaminated center portion, but moved little along the plume edge. This occurred in anisotropic situations where the saturated thickness varied from 15 meters, up-gradient of the source, to 12 meters, down-gradient of the source. No explanation is offered.

Comparisons were also performed to demonstrate the degree to which heads predicted by the optimzation model agreed with those computed by MOC. The heads predicted by the model correspond within 0.23 meters (0.75 ft) of the heads predicted by the MOC model. In general, the calculated drawdowns from the model exceed those predicted by MOC; thus producing a steeper final gradient. This may indicate that the final gradients that would be produced in the field by the optimal pumping strategy would actually be closer to horizontal than that shown by the post-processor. On the other hand, the model's use of analytic solutions and superposition may be more accurate than the finite difference simulation of the MOC model.

Comparisons are made between using the original 8-day time frame versus using a reduced 5-day time frame. They were accomplished using the hypothetical contamination problem and the parameters of model run 1d. The pumping strategies for the 8-day time frame, in general, showed no pumping during the final 3 days (when the hydraulic objectives are emphasized over the economic objectives) so it seemed logical that the final 3 days are not needed. The results indicate that the shorter time frame does produce a more economical unsteady pumping strategy; i.e. the operating and maintenance (O&M) costs are less. However, the resulting final gradient for the 5-day scenario is poorer than that for the 8-day. In addition, the steeper final gradient produced by the 5-day strategy requires much larger steady pumping values to maintain that gradient. Therefore, these results indicate that it is best to use as much time as is available for the optimal unsteady pumping phase if it is foreseen that there will be a period of steady pumping needed to keep the plume stabilized. Using a longer time frame for the optimal unsteady pumping phase does produce larger O&M costs but also develops a more horizontal

hydraulic gradient. The final 3 days without pumping actually allow time for the hydraulic heads at the observation wells to react to the stimulus at the pumping wells. This produces a much more uniform potentiometric surface. The additional cost is more than compensated for because the steady pumping values required to maintain this smaller gradient are much less (as much as two or three times) than for the shorter time frame which produces the steeper gradient. Therefore, in the long run, the total cost of optimal pumping plus steady pumping is much less.

To demonstrate applicability of the models for a significantly elongated contaminant plumes, data for a hazardous waste site at Otis Air Base, Massachusetts, is used. Without management there is significant movement of the plume within a 40-week period. With management (8 weeks of optimal unsteady pumping and 32 weeks of steady pumping) plume movement is negligible. However, because an elongated plume octagon is used, three times as much injection water is needed by the optimal strategy as was provided by the extraction wells. In addition, the assumption that the operating and maintenance costs are constant for the entire time frame may not be valid for an 8-week period. Eight weeks of continuous pumping would result in clogging of the wellscreens (especially in the injection wells), resulting in increased head losses and higher operating and maintenance costs. Filtration of the extracted water before it is used in the injection wells would delay the clogging process. However, time-varying unit O & M costs should be used in a strategy of this duration.

Preliminary work by H. H. Suguino and R. C. Peralta compared parallel versus octagonal configurations of extraction and injection wells. In both systems, there were three injection wells initially downgradient and three extraction wells initially upgradient of the source. They reported that the octagonal configuration required 5 to 20 percent less pumping to halt the plume than did the parallel system, depending on the scenario.

Because of the unusual quadratic form of the objective function, global optimality of the solution for the deterministic version of the model cannot be assured. When the optimization program is run, initial values for pumping can be given as starting points in the iteration process. Therefore, the only way to obtain some assurance of global optimality is to make systematic runs using the upper or lower bounds on pumping as starting points. For example, an optimization run would be made setting the lower bound on pumping as the starting point for time period one and the upper bound as the starting point for all other time periods. The second run would have the lower bound on pumping as the starting point for time period two and the upper bound as the starting point for all other time periods. Runs would then be made for all combinations of time periods and starting points. This was done with the isotropic hypothetical problem and it was found that all runs gave the same optimal unsteady pumping values. However, this does not guarantee global optimality for any other contaminant problem which has a different solution space.

2. Stochastic version

To better consider uncertain knowledge of aquifer parameters, a stochastic version of the original deterministic optimization model is developed. To accomplish this, original and modified versions of a procedure developed by Tung (Reference 42) are used. Stochastic influence coefficients (E values) are developed using mean and coefficient of variation (CV) values of aquifer transmissivity and effective porosity as well as a required reliaibility for the solution. These coefficients are used in the same manner as the unit response functions in the deterministic model.

The drawdown (change in head) at observation wells (which affects the objective function and gradient constraints) must be treated differently than drawdown at the pumping wells (which affect the drawdown constraints). For example, if a reliability of 95 per cent is specified for our solution, an E value corresponding to a reliabilty of .95 is used for the drawdown constraint. The user wants to be 95 per cent confident that the resulting drawdown produced by the optimal pumping at the pumping wells does not exceed the calculated value. On the other hand, the E value corresponding to a reliability of .05 is used to determine drawdown at the observation wells. In that case, the user wants to be 95 per cent confident the drawdown (produced by the optimal pumping) at the observation wells is not less than the calculated value. Thus E values corresponding to a reliability of .95 are used for the drawdown constraint and values corresponding to a reliability of .05 are used with the objective function and gradient constraints.

This approach guarantees the user a 95 per cent confidence level for the drawdown constraint. However, because the objective function minimizes the head differences between the observation wells (whose values are stochastic) and the source (whose value is also stochastic) a joint 95 per cent confidence level cannot be guaranteed. It would be some value slightly less than 95 per cent and cannot readily be determined.

The major differences between Tung's work and this study are:

1. Tung used the Cooper-Jacob equation to derive the stochastic coefficients.
2. Tung's objective function maximized pumping and did not incorporate stochastic coefficients.

The effect of uncertain knowledge of aquifer parameters on optimal pumping and objective functions values agree, in general, with the conclusions of Tung. As the reliability level decreases or aquifer parameter variance decreases pumping for each time period increases and the objective function improves.

The effect of changes in uncertainty of effective porosity on the pumping pattern and final hydraulic gradient differ from those observed by Tung. Tung derived the $\bar{P}$ coefficients (the partial derivative of drawdown with respect to effective porosity; equation 50) using the Cooper-Jacob equation. He computed $\bar{P}$ to have a value of 0 for all except

the first time period. Therefore, changes in uncertainty of effective porosity had almost no effect on the optimal pumping values. This may be due to the fact that the Cooper-Jacob equation is only valid for small values of the Boltzman variable ($u \leq .01$). Our study shows the $\bar{P}$ coefficient to have values for all time periods. For equal changes in CV, effective porosity produces smaller changes in pumping than does transmissivity. However, the resulting final gradients produced by these small changes in pumping are much poorer than the final gradient produced by a comparable change in CV of transmissivity. Uncertainty in effective porosity has little effect on allowable pumping, as Tung concluded, but does adversely affect the final gradient.

Four general statements can be made concerning the stochastic version of this model:

1. Increases in reliability level result in decreased pumping and O&M cost, and produce a poorer final gradient. Any reliability over 0.50 results in a larger objective function value than a strictly deterministic run.
2. Reductions in reliability level result in increased pumping and O&M cost, and produce an improved final gradient.
3. Increases in uncertainty of transmissivity and effective porosity both reduce optimal daily pumping values and produce a steeper final gradient .
4. Increases in uncertainty of transmissivity and effective porosity produce opposite affects on the total optimal pumping required. Transmissivity reduces total pumping; effective porosity increases total pumping.

Over an extended period, operating and maintenance costs would not remain constant as has been assumed. As a result, a proposed injection/extraction strategy may not be economically practical for extended operation. It would, however, be an economical and efficient method for short term containment.

C. RECOMMENDATIONS FOR FUTURE RESEARCH

This report is the first in a series of envisoned methodologies for optimizing remediation of a groundwater contamination problem. The following enhancements are possible:

1. Incorporate integer programming to allow the model to decide which wells should be used. This would allow the model to select optimal well spacings rather than requiring the user to try many different placements.
2. Provide a model that is more flexible in its handling of well configuration. It would be able to size an octagon or a different shape configuration for a specific plume shape.
3. Include pumping recommendations for optimal extraction of contaminant as well as for containment.

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APPENDIX I Derivation of coefficients for equation (18)

The derivation of the coefficients of equation (18) begins with the objective function:

$$\min: z = \sum_{t=1}^{T} \sum_{i=1}^{I} \left[c'_t \left(h_{i,g} - h_{i,o} + s_{i,t} \right) q_t + c''_t q_t \right]$$

$$+ W_f \sum_{j=1}^{J} \left[h_{o,TT} - h_{j,TT} \right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (7)$$

$h_{i,o}$ = original groundwater elevation at pump i

$s_{i,t}$ = dynamic drawdown at pump i at time t

$$= \sum_{k=1}^{t} \sum_{j=1}^{I} \hat{\delta}_{i,j,t-k+1} \, q_k \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (11)$$

$h_{o,TT}$ = groundwater elevation at contaminant source at time TT.

$$= h_{o,0} - \sum_{t=1}^{TT} \sum_{i=1}^{I} \left[\hat{\delta}_{o,i,TT-t+1} q_t \right] \quad \ldots\ldots\ldots\ldots \quad (36)$$

$h_{j,TT}$ = groundwater elevation at observation well at time TT.

$$= h_{j,0} - \sum_{t=1}^{TT} \sum_{i=1}^{I} \left[\hat{\delta}_{j,i,TT-t+1} q_t \right] \quad \ldots\ldots\ldots\ldots \quad (37)$$

Making these substitutions and squaring the head difference term we

obtain:

$$\min: \sum_{t=1}^{TT} \sum_{i=1}^{I} \left\{ c_t' \left[h_{i,g} - h_{i,o} + \sum_{k=1}^{t} \sum_{j=1}^{I} (\hat{\delta}_{i,j,t-k+1} q_k) \right] q_t + c_t'' q_t \right\}$$

$$+ W_f \sum_{j=1}^{J} \left\{ \sum_{t=1}^{TT} \left[\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1} \right] q_t \right\}$$

$$+ 2W_f \sum_{j=1}^{J} \left\{ (h_{o,0} - h_{j,0}) \left[\sum_{t=1}^{TT} \left(\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1} \right) q_t \right] \right\}$$

$$+ W_f \sum_{j=1}^{J} \left[h_{o,0} - h_{j,0} \right]^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (38)$$

Gathering linear terms and quadratic terms yields:

$$\min: \sum_{t=1}^{TT} \left\{ \sum_{i=1}^{I} \left[c_t' (h_{i,g} - h_{i,o}) + c_t'' \right] q_t \right\}$$

$$+ \sum_{t=1}^{TT} \left\{ \sum_{i=1}^{I} \left[c_t' \sum_{k=1}^{t} \sum_{j=1}^{I} (\hat{\delta}_{i,j,t-k+1} q_k) \right] q_t \right\}$$

$$+ \sum_{t=1}^{TT} \left\{ \sum_{j=1}^{J} 2W_f (h_{o,0} - h_{j,0}) \left[\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1} \right] q_t \right\}$$

$$+ W_f \sum_{j=1}^{J} \left\{ \sum_{t=1}^{TT} \left[\left(\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1} \right) q_t \right] \right\}^2 + Y \quad \ldots \quad (39)$$

Finally, coefficients in the objective function arrays (equation 18) are:
For each element corresponding to a given pumping well i and time period t;

$$C_e = c'_t(h_{i,g} - h_{i,o}) + c''_t \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (40)$$

$$K_e = c'_t \sum_{k=1}^{t} \sum_{j=1}^{I} \hat{\delta}_{i,j,t-k+1} q_k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (41)$$

For each element corresponding to a specific observation well j and time period t;

$$C_h = 2W_f\,(h_{o,0} - h_{j,0})\left[\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1}\right] \quad \ldots\ldots (42)$$

$$K_h = W_f\left[\sum_{i=1}^{I} \hat{\delta}_{j,i,TT-t+1} - \sum_{i=1}^{I} \hat{\delta}_{o,i,TT-t+1}\right] \quad \ldots\ldots\ldots\ldots (43)$$

In addition, the single value Y is defined as:

$$Y = W_f \sum_{j=1}^{J} (h_{o,0} - h_{j,0})^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (44)$$

APPENDIX II - Analysis of uncertainty in drawdown

Discrete formulation of drawdown at observation point j at the end of the nth period is given by equation (11) as:

$$s_{i,t} = \sum_{i=1}^{I} \sum_{k=1}^{t} \hat{\delta}_{i,j,t-k+1} q_k \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

where $\hat{\delta}_{i,j,t-k+1}$ = the unit response function which can be derived from the Theis equation as:

$$\hat{\delta}_{i,j,k} = \frac{1}{4\pi T}\{W[u_{i,j,k}] - W[u_{i,j,k-1}]\} \quad \ldots\ldots\ldots\ldots\ldots (13)$$

where:

$$u_{i,j,k} = \frac{r^2 \phi}{4Tk} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and

$$W[u_{i,j,k}] = \int_u^{\infty} \left[\frac{e^{-v}}{v} \right] dv \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

Since T (transmissivity) and ϕ (effective porosity) are random variables, the unit response function as well as drawdown are both random variables because they are functions of random variables.

To estimate statistical properties of random variables, the first-order analysis of uncertainty is employed. Taylor's expansion of drawdown about the mean values of T and ϕ can be expressed as:

$$s_{i,t} = \sum_{i=1}^{I} \sum_{k=1}^{t} \bar{B}_{i,j,k} q_{t-k+1} + \left.\frac{\partial s_{i,t}}{\partial T}\right|_{\bar{T}} (T-\bar{T}) + \left.\frac{\partial s_{i,t}}{\partial \phi}\right|_{\bar{\phi}} (\phi-\bar{\phi}) + HOT \quad \ldots\ldots\ldots (45)$$

where $\bar{B}_{i,j,k}$ is computed using mean values $\bar{T}$ and $\bar{\phi}$ and HOT represents the higher order terms. The time increments of k and t-k+1 are reversed from those in eq. 11 but they produce the same result.

First, we compute the middle term on the right hand side. The first order partial derivative of $s_{i,t}$ with respect to T can be obtained by Leibnitz rule for differentiating an integral (Reference 44, page 18):

$$I'(c) = \int_{a(c)}^{b(c)} \frac{\partial f(x,c)}{\partial c}\, dx + f[b(c),c]\frac{db}{dc} - f[a(c),c]\frac{da}{dc} \quad \ldots\ldots\ldots (46)$$

Performing the mathematics of the differentiation in three parts we define:

$$I'(c) = \frac{\partial s_{j,t}}{\partial T} = \frac{\partial}{\partial T}\left[\sum_{i=1}^{I}\sum_{k=1}^{t} \hat{\delta}_{i,j,k} q_{t-k+1}\right]$$

For the first term on the right hand side of equation (46):

$$\int_{a(c)}^{b(c)} \frac{\partial f(x,c)}{\partial c}\, dx = \int_{u}^{\infty} \left\{\partial\left[\frac{1}{4\pi T}\left(\frac{e^{-v}}{v}\right)\right]/\partial T\right\} dv$$

$$= -\frac{1}{4\pi T^2}\int_{u}^{\infty}\left[\frac{e^{-v}}{v}\right] dv$$

Second term:

$$f[b(c),c]\frac{db}{dT} = 0 \text{ because } b = \text{constant } (\infty)$$

Third term:

$$f[a(c),c]\frac{da}{dT} = \frac{1}{4\pi T}\left[\frac{e^{-u}}{u}\right]\frac{du}{dT} \quad \text{where } \frac{du}{dT} = \frac{d\left[\frac{r^2\phi}{4Tk}\right]}{dT} = -\frac{r^2\phi}{4T^2k} = -\frac{u}{T}$$

therefore:

$$f[a(c),c]\frac{da}{dT} = \frac{1}{4\pi T}\left[\frac{e^{-u}}{u}\right]\frac{-u}{T} = -\frac{1}{4\pi T^2}\,e^{-u}$$

Adding the three terms:

$$\frac{\partial s_{j,t}}{\partial T} = \sum_{i=1}^{I}\sum_{k=1}^{t}\bar{A}_{i,j,k}q_{t-k+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (47)$$

in which:

$$\bar{A}_{i,j,k} = \frac{1}{4\pi T^2}\left\{e^{-u_k} - \int_{u_k}^{\infty}\left[\frac{e^{-v}}{v}\right]dv\right\} \qquad \text{at } k = 1;$$

$$= \frac{1}{4\pi T^2}\left\{e^{-u_k} - e^{-u_{k-1}} + \int_{u_k}^{u_{k-1}}\left[\frac{e^{-v}}{v}\right]dv\right\} \qquad \text{at } k > 1 \quad \ldots (48)$$

Similarly, the first-order partial derivative of drawdown with respect to the effective porosity can be obtained in three parts from Leibnitz rule:

For the first term on the right hand side of equation (46):

$$\int_{a(c)}^{b(c)} \frac{\partial f(x,c)}{\partial c}\, dx = \int_{u}^{\infty} \left\{ \partial \left[\frac{1}{4\pi T} \left(\frac{e^{-v}}{v} \right) \right] / \partial \phi \right\} dv = 0$$

Second term:

$$f[b(c),c]\frac{db}{d\phi} = 0 \text{ because } b = \text{constant } (\infty)$$

Third term:

$$f[a(c),c]\frac{da}{d\phi} = \frac{1}{4\pi T}\left[\frac{e^{-u}}{u}\right]\frac{du}{d\phi} \quad \text{where } \frac{du}{d\phi} = \frac{d\left[\frac{r^2\phi}{4Tk}\right]}{d\phi} = \frac{r^2}{4Tk} = \frac{u}{\phi}$$

therefore:

$$f[a(c),c]\frac{da}{d\phi} = \frac{1}{4\pi T}\left[\frac{e^{-u}}{u}\right]\frac{u}{\phi} = \frac{1}{4\pi T\phi}\, e^{-u}$$

Only term three has a value and:

$$\frac{\partial s_{i,t}}{\partial \phi} = \sum_{i=1}^{l} \sum_{k=1}^{t} \bar{P}_{i,j,k} q_{t-k+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (49)$$

where:

$$\bar{P}_{i,j,k} = -\frac{1}{4\pi T\phi}\, e^{-u_k} \qquad \text{at } k = 1$$

$$= -\frac{1}{4\pi T\phi}\left(e^{-u_k} - e^{-u_{k-1}}\right) \qquad \text{at } k > 1 \quad \ldots\ldots\ldots\ldots (50)$$

The partial derivatives of drawdown with respect to transmissivity and effective porosity agree with those shown by McElwee and Yukler (Reference 25).

Ignoring the higher order terms in equation (45) the expectation of drawdown can be approximated by equation (25):

$$E(s_{j,t}) = \sum_{i=1}^{I} \sum_{k=1}^{t} \bar{B}_{i,j,k} q_{t-k+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

Furthermore, assuming independency of T and ϕ the variance of drawdown can be approximated as equation (26):

$$\mathrm{var}(s_{j,t}) = \left[\frac{\partial s_{j,t}}{\partial T}\right]^2 sdt^2 + \left[\frac{\partial s_{j,t}}{\partial \phi}\right]^2 sds^2$$

$$= \left[\sum_{i=1}^{I} \sum_{k=1}^{t} \bar{A}_{i,j,k} q_{t-k+1}\right]^2 sdt^2$$

$$+ \left[\sum_{i=1}^{I} \sum_{k=1}^{t} \bar{P}_{i,j,k} q_{t-k+1}\right]^2 sds^2 \quad \ldots\ldots\ldots\ldots \quad (26)$$

where sdt and sds are the standard deviations of the transmissivity and effective porosity, respectfully.

APPENDIX III - Derivation of equation (30)

Substituting equation (26) into equation (29), we can express $\sqrt{var(s_{j,t})}$ in terms of unknown pumping Q's more explicitly as:

$$f(q) = \sqrt{var(s_{j,t})} = \sqrt{ft(q)^2 + fs(q)^2} \quad \ldots\ldots\ldots\ldots (51)$$

where:

$$ft(q) = \sum_{i=1}^{I} \sum_{k=1}^{t} \left[\bar{A}_{i,j,k} q_{t-k+1} \right] sdt$$

and:

$$fs(q) = \sum_{i=1}^{I} \sum_{k=1}^{t} \left[\bar{P}_{i,j,k} q_{t-k+1} \right] sds$$

Equation (29) is a first order Taylor expansion of equation (51). The first term on the right-hand side of equation (29), f(Qo), is the value of the function f(q) calculated (with equation (51)) by using arbitrarily assumed pumping values, Qo's, in equation (51). The partial derivative in the second terms of equation (29) can be found by taking the derivative of equation (51) with respect to q and is expressed as:

$$\left. \frac{\partial f(q)}{\partial q_{t-k+1}} \right|_{Qo} = \frac{1}{f(Qo)} \left[ft(Qo)\bar{A}_{i,j,k}(sdt) + fs(Qo)\bar{P}_{i,j,k}(sds) \right] \quad \ldots (52)$$

Substituting equation (52) into equation (29) and multiplying it with q_{t-k+1} and Qo_{t-k+1}, respectively we obtain:

$$f(q) = f(Qo) \qquad \text{(term 1)}$$

$$- \frac{1}{f(Qo)} \sum_{i=1}^{I} \sum_{k=1}^{t} [ft(Qo)\bar{A}_{i,j,k}(sdt) + fs(Qo)\bar{P}_{i,j,k}(sds)]Qo_{t-k+1}$$

$$\text{(term 2)}$$

$$+ \frac{1}{f(Qo)} \sum_{i=1}^{I} \sum_{k=1}^{t} [ft(Qo)\bar{A}_{i,j,k}(sdt) + fs(Qo)\bar{P}_{i,j,k}(sds)]q_{t-k+1}$$

$$\text{(term 3)}$$

$$+ \text{HOT} \quad \ldots \quad (53)$$

The second term of equation (53) cancels the first term as shown. First, the second term reduces to f(Qo) as shown:

$$\frac{1}{f(Qo)} \sum_{i=1}^{I} \sum_{k=1}^{t} [\underbrace{ft(Qo)\bar{A}_{i,j,k}(sdt)}_{\frac{ft(Qo)}{Qo}} + \underbrace{fs(Qo)\bar{P}_{i,j,k}(sds)}_{\frac{fs(Qo)}{Qo}}]Qo_{t-k+1}$$

reduces to

reduces to $\underbrace{ft(Qo)^2 + fs(Qo)^2}$

reduces to $f(Qo)^2$

and $\frac{1}{f(Qo)} \bullet f(Qo)^2 = f(Qo)$

Therefore, term 1 + term 2 = f(Qo) - f(Qo) = 0

By dropping the higher order terms (HOT) the third term of equation (53) can be written as equation (30).

APPENDIX IV EXAMPLE OF EXPERT SYSTEM ANALYSIS (system questions with large bold figures corresponding to user responses)

EXPERT PROGRAM TO DETERMINE ECONOMIC METHOD
FOR CONTAINING A CONTAMINANT PLUME

This system will determine the best possible technique to contain a contaminant plume based on input from you and your confidence in that input. There are three possible answers for any one question.

1. (W)hy; if you wish to know the reason a question is asked.
2. (U)nknown; if you do not know an answer and wish the program to estimate an answer.
3. (Y)es followed by the answer to the question and a confidence level for your answer.

ALL RESPONSES SHOULD BE IN CAPITAL LETTERS.

Execution suspended : Hit ENTER when you are ready to continue.

This system analyzes three possible containment techniques; slurry wall, sheet piling and pumping. All three strategies are based on the assumptions that:

1. The contaminant is from a source which forms an elliptically shaped plume.
2. All containment techniques are configured in the shape of a regular octagon centered on the contaminant source.
   a. The pumping technique is based on wells located on all eight sides of the octagon.
   b. The other two techniques are based on forming only the five down-gradient sides of the octagon.

The following questions are intended to characterize the soil enviroment. If you are ready to continue type CLS and hit ENTER to clear the screen and hit ENTER again to begin the questions. If at any time, you wish to quit in the middle of the program simply hit Ctrl C.

Execution suspended :

Do you understand that the system assumes the soil is homogeneous in the area of contamination when it estimates the size of the plume? Answer (W)hy, (Y)es or (N)o.

Y

Do you have an estimate of rock in the soil? Answer (W)hy, (U)nknown or (Y)es.

1. None (0-10% by volume)
2. Small (11-30%)
3. Large (> 30%)

Y

Give your answer (Type 1,2, or 3) and your confidence (in per cent) in the answer. Separate each response with a space.

2 85

You have input no. 2 as your answer and 85 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Do you know the condition of the boundary between the aquifer material and the bedrock? Answer (W)hy, (U)nknown or (Y)es.

1. Very irregular (large irregular change in depth to bedrock or bedrock is highly fractured)
2. Slightly irregular (small regular change in depth to bedrock or bedrock has small fractures)
3. Regular (little change in depth to bedrock or bedrock has no fractures

W

If the boundary between the aquifer material and the bedrock is very irregular in shape or the bedrock has fractures in it there is a good chance of groundwater leakage and it would be necessary to key the slurry wall into the bedrock to provide an impermeable barrier. This causes the slurry wall to be cost prohibitive and not a viable solution.

Do you know the condition of the boundary between the aquifer material and the bedrock? Answer (W)hy, (U)nknown or (Y)es.

1. Very irregular (large irregular change in depth to bedrock or bedrock is highly fractured)
2. Slightly irregular (small regular change in depth to bedrock or bedrock has small fractures)
3. Regular (little change in depth to bedrock or bedrock has no fractures

U

Since unknown was given as the answer the model will assume a slurry trench is a viable solution.

Do you know what soil type best describes the aquifer material? An answer must be given (U is unacceptable). Answer (W)hy or (Y)es.

SOIL TYPE	% CLAY	% SAND	%SILT
1. Sand	<10%	>90%	>90%
2. Sandy-loam	<20%	>85%	50-70%
3. Sandy-clay	35-55%	60-85%	50-65%
4. Silty-clay	40-60%	20-40%	40-60%
5. Clay	>40%	30-75%	<60%
6. Loam	5-25%	40-80%	75-95%

Y

Give your answer (Type 1-6) and your confidence (in per cent) in the answer. Separate each response with a space.

2 80

You have input no. 2 as your answer and 80 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Do you have any field data of hydraulic conductivity (ft/d)? Answer with (W)hy, (N)o or (Y)es.

Y

How many field values do you have for hydraulic conductivity.

4

You have declared that you have 4 hydraulic conductivity values. Do you wish to change this? Only (Y)es will allow you to change this input.

N

Enter all hydraulic conductivity values (ft/d). Type all values on one line with a space between each value and then press ENTER. Decimals are accepted but not required.

265.35 270 270 274.65

You have input these hydraulic conductivity values:

0.265E+03 0.270E+03 0.270E+03 0.275E+03

Do you wish to change any of these values? Only (Y)es will allow you to change this input.

N

Do you have any field data of effective porosity for this aquifer? Answer with (W)hy, (N)o or (Y)es.

Y

How many field values do you have for effective porosity.

4

You have declared that you have 4 effective porosity values. Do you wish to change this? Only (Y)es will allow you to change this input.

N

Enter all effective porosity values (in decimal). Type all values on one line with a space between each value and then press ENTER.

.24 .26 .34 .36

You have input these effective porosity values:

0.240E+00 0.260E+00 0.340E+00 0.360E+00

Do you wish to change any of these values? Only (Y)es will allow you to change this input.

N

Based on soil type, field or lab data or a combination of both:

the mean hydraulic conductivity is 270.0072 ft/d
with a standard error of 3.8382
the mean effective porosity is 0.30
with a standard error of 0.06

** Soil Characterisation Complete **

The following questions are intended to characterize the site enviroment. All questions require an answer. (U)nknown is unacceptable. If you are ready to continue type CLS and hit ENTER to clear the screen and hit ENTER again to begin the questions.

Execution suspended :

Do you understand the system assumption that constant enviromental conditions exist (and no other remedial action has been attempted) throughout the containment period? Answer (W)hy, (Y)es or (N)o.

Y

The following are acceptable estimates of average monthly precipitation (in/m) at the site during the entire pumping period. Can you estimate the average monthly precip. at your site for the time period that includes the optimal pumping and the steady pumping periods. Answer (W)hy or (Y)es.

1. 0 - 2
2. 2 - 4
3. > 4

Y

Give your answer (Type 1,2, or 3) and your confidence (in per cent) in the answer. Separate each response with a space.

2 90

You have input no. 2 as your answer and 90 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Below are common descriptions of drainage classes.Can you describe drainage at the site? Answer (W)hy or (Y)es.

Drainage Class	Observable action
1.Very poorly drained	Water remains at or on the surface most of the year
2.Poorly drained	Water remains at or on the surface some of the year
3.Somewhat poorly drained	Soils are wet for significant portions of the year
4.Moderately well drained	Soils are seasonably wet (high spring water table)
5.Well drained	Water readily removed from the soil
6.Somewhat excessively	Water is rapidly removed from the soil (i.e. uniform drained sands)
7.Excessively drained	Very rapid removal of water, little or no retention

Y

Give your answer (Type 1-7) and your confidence (in per cent) in the answer. Separate each response with a space.

6 80

You have input no. 6 as your answer and 80 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Can you estimate the average depth (ft) to the base of the aquifer? Answer (W)hy or (Y)es.

Y

Give your answer and your confidence (in per cent) in the answer. Separate each response with a space.

70 70

You have input 70.00 ft as your answer and 70% as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Can you estimate the average saturated thickness (ft) of the aquifer? Answer (W)hy or (Y)es.

Y

Give your answer and your confidence (in per cent) in the answer. Separate each response with a space.

50 70

You have input 50.00 ft as your answer and 70% as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Can you estimate the average hydraulic gradient (0.0-0.99) of the potentiometric surface of the aquifer in the direction of plume movement? Answer (W)hy or (Y)es.

Y

Give your answer and your confidence (in per cent) in the answer. Separate each response with a space.

.0044 60

You have input 0.004 as your answer and 60 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

** Site Characterization Complete **

The following questions are intended to characterize the contaminant. All questions require an answer. (U)nknown is unacceptable. If you are ready to continue type CLS and hit ENTER to clear the screen and hit ENTER again to begin the questions.

Execution suspended :

Do you understand the system assumption that water is the contaminant carrier and that advection is the major mechanism of contaminant movement? Answer (W)hy, (Y)es or (N)o.

N

These are assumptions that greatly simplify the prediction of plume movement. A more sophisticated model is needed if mechanical dispersion or molecular diffusion are also mechanisms of contaminant transport. The safety factor used in the calculation of plume extent provides for enough margin to account for dispersion.

Do you understand the system assumption that water is the contaminant carrier and that advection is the major mechanism of contaminant movement? Answer (W)hy, (Y)es or (N)o.

Y

Does the contaminant contain any of the following compounds? Answer (W)hy, (N)o or (Y)es.

Alcohol	Sulfuric acid	Calcium hydroxide
Hydrochloric acid	Sodium hydroxide	Brine (sp. gravity > 1.2)
Aldehydes	Ketones	Hydrocarbons (aliphatic and aromatic)
Heterocyclics	Organic acids	Acid chlorides
Phenols	Glycols	

N

What is your confidence (in per cent) in this answer?

90

You have input N as your answer and 90 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Can you estimate the length of time (days) from the present until a containment strategy can be implemented? Answer (W)hy or (Y)es.

Y

Give your answer and your confidence (in per cent) in the answer. Separate each response with a space.

30 90

You have input 30 days as your answer and 90 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

Can you estimate the present detectable maximum extent of the contaminant plume (ft) from its source point? Answer with (W)hy or (Y)es.

Y

Give your answer and your confidence (in per cent) in the answer. Separate each response with a space.

645 70

You have input 645.0 ft. as your answer and 70 % as your confidence in that answer. Do you wish to change either one of these values? Only (Y)es will allow you to change this input.

N

** Contaminant Characterization Complete **

Execution suspended : When you are ready to continue hit ENTER

What is the upper limit on pumping (cu.ft./pump/day) for a maximum lift of 45.00 ft. based on available equipment? This will be used to estimate the operating costs of the pumps for preliminary economic comparison between the systems under consideration.

W or Y are not necessary. Just input an answer.

.4E+6

You have input 400000.00 cu.ft./pump/day as your answer. Do you wish to change this input? Only (Y)es will allow you to change this.

N

A safety factor is used to estimate the future extent of the contaminant plume to ensure that the proposed octagonal containment strategy is outside the limits of the plume at the time of strategy implementation. After many simulation runs of hypothetical contamination problems it has been determined that this safety factor is most influenced by:

1. The uncertainty of the transmissivity value used; this uncertainty is measured by the coefficient of variation which is the standard error divided by the mean. This is based on a log-normal distribution for hydraulic conductivity.
2. The amount of precipitation in the contaminated area.
3. The drainage in the contaminated area.
4. A dispersivity value of 100 ft.

However, it was determined that any safety factor greater than 2 serves no purpose. Therefore, the largest safety factor used is 2.

Execution suspended : When you are ready to continue hit ENTER

The safety factor that will be used to estimate future plume extent is 1.429. This safety factor is based on an increase of 0.404 due to the transmissivity coefficient of variation and an additional increase of .025 due to your input of:

1. 2-4 in/month of precipitation and

2. Area is somewhat excessively drained.

Execution suspended : When you are ready to continue press ENTER

The system estimate for the present extent of the contaminant plume is 921.430 ft.
The system estimate for the future extent of the plume at containment implementation is 1091.149 ft. This is based on:

1. A hydraulic conductivity of 270.007 ft/d
2. A hydraulic gradient of 0.0044
3. An effective porosity of 0.300
4. Time to containment implementation of 30 days and
5. A safety factor of 1.429

Execution suspended : When you are ready to continue press ENTER

Based on a predicted plume extent of 1091.15 ft. each side of the regular octagon will be 904.10 ft. long. The capital cost estimate for the pumping scheme will be based on a well spacing of one-fourth of the side length (2 pump wells per side located at the 1/4 and 3/4 points), well holes drilled are 24" in diameter and fully penetrate the aquifer and a 1986 purchase price of $1500/pump.

The program will now calculate the capital costs for the three containment schemes.

Execution suspended : When you are ready to continue hit ENTER

The unit costs for the economic comparison are based on 1986 prices. Enter a coefficient to update these costs (Enter 1.00 if 1986 costs are acceptable).

1

You have input 1.00 as the coefficient to update the 1986 capital costs. Do you wish to change this? Only (Y)es will allow you to change this input.

N

The system recommends a pumping containment strategy. Its confidence in this recommendation is 59%.

Do you have any questions about:

1. Recommendation
2. Confidence value
3. None

Indicate by number.

2

The system confidence of 59% is based on:

The user confidence of 60% in the hydraulic gradient.

In addition, the confidence factor was further reduced because:

The user was uncertain about the amount of irregularity in the aquifer-bedrock interface.

Execution suspended : If you are ready to continue hit ENTER

Do you have any questions about:

1. Recommendation
2. Confidence value
3. None

Indicate by number.

1

The pumping capital cost was the smallest of the techniques considered. The costs were:

Pumping	569246.19
Slurry wall	63653780.00
Sheet Piling	3654844.25

However, it should be kept in mind that operation and maintenance (O&M) costs were not considered in this capital cost comparison. If the pumping technique is to be utilized for a long period of time the O&M costs for pumping become a major part of the economic analysis and need to be considered.

Therefore, based on these assumptions:

1. Pumping at all wells is at the upper limit specified.
2. Pumping lift is the maximum allowed (1/2 of the saturated thickness).
3. Pumps are replaced every 10 years.
4. Operating costs are $4.13 per 1 million cu.ft/ft and increase by 1.5 times every 10 yrs.
5. Maintenance costs are $38 per 1 million cu.ft and triple over a 10 yr. period.

If the pumping strategy exceeds 0.85E+00 years, the next least capital cost technique is the most economic.
Execution suspended : If you are ready to continue hit ENTER

How much time (days) should be allowed to stabilize the plume once the pumping strategy is begun (assuming pumping is begun immediately following installation of the wells? (Y)es is unnecessary. Just input a value.

8

You have input 8 days as your answer.Do you wish to change this? Only (Y)es will allow you to change this input.

N

The user can now run either the deterministic version or the stochastic version of the optimization program. If field or lab data is plentiful for this aquifer then it is recommended that the deterministic version be run because it develops optimal pumping values that are more predictable for the situation. The deterministic version is run by developing an input file, MODEL2.DAT, as described in Section VI and Appendix V.

The stochastic version is normally run if field or lab data is scarce. Because of the uncertainty involved in the data and the required reliability in the solution, the optimal pumping allowed within the constraints is less and the resulting heads at all wells are at higher estimated elevations as compared to the deterministic version. The input file and running of the stochastic version are also described in Section VI and Appendix V. In many cases it is advantageous to run both deterministic and stochastic versions and compare the results.

When you are ready to continue hit ENTER.
Execution suspended :

If you wish to create the input file, SMODEL.DAT, for the stochastic version the suggested input to the optimisation program is:

Transmissivity
mean 13500.360 ft.sqd./d
coefficient of variation 0.404

Effective porosity
mean 0.300
coefficient of variation 0.196

Octagon side length 904.10 ft.

Time period to stabilize plume 8 days

Well spacing - 1/2, 1/4, 1/8 of side length
Due to memory limitations, the stochastic optimization model cannot compute a strategy if wells are spaced at 1/8 of the side length. However, the pump spacing should never exceed the "effective radius of influence" of the pump for the 8 day time period specified

Would you like a hard copy of this information? (Make sure your printer is turned on.) Answer (Y)es or (N)o.

N

This program has the ability to develop an input file, SMODEL.DAT, for use with the stochastic version of the optimization model. This input is based on the mean and coefficient of variation for transmissivity and effective porosity calculated previously. The well configuration is based on 1 ft. radius pump wells located at the 1/4 and 3/4 points of each side of the octagon. The user will input an average ground slope and direction of that slope. The program assumes the hydraulic gradient to be symmetrical to the x-axis of the octagon and that the saturated thickness is constant.

Do you wish the program to develop this input file for you? Answer (Y)es or (N)o.

Y

You have asked the program to develop a data file to be used with the stochastic optimization model. Do you wish to change this input? Only (Y)es will allow you to make a change.

N

A maximum of 10 "time periods" is allowed in the optimization program for the pumping strategy to stabilize the plume. Select the units you wish to use for each time period (1,2 or 3).

1. Day
2. Week
3. Month

1

How many DAY (s) will you allow for the pumping strategy to stabilize movement of the plume once the wells are in place and functioning?

8

You have input 8 DAY (s) as your answer. Do you wish to change this input? Only (Y)es will allow you to change this.

N

How confident do you want to be in the final heads at the observation wells and the drawdowns at the pumping wells that are generated by the optimization program (This is referred to as a reliability level)? Answer 1,2,3,4 or 5

1. 99%
2. 95%
3. 90%
4. 85%
5. 80%

2

You have input 0.95 as the required confidence level for the optimization program. Do you wish to change this input? Only (Y)es will allow you to change this.

N

Input the average ground slope (ft/ft) in the area of contamination and the counter clockwise angle (degrees) from the positive x-axis to a line in the direction of the DOWNWARD slope. The positive x-axis is in the direction of the downward hydraulic gradient and the octagon of wells is symmetrical with respect to it. Separate the two values with a space.

0 0

You have input 0.0000 as the average slope of the ground and 0.0 degrees as the angle the downward slope makes with the direction of the hydraulic gradient (the x-axis). Do you wish to change this input? Only (Y)es will allow you to change this.

N

Input the ground elevation (ft) and the potentiometric surface elevation (ft) at the contaminant source. Separate the two values with a space.

120 101

You have input 120.00 as the ground elevation and 101.00 as the potentiometric surface elevation at the contaminant source. Do you wish to change this input? Only (Y)es will allow you to change this.

N

As described in Volume I, one must usually run the stochastic model several times to assure validity of results. This iterative process is performed until assumed pumping values input into the model are within about 5% of the optimal values subsequently computed by the model.

You are now ready to input assumed pumping values for SMODEL.DAT in cu.ft./ DAY /pump. If this data is for the first optimization, simply guess values for each DAY . For all others use the optimal values from the previous optimization as assumed values.

Input 8 pumping values with a space between each value (only 5 values per line, then hit return). These values must be less than the upper limit on pumping input previously.

.25E+6 .25E+6 .25E+6 .25E+6 .25E+6
.25E+6 .25E+6 .25E+6

You have input the following initial pumping values:

.25E+06 .25E+06 .25E+06 .25E+06 .25E+06 .25E+06 .25E+06 .25E+06

Do you wish to change this input? Only (Y)es will allow you to change this.

N

The input data file, SMODEL.DAT, has been created for running the stochastic version of the optimization program. Follow the detailed instructions in Section VI to run the program.

This program is complete. We hope it has been an aid in analyzing your contamination problem.
Execution terminated : 0

C:\BW>

APPENDIX V Data Input Format (for MODEL2.DAT or SMODEL.DAT, to be read by MODEL2.FOR or SMODEL.FOR respectively)

Card	Column	Format	Variable	Description
1	1-5	I5	I	Total number of pumping wells (max = 32 & 20)*
	6-10	I5	L	Total number of wells = 2I+1 (max = 65 & 41)
	11-15	I5	IT	Number of time periods (max = 10)
	16-20	F5.2	R	Radius of pumping well
	21-30	F10.2	AA	Distance from source point to octagon along x-axis
	32-35	A4	Time	Unit of time being used(skip col. 31)
	37-40	A4	Length	Unit of length being used(skip col 36)
	41-45	I5	Model	Indicates which model is being run; deterministic is 1, stochastic is 2
2	1-10	F10.2	QU	Upper limit on pumping (10^3 ft^3/Time)
	11-15	F5.2	EP	Effective porosity
deter.**	16-25	F10.2	Kmin	Minimum hydraulic conductivity (assumed at 90° to Kmax)
deter.	26-35	F10.2	Kmax	Maximum hydraulic conductivity
deter.	36-40	F5.2	Angl	Angle counterclockwise (CCW) from x-axis to Kmax
stoc.	16-20	F5.2	Covs	Coefficient of variation for effective porosity data (equal to standard error divided by the mean)

* First number is maximum for deterministic model and second is for stochastic model; if only one number is shown it is the maximum for both models.

** Card 2 is different for each model from column 16 to the right.

Card	Column	Format	Variable	Description
stoc.	21-25	F5.2	Covt	Coefficient of variation for transmissivity data
stoc.	26-30	F5.2	CL	Reliability as a decimal
stoc.	31-35	F5.2	F1	Standard normal deviate corresponding to reliability (Table 9)
stoc.	36-45	F10.2	TRANS	Transmissivity

Data set	Number of cards	Format	Variable	Description
1	8	F10.2,I5	SL,NP	Length of each side and total number of wells on each side (=2I; begin with side farthest down-gradient and go CCW)
2	I	2F10.2	HP(I,2)	Ground elev. & potentiometric surface elev. of each pump well (begin with pump well 1P, figure 5, and go CCW)
3	L-I	F10.2	HO(L-I)	Potentiometric surface elev. of each observation well (begin with source, go to well 1, figure 5, and go CCW)
4	L	F10.2	ST(L)	Saturated thickness of all wells (begin w/source,go to well 1, figure 5, and go CCW)
5	IT	F10.2	Q(IT)	Pumping values for each time period. These are used as the initial values for the stochastic model or are the optimal pumping values from GAMS to be used in HEAD.FOR. They are not needed w/deterministic model (10^3 ft^3/ Time).

APPENDIX VI Output File MODEL2.LST from GAMS-MINOS using MODEL2.DAT

```
GAMS 2.04 PC AT/XT                                    87/10/29 13:16:15  PAGE
  1
G E N E R A L  A L G E B R A I C  M O D E L I N G  S Y S T E M
C O M P I L A T I O N

  2  * FOR SIDE 1
  3  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
  4  * FOR SIDE 2
  5  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
  6  * FOR SIDE 3
  7  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
  8  * FOR SIDE 4
  9  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
 10  * FOR SIDE 5
 11  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
 12  * FOR SIDE 6
 13  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
 14  * FOR SIDE 7
 15  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
 16  * FOR SIDE 8
 17  *   THE L=  900.00 ;NO. PUMPS=   4 ;SPACING=  450.00
 18  * WELL RADIUS IS 1.00
 19  * EFFECTIVE POROSITY IS    0.30
 20  * HCMIN IS  180.00
 21  * HCMAX IS  270.00
 22  * TIME PERIOD IS A   DAY
 23  * LENGTH DIMENSION IS    FT
 24  * LOW LIMIT ON DD AT PUMP WELLS = 1/2(SAT. THICK.)
 25  * HIGH LIMIT ON DD AT PUMP WELLS = GROUND ELEV.
 26
 27  SETS
 28      I PUMPING WELLS /1*  16/
 29      T TIME STEPS/1* 8/
 30      J DUMMY SET /1/
 31      N DUMMY SET /1*2/
 32  SCALAR
 33        QU UPPER PUMPING /  400.00/
 34        CON CONSTANT TERM IN SQD HEAD DIFF /   296.0/
 35        WF WEIGHT FACTOR /    1.00/
 36        HS SOURCE PIEZ. ELEV. /  101.00/
 37        FT FINAL TIME PERIOD /   8/
 38  PARAMETER
 39            HOB(I) INITIAL HEAD AT EACH OBS WELL
 40            / 1     95.00
 41              2     97.00
 42              3     99.00
 43              4    101.00
 44              5    103.00
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45          6     105.00
46          7     107.00
47          8     107.00
48          9     107.00
49         10     105.00
50         11     103.00
51         12     101.00
52         13      99.00
53         14      97.00
54         15      95.00
55         16      95.00/
56
57        ST(I) SATURATED THICK. AT EACH PUMP WELL
58        / 1      50.00
59          2      50.00
60          3      50.00
61          4      50.00
62          5      50.00
63          6      50.00
64          7      50.00
65          8      50.00
66          9      50.00
67         10      50.00
68         11      50.00
69         12      50.00
70         13      50.00
71         14      50.00
72         15      50.00
73         16      50.00/
74
75        SC(T) INFLUENCE COEFS. FOR SOURCE WELL
76        / 1 -0.3315E-07
77          2 -0.1337E-05
78          3 -0.3576E-05
79          4 -0.4572E-05
80          5 -0.4669E-05
81          6 -0.4396E-05
82          7 -0.4010E-05
83          8 -0.3615E-05/
84
85        TI(I) SPECIFIES OBS WELLS DOWN GRADIENT OF SOURCE
86        / 1     1
87          2     1
88          3     1
89          4     0
90          5     0
91          6     0
92          7     0
93          8     0
94          9     0
```

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 95            10     0
 96            11     0
 97            12     0
 98            13     1
 99            14     1
100            15     1
101            16     1/
102
103           TT(T) TIME VECTOR TO PROVIDE PARTIAL SUMS
104           / 1     1
105             2     2
106             3     3
107             4     4
108             5     5
109             6     6
110             7     7
111             8     8/

112  TABLE HO(I,N) GROUND EL. & INIT. HEAD AT EACH PUMP WELL
113                 1          2
114        1     120.00      96.00
115        2     120.00      98.00
116        3     120.00     100.00
117        4     120.00     102.00
118        5     120.00     104.00
119        6     120.00     106.00
120        7     120.00     107.00
121        8     120.00     107.00
122        9     120.00     106.00
123       10     120.00     104.00
124       11     120.00     102.00
125       12     120.00     100.00
126       13     120.00      98.00
127       14     120.00      96.00
128       15     120.00      95.00
129       16     120.00      95.00

130  TABLE B(I,T) INFLUENCE COEF.AT PUMP WELLS
131                 1            2            3            4            5
132        1 -0.6979E-01 -0.8494E-02 -0.6710E-02 -0.5616E-02 -0.4833E-02
133        2 -0.7005E-01 -0.7953E-02 -0.5578E-02 -0.4289E-02 -0.3483E-02
134        3 -0.6816E-01 -0.4295E-02 -0.2504E-02 -0.1776E-02 -0.1381E-02
135        4  0.6816E-01  0.4295E-02  0.2504E-02  0.1776E-02  0.1381E-02
136        5  0.7005E-01  0.7953E-02  0.5578E-02  0.4289E-02  0.3483E-02
137        6  0.6979E-01  0.8494E-02  0.6710E-02  0.5616E-02  0.4833E-02
138        7  0.6960E-01  0.8435E-02  0.6885E-02  0.5979E-02  0.5319E-02
139        8  0.6960E-01  0.8435E-02  0.6885E-02  0.5979E-02  0.5319E-02
140        9  0.6979E-01  0.8494E-02  0.6710E-02  0.5616E-02  0.4833E-02
141       10  0.7005E-01  0.7953E-02  0.5578E-02  0.4289E-02  0.3483E-02
142       11  0.6816E-01  0.4295E-02  0.2504E-02  0.1776E-02  0.1381E-02
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143        12 -0.6816E-01 -0.4295E-02 -0.2504E-02 -0.1776E-02 -0.1381E-02
144        13 -0.7005E-01 -0.7953E-02 -0.5578E-02 -0.4289E-02 -0.3483E-02
145        14 -0.6979E-01 -0.8494E-02 -0.6710E-02 -0.5616E-02 -0.4833E-02
146        15 -0.6960E-01 -0.8435E-02 -0.6885E-02 -0.5979E-02 -0.5319E-02
147        16 -0.6960E-01 -0.8435E-02 -0.6885E-02 -0.5979E-02 -0.5319E-02
148
149  +            6            7            8
150         1 -0.4234E-02 -0.3756E-02 -0.3365E-02
151         2 -0.2932E-02 -0.2531E-02 -0.2223E-02
152         3 -0.1133E-02 -0.9611E-03 -0.8351E-03
153         4  0.1133E-02  0.9611E-03  0.8351E-03
154         5  0.2932E-02  0.2531E-02  0.2223E-02
155         6  0.4234E-02  0.3756E-02  0.3365E-02
156         7  0.4783E-02  0.4327E-02  0.3930E-02
157         8  0.4783E-02  0.4327E-02  0.3930E-02
158         9  0.4234E-02  0.3756E-02  0.3365E-02
159        10  0.2932E-02  0.2531E-02  0.2223E-02
160        11  0.1133E-02  0.9611E-03  0.8351E-03
161        12 -0.1133E-02 -0.9611E-03 -0.8351E-03
162        13 -0.2932E-02 -0.2531E-02 -0.2223E-02
163        14 -0.4234E-02 -0.3756E-02 -0.3365E-02
164        15 -0.4783E-02 -0.4327E-02 -0.3930E-02
165        16 -0.4783E-02 -0.4327E-02 -0.3930E-02

166  TABLE OB(I,T) INFLUENCE COEF.AT OBS WELLS
167                 1            2            3            4            5
168         1 -0.1184E-01 -0.9571E-02 -0.7389E-02 -0.6220E-02 -0.5419E-02
169         2 -0.1192E-01 -0.9395E-02 -0.6981E-02 -0.5572E-02 -0.4634E-02
170         3 -0.1138E-01 -0.7140E-02 -0.4599E-02 -0.3405E-02 -0.2710E-02
171         4 -0.4102E-18 -0.3388E-19 -0.8132E-19 -0.1084E-18  0.0000E+00
172         5  0.1138E-01  0.7140E-02  0.4599E-02  0.3405E-02  0.2710E-02
173         6  0.1192E-01  0.9395E-02  0.6981E-02  0.5572E-02  0.4634E-02
174         7  0.1184E-01  0.9571E-02  0.7389E-02  0.6220E-02  0.5419E-02
175         8  0.1196E-01  0.1039E-01  0.8155E-02  0.6934E-02  0.6098E-02
176         9  0.1184E-01  0.9571E-02  0.7389E-02  0.6220E-02  0.5419E-02
177        10  0.1192E-01  0.9395E-02  0.6981E-02  0.5572E-02  0.4634E-02
178        11  0.1138E-01  0.7140E-02  0.4599E-02  0.3405E-02  0.2710E-02
179        12  0.2954E-18 -0.1876E-18  0.1221E-18  0.2778E-18  0.8132E-19
180        13 -0.1138E-01 -0.7140E-02 -0.4599E-02 -0.3405E-02 -0.2710E-02
181        14 -0.1192E-01 -0.9395E-02 -0.6981E-02 -0.5572E-02 -0.4634E-02
182        15 -0.1184E-01 -0.9571E-02 -0.7389E-02 -0.6220E-02 -0.5419E-02
183        16 -0.1196E-01 -0.1039E-01 -0.8155E-02 -0.6934E-02 -0.6098E-02
184
185  +            6            7            8
186         1 -0.4804E-02 -0.4306E-02 -0.3890E-02
187         2 -0.3962E-02 -0.3455E-02 -0.3058E-02
188         3 -0.2255E-02 -0.1932E-02 -0.1692E-02
189         4 -0.5421E-19 -0.1084E-18 -0.1355E-18
190         5  0.2255E-02  0.1932E-02  0.1692E-02
191         6  0.3962E-02  0.3455E-02  0.3058E-02
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7	0.4804E-02	0.4306E-02	0.3890E-02
8	0.5445E-02	0.4904E-02	0.4440E-02
9	0.4804E-02	0.4306E-02	0.3890E-02
10	0.3962E-02	0.3455E-02	0.3058E-02
11	0.2255E-02	0.1932E-02	0.1692E-02
12	0.8132E-19	0.2372E-18	0.1897E-18
13	-0.2255E-02	-0.1932E-02	-0.1692E-02
14	-0.3962E-02	-0.3455E-02	-0.3058E-02
15	-0.4804E-02	-0.4306E-02	-0.3890E-02
16	-0.5445E-02	-0.4904E-02	-0.4440E-02

TABLE C(I,T) LINEAR HYDR. COEFS. OF OBJ. FUNC.

	1	2	3	4	5
1	-0.4663E-01	-0.5162E-01	-0.5760E-01	-0.6497E-01	-0.7459E-01
2	-0.2443E-01	-0.2761E-01	-0.3166E-01	-0.3704E-01	-0.4454E-01
3	-0.6752E-02	-0.7713E-02	-0.9001E-02	-0.1082E-01	-0.1360E-01
4	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
5	-0.6781E-02	-0.7745E-02	-0.9036E-02	-0.1086E-01	-0.1364E-01
6	-0.2449E-01	-0.2767E-01	-0.3173E-01	-0.3711E-01	-0.4462E-01
7	-0.4672E-01	-0.5172E-01	-0.5771E-01	-0.6508E-01	-0.7470E-01
8	-0.5332E-01	-0.5890E-01	-0.6540E-01	-0.7323E-01	-0.8326E-01
9	-0.4672E-01	-0.5172E-01	-0.5771E-01	-0.6508E-01	-0.7470E-01
10	-0.2449E-01	-0.2767E-01	-0.3173E-01	-0.3711E-01	-0.4462E-01
11	-0.6781E-02	-0.7745E-02	-0.9036E-02	-0.1086E-01	-0.1364E-01
12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
13	-0.6752E-02	-0.7713E-02	-0.9001E-02	-0.1082E-01	-0.1360E-01
14	-0.2443E-01	-0.2761E-01	-0.3166E-01	-0.3704E-01	-0.4454E-01
15	-0.4663E-01	-0.5162E-01	-0.5760E-01	-0.6497E-01	-0.7459E-01
16	-0.5324E-01	-0.5880E-01	-0.6529E-01	-0.7312E-01	-0.8315E-01

+	6	7	8
1	-0.8863E-01	-0.1148E+00	-0.1421E+00
2	-0.5582E-01	-0.7515E-01	-0.9539E-01
3	-0.1838E-01	-0.2856E-01	-0.4553E-01
4	0.0000E+00	0.0000E+00	0.0000E+00
5	-0.1841E-01	-0.2857E-01	-0.4553E-01
6	-0.5586E-01	-0.7517E-01	-0.9539E-01
7	-0.8871E-01	-0.1149E+00	-0.1421E+00
8	-0.9790E-01	-0.1247E+00	-0.1435E+00
9	-0.8871E-01	-0.1149E+00	-0.1421E+00
10	-0.5588E-01	-0.7517E-01	-0.9539E-01
11	-0.1841E-01	-0.2857E-01	-0.4553E-01
12	0.0000E+00	0.0000E+00	0.0000E+00
13	-0.1838E-01	-0.2856E-01	-0.4553E-01
14	-0.5582E-01	-0.7515E-01	-0.9539E-01
15	-0.8863E-01	-0.1148E+00	-0.1421E+00
16	-0.9781E-01	-0.1247E+00	-0.1435E+00

TABLE CT(I,T) LINEAR ECON COEFS. OF OBJ. FUNC.

1	2	3	4	5

240	1	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
241	2	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
242	3	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
243	4	0.1123E-03	0.1123E-03	0.1123E-03	0.1123E-03	0.1123E-03
244	5	0.1040E-03	0.1040E-03	0.1040E-03	0.1040E-03	0.1040E-03
245	6	0.9573E-04	0.9573E-04	0.9573E-04	0.9573E-04	0.9573E-04
246	7	0.9160E-04	0.9160E-04	0.9160E-04	0.9160E-04	0.9160E-04
247	8	0.9160E-04	0.9160E-04	0.9160E-04	0.9160E-04	0.9160E-04
248	9	0.9573E-04	0.9573E-04	0.9573E-04	0.9573E-04	0.9573E-04
249	10	0.1040E-03	0.1040E-03	0.1040E-03	0.1040E-03	0.1040E-03
250	11	0.1123E-03	0.1123E-03	0.1123E-03	0.1123E-03	0.1123E-03
251	12	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
252	13	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
253	14	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
254	15	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
255	16	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04	0.3788E-04
256						
257	+	6	7	8		
258	1	0.3788E-04	0.3788E-04	0.3788E-04		
259	2	0.3788E-04	0.3788E-04	0.3788E-04		
260	3	0.3788E-04	0.3788E-04	0.3788E-04		
261	4	0.1123E-03	0.1123E-03	0.1123E-03		
262	5	0.1040E-03	0.1040E-03	0.1040E-03		
263	6	0.9573E-04	0.9573E-04	0.9573E-04		
264	7	0.9160E-04	0.9160E-04	0.9160E-04		
265	8	0.9160E-04	0.9160E-04	0.9160E-04		
266	9	0.9573E-04	0.9573E-04	0.9573E-04		
267	10	0.1040E-03	0.1040E-03	0.1040E-03		
268	11	0.1123E-03	0.1123E-03	0.1123E-03		
269	12	0.3788E-04	0.3788E-04	0.3788E-04		
270	13	0.3788E-04	0.3788E-04	0.3788E-04		
271	14	0.3788E-04	0.3788E-04	0.3788E-04		
272	15	0.3788E-04	0.3788E-04	0.3788E-04		
273	16	0.3788E-04	0.3788E-04	0.3788E-04		

274 TABLE K(I,T) HYDR QUAD COEFS OF OBJ FUNC(I=OBSER WELL)

		1	2	3	4	5
276	1	-0.3886E-02	-0.4302E-02	-0.4800E-02	-0.5414E-02	-0.6216E-02
277	2	-0.3054E-02	-0.3451E-02	-0.3958E-02	-0.4630E-02	-0.5568E-02
278	3	-0.1688E-02	-0.1928E-02	-0.2250E-02	-0.2705E-02	-0.3400E-02
279	4	0.3615E-05	0.4010E-05	0.4396E-05	0.4669E-05	0.4572E-05
280	5	0.1695E-02	0.1936E-02	0.2259E-02	0.2714E-02	0.3409E-02
281	6	0.3061E-02	0.3459E-02	0.3967E-02	0.4639E-02	0.5577E-02
282	7	0.3893E-02	0.4310E-02	0.4809E-02	0.5423E-02	0.6225E-02
283	8	0.4444E-02	0.4908E-02	0.5450E-02	0.6103E-02	0.6938E-02
284	9	0.3893E-02	0.4310E-02	0.4809E-02	0.5423E-02	0.6225E-02
285	10	0.3061E-02	0.3459E-02	0.3967E-02	0.4639E-02	0.5577E-02
286	11	0.1695E-02	0.1936E-02	0.2259E-02	0.2714E-02	0.3409E-02
287	12	0.3615E-05	0.4010E-05	0.4396E-05	0.4669E-05	0.4572E-05
288	13	-0.1688E-02	-0.1928E-02	-0.2250E-02	-0.2705E-02	-0.3400E-02

14	-0.3054E-02	-0.3451E-02	-0.3958E-02	-0.4630E-02	-0.5568E-02
15	-0.3886E-02	-0.4302E-02	-0.4800E-02	-0.5414E-02	-0.6216E-02
16	-0.4436E-02	-0.4900E-02	-0.5441E-02	-0.6094E-02	-0.6929E-02

+	6	7	8
1	-0.7386E-02	-0.9570E-02	-0.1184E-01
2	-0.6978E-02	-0.9393E-02	-0.1192E-01
3	-0.4595E-02	-0.7139E-02	-0.1138E-01
4	0.3576E-05	0.1337E-05	0.3315E-07
5	0.4603E-02	0.7142E-02	0.1138E-01
6	0.6985E-02	0.9396E-02	0.1192E-01
7	0.7393E-02	0.9572E-02	0.1184E-01
8	0.8158E-02	0.1040E-01	0.1196E-01
9	0.7393E-02	0.9572E-02	0.1184E-01
10	0.6985E-02	0.9396E-02	0.1192E-01
11	0.4603E-02	0.7142E-02	0.1138E-01
12	0.3576E-05	0.1337E-05	0.3315E-07
13	-0.4595E-02	-0.7139E-02	-0.1138E-01
14	-0.6978E-02	-0.9393E-02	-0.1192E-01
15	-0.7386E-02	-0.9570E-02	-0.1184E-01
16	-0.8151E-02	-0.1039E-01	-0.1196E-01

TABLE KT(I,T) ECONOMIC QUADRATIC COEFS. OF OBJ. FUNC.

	1	2	3	4	5
1	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
2	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
3	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
4	0.2816E-06	0.1775E-07	0.1035E-07	0.7339E-08	0.5707E-08
5	0.2895E-06	0.3286E-07	0.2305E-07	0.1772E-07	0.1439E-07
6	0.2884E-06	0.3510E-07	0.2773E-07	0.2321E-07	0.1997E-07
7	0.2876E-06	0.3485E-07	0.2845E-07	0.2471E-07	0.2198E-07
8	0.2876E-06	0.3485E-07	0.2845E-07	0.2471E-07	0.2198E-07
9	0.2884E-06	0.3510E-07	0.2773E-07	0.2321E-07	0.1997E-07
10	0.2895E-06	0.3286E-07	0.2305E-07	0.1772E-07	0.1439E-07
11	0.2816E-06	0.1775E-07	0.1035E-07	0.7339E-08	0.5707E-08
12	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
13	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
14	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
15	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00
16	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00	0.0000E+00

+	6	7	8
1	0.0000E+00	0.0000E+00	0.0000E+00
2	0.0000E+00	0.0000E+00	0.0000E+00
3	0.0000E+00	0.0000E+00	0.0000E+00
4	0.4682E-08	0.3971E-08	0.3451E-08
5	0.1212E-07	0.1046E-07	0.9187E-08
6	0.1750E-07	0.1552E-07	0.1390E-07
7	0.1976E-07	0.1788E-07	0.1624E-07
8	0.1976E-07	0.1788E-07	0.1624E-07

```
338 ,       9  0.1750E-07  0.1552E-07  0.1390E-07
339        10  0.1212E-07  0.1046E-07  0.9187E-08
340        11  0.4682E-08  0.3971E-08  0.3451E-08
341        12  0.0000E+00  0.0000E+00  0 0000E+00
342        13  0.0000E+00  0.0000E+00  0.0000E+00
343        14  0.0000E+00  0.0000E+00  0.0000E+00
344        15  0.0000E+00  0.0000E+00  0.0000E+00
345        16  0.0000E+00  0.0000E+00  0.0000E+00
346
347  ALIAS (T,L,M);
348
349  TABLE IND(L,M) INDICE MATRIX FOR SUMMING B(T-T+1)*QT
350        .  1  2  3  4  5  6  7  8
351       1  1  2  3  4  5  6  7  8
352       2  2  3  4  5  6  7  8
353       3  3  4  5  6  7  8
354       4  4  5  6  7  8
355       5  5  6  7  8
356       6  6  7  8
357       7  7  8
358       8  8
359
360  VARIABLE  Q(T,J)   PUMPING DURING EACH TIME PERIOD
361            MIN      SYMBOL FOR OBJECTIVE FUNCTION
362            S1       LINEAR HYDRAULIC PORTION OF OBJEC. FUNC.
363            S2       LINEAR ECONOMIC PORTION OF OBJEC FUNC
364            S3       HYDRAULIC QUAD. PORTION OF OB. FUNC.
365            S4       ECONOMIC QUAD. PORTION OF OB. FUNC.
366            ;
367  POSITIVE VARIABLE Q(T,J);
368  FREE VARIABLE MIN;
369
370  EQUATIONS  WTL     LOWER WATER TABLE LIMIT
371             WTH     UPPER WATER TABLE LIMIT
372             OBJ     OBJECTIVE FUNCTION
373             OB1     LINEAR HYDRAULIC PORTION OF OB. FUNC.
374             OB2     LINEAR ECONOMIC PORTION OF OB FUNC
375             OB3     HYDRAULIC QUAD.PORTION OF OB. FUNC.
376             OB4     ECONOMIC QUAD. PORTION OF OB. FUNC.
377             GRAD    CAUSES DOWM GRAD OBS WELLS TO BE HIGHER THAN SOUR
378             ;
379  WTL(I,T,J)..
380         0.5*ST(I)-SUM((L,M),B(I,L)*Q(M,J)$(IND(L,M) EQ TT(T))) =G= 0
381  WTH(I,T,J)..
382   HO(I,'1')-(HO(I,'2')-SUM((L,M),B(I,L)*Q(M,J)$(IND(L,M) EQ TT(T))))
=

.0;
383  GRAD(I,J)$(TI(I))..
384           HOB(I)-SUM((L,M),OB(I,L)*Q(M,J)$(IND(L,M) EQ FT))
```

```
385              -(HS-SUM((L,M),SC(L)*Q(M,J)$(IND(L,M) EQ FT))) =G= 0.0;
386  OB1..          SUM((I,T,J),C(I,T)*Q(T,J)) =E= S1;
387  OB2..          SUM((I,T,J),CT(I,T)*Q(T,J)) =E= S2;
388  OB3..          SUM((I),WF*SQR(SUM((T,J),K(I,T)*(Q(T,J))))) =E= S3;
389  OB4..   SUM((I,T,J),SUM((L,M),KT(I,L)*Q(M,J)$(IND(L,M) EQ TT(T)))
390              *Q(T,J)) =E= S4;
391
392  OBJ..          S1+S2+S3+S4+CON =E= MIN;
393
394  Q.UP(T,J)=QU;
395  Q.LO(T,J)=0.00;
396  Q.L(T,J)=105.00;
397
398  MODEL CONTAM /ALL/;
399
400  OPTION ITERLIM = 2000;
401  OPTION LIMROW = 0;
402  OPTION LIMCOL = 0;
403  *PTION SOLPRINT = OFF;
404
405  SOLVE CONTAM USING NLP MINIMIZING MIN;
406
407  DISPLAY Q.L, Q.M, Q.LO, Q.UP, MIN.L;
408
409  * THE INDICE MATRIX (L,M) IS A DUMMY MATRIX USED TO ALLOW THE CORRE
410  * MULTIPLICATION OF KT(I,T)*Q(T,J) (Q(T,J) IS ACTUALLY A COLUMN VEC
411  * BUT THE DUMMY J=1 IS NEEDED BECAUSE ALL MATRICES MUST BE AT LEAST
)
412  * I.E. FOR TIME PERIOD 2 TT(T)=2; THEREFORE IN THE INDICE MATRIX FO
413  * ALL TWOS THE MULTIPLICATIONS TAKE PLACE(WHEN L=2,M=1 AND WHEN L=1
414  * M=2) SO KT(I,2)*Q(1,1)+KT(I,1)*Q(2,1) IS THE RESULT.
415
416  * THE ALIAS FUNCTION ALLOWS US TO SAY THAT L OR M CAN BE SUBSTITUTE
417  * FOR T IN ANY MATRIX.
418
419  * BECAUSE T IS COMPARED TO OTHER VALUES IT MUST BE SET AS A PARAMET
420
421  * THE OB3 EQUATION IS MULTIPLYING EACH ROW OF THE K MATRIX
422  * BY THE COLUMN VECTOR Q,THEN SQUARING THE ROW TIMES THE Q VECTOR A
423  * THEN SUMMING THESE.
424
425  * THE OB4 EQUATION ONLY USES THAT PART OF THE KT MATRIX THAT IT
426  * NEEDS DEPENDING ON THE TIME PERIOD BEING ANALYZED. BY ONLY USING
427  * THE L AND M VALUES FOR WHICH THERE IS A T VALUE INSIDE THE MATRIX
428  * ALLOWS THIS TO BE DONE. (SEE EXPLANATION OF INDICE MATRIX)
429  * EXAMPLE: FOR 4 TIME PERIODS THE TOTAL ECONOMIC VALUE FOR THE
430  *          QUADRATIC PORTION WOULD EQUAL-
431  * KT(I,4)Q(1)+KT(I,3)Q(2)+KT(I,2)Q(3)+KT(I,1)Q(4)+KT(I,3)Q(1)+
432  * KT(I,2)Q(2)+KT(I,1)Q(3)+KT(I,2)Q(1)+KT(I,1)Q(2)+KT(I,1)Q(1)
433  * SUMMED OVER ALL I (PUMPING WELLS)
```

```
COMPILATION TIME     =        0.485 MINUTES

MODEL STATISTICS

BLOCKS OF EQUATIONS      8       SINGLE EQUATIONS   268
BLOCKS OF VARIABLES      6       SINGLE VARIABLES    13
NON ZERO ELEMENTS     1249       NON LINEAR N-Z      16
DERIVATIVE POOL         20       CONSTANT POOL      176
CODE LENGTH           4241

GENERATION TIME      =        3.410 MINUTES


EXECUTION TIME       =        3.631 MINUTES

               S O L V E      S U M M A R Y

     MODEL   CONTAM                OBJECTIVE  MIN
     TYPE    NLP                   DIRECTION  MINIMIZE
     SOLVER  MINOS5                FROM LINE  405

**** SOLVER STATUS     1 NORMAL COMPLETION
**** MODEL STATUS      2 LOCALLY OPTIMAL
**** OBJECTIVE VALUE                10.6164

 RESOURCE USAGE, LIMIT          2.083      1000.000
 ITERATION COUNT, LIMIT        15          2000
 EVALUATION ERRORS              0             0

     M I N O S  ---  VERSION  5.1   Jun  1987
     = = = = =

     B. A. Murtagh, University of New South Wales
       and
     P. E. Gill,  W. Murray,  M. A. Saunders and M. H. Wright
     Systems Optimization Laboratory, Stanford University.


 WORK SPACE NEEDED (ESTIMATE)  --  124410 WORDS.
 WORK SPACE AVAILABLE          --   30618 WORDS.


 EXIT -- OPTIMAL SOLUTION FOUND
 MAJOR ITNS, SUPERBASICS        8        0
 FUNOBJ, FUNCON CALLS           0       49
 INTERPRETER USAGE            .43
 NORM RG / NORM PI      0.000E+00
```

---- EQU WTL LOWER WATER TABLE LIMIT

	LOWER	LEVEL	UPPER	MARGINAL
1 .1.1	-25.000	20.478	+INF	.
1 .2.1	-25.000	21.680	+INF	.
1 .3.1	-25.000	22.413	+INF	.
1 .4.1	-25.000	22.777	+INF	.
1 .5.1	-25.000	19.205	+INF	.
1 .6.1	-25.000	7.183	+INF	.
1 .7.1	-25.000	6.089	+INF	.
1 .8.1	-25.000	5.301	+INF	.
2 .1.1	-25.000	20.555	+INF	.
2 .2.1	-25.000	21.593	+INF	.
2 .3.1	-25.000	22.000	+INF	.
2 .4.1	-25.000	22.000	+INF	.
2 .5.1	-25.000	18.064	+INF	.
2 .6.1	-25.000	5.712	+INF	.
2 .7.1	-25.000	4.495	+INF	.
2 .8.1	-25.000	3.725	+INF	.
3 .1.1	-25.000	20.000	+INF	.

---- EQU WTH UPPER WATER TABLE LIMIT

	LOWER	LEVEL	UPPER	MARGINAL
1 .1.1	-24.000	-20.478	+INF	.
1 .2.1	-24.000	-21.680	+INF	.
1 .3.1	-24.000	-22.413	+INF	.
1 .4.1	-24.000	-22.777	+INF	.
1 .5.1	-24.000	-19.205	+INF	.
1 .6.1	-24.000	-7.183	+INF	.
1 .7.1	-24.000	-6.089	+INF	.
1 .8.1	-24.000	-5.301	+INF	.
2 .1.1	-22.000	-20.555	+INF	.
2 .2.1	-22.000	-21.593	+INF	.
2 .3.1	-22.000	-22.000	+INF	0.055
2 .4.1	-22.000	-22.000	+INF	0.040
2 .5.1	-22.000	-18.064	+INF	.
2 .6.1	-22.000	-5.712	+INF	.
2 .7.1	-22.000	-4.495	+INF	.
2 .8.1	-22.000	-3.725	+INF	.
3 .1.1	-20.000	-20.000	+INF	0.061
3 .2.1	-20.000	-20.000	+INF	0.060
3 .3.1	-20.000	-19.602	+INF	.
3 .4.1	-20.000	-19.006	+INF	.
3 .5.1	-20.000	-14.727	+INF	.
3 .6.1	-20.000	-2.550	+INF	.

3 .7.1	-20.000	-1.832	+INF	.
3 .8.1	-20.000	-1.457	+INF	.

	LOWER	LEVEL	UPPER	MARGINAL
---- EQU OBJ	-296.000	-296.000	-296.000	-1.000
---- EQU OB1	.	.	.	-1.000
---- EQU OB2	.	.	.	-1.000
---- EQU OB3	.	.	.	-1.000
---- EQU OB4	.	.	.	-1.000

OBJ OBJECTIVE FUNCTION

OB1 LINEAR HYDRAULIC PORTION OF OB. FUNC.
OB2 LINEAR ECONOMIC PORTION OF OB FUNC
OB3 HYDRAULIC QUAD.PORTION OF OB. FUNC.
OB4 ECONOMIC QUAD. PORTION OF OB. FUNC.

---- EQU GRAD CAUSES DOWM GRAD OBS WELLS TO BE HIGHER THAN SOURCE

	LOWER	LEVEL	UPPER	MARGINAL
1 .1	6.000	6.000	+INF	14.954
2 .1	4.000	4.996	+INF	.
3 .1	2.000	2.877	+INF	.
13.1	2.000	2.877	+INF	.
14.1	4.000	4.996	+INF	.
15.1	6.000	6.000	+INF	.
16.1	6.000	6.785	+INF	.

---- VAR Q PUMPING DURING EACH TIME PERIOD

	LOWER	LEVEL	UPPER	MARGINAL
1.1	.	293.427	400.000	.
2.1	.	274.937	400.000	.
3.1	.	259.482	400.000	.
4.1	.	244.743	400.000	.
5.1	.	177.996	400.000	.
6.1	.	.	400.000	0.005
7.1	.	.	400.000	0.017
8.1	.	.	400.000	0.038

	LOWER	LEVEL	UPPER	MARGINAL
---- VAR MIN	-INF	10.616	+INF	.

```
---- VAR S1              -INF   -657.381     +INF        .
---- VAR S2              -INF      1.389     +INF        .
---- VAR S3              -INF    369.752     +INF        .
---- VAR S4              -INF      0.856     +INF        .

  MIN          SYMBOL FOR OBJECTIVE FUNCTION
  S1           LINEAR HYDRAULIC PORTION OF OBJEC. FUNC.
  S2           LINEAR ECONOMIC PORTION OF OBJEC FUNC
  S3           HYDRAULIC QUAD. PORTION OF OB. FUNC.
  S4           ECONOMIC QUAD. PORTION OF OB. FUNC.


**** REPORT SUMMARY :          0     NONOPT
                               0 INFEASIBLE
                               0  UNBOUNDED
                               0     ERRORS


----     407 VARIABLE  Q.L            PUMPING DURING EACH TIME PERIOD

             1

1      293.427
2      274.937
3      259.482
4      244.743
5      177.996


----     407 VARIABLE  Q.M            PUMPING DURING EACH TIME PERIOD

             1

6        0.005
7        0.017
8        0.038


----     407 VARIABLE  Q.LO           PUMPING DURING EACH TIME PERIOD

                       ALL       0.000


----     407 VARIABLE  Q.UP           PUMPING DURING EACH TIME PERIOD

             1

1      400 000
2      400.000
3      400 000
```

```
4       400.000
5       400.000
6       400.000
7       400.000
8       400.000


----     407 VARIABLE  MIN.L              =          10.616 SYMBOL FOR OBJECTIVE
                                                           FUNCTION


**** FILE SUMMARY

INPUT    C:\BW\MODEL2.GMS
OUTPUT   C:\BW\MODEL2.LST

EXECUTION TIME       =          0.410 MINUTES
```

APPENDIX VII Output File MODEL2.CAL from HEAD.FOR using MODEL2.DAT

```
Q=   293.427 x 1000 CU.   FT/ DAY
Q=   274.937 x 1000 CU.   FT/ DAY
Q=   259.482 x 1000 CU.   FT/ DAY
Q=   244.743 x 1000 CU.   FT/ DAY
Q=   177.996 x 1000 CU.   FT/ DAY
Q=     0.000 x 1000 CU.   FT/ DAY
Q=     0.000 x 1000 CU.   FT/ DAY
Q=     0.000 x 1000 CU.   FT/ DAY

TARGET ELEV IS  101.0053   FT

OBSER WELL   1  ELEV. IS  101.0053   FT
OBSER WELL   2  ELEV. IS  102.0012   FT
OBSER WELL   3  ELEV. IS  101.8821   FT
OBSER WELL   4  ELEV. IS  101.0000   FT
OBSER WELL   5  ELEV. IS  100.1179   FT
OBSER WELL   6  ELEV. IS   99.9988   FT
OBSER WELL   7  ELEV. IS  100.9947   FT
OBSER WELL   8  ELEV. IS  100.2094   FT
OBSER WELL   9  ELEV. IS  100.9947   FT
OBSER WELL  10  ELEV. IS   99.9988   FT
OBSER WELL  11  ELEV. IS  100.1179   FT
OBSER WELL  12  ELEV. IS  101.0000   FT
OBSER WELL  13  ELEV. IS  101.8821   FT
OBSER WELL  14  ELEV. IS  102.0012   FT
OBSER WELL  15  ELEV. IS  101.0053   FT
OBSER WELL  16  ELEV. IS  101.7996   FT

SUM OF ELEV DIFFERENCES SQD. IS    8.3730   FT**2

PUMPING WELL  1 STEADY STATE PUMPING IS          -31960.2109 CU   FT/ DAY

PUMPING WELL  2 STEADY STATE PUMPING IS          -21987.4180 CU   FT/ DAY

PUMPING WELL  3 STEADY STATE PUMPING IS          -11544.8115 CU   FT/ DAY

PUMPING WELL  4 STEADY STATE PUMPING IS           11544.8115 CU   FT/ DAY

PUMPING WELL  5 STEADY STATE PUMPING IS           21987.4180 CU   FT/ DAY

PUMPING WELL  6 STEADY STATE PUMPING IS           31960.2109 CU   FT/ DAY

PUMPING WELL  7 STEADY STATE PUMPING IS           28734.0625 CU   FT/ DAY

PUMPING WELL  8 STEADY STATE PUMPING IS           28734.0625 CU   FT/ DAY

PUMPING WELL  9 STEADY STATE PUMPING IS           31960.2109 CU   FT/ DAY

PUMPING WELL 10 STEADY STATE PUMPING IS           21987.4180 CU   FT/ DAY

PUMPING WELL 11 STEADY STATE PUMPING IS           11544.8115 CU   FT/ DAY

PUMPING WELL 12 STEADY STATE PUMPING IS          -11544.8115 CU   FT/ DAY
```

PUMPING WELL 13 STEADY STATE PUMPING IS	-21987.4180 CU	FT/ DAY
PUMPING WELL 14 STEADY STATE PUMPING IS	-31960.2109 CU	FT/ DAY
PUMPING WELL 15 STEADY STATE PUMPING IS	-28734.0625 CU	FT/ DAY
PUMPING WELL 16 STEADY STATE PUMPING IS	-28734.0625 CU	FT/ DAY

Program 1 BW.BAT

```
CD\BW
PATH C:\
```

Program 2 FORT2.BAT

```
GAMS %1
PATH C:\
BW
```

Program 3 GM.BAT

```
CD\GAMSLIB
PATH C:\GAMS2.04;C:\
```

Program 4 GAMS.BAT

```
ECHO OFF
ECHO PW 79 PS 60 A CE SYSDIR C:\GAMS2.04 > GAMSSCRA.PRM
ECHO I %1 %2 %3 %4 %5 %6 %7 %8 %9 >> GAMSSCRA.PRM
:AGANE
SHIFT
IF A%9 == A GOTO DONE
FOR %%I IN (1 2 3 4 5 6 7 8 9) DO SHIFT
ECHO %1 %2 %3 %4 %5 %6 %7 %8 %9 >> GAMSSCRA.PRM
GOTO AGANE
:DONE
GAMSNVRN
GAMSSCR1.BAT
: ONLY NEED ONE BATCH FILE: CHECK OPEN HAS ERASED PREVIOUS O/P FILE
```

Program 5 FORT.BAT

```
REM - FILE NAME IS FORT.BAT
PATH C:
ERASE KERNEL.OUT
ERASE TRANS.OUT
ERASE %1.OUT
IF %3 == NO GOTO ABC
SET PROFORT.ERR=C:PROFORT.ERR
PROFORT %1,,NUL,\PROFORT.LIB
:ABC
%1
COPY %1.OUT+%2.GMS %1.GMS
COPY %1.GMS C:\GAMSLIB
PATH C:\GAMS2.04;C:\GAMSLIB;C:\
GAMS %1
```

Program 6 FORT1.BAT

```
REM - FILE NAME IS FORT1.BAT
PATH C:
SET PROFORT.ERR=C:PROFORT.ERR
ERASE MODEL2.CAL
PROFORT %1 /L %2 > $1.LST
LINK %1,,NUL,\PROFORT.LIB
%1
```

Program 7 EXP.BAT

```
REM - FILE NAME IS EXP.BAT
PATH C:
ERASE SMODEL.DAT
IF %2 == NO GOTO ABC
SET PROFORT.ERR=C:PROFORT.ERR
PROFORT %1 \L > %1.LST
LINK %1
ABC
%1
```

Program 8 MODEL2.FOR

```
C  CALCULATING THE OBJECTIVE FUNCTION COEFFICIENTS
C    USES ADDITIONAL CONSTRAINT OF OBS HEADS < SOURCE HEAD
C  CALCULATING THE COORDINATES AND INFLUENCE COEFFICIENTS FOR ALL
C  WELLS USING SUBROUTINE CALCULATION
C   INFLUENCE COEFS. ARE BASED ON INFINITE SERIES FOR THEIS WITH U<5.1
C      AND THE NEG. POWER SERIES FOR U>5.1
C
C  TERMS:
C  A= DISTANCE FROM SOURCE TO DOWN GRADIENT SIDE OF OCTAGON
C  X(L)=VECTOR OF X COORDS. FOR ALL WELLS
C  Y(L)=VECTOR OF Y COORDS. FOR ALL WELLS
C  SL= LENGTH OF A SIDE OF THE OCTAGON
C  SL2=LENGTH OF SIDES PARALLEL TO GRADIENT
C  SP= SPACING OF PUMPING WELLS (MUST BE EVEN MULTIPLE OF SL)
C  SP2=SPACING OF PUMP WELLS FOR SIDES PARALLEL TO GRADIENT
C  I= TOTAL PUMPING WELLS
C  L= TOTAL OBSERVATION WELLS(2*I+1) ALL PUMPING WELLS ARE ALSO OBSER.
C     WELLS. OBSER. WELL ALSO AT SOURCE(REASON FOR +1)
C  LL=ONLY ACTUAL OBSER. WELLS (NOT PUMPING OR SOURCE)
C  W(T)= VECTOR OF WELL COEFS. FOR ALL TIME PERIODS FOR A WELL J ON A
C        WELL I
C  IT= NUMBER OF TIME PERIODS
C  ST(L)= SATURATED THICK. VALUES FOR EACH WELL. THESE VALUES ARE KEPT
C    IN FILE TRANS_.DAT. THEY BEGIN WITH THE SOURCE(OBS) WELL, GO TO THE
C    OBS WELL AT X=A,Y=SL/2 AND THEN PROGRESS CCW AROUND THE OCTAGON
C    ALTERNATING PUMP WELL, OBS WELL, ETC. TO TOTAL WELLS=L
C  EP= EFFECTIVE POROSITY
C  R= RADIUS OF PUMPING WELL
C  NP= NO. OF WELLS ON A SIDE= SL/(SP/2)
C  HCMAX= MAX. HYDRAULIC CONDUC. (ASSUMED ALONG X-AXIS)
C  HCMIN= MIN. HYDRAULIC COND. (ASSUMED ALONG Y-AXIS)
C  ANGL= ANGLE CCW FROM X-AXIS TO DIRECTION OF HCMAX
C  HYCON=CALCULATED HYDR. CON. BASED ON DIRECTION OF FLOW
C  TERML,TERMS,TERM1 ARE USED WITH THE NEG. POWER SERIES
C  QU=UPPER LIMIT ON PUMPING(USER INPUT)
C  HL=LOWER LIMIT IN HEAD AT WELLS (USER INPUT)
C  HL=UPPER LIMIT IN HEAD AT WELLS (USER INPUT)
C
C
C  CALCULATION OF COORDINATES OF ALL WELLS(OBS.& PUMP) STARTING WITH
C    SOURCE WELL AND THEN TO WELL(A,0) ON X-AXIS AND THEN CCW
C
      DIMENSION HP(36,2),HO(34),IDUM(20),ST(65)
      DIMENSION SL(8),NP(8),SP(8),SPS(8)
      DOUBLE PRECISION BP(35,10,65), SUMBP, B(35,10),CT(35,10)
      DOUBLE PRECISION SUMBOB(35,10),C(35,10),K(35,10),KT(35,10)
      DOUBLE PRECISION X(65),Y(65)
      DOUBLE PRECISION PUMPOB(35,10),PUMPSC(10),R,PI,TRANS,THETA,Z,HYCON
      COMMON/CARD1/ IT, I, L, BP, LL,A,R,EP,PI,ST,HCMAX,HCMIN
      COMMON/CARD2/ SL,SP,NP,HYCON,X,Y,ANGL
      OPEN(5, FILE='MODEL2.DAT', STATUS='OLD', ERR=1201)
```

```
      OPEN(4, FILE='MODEL2.OUT', STATUS='NEW', ERR=1202)
      OPEN(6, FILE='KERNEL.OUT', STATUS='NEW', ERR=1203)
      OPEN(7, FILE='TRANS.OUT', STATUS='NEW', ERR=1204)
C
      READ(5,2)I,L,IT,R,A,TIME,LENGTH
2     FORMAT(3I5,F5.2,F10.2,2A5)
      READ(5,4)QU,EP,HCMIN,HCMAX,ANGL
4     FORMAT(F10.2,F5.2,2F10.2,F5.2)
C  READING LENGTH OF EACH SIDE OF OCTAGON AND NO. OF
C   WELLS ON A SIDE (2 * PUMP WELLS)
      DO 88 II=1,8
      READ(5,3)SL(II),NP(II)
3     FORMAT(F10.2,I5)
88    CONTINUE
C  READING GROUNDWATER TABLE AND GROUND SURFACE ELEVATIONS
C  FOR PUMPING WELLS HP(I,2) FROM FILE MODEL2.DAT
C    *SOURCE GW TABLE ELEV. IS FIRST AFTER PUMP WELL DATA*
C
      DO 100 II=1,I
        READ(5,95)(HP(II,J),J=1,2)
100   CONTINUE
95    FORMAT(2F10.2)
C  READING GROUNDWATER TABLE ELEVATIONS FOR OBSERVATION WELLS
C  HO(L-I)-FROM FILE MODEL2.DAT
C
      DO 200 II=1,L-I
       READ(5,95) HO(II)
200   CONTINUE
C  READ THE SATURATED THICK. VALUES FOR EACH WELL FROM FILE MODEL2.DAT
C   START W/SOURCE, THEN TO OBS WELL (X=A,Y=SL/2), THEN CCW
      DO 250 II=1,L
      READ(5,96) ST(II)
96    FORMAT(F10.2)
C     WRITE(6,96) ST(II)
250   CONTINUE
      LL= (L-1)/2
      PI=22./7.
C  CALCULATE THE WELL SPACING ON EACH SIDE
      DO 9 II=1,8
      SP(II)=SL(II)/(NP(II)/2)
      WRITE(7,1)SP(II)
1     FORMAT(2F10.2)
9     CONTINUE
      CALL CALC
C  CALCULATION OF TABLE VALUES FOR GAMS
C  SETTING COST OF PUMPING ONE UNIT VOLUME A UNIT DISTANCE($/CU-FT/FT)
C  EQUIVALENT TO $.18 AC-FT/FT
►     CK=4.1322E-6
►C    CK=0.
C  SETTING COST OF MAINTENANCE OF PUMP FOR ONE VOLUME DELIVERED($/CU-FT)
C  EQUIVALENT TO $1.65/AC-FT
►     CKK=3.7879E-5
►C    CKK=0.
```

```
C  WEIGHT FACTOR TO CONVERT H(SOURCE)-H(OBS.) FROM LENGTH TO $
►     WF=1.
C  DO LOOP FOR ALL PUMPING WELLS(ODD WELLS ARE PUMP WELLS)
      DO 850 II=3,L,2
C  DO LOOP FOR ALL TIME PERIODS
       DO 850 IJ=1,IT
C  CALCULATION OF B(I,T) TABLE(SUM OF INFLUENCE COEFS. FROM ALL
C    PUMP WELLS IP ON A PUMP WELL II DURING TIME IJ (CORRECT TIME ORDER)
        SUMBP=0.0
        DO 500 IP=1,I
C   IP=PUMP WELLS, IJ=TIME STEPS, II=ODD(PUMP)OBSER WELLS
         SUMBP = SUMBP + BP(IP,IJ,II)
500     CONTINUE
C   JT CHANGES ODD NUMBERED PUMP WELLS TO 1,2,3 ORDER
        JT= (II-1)/2
C   FOR GAMS TABLE B(PUMP,TIME)CORRECT TIME ORDER
        B(JT,IJ) = SUMBP
C  STORE B VALUES IN KERNEL.OUT TO CALC PUMP WELL HEADS
        WRITE(6,402) B(JT,IJ)
850     CONTINUE
C  CALC OF CONSTANT TERM(LAST TERM OF SQD HEAD DIFF)
        CONST = 0.
        DO 600 IO=2,L-I
         CONST = CONST + WF*(HO(1)-HO(IO))**2
600     CONTINUE
C  SUMMATION OF INFLUENCE COEFFICIENTS OVER ALL OBSERVATION WELLS
C  FOR EACH PUMPING WELL (I+1 IS THE SOURCE WELL+ALL PUMP WELLS)
        DO 705 II= 1,I
        DO 705 IJ= 1,IT
        SUMBOB(II,IJ) = 0.
C   IL ONLY SUMS THE EVEN (OBSER) WELLS
        DO 700 IL=2, L-1, 2
         SUMBOB(II,IJ) = SUMBOB(II,IJ) + BP(II,IJ,IL)
700     CONTINUE
705     CONTINUE
C  CALCULATION OF LINEAR ECONOMIC COEFFICIENTS CT(I,T) FOR GAMS
C     (IN CORRECT TIME ORDER)
C  NN KEEPS TRACK OF PUMP WELL NOS. IN RELATION TO ALL WELLS
      NN=1
      DO 400 II= 1,I
      NN=NN+2
      DO 400 IJ= 1,IT
►     CK=4.1322E-6
►C    CK=0.
C  NO COST FOR INJECTION PUMPING
      IF(X(NN).GT.0.0) CK=0.
        CT(II,IJ) = CK*(HP(II,1)-HP(II,2)) + CKK
C     WRITE(6,403)NN,X(NN), HP(II,1), HP(II,2), CT(II,IJ)
C03   FORMAT(I3,3F10.4, D10.4)
C  CALCULATION OF ECONOMIC QUADRATIC COEFFICIENTS KT(I,T) FOR GAMS
C     (IN CORRECT TIME ORDER)
        KT(II,IJ) = CK*B(II,IJ)
400   CONTINUE
```

```
C       DO 233 KK=1,IT
C       WRITE(6,203)(CT(M,KK),M=1,I)
C03     FORMAT(30D10.2)
C33     CONTINUE
C  SUMMATION OF INFLUENCE COEFFICIENTS OVER ALL PUMP WELLS
C     ON THE SOURCE
        DO 703 IJ= 1,IT
        PUMPSC(IJ)=0.
C  II SUMS OVER ALL PUMP WELLS
        DO 702 II=1,I
        PUMPSC(IJ)= PUMPSC(IJ)+BP(II,IJ,1)
702     CONTINUE
C  STORE PUMPSC IN FILE KERNEL.OUT TO CALCULATE SOURCE WELL HEAD
        WRITE(6,402) PUMPSC(IJ)
402     FORMAT(D15.4)
703     CONTINUE
C  SUM OF INFLUENCE COEFFICIENTS OVER ALL PUMPING WELLS
C     FOR EACH OBSER. WELL
C  IL IS THE EVEN(OBSER) WELLS (NOT INCLUDE SOURCE WELL #1)
        N1=0
        DO 704 IL=2,L-1,2
        IF(X(IL).GT.0.0.AND.Y(IL).LT.0.0) N1=N1+1
        DO 704 IJ= 1,IT
        PUMPOB(IL,IJ)=0.
C  II SUMS OVER ALL PUMP WELLS
        DO 404 II=1,I
        PUMPOB(IL,IJ)= PUMPOB(IL,IJ)+BP(II,IJ,IL)
404     CONTINUE
C  STORE PUMPOB IN FILE KERNEL.OUT TO CALCULATE OBS WELL HEADS
        WRITE(6,402) PUMPOB(IL,IJ)
704     CONTINUE
        DO 710 IL=2,L-1,2
        DO 710 IJ= 1,IT
C  IO PUTS K(IO,IJ) FOR OBSER WELLS INTO 1,2,3 ORDER
        IO=(IL)/2
C  KR REVERSES THE TIME ORDER OF IT
        KR=IT-IJ+1
C  IG CHANGES IO INDICE TO OBSER WELL GW TABLE INDICE HO(IG)
C       BECAUSE HO(1) IS THE SOURCE
        IG= IO+1
C  CALCULATION OF HYDRAULIC QUADRATIC COEFFICIENTS K(I,T) FOR GAMS
C     (IN REVERSE TIME ORDER)
          K(IO,IJ) = (PUMPOB(IL,KR)-PUMPSC(KR))
C  CALCULATION OF LINEAR HYDRAULIC COEF. C(J,T) IN REVERSE ORDER
        C(IO,IJ)= 2*WF*K(IO,IJ)*(HO(1)-HO(IG))
710     CONTINUE
C       PRINT 1410
C410    FORMAT('I AM AT THE WRITE PORTION')
C
C  WRITING DATA IN GAMS/MINOS FORMAT INTO FILE MODEL2.OUT
C       DO 333 KK=1,IT
C       WRITE(6,303)(CT(M,KK),M=1,I)
C03     FORMAT(30D10.2)
```

```
C33   CONTINUE
C
      WRITE(4,444)
444   FORMAT('$OFFSYMLIST OFFSYMXREF')
      DO 6 II=1,8
      WRITE(4,7)II,SL(II),NP(II),SP(II)
7     FORMAT('* FOR SIDE',I2,/,'*   THE L=',F8.2,' ;NO. PUMPS=',I4,
     !' ;SPACING=',F8.2)
6     CONTINUE
      WRITE(4,8)R,EP,HCMIN,HCMAX,ANGL,TIME,LENGTH
8     FORMAT('* WELL RADIUS IS',F5.2,/,
     !'* EFFECTIVE POROSITY IS',F8.2,/,
     !'* HCMIN IS',F8.2,/,
     !'* HCMAX IS',F8.2,/,
     !'* ANGLE FROM X-AXIS TO HCMAX IS',F6.2,/,
     !'* TIME PERIOD IS A ',A5,/,
     !'* LENGTH DIMENSION IS ',A5,/,
     !'* LOW LIMIT ON DD AT PUMP WELLS = 1/2(SAT. THICK.)',/,
     !'* HIGH LIMIT ON DD AT PUMP WELLS = GROUND ELEV.',/)
C
      WRITE(4,550)I,IT,QU,CONST, WF,HO(1),IT
550   FORMAT( SETS',/,4X,'I PUMPING WELLS /1*',I4,'/',/,4X,'T TIME STEPS
     !/1*',I2,'/',/,4X,'J DUMMY SET /1/',/,4X,'N DUMMY SET /1*2/',/,
     !'SCALAR ,/,6X,'QU UPPER PUMPING /',F8.2,'/',/,6X,
     !'CON CONSTANT TERM IN SQD HEAD DIFF /',F8.1,'/',/,6X,
     !'WF WEIGHT FACTOR /',F8.2,'/',/,6X,
     !'HS SOURCE PIEZ. ELEV. /',F8.2,'/',/,6X,
     !'FT FINAL TIME PERIOD /', I4,'/')
      WRITE(4,751)
751   FORMAT('PARAMETER',/,9X,'HOB(I) INITIAL HEAD AT EACH OBS WELL')
      DO 901 J=2,L-I
      JJ = J-1
      IF(J.EQ.2) WRITE(4,911) JJ, HO(J)
911   FORMAT(9X,'/',I2,F10.2)
      IF(J.EQ.L-I) WRITE(4,921) JJ, HO(J)
921   FORMAT(10X,I2,F10.2,'/')
      IF(J.NE.2.AND.J.NE.L-I) WRITE(4,931) JJ, HO(J)
931   FORMAT(10X,I2,F10.2)
901   CONTINUE
      WRITE(4,651)
651   FORMAT(/,9X,'ST(I) SATURATED THICK. AT EACH PUMP WELL')
      N=0
      DO 601 J=3,L,2
      N=N+1
      IF(J.EQ.3) WRITE(4,611) N,ST(J)
611   FORMAT(9X,'/',I2,F10.2)
      IF(J.EQ.L) WRITE(4,621) N, ST(J)
621   FORMAT(10X,I2,F10.2,'/')
      IF(J.NE.3.AND.J.NE.L) WRITE(4,631) N, ST(J)
631   FORMAT(10X,I2,F10.2)
601   CONTINUE
      WRITE(4,753)
753   FORMAT(/,9X,'SC(T) INFLUENCE COEFS. FOR SOURCE WELL')
```

```
      DO 801 J=1,IT
      IF(J.EQ.1) WRITE(4,811) J,PUMPSC(J)
811   FORMAT(9X,'/',I2,E12.4)
      IF(J.EQ.IT) WRITE(4,821) J, PUMPSC(J)
821   FORMAT(10X,I2,E12.4,'/')
      IF(J.NE.1.AND.J.NE.IT) WRITE(4,831) J, PUMPSC(J)
831   FORMAT(10X,I2,E12.4)
801   CONTINUE
      WRITE(4,7521)
7521  FORMAT(/,9X,'TI(I) SPECIFIES OBS WELLS DOWN-GRADIENT FROM SOURCE'
     !)
C  NN KEEPS TRACK OF OBS WELL NUMBER AS A PART OF ALL WELLS
C     I.E. PUMP WELLS ARE 3,5... ;OBS. WELLS ARE 2,4.....
      NN=0
      DO 8001 J=1,I
      NN=NN+2
      IF(X(NN).GT.0.0) THEN
         KOBS=1
      ELSE
         KOBS=0
      ENDIF
C     WRITE(6,66)NN,X(NN),KOBS
C6    FORMAT(I3,F10.2,I2)
      IF(J.EQ.1) WRITE(4,8101) J, KOBS
8101  FORMAT(9X,'/',I2,I5)
      IF(J.EQ.I) WRITE(4,8201) J, KOBS
8201  FORMAT(10X,I2,I5,'/')
      IF(J.NE.1.AND.J.NE.I) WRITE(4,8301) J, KOBS
8301  FORMAT(10X,I2,I5)
8001  CONTINUE
      WRITE(4,752)
752   FORMAT(/,9X,'TT(T) TIME VECTOR TO PROVIDE PARTIAL SUMS')
      DO 800 J=1,IT
      IF(J.EQ.1) WRITE(4,810) J, J
810   FORMAT(9X,'/',I2,I5)
      IF(J.EQ.IT) WRITE(4,820) J, J
820   FORMAT(10X,I2,I5,'/')
      IF(J.NE.1.AND.J.NE.IT) WRITE(4,830) J, J
830   FORMAT(10X,I2,I5)
800   CONTINUE
      WRITE(4,750)
750   FORMAT('TABLE HO(I,N) GROUND EL. & INIT. HEAD AT EACH PUMP WELL')
      WRITE(4,650)(J, J=1,2)
650   FORMAT(5X,I10,I10)
      DO 900 J=1,I
      WRITE(4,660)J,(HP(J,M), M=1,2)
660   FORMAT(5X,I2,2F10.2)
900   CONTINUE
      WRITE(4,940)
940   FORMAT('TABLE B(I,T) INFLUENCE COEF.AT PUMP WELLS')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1501
      NIT= IT/5 + 1
```

```
      DO 1002 KK= 1,NIT
      IF(KK.GT.1) GO TO 7010
      WRITE(4,950)(J, J=1,5)
950   FORMAT(1X,10I12)
      N=1
      DO 1001 J=1,I
      WRITE(4,960)J,(B(J,M), M=1,5)
960   FORMAT(5X,I2,10E12.4)
1001  CONTINUE
      GO TO 1002
7010  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7002
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1002
      WRITE(4,951)(J,J=JB,JE)
951   FORMAT(/,'+',10I12)
      DO 1000 JJ=1,I
      WRITE(4,960)JJ,(B(JJ,M), M=JB,JE)
1000  CONTINUE
      GOTO 1002
7002  WRITE(4,951)(J,J=JB,IT)
      DO 1008 J=1,I
      WRITE(4,960)J,(B(J,M),M=JB,IT)
1008  CONTINUE
1002  CONTINUE
      GOTO 15031
1501  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1502 J=1,I
      WRITE(4,960)J,(B(J,M), M=1,IT)
1502  CONTINUE
15031 WRITE(4,9401)
9401  FORMAT('TABLE OB(I,T) INFLUENCE COEF.AT OBS WELLS')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 15011
      NIT= IT/5 + 1
      DO 10021 KK= 1,NIT
      IF(KK.GT.1) GO TO 70101
      WRITE(4,9501)(J, J=1,5)
9501  FORMAT(1X,10I12)
      N=1
      DO 10011 J=1,I
      WRITE(4,9601)J,(PUMPOB(2*J,M), M=1,5)
9601  FORMAT(5X,I2,10E12.4)
10011 CONTINUE
      GO TO 10021
70101 JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 70021
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 10021
      WRITE(4,9511)(J,J=JB,JE)
```

```
9511  FORMAT(/,'+',10I12)
      DO 10001 JJ=1,I
      WRITE(4,9601)JJ,(PUMPOB(2*JJ,M), M=JB,JE)
10001 CONTINUE
      GOTO 10021
70021 WRITE(4,9511)(J,J=JB,IT)
      DO 10081 J=1,I
      WRITE(4,9601)J,(PUMPOB(2*J,M),M=JB,IT)
10081 CONTINUE
10021 CONTINUE
      GOTO 1503
15011 WRITE(4,9501)(J, J=1,IT)
      N=1
      DO 15021 J=1,I
      WRITE(4,9601)J,(PUMPOB(2*J,M), M=1,IT)
15021 CONTINUE
1503  WRITE(4,970)
970   FORMAT('TABLE C(I,T) LINEAR HYDR. COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1601
      NIT= IT/5 + 1
      DO 1102 KK= 1,NIT
      IF(KK.GT.1) GO TO 7011
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1101 J=1,I
      WRITE(4,960)J,(C(J,M), M=1,5)
1101  CONTINUE
      GO TO 1102
7011  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7102
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1102
      WRITE(4,951)(J,J=JB,JE)
      DO 1100 JJ=1,I
      WRITE(4,960)JJ,(C(JJ,M), M=JB,JE)
1100  CONTINUE
      GOTO 1102
7102  WRITE(4,951)(J,J=JB,IT)
      DO 1108 J=1,I
      WRITE(4,960)J,(C(J,M),M=JB,IT)
1108  CONTINUE
1102  CONTINUE
      GOTO 1603
1601  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1602 J=1,I
      WRITE(4,960)J,(C(J,M), M=1,IT)
1602  CONTINUE
1603  WRITE(4,975)
975   FORMAT('TABLE CT(I,T) LINEAR ECON COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
```

```
      IF(IT.LT.5) GOTO 1701
      NIT= IT/5 +1
C     DO 433 KK=1,IT
C     WRITE(6,403)(CT(M,KK),M=1,I)
C03   FORMAT(30D10.2)
C33   CONTINUE
      DO 1112 KK= 1,NIT
      IF(KK.GT.1) GO TO 7012
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1111 J=1,I
      WRITE(4,960)J,(CT(J,M), M=1,5)
1111  CONTINUE
      GO TO 1112
7012  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7112
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1112
      WRITE(4,951)(J,J=JB,JE)
      DO 1110 JJ=1,I
      WRITE(4,960)JJ,(CT(JJ,M), M=JB,JE)
1110  CONTINUE
      GOTO 1112
7112  WRITE(4,951)(J,J=JB,IT)
      DO 1118 J=1,I
      WRITE(4,960)J,(CT(J,M),M=JB,IT)
1118  CONTINUE
1112  CONTINUE
      GOTO 1703
1701  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1702 J=1,I
      WRITE(4,960)J,(CT(J,M), M=1,IT)
1702  CONTINUE
1703  WRITE(4,980)
980   FORMAT('TABLE K(I,T) HYDR QUAD COEFS OF OBJ FUNC(I=OBSER WELL)')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1801
      NIT= IT/5 + 1
      DO 1122 KK= 1,NIT
      IF(KK.GT.1) GO TO 7013
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1121 J=1,I
      WRITE(4,960)J,(K(J,M), M=1,5)
1121  CONTINUE
      GO TO 1122
7013  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7122
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1122
```

```
      WRITE(4,951)(J,J=JB,JE)
      DO 1120 JJ=1,I
      WRITE(4,960)JJ,(K(JJ,M), M=JB,JE)
1120  CONTINUE
      GOTO 1122
7122  WRITE(4,951)(J,J=JB,IT)
      DO 1128 J=1,I
      WRITE(4,960)J,(K(J,M),M=JB,IT)
1128  CONTINUE
1122  CONTINUE
      GOTO 1803
1801  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1802 J=1,I
      WRITE(4,960)J,(K(J,M), M=1,IT)
1802  CONTINUE
1803  WRITE(4,985)
985   FORMAT('TABLE KT(I,T) ECONOMIC QUADRATIC COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1901
      NIT= IT/5 + 1
      DO 1222 KK= 1,NIT
      IF(KK.GT.1) GO TO 7014
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1220 J=1,I
      WRITE(4,960)J,(KT(J,M), M=1,5)
1220  CONTINUE
      GO TO 1222
7014  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7222
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1222
      WRITE(4,951)(J,J=JB,JE)
      DO 1130 JJ=1,I
      WRITE(4,960)JJ,(KT(JJ,M), M=JB,JE)
1130  CONTINUE
      GOTO 1222
7222  WRITE(4,951)(J,J=JB,IT)
      DO 1228 J=1,I
      WRITE(4,960)J,(KT(J,M),M=JB,IT)
1228  CONTINUE
1222  CONTINUE
      GOTO 1903
1901  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1902 J=1,I
      WRITE(4,960)J,(KT(J,M), M=1,IT)
1902  CONTINUE
1903  WRITE(4,1960)
1960  FORMAT(/,'ALIAS (T,L,M);')
      WRITE(4,1965)
```

```
1965  FORMAT(/,'TABLE IND(L,M) INDICE MATRIX FOR SUMMING B(T-T+1)*QT')
      WRITE(4,1970)(J,J=1,IT)
1970  FORMAT(5X,10I3)
      DO 1975 M=1,IT
      WRITE(4,1980)M,(N,N=M,IT)
1980  FORMAT(2X,12I3)
1975  CONTINUE
      CLOSE (5, ERR=1004, STATUS='KEEP')
      CLOSE (4, ERR=1005, STATUS='KEEP')
      CLOSE (6, ERR=1006, STATUS='KEEP')
      CLOSE (7, ERR=1007, STATUS='KEEP')
      GOTO 40
1201  PRINT 30
30    FORMAT('  AHA! ERROR FROM OPEN 5 ')
      GOTO 40
1202  PRINT 32
32    FORMAT('  AHA! ERROR FROM OPEN 4 ')
      GOTO 40
1203  PRINT 33
33    FORMAT('  AHA! ERROR FROM OPEN 6 ')
      GOTO 40
1204  PRINT 34
34    FORMAT('  AHA! ERROR FROM OPEN 7 ')
      GOTO 40
1004  PRINT 37
37    FORMAT('  AHA! ERROR FROM CLOSE 5 ')
      GOTO 40
1006  PRINT 31
31    FORMAT('  AHA! ERROR FROM CLOSE 6 ')
      GOTO 40
1007  PRINT 36
36    FORMAT('  AHA! ERROR FROM CLOSE 7 ')
      GOTO 40
1005  PRINT 35
35    FORMAT('  AHA! ERROR FROM CLOSE 4 ')
40    STOP
      END
C
      SUBROUTINE CALC
      DIMENSION SL(8),SP(8),NP(8),ST(65)
      DOUBLE PRECISION X(65),Y(65),R,U,W(10),TERM,BP(35,10,65)
      DOUBLE PRECISION S(65,65), WMINK, PI, ANF
      DOUBLE PRECISION Z,THETA,TRANS,UN,WU,TERML,TERM1,TERMS,HYCON
      COMMON/CARD1/ IT, I, L, BP, LL,A,R,EP,PI,ST,HCMAX,HCMIN
      COMMON/CARD2/ SL,SP,NP,HYCON,X,Y,ANGL
C
C  CALCULATION OF COORDINATES OF ALL WELLS (OBS & PUMP) STARTING WITH
C  SOURCE WELL AND THEN TO WELL (A,SL/2) AND THEN CCW
C
      LNP=3
      MNP=NP(2)+2
      X(1)= 0.
      Y(1)= 0.
```

```
C       WRITE(6,13)X(1),Y(1)
        X(2)= A
        Y(2)= SL(1)/2.
C       WRITE(6,13)X(2),Y(2)
13      FORMAT(2F10.2)
        DO 300 II=LNP,MNP
        X(II)=X(II-1)-(SP(2)/2)*DSIN(PI/4.)
        Y(II)=Y(II-1)+(SP(2)/2)*DCOS(PI/4.)
C       WRITE(6,13)X(II),Y(II)
300     CONTINUE
        LNP=LNP+NP(2)
        MNP=MNP+NP(3)
        DO 400 II=LNP,MNP
        X(II)=X(II-1)-(SP(3)/2)
        Y(II)=Y(II-1)
C       WRITE(6,13)X(II),Y(II)
400     CONTINUE
        LNP=LNP+NP(3)
        MNP=MNP+NP(4)
        DO 500 II=LNP,MNP
        X(II)=X(II-1)-(SP(4)/2)*DSIN(PI/4.)
        Y(II)=Y(II-1)-(SP(4)/2)*DCOS(PI/4.)
C       WRITE(6,13)X(II),Y(II)
500     CONTINUE
        LNP=LNP+NP(4)
        MNP=MNP+NP(5)
        DO 600 II=LNP,MNP
        X(II)= X(II-1)
        Y(II)=Y(II-1)-(SP(5)/2)
C       WRITE(6,13)X(II),Y(II)
600     CONTINUE
        LNP=LNP+NP(5)
        MNP=MNP+NP(6)
        DO 1500 II=LNP,MNP
        X(II)=X(II-1)+(SP(6)/2)*DSIN(PI/4.)
        Y(II)=Y(II-1)-(SP(6)/2)*DCOS(PI/4.)
C       WRITE(6,13)X(II),Y(II)
1500    CONTINUE
        LNP=LNP+NP(6)
        MNP=MNP+NP(7)
        DO 1600 II=LNP,MNP
        X(II)=X(II-1)+(SP(7)/2)
        Y(II)=Y(II-1)
C       WRITE(6,13)X(II),Y(II)
1600     CONTINUE
        LNP=LNP+NP(7)
        MNP=MNP+NP(8)
        DO 1700 II=LNP,MNP
        X(II)=X(II-1)+(SP(8)/2)*DSIN(PI/4.)
        Y(II)=Y(II-1)+(SP(8)/2)*DCOS(PI/4.)
C       WRITE(6,13)X(II),Y(II)
1700     CONTINUE
        LNP=LNP+NP(8)
```

```
      DO 1800 II= LNP, L
      X(II)= X(II-1)
      Y(II)=Y(II-1)+(SP(1)/2)
C     WRITE(6,13)X(II),Y(II)
1800   CONTINUE
C
C     CALCULATION OF INFLUENCE COEFFICIENTS FOR Q= 1000 CU-FT/DAY
C
C  ODD NUMBERED WELLS ARE PUMP WELLS
      DO 1300 J=3,L,2
C  ALL WELLS ARE OBSER WELLS
      DO 1250 M=1,L
C  CALCULATE HYCON BASED ON HCMAX AND HCMIN
      IF(X(M).EQ.X(J)) HYCON=HCMIN
      IF(Y(M).EQ.Y(J).OR.M.EQ.J) HYCON=HCMAX
      IF(X(M).EQ.X(J).OR.Y(M).EQ.Y(J)) GOTO 604
      Z = (Y(M)-Y(J))/(X(M)-X(J))
      THETA= DATAN(Z)
      HYCON= (HCMAX*HCMIN)/(HCMIN*(DCOS(THETA-ANGL))**2+HCMAX*
     !(DSIN(THETA-ANGL))**2)
C     IF(M.EQ.(J+1)) WRITE(6,605) J,M,X(J),X(M),Y(J),Y(M),THETA,HYCON
C     IF(J.EQ.L.AND.M.EQ.2) WRITE(6,605)J,M,X(J),X(M),Y(J),Y(M),THETA,
C    ^HYCON
C05   FORMAT(' J=',I3,2X,'M=',I3,2X,'X=',2D12.4,2X,'Y=',2D12.4,/,
C    ^' THETA=',D12.4,' HYCON=',D12.4)
C  S= DISTANCE BETWEEN PUMP WELL J & OBSER. WELL M
604   IF(ABS(X(J)-X(M)).LT.1.0.AND.ABS(Y(J)-Y(M)).LT.1.0) GOTO 505
      IF(ABS(X(J)-X(M)).GT.1.0.AND.ABS(Y(J)-Y(M)).GT.1.0) GOTO 560
      IF(ABS(X(J)-X(M)).LT.1.) S(J,M)= ABS(Y(J)-Y(M))
      IF(ABS(Y(J)-Y(M)).LT.1.) S(J,M)= ABS(X(J)-X(M))
      GOTO 510
560   S(J,M)=DSQRT(((X(J)-X(M))**2)+((Y(J)-Y(M))**2))
      GOTO 510
505   S(J,M)=R
C USING THE LOWER SAT. THICK. FOR PUMP WELL INFLUENCE AND THE HIGH
C   SAT. TH. FOR THE OBS WELL (THE LOW TR PROVIDES THE HIGHEST INFLUENCE
C   AND THE HIGH PROVIDES THE LEAST INFLUENCE)
C11   IF(M.EQ.1.AND.ST(J).LT.ST(M)) THEN
C               TRANS=ST(M)*HYCON
C        ELSE
C               TRANS=ST(J)*HYCON
C        ENDIF
C     IF(M.EQ.1) GO TO 510
C     IF(MOD(M,2)) 5001,5002,5001
C001  IF(ST(J).LT.ST(M)) THEN
C               TRANS=ST(J)*HYCON
C        ELSE
C               TRANS=ST(M)*HYCON
C        ENDIF
C     GO TO 510
C002  IF(ST(J).LT.ST(M)) THEN
C               TRANS=ST(M)*HYCON
C        ELSE
```

```
C                TRANS=ST(J)*HYCON
C          ENDIF
C  AVERAGE THE SAT. THICK. FOR OBS & PUMP WELLS TO CALC TRANS
510    TRANS=HYCON*(ST(J)+ST(M))/2.
C10    IF(M.EQ.(J+1)) WRITE(7,42) TRANS
C      IF(J.EQ.L.AND.M.EQ.2) WRITE(7,42) TRANS
C1     FORMAT(D20.10)
C  WRITE TRANS FROM PUMP WELL TO OBS WELL DIRECTLY CCW FROM IT
C      INTO FILE TRANS.OUT
       IF(M.EQ.(J+1)) WRITE(7,41) TRANS
       IF(J.EQ.L.AND.M.EQ.2) WRITE(7,41) TRANS
41     FORMAT(D20.10)
       DO 1200 K=1, IT
C  CALCULATE BOLTZMAN VARIABLE, U
C      WRITE(6,1888)J,TR(J),M,TR(M),AVGTR
C888   FORMAT('TR',I3,'=',F10.2,'TR',I3,'=',F10.2,'AVGTR=',F10.2)
       U= (S(J,M)**2)*EP/(4*TRANS*K)
C      IF(J.EQ.3.AND.M.EQ.33)
C        WRITE(6,515)J,M,S(J,M),TRANS,U
C15    FORMAT(' J=',I3,2X,'M=',I3,2X,'S=',D12.4,2X,'TRANS=',D12.4,2X,
C     ^'U=',D12.4)
       IF(K.EQ.1) WMINK=0.
C  CALCULATE WELL COEFFICIENT, W(U). USE ALT. SERIES FOR U<5.0 AND USE
C      NEG. POWER SERIES FOR U>5.0
C  TERML IS THE e(-X)/X TERM OF NEG. POWER SERIES
C  TERM IS EACH TERM OF NEG. POWER SERIES
C  TERMS IS THE SUM OF THE TERM
       IF(U.GT.5.0) THEN
          TERML=(DEXP(-U/2)*DEXP(-U/2))/U
       ELSE
C   FIRST 2 TERMS OF INFINITE SERIES FOR W(U)
          WU=-0.5772-(DLOG(U))
       ENDIF
       TERMS=0.0
       TERM1=100.
       DO 900 N=1,1000
       ANF=N
C  LOOP TO CALCULATE N FACTORIAL
       NN= N-1
       DO 800 JB=1,NN
       ANF= ANF*(N-JB)
800    CONTINUE
C  CALCULATE ADDITIONAL TERMS OF W(U)
       IF(U.GT.5.0) THEN
          TERM=(-1.)**N*ANF/U**N
       ELSE
          TERM= (-1.)**(N)*(U**N)/(N*ANF)
       ENDIF
C IN POWER SERIES CHECK IF N+1 TERM > N TERM. IF SO; STOP.
       IF(ABS(TERM).GT.ABS(TERM1)) GOTO 910
C  IN POWER SERIES SUM THE TERMS IF THEY ARE GETTING SMALLER
       IF(U.GT.5.0) TERMS=TERMS+TERM
       IF(U.GT.5.0) TERM1=TERM
```

```
C  CHECKING IF LAST TERM OF W(U) < .0001
      IF(ABS(TERM).LT.1.0D-10) GOTO 910
C  CALCULATING THE WELL FUNCTION BASED ON SMALL U OR LARGE U
      IF(U.GT.5.0) THEN
         WU=TERML*(1.+TERMS)
      ELSE
         WU= WU-TERM
      ENDIF
C     IF(U.LT.4.5.AND.M.EQ.7) WRITE(6,802) U,K,N,ANF,TERM,WU
C02   FORMAT(' U=',D12.4,' K=',I2,2X,'N=',I4,2X,'ANF=',D12.4,
C    !' TERM=',D12.4,2X,'W(K)=',D12.4)
C     IF(U.GT.4.5.AND.M.EQ.7) WRITE(6,803) U,K,N,ANF,TERM,WU,TERMS
C03   FORMAT(' U=',D12.4,' K=',I2,2X,'N=',I4,2X,'ANF=',D12.4,
C    !' TERM=',D12.4,2X,'W(K)=',D12.4,D12.4)
C     IF(J.EQ.3.AND.M.EQ.33) WRITE(6,704) W(K)
C04   FORMAT('W(K)=',D12.4)
900   CONTINUE
C10   WRITE(4,805) TERM, W(K)
C05   FORMAT(' TERM=',D12.4,2X,'W(K)=',D12.4)
C  JT PUTS PUMP WELLS IN 1,2,3 ORDER
910   JT= (J-1)/2
C  BP(PUMP WELL, TIME, ALL WELLS)
      BP(JT,K,M)=((WU-WMINK)/(4.*PI*TRANS))*1000.
C  CHANGING INJECTION WELL COEFS. TO NEGATIVE
C     IF(M.EQ.1.AND.K.EQ.1)WRITE(6,14)J,X(J)
14    FORMAT(I5,F10.2)
915   IF(X(J).GT.0.0) BP(JT,K,M)=-BP(JT,K,M)
C IF PUMP WELL IS ON Y-AXIS ELIMINATE IT
915   IF(X(J).EQ.0.0) BP(JT,K,M)= 0.0
C     IF(M.EQ.7.OR.M.EQ.9)
C    !WRITE(6,902) TERM,W(K),WMINK,JT,K,M,BP(JT,K,M)
C02   FORMAT(' TERM=',D12.4,2X,'W(K)=',D12.4,2X,'WMINK=',D12.4,2X,/,
C    !3I4,2X,'B=',D12.4)
      WMINK= WU
1200  CONTINUE
1250  CONTINUE
1300  CONTINUE
C     DO 1650 J=1,L
C650  WRITE(6,14) J,X(J)
C305  FORMAT('PUMPING WELL NO.',I5)
C     DO 1550 LT=1,IT
C     WRITE(6,1310) LT
C310  FORMAT('TIME',I5)
C     WRITE(6,1315)(BP(J,LT,M),M=1,L)
C315  FORMAT(5D15.4)
C550  CONTINUE
C650  CONTINUE
C     PRINT 1400,I
C400  FORMAT('I MADE IT TO END OF SUB.I2=',I10)
      RETURN
      END
```

Program 9 SMODEL.FOR

```
C  CALCULATING THE STOCHASTIC OBJECTIVE FUNCTION COEFFICIENTS
C    USES ADDITIONAL CONSTRAINT OF OBS HEADS < SOURCE HEAD
C  CALCULATING THE COORDINATES AND INFLUENCE COEFFICIENTS FOR ALL
C  WELLS USING SUBROUTINE CALCULATION
C   INFLUENCE COEFS. ARE BASED ON INFINITE SERIES FOR THEIS WITH U<5.1
C      AND THE NEG. POWER SERIES FOR U>5.1
C
C  TERMS:
C  A= DISTANCE FROM SOURCE TO DOWN GRADIENT SIDE OF OCTAGON
C  X(L)=VECTOR OF X COORDS. FOR ALL WELLS
C  Y(L)=VECTOR OF Y COORDS. FOR ALL WELLS
C  SL= LENGTH OF A SIDE OF THE OCTAGON
C  SL2=LENGTH OF SIDES PARALLEL TO GRADIENT
C  SP= SPACING OF PUMPING WELLS (MUST BE EVEN MULTIPLE OF SL)
C  SP2=SPACING OF PUMP WELLS FOR SIDES PARALLEL TO GRADIENT
C  I= TOTAL PUMPING WELLS
C  L= TOTAL OBSERVATION WELLS(2*I+1) ALL PUMPING WELLS ARE ALSO OBSER.
C     WELLS. OBSER. WELL ALSO AT SOURCE(REASON FOR +1)
C  LL=ONLY ACTUAL OBSER. WELLS (NOT PUMPING OR SOURCE)
C  W(T)= VECTOR OF WELL COEFS. FOR ALL TIME PERIODS FOR A WELL J ON A
C        WELL I
C  IT= NUMBER OF TIME PERIODS
C  ST(L)= SATURATED THICK. VALUES FOR EACH WELL. THESE VALUES ARE KEPT
C    IN FILE TRANS_.DAT. THEY BEGIN WITH THE SOURCE(OBS) WELL, GO TO THE
C    OBS WELL AT X=A,Y=SL/2 AND THEN PROGRESS CCW AROUND THE OCTAGON
C    ALTERNATING PUMP WELL, OBS WELL, ETC. TO TOTAL WELLS=L
C  EP= EFFECTIVE POROSITY
C  R= RADIUS OF PUMPING WELL
C  NP= NO. OF WELLS ON A SIDE= SL/(SP/2)
C  HCMAX= MAX. HYDRAULIC CONDUC. (ASSUMED ALONG X-AXIS)
C  HCMIN= MIN. HYDRAULIC COND. (ASSUMED ALONG Y-AXIS)
C  HYCON=CALCULATED HYDR. CON. BASED ON DIRECTION OF FLOW
C  TERML,TERMS,TERM1 ARE USED WITH THE NEG. POWER SERIES
C  QU=UPPER LIMIT ON PUMPING(USER INPUT)
C  HL=LOWER LIMIT IN HEAD AT WELLS (USER INPUT)
C  HL=UPPER LIMIT IN HEAD AT WELLS (USER INPUT)
C
C
C  CALCULATION OF COORDINATES OF ALL WELLS(OBS.& PUMP) STARTING WITH
C    SOURCE WELL AND THEN TO WELL(A,0) ON X-AXIS AND THEN CCW
C
      DIMENSION HP(20,2),HO(41),IDUM(20),ST(41)
      DIMENSION SL(8),NP(8),SP(8),SPS(8),Q(8)
      DOUBLE PRECISION SUMBP, CT(20,10),E(20,10,41),E2(20,10,14)
      DOUBLE PRECISION C(20,10),K(20,10),KT(20,10),COVT
      DOUBLE PRECISION X(41),B1(20,10),B2(20,10),EP,COVS
      DOUBLE PRECISION PUMPOB(20,10),PUMPSC(10),PI,TRANS,Z
      COMMON/CARD1/ IT,I,L,LL,R,ST,AA,QU
      COMMON/CARD2/ SL,SP,NP,X
      COMMON/CARD3/ Q
      COMMON/CARD4/ E
```

```
      COMMON/CARD5/ E2
      COMMON/CARD6/ CL,F1
      COMMON/CARD7/ EP,TRANS,COVT,COVS
      OPEN(5, FILE='SMODEL.DAT', STATUS='OLD', ERR=1201)
      OPEN(4, FILE='SMODEL.OUT', STATUS='NEW', ERR=1202)
      OPEN(6, FILE='KERNEL.OUT', STATUS='NEW', ERR=1203)
      OPEN(7, FILE='TRANS.OUT', STATUS='NEW', ERR=1204)
C
      READ(5,2)I,L,IT,R,AA,TIME,LENGTH
2     FORMAT(3I5,F5.2,F10.2,2A5)
      READ(5,4)QU,EP,COVS,COVT,CL,F1,TRANS
4     FORMAT(F10.2,5F5.2,F10.2)
C  READING LENGTH OF EACH SIDE OF OCTAGON AND NO. OF
C   WELLS ON A SIDE (2 * PUMP WELLS)
      DO 88 II=1,8
      READ(5,3)SL(II),NP(II)
3     FORMAT(F10.2,I5)
88    CONTINUE
C  READING GROUNDWATER TABLE AND GROUND SURFACE ELEVATIONS
C  FOR PUMPING WELLS HP(I,2) FROM FILE SMODEL.DAT
C    *SOURCE GW TABLE ELEV. IS FIRST AFTER PUMP WELL DATA*
C
      DO 100 II=1,I
        READ(5,95)(HP(II,J),J=1,2)
100   CONTINUE
95    FORMAT(2F10.2)
C  READING GROUNDWATER TABLE ELEVATIONS FOR OBSERVATION WELLS
C  HO(L-I)-FROM FILE SMODEL.DAT
C
      DO 200 II=1,L-I
       READ(5,95) HO(II)
200   CONTINUE
C  READ THE SATURATED THICK. VALUES FOR EACH WELL FROM FILE SMODEL.DAT
C   START W/SOURCE, THEN TO OBS WELL (X=A,Y=SL/2), THEN CCW
      DO 250 II=1,L
      READ(5,96) ST(II)
96    FORMAT(F10.2)
C     WRITE(6,96) ST(II)
250   CONTINUE
C  READ THE PUMPING ESTIMATES FROM FILE SMODEL.DAT
      DO 201 II=1,IT
      READ(5,97) Q(II)
97    FORMAT(F10.4)
201   CONTINUE
      LL= (L-1)/2
      PI=22./7.
C  CALCULATE THE WELL SPACING ON EACH SIDE
      DO 9 II=1,8
      SP(II)=SL(II)/(NP(II)/2)
C  WRITE THE WELL SPACINGS INTO TRANS.OUT TO BE READ BY SHEAD.FOR
      WRITE(7,1)SP(II)
1     FORMAT(2F10.2)
9     CONTINUE
```

```
      CALL CALC
C      DO 1650 J=1,L
C      WRITE(7,1305) J
C1305  FORMAT('PUMPING WELL NO.',I5)
C      DO 1550 LT=1,4
C      WRITE(7,1310) LT
C1310  FORMAT('TIME',I5)
C      WRITE(7,1315)(E(J,LT,M),M=1,4)
C      WRITE(7,1315)(E2(J,LT,M),M=1,4)
C1315  FORMAT(5D15.4)
C1550  CONTINUE
C1650  CONTINUE
C  CALCULATION OF TABLE VALUES FOR GAMS
C  SETTING COST OF PUMPING ONE UNIT VOLUME A UNIT DISTANCE($/CU-FT/FT)
C  EQUIVALENT TO $.18 AC-FT/FT
►     CK=4.1322E-6
►C    CK=0.
C  SETTING COST OF MAINTENANCE OF PUMP FOR ONE VOLUME DELIVERED($/CU-FT)
C  EQUIVALENT TO $1.65/AC-FT
►     CKK=3.7879E-5
►C    CKK=0.
C  WEIGHT FACTOR TO CONVERT H(SOURCE)-H(OBS.) FROM LENGTH TO $
►     WF=1.
C  DO LOOP FOR ALL PUMPING WELLS(ODD WELLS ARE PUMP WELLS)
      DO 850 II=3,L,2
C  DO LOOP FOR ALL TIME PERIODS
       DO 850 IJ=1,IT
C  CALCULATION OF B(I,T) TABLE(SUM OF INFLUENCE COEFS. FROM ALL
C    PUMP WELLS IP ON A PUMP WELL II DURING TIME IJ (CORRECT TIME ORDER)
        SUMBP=0.0
        SUMBP2=0.0
        DO 500 IP=1,I
C   IP=PUMP WELLS, IJ=TIME STEPS, II=ODD(PUMP)OBSER WELLS
C   E2 IS FOR THE OBJ FUNC W/CL=5%
C   E IS FOR DD CONSTRAINTS  W/CL=95%
         SUMBP = SUMBP + E(IP,IJ,II)
         SUMBP2= SUMBP2 + E2(IP,IJ,II)
500     CONTINUE
C   JT CHANGES ODD NUMBERED PUMP WELLS TO 1,2,3 ORDER
        JT= (II-1)/2
C   FOR GAMS TABLE B(PUMP,TIME)CORRECT TIME ORDER
        B1(JT,IJ) = SUMBP
        B2(JT,IJ) = SUMBP2
C  STORE B1 VALUES IN KERNEL.OUT TO CALC PUMP WELL HEADS
        WRITE(6,402) B1(JT,IJ)
850     CONTINUE
C  CALC OF CONSTANT TERM(LAST TERM OF SQD HEAD DIFF)
        CONST = 0.
        DO 600 IO=2,L-I
         CONST = CONST + WF*(HO(1)-HO(IO))**2
600     CONTINUE
C  SUMMATION OF INFLUENCE COEFFICIENTS OVER ALL OBSERVATION WELLS
C  FOR EACH PUMPING WELL (I+1 IS THE SOURCE WELL+ALL PUMP WELLS)
```

```
C        DO 705 II= 1,I
C        DO 705 IJ= 1,IT
C        SUMBOB(II,IJ) = 0.
C    IL ONLY SUMS THE EVEN (OBSER) WELLS
C        DO 700 IL=2, L-1, 2
C         SUMBOB(II,IJ) = SUMBOB(II,IJ) + E2(II,IJ,IL)
C00      CONTINUE
C05      CONTINUE
C  CALCULATION OF LINEAR ECONOMIC COEFFICIENTS CT(I,T) FOR GAMS
C     (IN CORRECT TIME ORDER)
C  NN KEEPS TRACK OF PUMP WELL NOS. IN RELATION TO ALL WELLS
      NN=1
      DO 400 II= 1,I
      NN=NN+2
      DO 400 IJ= 1,IT
►     CK=4.1322E-6
►C    CK=0.
C  NO COST FOR INJECTION PUMPING
      IF(X(NN).GT.0.0) CK=0.
        CT(II,IJ) = CK*(HP(II,1)-HP(II,2)) + CKK
C     WRITE(6,403)NN,X(NN), HP(II,1), HP(II,2), CT(II,IJ)
C03   FORMAT(I3,3F10.4, D10.4)
C  CALCULATION OF ECONOMIC QUADRATIC COEFFICIENTS KT(I,T) FOR GAMS
C     (IN CORRECT TIME ORDER)
        KT(II,IJ) = CK*B2(II,IJ)
400   CONTINUE
C     DO 233 KK=1,IT
C     WRITE(6,203)(CT(M,KK),M=1,I)
C03   FORMAT(30D10.2)
C33   CONTINUE
C  SUMMATION OF INFLUENCE COEFFICIENTS OVER ALL PUMP WELLS
C     ON THE SOURCE
      DO 703 IJ= 1,IT
      PUMPSC(IJ)=0.
C  II SUMS OVER ALL PUMP WELLS
      DO 702 II=1,I
      PUMPSC(IJ)= PUMPSC(IJ)+E2(II,IJ,1)
702   CONTINUE
C  STORE PUMPSC IN FILE KERNEL.OUT TO CALCULATE OBS WELL HEADS
      WRITE(6,402) PUMPSC(IJ)
402   FORMAT(D15.4)
703   CONTINUE
C  SUM OF INFLUENCE COEFFICIENTS OVER ALL PUMPING WELLS
C     FOR EACH OBSER. WELL
C  IL IS THE EVEN(OBSER) WELLS (NOT INCLUDE SOURCE WELL #1)
      DO 704 IL=2,L-1,2
      DO 704 IJ= 1,IT
      IO = IL/2
      PUMPOB(IO,IJ)=0.
C  II SUMS OVER ALL PUMP WELLS
      DO 404 II=1,I
      PUMPOB(IO,IJ)= PUMPOB(IO,IJ)+E2(II,IJ,IL)
404   CONTINUE
```

```
C  STORE PUMPOB IN FILE KERNEL.OUT TO CALCULATE OBS WELL HEADS
      WRITE(6,402) PUMPOB(IO,IJ)
704   CONTINUE
      DO 710 IL=2,L-1,2
      DO 710 IJ= 1,IT
C  IO PUTS K(IO,IJ) FOR OBSER WELLS INTO 1,2,3 ORDER
      IO=(IL)/2
C  KR REVERSES THE TIME ORDER OF IT
      KR=IT-IJ+1
C  IG CHANGES IO INDICE TO OBSER WELL GW TABLE INDICE HO(IG)
C     BECAUSE HO(1) IS THE SOURCE
      IG= IO+1
C  CALCULATION OF HYDRAULIC QUADRATIC COEFFICIENTS K(I,T) FOR GAMS
C    (IN REVERSE TIME ORDER)
        K(IO,IJ) = (PUMPOB(IO,KR)-PUMPSC(KR))
C  CALCULATION OF LINEAR HYDRAULIC COEF. C(J,T) IN REVERSE ORDER
      C(IO,IJ)= 2*WF*K(IO,IJ)*(HO(1)-HO(IG))
710   CONTINUE
C     PRINT 1410
C410  FORMAT('I AM AT THE WRITE PORTION')
C
C  WRITING DATA IN GAMS/MINOS FORMAT INTO FILE MODEL1.OUT
C     DO 333 KK=1,IT
C     WRITE(6,303)(CT(M,KK),M=1,I)
C03   FORMAT(30D10.2)
C33   CONTINUE
C
      WRITE(4,444)
444   FORMAT('$OFFSYMLIST OFFSYMXREF')
      DO 6 II=1,8
      WRITE(4,7)II,SL(II),NP(II),SP(II)
7     FORMAT('* FOR SIDE',I2,/,'*   THE L=',F8.2,' ;NO. PUMPS=',I4,
     !' ;SPACING=',F8.2)
6     CONTINUE
      WRITE(4,548)TRANS,COVT,EP,COVS,F1,CL
548   FORMAT('* TRANSMISSIVITY IS',F10.2,/,'* TRANS COV IS ',F3.2,/,
     ^'* EFFECTIVE POROSITY IS ',F3.2,/,'* EFF PORO COV IS ',F3.2,/,
     ^'* F1 IS ',F4.2,/,'* RELIABILITY IS ',F3.2,/,/,
     ^'* ESTIMATED PUMPING')
      DO 551 JJ=1,IT
      WRITE(4,549) JJ,Q(JJ)
549   FORMAT('* Q',I2,' IS ',F10.3)
551   CONTINUE
      WRITE(4,8)R,TIME,LENGTH
8     FORMAT('* WELL RADIUS IS',F5.2,/,
     !'* TIME PERIOD IS A ',A6,/,
     !'* LENGTH DIMENSION IS ',A6,/,
     !'* LOW LIMIT ON DD AT PUMP WELLS = 1/2(SAT. THICK.)',/,
     !'* HIGH LIMIT ON DD AT PUMP WELLS = GROUND ELEV.',/)
C
      WRITE(4,550)I,IT,QU,CONST, WF,HO(1),IT
550   FORMAT('SETS',/,4X,'I PUMPING WELLS /1*',I4,'/',/,4X,'T TIME STEPS
     !/1*',I2,'/',/,4X,'J DUMMY SET /1/',/,4X,'N DUMMY SET /1*2/',/,
```

```
     !'SCALAR',/,6X,'QU UPPER PUMPING /',F8.2,'/',/,6X,
     !'CON CONSTANT TERM IN SQD HEAD DIFF /',F8.1,'/',/,6X,
     !'WF WEIGHT FACTOR /',F8.2,'/',/,6X,
     !'HS SOURCE PIEZ. ELEV. /',F8.2,'/',/,6X,
     !'FT FINAL TIME PERIOD /', I4,'/')
      WRITE(4,751)
751   FORMAT('PARAMETER',/,9X,'HOB(I) INITIAL HEAD AT EACH OBS WELL')
      DO 901 J=2,L-I
      JJ = J-1
      IF(J.EQ.2) WRITE(4,911) JJ, HO(J)
911   FORMAT(9X,'/',I2,F10.2)
      IF(J.EQ.L-I) WRITE(4,921) JJ, HO(J)
921   FORMAT(10X,I2,F10.2,'/')
      IF(J.NE.2.AND.J.NE.L-I) WRITE(4,931) JJ, HO(J)
931   FORMAT(10X,I2,F10.2)
901   CONTINUE
      WRITE(4,651)
651   FORMAT(/,9X,'ST(I) SATURATED THICK. AT EACH PUMP WELL')
      N=0
      DO 601 J=3,L,2
      N=N+1
      IF(J.EQ.3) WRITE(4,611) N,ST(J)
611   FORMAT(9X,'/',I2,F10.2)
      IF(J.EQ.L) WRITE(4,621) N, ST(J)
621   FORMAT(10X,I2,F10.2,'/')
      IF(J.NE.3.AND.J.NE.L) WRITE(4,631) N, ST(J)
631   FORMAT(10X,I2,F10.2)
601   CONTINUE
      WRITE(4,753)
753   FORMAT(/,9X,'SC(T) INFLUENCE COEFS. FOR SOURCE WELL')
      DO 801 J=1,IT
      IF(J.EQ.1) WRITE(4,811) J,PUMPSC(J)
811   FORMAT(9X,'/',I2,E12.4)
      IF(J.EQ.IT) WRITE(4,821) J, PUMPSC(J)
821   FORMAT(10X,I2,E12.4,'/')
      IF(J.NE.1.AND.J.NE.IT) WRITE(4,831) J, PUMPSC(J)
831   FORMAT(10X,I2,E12.4)
801   CONTINUE
      WRITE(4,7521)
7521  FORMAT(/,9X,'TI(I) SPECIFIES OBS WELLS HEAD ABOVE SOURCE HEAD')
C  NN KEEPS TRACK OF OBS WELL NUMBER AS A PART OF ALL WELLS
C     I.E. PUMP WELLS ARE 3,5... ;OBS. WELLS ARE 2,4.....
      NN=0
      DO 8001 J=1,I
      NN=NN+2
      IF(X(NN).GT.0.0) THEN
         KOBS=1
      ELSE
         KOBS=0
      ENDIF
C     WRITE(6,66)NN,X(NN),KOBS
C6    FORMAT(I3,F10.2,I2)
      IF(J.EQ.1) WRITE(4,8101) J, KOBS
```

```
8101  FORMAT(9X,'/',I2,I5)
      IF(J.EQ.I) WRITE(4,8201) J, KOBS
8201  FORMAT(10X,I2,I5,'/')
      IF(J.NE.1.AND.J.NE.I) WRITE(4,8301) J, KOBS
8301  FORMAT(10X,I2,I5)
8001  CONTINUE
      WRITE(4,752)
752   FORMAT(/,9X,'TT(T) TIME VECTOR TO PROVIDE PARTIAL SUMS')
      DO 800 J=1,IT
      IF(J.EQ.1) WRITE(4,810) J, J
810   FORMAT(9X,'/',I2,I5)
      IF(J.EQ.IT) WRITE(4,820) J, J
820   FORMAT(10X,I2,I5,'/')
      IF(J.NE.1.AND.J.NE.IT) WRITE(4,830) J, J
830   FORMAT(10X,I2,I5)
800   CONTINUE
      WRITE(4,750)
750   FORMAT('TABLE HO(I,N) GROUND EL. & INIT. HEAD AT EACH PUMP WELL')
      WRITE(4,650)(J, J=1,2)
650   FORMAT(5X,I10,I10)
      DO 900 J=1,I
      WRITE(4,660)J,(HP(J,M), M=1,2)
660   FORMAT(5X,I2,2F10.2)
900   CONTINUE
      WRITE(4,940)
940   FORMAT('TABLE B(I,T) INFLUENCE COEF.AT PUMP WELLS')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1501
      NIT= IT/5 + 1
      DO 1002 KK= 1,NIT
      IF(KK.GT.1) GO TO 7010
      WRITE(4,950)(J, J=1,5)
950   FORMAT(1X,10I12)
      N=1
      DO 1001 J=1,I
      WRITE(4,960)J,(B1(J,M), M=1,5)
960   FORMAT(5X,I2,10E12.4)
1001  CONTINUE
      GO TO 1002
7010  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7002
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1002
      WRITE(4,951)(J,J=JB,JE)
951   FORMAT(/,'+',10I12)
      DO 1000 JJ=1,I
      WRITE(4,960)JJ,(B1(JJ,M), M=JB,JE)
1000  CONTINUE
      GOTO 1002
7002  WRITE(4,951)(J,J=JB,IT)
      DO 1008 J=1,I
      WRITE(4,960)J,(B1(J,M),M=JB,IT)
```

```
1008  CONTINUE
1002  CONTINUE
      GOTO 15031
1501  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1502 J=1,I
      WRITE(4,960)J,(B1(J,M), M=1,IT)
1502  CONTINUE
15031 WRITE(4,9401)
9401  FORMAT('TABLE OB(I,T) INFLUENCE COEF.AT OBS WELLS')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 15011
      NIT= IT/5 + 1
      DO 10021 KK= 1,NIT
      IF(KK.GT.1) GO TO 70101
      WRITE(4,9501)(J, J=1,5)
9501  FORMAT(1X,10I12)
      N=1
      DO 10011 J=1,I
      WRITE(4,9601)J,(PUMPOB(J,M), M=1,5)
9601  FORMAT(5X,I2,10E12.4)
10011 CONTINUE
      GO TO 10021
70101 JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 70021
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 10021
      WRITE(4,9511)(J,J=JB,JE)
9511  FORMAT(/,'+',10I12)
      DO 10001 JJ=1,I
      WRITE(4,9601)JJ,(PUMPOB(JJ,M), M=JB,JE)
10001 CONTINUE
      GOTO 10021
70021 WRITE(4,9511)(J,J=JB,IT)
      DO 10081 J=1,I
      WRITE(4,9601)J,(PUMPOB(J,M),M=JB,IT)
10081 CONTINUE
10021 CONTINUE
      GOTO 1503
15011 WRITE(4,9501)(J, J=1,IT)
      N=1
      DO 15021 J=1,I
      WRITE(4,9601)J,(PUMPOB(J,M), M=1,IT)
15021 CONTINUE
1503  WRITE(4,970)
970   FORMAT('TABLE C(I,T) LINEAR HYDR. COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1601
      NIT= IT/5 + 1
      DO 1102 KK= 1,NIT
      IF(KK.GT.1) GO TO 7011
      WRITE(4,950)(J, J=1,5)
```

```
      N=1
      DO 1101 J=1,I
      WRITE(4,960)J,(C(J,M), M=1,5)
1101  CONTINUE
      GO TO 1102
7011  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7102
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1102
      WRITE(4,951)(J,J=JB,JE)
      DO 1100 JJ=1,I
      WRITE(4,960)JJ,(C(JJ,M), M=JB,JE)
1100  CONTINUE
      GOTO 1102
7102  WRITE(4,951)(J,J=JB,IT)
      DO 1108 J=1,I
      WRITE(4,960)J,(C(J,M),M=JB,IT)
1108  CONTINUE
1102  CONTINUE
      GOTO 1603
1601  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1602 J=1,I
      WRITE(4,960)J,(C(J,M), M=1,IT)
1602  CONTINUE
1603  WRITE(4,975)
975   FORMAT('TABLE CT(I,T) LINEAR ECON COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1701
      NIT= IT/5 +1
C     DO 433 KK=1,IT
C     WRITE(6,403)(CT(M,KK),M=1,I)
C03   FORMAT(30D10.2)
C33   CONTINUE
      DO 1112 KK= 1,NIT
      IF(KK.GT.1) GO TO 7012
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1111 J=1,I
      WRITE(4,960)J,(CT(J,M), M=1,5)
1111  CONTINUE
      GO TO 1112
7012  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7112
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1112
      WRITE(4,951)(J,J=JB,JE)
      DO 1110 JJ=1,I
      WRITE(4,960)JJ,(CT(JJ,M), M=JB,JE)
1110  CONTINUE
      GOTO 1112
```

```
7112  WRITE(4,951)(J,J=JB,IT)
      DO 1118 J=1,I
      WRITE(4,960)J,(CT(J,M),M=JB,IT)
1118  CONTINUE
1112  CONTINUE
      GOTO 1703
1701  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1702 J=1,I
      WRITE(4,960)J,(CT(J,M), M=1,IT)
1702  CONTINUE
1703  WRITE(4,980)
980   FORMAT('TABLE K(I,T) HYDR QUAD COEFS OF OBJ FUNC(I=OBSER WELL)')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1801
      NIT= IT/5 + 1
      DO 1122 KK= 1,NIT
      IF(KK.GT.1) GO TO 7013
      WRITE(4,950)(J, J=1,5)
      N=1
      DO 1121 J=1,I
      WRITE(4,960)J,(K(J,M), M=1,5)
1121  CONTINUE
      GO TO 1122
7013  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7122
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1122
      WRITE(4,951)(J,J=JB,JE)
      DO 1120 JJ=1,I
      WRITE(4,960)JJ,(K(JJ,M), M=JB,JE)
1120  CONTINUE
      GOTO 1122
7122  WRITE(4,951)(J,J=JB,IT)
      DO 1128 J=1,I
      WRITE(4,960)J,(K(J,M),M=JB,IT)
1128  CONTINUE
1122  CONTINUE
      GOTO 1803
1801  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1802 J=1,I
      WRITE(4,960)J,(K(J,M), M=1,IT)
1802  CONTINUE
1803  WRITE(4,985)
985   FORMAT('TABLE KT(I,T) ECONOMIC QUADRATIC COEFS. OF OBJ. FUNC.')
C  TO BE ABLE TO WRITE TABLE VALUES FOR TIMES > 5 UNDER ORIGINAL
      IF(IT.LT.5) GOTO 1901
      NIT= IT/5 + 1
      DO 1222 KK= 1,NIT
      IF(KK.GT.1) GO TO 7014
      WRITE(4,950)(J, J=1,5)
```

```
      N=1
      DO 1220 J=1,I
      WRITE(4,960)J,(KT(J,M), M=1,5)
1220  CONTINUE
      GO TO 1222
7014  JB= (KK-1)*5+1
      JE= KK*5
      JX=(KK-1)*5
      IF(KK.EQ.NIT.AND.JX.LT.IT) GOTO 7222
      IF(KK.EQ.NIT.AND.JX.EQ.IT) GOTO 1222
      WRITE(4,951)(J,J=JB,JE)
      DO 1130 JJ=1,I
      WRITE(4,960)JJ,(KT(JJ,M), M=JB,JE)
1130  CONTINUE
      GOTO 1222
7222  WRITE(4,951)(J,J=JB,IT)
      DO 1228 J=1,I
      WRITE(4,960)J,(KT(J,M),M=JB,IT)
1228  CONTINUE
1222  CONTINUE
      GOTO 1903
1901  WRITE(4,950)(J, J=1,IT)
      N=1
      DO 1902 J=1,I
      WRITE(4,960)J,(KT(J,M), M=1,IT)
1902  CONTINUE
1903  WRITE(4,1960)
1960  FORMAT(/,'ALIAS (T,L,M);')
      WRITE(4,1965)
1965  FORMAT(/,'TABLE IND(L,M) INDICE MATRIX FOR SUMMING B(T-T+1)*QT')
      WRITE(4,1970)(J,J=1,IT)
1970  FORMAT(5X,10I3)
      DO 1975 M=1,IT
      WRITE(4,1980)M,(N,N=M,IT)
1980  FORMAT(2X,12I3)
1975  CONTINUE
      CLOSE (5, ERR=1004, STATUS='KEEP')
      CLOSE (4, ERR=1005, STATUS='KEEP')
      CLOSE (6, ERR=1006, STATUS='KEEP')
      CLOSE (7, ERR=1007, STATUS='KEEP')
      GOTO 40
1201  PRINT 30
30    FORMAT('  AHA! ERROR FROM OPEN 5 ')
      GOTO 40
1202  PRINT 32
32    FORMAT('  AHA! ERROR FROM OPEN 4 ')
      GOTO 40
1203  PRINT 33
33    FORMAT('  AHA! ERROR FROM OPEN 6 ')
      GOTO 40
1204  PRINT 34
34    FORMAT('  AHA! ERROR FROM OPEN 7 ')
      GOTO 40
```

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1004  PRINT 37
37    FORMAT('  AHA! ERROR FROM CLOSE 5 ')
      GOTO 40
1006  PRINT 31
31    FORMAT('  AHA! ERROR FROM CLOSE 6 ')
      GOTO 40
1007  PRINT 36
36    FORMAT('  AHA! ERROR FROM CLOSE 7 ')
      GOTO 40
1005  PRINT 35
35    FORMAT('  AHA! ERROR FROM CLOSE 4 ')
40    STOP
      END
C
      SUBROUTINE CALC
      DIMENSION SL(8),SP(8),NP(8),ST(41),Q(10)
      DOUBLE PRECISION B(20,10,41),A(20,10,41),EP,COVT,COVS
      DOUBLE PRECISION E(20,10,41),D(20,10,41),E2(20,10,41)
      DOUBLE PRECISION X(41),Y(41),W(10),U,TERM,BP(20,10,41)
      DOUBLE PRECISION S(41,41), WMINK, PI, ANF,U1,FTQ,FSQ,FQO
      DOUBLE PRECISION Z,THETA,TRANS,UN,WU,TERML,TERM1,TERMS,EU,EU1
      COMMON/CARD1/ IT,I,L,LL,R,ST,AA,QU
      COMMON/CARD2/ SL,SP,NP,X
      COMMON/CARD3/ Q
      COMMON/CARD4/ E
      COMMON/CARD5/ E2
      COMMON/CARD6/ CL,F1
      COMMON/CARD7/ EP,TRANS,COVT,COVS
C
C  CALCULATION OF COORDINATES OF ALL WELLS (OBS & PUMP) STARTING WITH
C  SOURCE WELL AND THEN TO WELL (A,SL/2) AND THEN CCW
C
C      WRITE(6,2)I,L,IT,R,AA,TIME,LENGTH
C2     FORMAT(3I5,F5.2,F10.2,2A5)
C      WRITE(6,4)QU,EP,COVS,COVT,CL,F1,TRANS
C4     FORMAT(F10.2,5F5.2,F10.2)
      PI = 22./7.
      SDT=COVT*TRANS
      SDS=COVS*EP
C      WRITE(6,11)TRANS,COVT,SDT
C      WRITE(6,11)EP,COVS,SDS
11    FORMAT(3D15.4)
      LNP=3
      MNP=NP(2)+2
      X(1)= 0.
      Y(1)= 0.
C     WRITE(6,13)X(1),Y(1)
      X(2)= AA
      Y(2)= SL(1)/2.
C     WRITE(6,13)X(2),Y(2)
13    FORMAT(2F10.2)
      DO 300 II=LNP,MNP
      X(II)=X(II-1)-(SP(2)/2)*DSIN(PI/4.)
```

```
      Y(II)=Y(II-1)+(SP(2)/2)*DCOS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
300   CONTINUE
      LNP=LNP+NP(2)
      MNP=MNP+NP(3)
      DO 400 II=LNP,MNP
      X(II)=X(II-1)-(SP(3)/2)
      Y(II)=Y(II-1)
C     WRITE(6,13)X(II),Y(II)
400   CONTINUE
      LNP=LNP+NP(3)
      MNP=MNP+NP(4)
      DO 500 II=LNP,MNP
      X(II)=X(II-1)-(SP(4)/2)*DSIN(PI/4.)
      Y(II)=Y(II-1)-(SP(4)/2)*DCOS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
500   CONTINUE
      LNP=LNP+NP(4)
      MNP=MNP+NP(5)
      DO 600 II=LNP,MNP
      X(II)= X(II-1)
      Y(II)=Y(II-1)-(SP(5)/2)
C     WRITE(6,13)X(II),Y(II)
600   CONTINUE
      LNP=LNP+NP(5)
      MNP=MNP+NP(6)
      DO 1500 II=LNP,MNP
      X(II)=X(II-1)+(SP(6)/2)*DSIN(PI/4.)
      Y(II)=Y(II-1)-(SP(6)/2)*DCOS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1500  CONTINUE
      LNP=LNP+NP(6)
      MNP=MNP+NP(7)
      DO 1600 II=LNP,MNP
      X(II)=X(II-1)+(SP(7)/2)
      Y(II)=Y(II-1)
C     WRITE(6,13)X(II),Y(II)
1600   CONTINUE
      LNP=LNP+NP(7)
      MNP=MNP+NP(8)
      DO 1700 II=LNP,MNP
      X(II)=X(II-1)+(SP(8)/2)*DSIN(PI/4.)
      Y(II)=Y(II-1)+(SP(8)/2)*DCOS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1700   CONTINUE
      LNP=LNP+NP(8)
      DO 1800 II= LNP, L
      X(II)= X(II-1)
      Y(II)=Y(II-1)+(SP(1)/2)
C     WRITE(6,13)X(II),Y(II)
1800   CONTINUE
C
C     CALCULATION OF INFLUENCE COEFFICIENTS FOR Q= 1000 CU-FT/DAY
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```
C
C  ODD NUMBERED WELLS ARE PUMP WELLS
      DO 1300 J=3,L,2
C  ALL WELLS ARE OBSER WELLS
      DO 1250 M=1,L
C  CALCULATE HYCON BASED ON HCMAX AND HCMIN
C     IF(X(M).EQ.X(J)) HYCON=HCMIN
C     IF(Y(M).EQ.Y(J).OR.M.EQ.J) HYCON=HCMAX
C     IF(X(M).EQ.X(J).OR.Y(M).EQ.Y(J)) GOTO 604
C     Z = (Y(M)-Y(J))/(X(M)-X(J))
C     THETA= DATAN(Z)
C     HYCON= (HCMAX*HCMIN)/(HCMIN*(DCOS(THETA))**2+HCMAX*
C    !(DSIN(THETA))**2)
C     IF(M.EQ.(J+1)) WRITE(6,605) J,M,X(J),X(M),Y(J),Y(M),THETA,HYCON
C     IF(J.EQ.L.AND.M.EQ.2) WRITE(6,605)J,M,X(J),X(M),Y(J),Y(M),THETA,
C    ^HYCON
C05   FORMAT(' J=',I3,2X,'M=',I3,2X,'X=',2D12.4,2X,'Y=',2D12.4,/,
C    ^' THETA=',D12.4,' HYCON=',D12.4)
C  S= DISTANCE BETWEEN PUMP WELL J & OBSER. WELL M
604   IF(ABS(X(J)-X(M)).LT.1.0.AND.ABS(Y(J)-Y(M)).LT.1.0) GOTO 505
      IF(ABS(X(J)-X(M)).GT.1.0.AND.ABS(Y(J)-Y(M)).GT.1.0) GOTO 560
      IF(ABS(X(J)-X(M)).LT.1.) S(J,M)= ABS(Y(J)-Y(M))
      IF(ABS(Y(J)-Y(M)).LT.1.) S(J,M)= ABS(X(J)-X(M))
      GOTO 510
560   S(J,M)=DSQRT(((X(J)-X(M))**2)+((Y(J)-Y(M))**2))
      GOTO 510
505   S(J,M)=R
C USING THE LOWER SAT. THICK. FOR PUMP WELL INFLUENCE AND THE HIGH
C   SAT. TH. FOR THE OBS WELL (THE LOW TR PROVIDES THE HIGHEST INFLUENCE
C   AND THE HIGH PROVIDES THE LEAST INFLUENCE)
C11   IF(M.EQ.1.AND.ST(J).LT.ST(M)) THEN
C               TRANS=ST(M)*HYCON
C        ELSE
C               TRANS=ST(J)*HYCON
C        ENDIF
C     IF(M.EQ.1) GO TO 510
C     IF(MOD(M,2)) 5001,5002,5001
C001  IF(ST(J).LT.ST(M)) THEN
C               TRANS=ST(J)*HYCON
C        ELSE
C               TRANS=ST(M)*HYCON
C        ENDIF
C     GO TO 510
C002  IF(ST(J).LT.ST(M)) THEN
C               TRANS=ST(M)*HYCON
C        ELSE
C               TRANS=ST(J)*HYCON
C        ENDIF
C  AVERAGE THE SAT. THICK. FOR OBS & PUMP WELLS TO CALC TRANS
C10   TRANS=HYCON*(ST(J)+ST(M))/2.
C10   IF(M.EQ.(J+1)) WRITE(7,42) TRANS
C     IF(J.EQ.L.AND.M.EQ.2) WRITE(7,42) TRANS
C1    FORMAT(D20.10)
```

```
C  WRITE TRANS FROM PUMP WELL TO OBS WELL DIRECTLY CCW FROM IT
C      INTO FILE TRANS.OUT
510    IF(M.EQ.(J+1)) WRITE(7,41) TRANS
       IF(J.EQ.L.AND.M.EQ.2) WRITE(7,41) TRANS
41     FORMAT(D20.10)
       DO 1200 K=1, IT
C  CALCULATE BOLTZMAN VARIABLE, U
C      WRITE(6,1888)J,TR(J),M,TR(M),AVGTR
C888   FORMAT('TR',I3,'=',F10.2,'TR',I3,'=',F10.2,'AVGTR=',F10.2)
       U= (S(J,M)**2)*EP/(4*TRANS*K)
C      IF(J.EQ.3.AND.M.EQ.33)
C        WRITE(6,515)J,M,S(J,M),TRANS,U
C15    FORMAT(' J=',I3,2X,'M=',I3,2X,'S=',D12.4,2X,'TRANS=',D12.4,2X,
C     ^'U=',D12.4)
       IF(K.EQ.1) THEN
          WMINK=0.
          U1 = 1.
       ENDIF
C  CALCULATE WELL COEFFICIENT, W(U). USE ALT. SERIES FOR U<5.0 AND USE
C      NEG. POWER SERIES FOR U>5.0
C  TERML IS THE e(-X)/X TERM OF NEG. POWER SERIES
C  TERM IS EACH TERM OF NEG. POWER SERIES
C  TERMS IS THE SUM OF THE TERM
       IF(U.GT.5.0) THEN
          TERML=(DEXP(-U/2)*DEXP(-U/2))/U
       ELSE
C   FIRST 2 TERMS OF INFINITE SERIES FOR W(U)
          WU=-0.5772-(DLOG(U))
       ENDIF
       TERMS=0.0
       TERM1=100.
       DO 900 N=1,1000
       ANF=N
C  LOOP TO CALCULATE N FACTORIAL
       NN= N-1
       DO 800 JB=1,NN
       ANF= ANF*(N-JB)
800    CONTINUE
C  CALCULATE ADDITIONAL TERMS OF W(U)
       IF(U.GT.5.0) THEN
          TERM=(-1.)**N*ANF/U**N
       ELSE
          TERM= (-1.)**(N)*(U**N)/(N*ANF)
       ENDIF
C IN POWER SERIES CHECK IF N+1 TERM > N TERM. IF SO; STOP.
       IF(ABS(TERM).GT.ABS(TERM1)) GOTO 910
C  IN POWER SERIES SUM THE TERMS IF THEY ARE GETTING SMALLER
       IF(U.GT.5.0) TERMS=TERMS+TERM
       IF(U.GT.5.0) TERM1=TERM
C  CHECKING IF LAST TERM OF W(U) < .0001
       IF(ABS(TERM).LT.1.0D-10) GOTO 910
C  CALCULATING THE WELL FUNCTION BASED ON SMALL U OR LARGE U
       IF(U.GT.5.0) THEN
```

```
         WU=TERML*(1.+TERMS)
      ELSE
         WU= WU-TERM
      ENDIF
C     IF(U.LT.4.5) WRITE(6,802) U,K,N,ANF,TERM,WU
C02   FORMAT(' U=',D12.4,' K=',I2,2X,'N=',I4,2X,'ANF=',D12.4,
C    !' TERM=',D12.4,2X,'W(K)=',D12.4)
C     IF(U.GT.4.5) WRITE(6,803) U,K,N,ANF,TERM,WU,TERMS
C03   FORMAT(' U=',D12.4,' K=',I2,2X,'N=',I4,2X,'ANF=',D12.4,
C    !' TERM=',D12.4,2X,'W(K)=',D12.4,D12.4)
C     IF(J.EQ.3.AND.M.EQ.33) WRITE(6,704) WU
C04   FORMAT('W(K)=',D12.4)
900   CONTINUE
C10   WRITE(4,805) TERM, WU
C05   FORMAT(' TERM=',D12.4,2X,'W(K)=',D12.4)
C  JT PUTS PUMP WELLS IN 1,2,3 ORDER
910   JT= (J-1)/2
C  BP(PUMP WELL, TIME, ALL WELLS)
      BP(JT,K,M)=((WU-WMINK)/(4.*PI*TRANS))*1000.
C  A AND B ARE STOCHASTIC COEFS. IN THE TUNG PAPER
      EU=DEXP(-U)
      EU1=DEXP(-U1)
      IF(K.EQ.1) EU1=0.
      A(JT,K,M)=1000*(EU-EU1-WU+WMINK)/(4*PI*TRANS**2)
      B(JT,K,M)=-1000*(EU-EU1)/(4*PI*TRANS*EP)
C  CHANGING INJECTION WELL COEFS. TO NEGATIVE
C     IF(M.EQ.1.AND.K.EQ.1)WRITE(6,14)J,X(J)
14    FORMAT(I5,F10.2)
915   IF(X(J).GT.0.0) BP(JT,K,M)=-BP(JT,K,M)
C     IF(M.EQ.7.OR.M.EQ.9)
C    !WRITE(6,902) TERM,WU,WMINK,JT,K,M,BP(JT,K,M)
C02   FORMAT(' TERM=',D12.4,2X,'W(K)=',D12.4,2X,'WMINK=',D12.4,2X,/,
C    !3I4,2X,'B=',D12.4)
      WMINK= WU
      U1 = U
1200  CONTINUE
1250  CONTINUE
1300  CONTINUE
C  USING TUNG'S METHOD TO DETERMINE THE STOCHASTIC INFLUENCE COEFS (E)
      DO 11000 II=1,I
      DO 11000 K=1,IT
      DO 11000 M=1,L
      IV = 2*II + 1
      FTQ=0.
      FTS=0.
      DO 12000 III=1,I
      DO 12000 KK=1,K
      FTQ=FTQ+A(III,KK,M)*Q(K-KK+1)*SDT
      FSQ=FSQ+B(III,KK,M)*Q(K-KK+1)*SDS
C      IF(II.EQ.1.AND.K.EQ.3.AND.M.EQ.1)
C     !WRITE(6,11001)A(III,KK,M),Q(K-KK+1),SDT,FTQ,B(III,KK,M),SDS,FSQ
C11001 FORMAT(' A=',D10.4,' Q=',D10.4,' ST=',D10.4,' FTQ=',D10.4,/,
C     !' B=',D10.4,' SS=',D10.4,' FSQ=',D10.4)
```

```
12000 CONTINUE
      FQO=DSQRT(FTQ**2+FSQ**2)
C      WRITE(6,12001)FQO
C12001 FORMAT(' FQO=',D10.4)
      D(II,K,M)=(FTQ*A(II,K,M)*SDT+FSQ*B(II,K,M)*SDS)/FQO
C     IF(X(II).GT.0.0) D(II,K,M) = -D(II,K,M)
C  NEG BP IS CHANGED BACK TO POSITIVE TO BE ADDED TO E AND THEN E IS
C   CHANGED INTO NEG.
      IF(X(IV).GT.0.0) BP(II,K,M) = -BP(II,K,M)
C  E2 IS MINUS BECAUSE FOR THE OBJ. FUNCTION WE WANT 5% CL
C  E1 IS FOR DD CONSTRAINTS (95% CL) JUST AS WITH TUNG'S DERIVATION
      E(II,K,M)=BP(II,K,M)+F1*D(II,K,M)
      E2(II,K,M)=BP(II,K,M)-F1*D(II,K,M)
C  CHANGING THE E COEF TO NEG IF THE PUMPS ARE INJECTION
      IF(X(IV).GT.0.0) THEN
          E(II,K,M)=-E(II,K,M)
          E2(II,K,M)=-E2(II,K,M)
      ENDIF
C IF PUMP WELL IS ON Y-AXIS ELIMINATE IT
      IF(X(IV).EQ.0.0) THEN
          E(II,K,M)= 0.0
          E2(II,K,M)= 0.0
      ENDIF
11000 CONTINUE
C      DO 1650 J=1,I
C      WRITE(6,1305) J,F1
C1305  FORMAT(' PUMPING WELL NO.',I5,F10.2)
C      DO 1550 LT=1,4
C      WRITE(6,1310) LT
C1310  FORMAT(' TIME',I5)
C      WRITE(6,1315)(D(J,LT,M),M=1,4)
C      WRITE(6,1315)(E(J,LT,M),M=1,4)
C      WRITE(6,1315)(E2(J,LT,M),M=1,4)
C1315  FORMAT(5D15.4)
C1550  CONTINUE
C1650  CONTINUE
      RETURN
      END
```

Program 10 BOB2.GMS

```
VARIABLE  Q(T,J)   PUMPING DURING EACH TIME PERIOD
          MIN      SYMBOL FOR OBJECTIVE FUNCTION
          S1       LINEAR HYDRAULIC PORTION OF OBJEC. FUNC.
          S2       LINEAR ECONOMIC PORTION OF OBJEC. FUNC
          S3       QUAD. HYDRAULIC PORTION OF OBJEC. FUNC.
          S4       QUAD. ECONOMIC PORTION OF OBJEC. FUNC.
          ;
POSITIVE VARIABLE Q(T,J);
FREE VARIABLE MIN;

EQUATIONS  WTL     LOWER WATER TABLE LIMIT
           WTH     UPPER WATER TABLE LIMIT
           OBJ     OBJECTIVE FUNCTION
           OB1     LINEAR HYDRAULIC PORTION OF OB. FUNC.
           OB2     LINEAR ECONOMIC PORTION OF OB FUNC
           OB3     QUADRATIC HYDRAULIC PORTION OF OB. FUNC.
           OB4     QUADRATIC ECONOMIC PORTION OF OB. FUNC.
           GRAD    CAUSES DOWM GRAD OBS WELLS TO BE HIGHER THAN SOURCE
           ;
WTL(I,T,J)..
       0.5*ST(I)-SUM((L,M),B(I,L)*Q(M,J)$(IND(L,M) EQ TT(T))) =G= 0.0;
WTH(I,T,J)..
 HO(I,'1')-(HO(I,'2')-SUM((L,M),B(I,L)*Q(M,J)$(IND(L,M) EQ TT(T)))) =G=
0.0;
GRAD(I,J)$(TI(I))..
         HOB(I)-SUM((L,M),OB(I,L)*Q(M,J)$(IND(L,M) EQ FT))
           -(HS-SUM((L,M),SC(L)*Q(M,J)$(IND(L,M) EQ FT))) =G= 0.0;
OB1..          SUM((I,T,J),C(I,T)*Q(T,J)) =E= S1;
OB2..          SUM((I,T,J),CT(I,T)*Q(T,J)) =E= S2;
OB3..          SUM(I),WF*SQR(SUM((T,J),K(I,T)*(Q(T,J))))) =E= S3;
OB4..  SUM((I,T,J),SUM((L,M),KT(I,L)*Q(M,J)$(IND(L,M) EQ TT(T)))
           *Q(T,J)) =E= S4;

OBJ..          S1+S2+S3+S4+CON =E= MIN;

Q.UP(T,J)=QU;
Q.LO(T,J)=0.00;
Q.L(T,J)=150.00;

MODEL CONTAM /ALL/;

OPTION ITERLIM = 2000;
OPTION LIMROW = 0;
OPTION LIMCOL = 0;
*PTION SOLPRINT = OFF;

SOLVE CONTAM USING NLP MINIMIZING MIN;

DISPLAY Q.L, Q.M, Q.LO, Q.UP, MIN.L;

* THE INDICE MATRIX (L,M) IS A DUMMY MATRIX USED TO ALLOW THE CORRECT
```

```
* MULTIPLICATION OF KT(I,T)*Q(T,J) (Q(T,J) IS ACTUALLY A COLUMN VECTOR
* BUT THE DUMMY J=1 IS NEEDED BECAUSE ALL MATRICES MUST BE AT LEAST 2D)
* i.e. FOR TIME PERIOD 2 TT(T)=2; THEREFORE IN THE INDICE MATRIX FOR
* ALL TWOS THE MULTIPLICATIONS TAKE PLACE(WHEN L=2,M=1 AND WHEN L=1,
* M=2) SO KT(I,2)*Q(1,1)+KT(I,1)*Q(2,1) IS THE RESULT.

* THE ALIAS FUNCTION ALLOWS US TO SAY THAT L OR M CAN BE SUBSTITUTED
* FOR T IN ANY MATRIX.

* BECAUSE T IS COMPARED TO OTHER VALUES IT MUST BE SET AS A PARAMETER

* THE OB3 EQUATION IS MULTIPLYING EACH ROW OF THE K MATRIX
* BY THE COLUMN VECTOR Q,THEN SQUARING THE ROW TIMES THE Q VECTOR AND
* THEN SUMMING THESE.

* THE OB4 EQUATION ONLY USES THAT PART OF THE KT MATRIX THAT IT
* NEEDS DEPENDING ON THE TIME PERIOD BEING ANALYZED. BY ONLY USING
* THE L AND M VALUES FOR WHICH THERE IS A T VALUE INSIDE THE MATRIX
* ALLOWS THIS TO BE DONE. (SEE EXPLANATION OF INDICE MATRIX)
* EXAMPLE: FOR 4 TIME PERIODS THE TOTAL ECONOMIC VALUE FOR THE
*          QUADRATIC PORTION WOULD EQUAL-
* KT(I,4)Q(1)+KT(I,3)Q(2)+KT(I,2)Q(3)+KT(I,1)Q(4)+KT(I,3)Q(1)+
* KT(I,2)Q(2)+KT(I,1)Q(3)+KT(I,2)Q(1)+KT(I,1)Q(2)+KT(I,1)Q(1)
* SUMMED OVER ALL I (PUMPING WELLS)
```

Program 11 HEAD2.FOR (or SHEAD.FOR)

```
C  CALCULATION OF FINAL TARGET AND OBS WELL HEADS
C    AND SS PUMPING TO RETAIN THE FINAL HEADS
C
      DIMENSION HO(40),Q(10)
      DIMENSION ELEVOB(40),QQ(20),PUMPEL(40)
      DIMENSION ST(40),HG(40,2),SL(8),SP(8),NP(8)
      DOUBLE PRECISION  PUMPOB(35,10),PUMPSC(10),SUMBP(35,10),TRANS(35)
C  MODEL2.DAT (OR SMODEL.DAT) HAS ALL WELL HEADS AND FINAL PUMPING VALUES
in head2.for  OPEN(3, FILE='MODEL2.DAT', STATUS='OLD', ERR=1003)
in shead.for  OPEN(3, FILE='SMODEL.DAT', STATUS='OLD', ERR=1003)
C  KERNEL.OUT HAS THE INFLUENCE COEF SUMS FOR TARGET & OBS WELLS
      OPEN(2,FILE='KERNEL.OUT', STATUS='OLD', ERR=1004)
C  MODEL2.CAL WILL STORE THE FINAL HEADS AT TARGET & OBS WELLS
      OPEN(8,FILE='MODEL2.CAL', STATUS='NEW', ERR=1005)
C  TRANS.OUT WILL STORE THE TRANSMISSIVITY AND WELL SPACING AT ALL
C       PUMPING WELLS IN .DAT FORMAT
      OPEN(9,FILE='TRANS.OUT', STATUS='OLD', ERR=1009)
C  TOTDD=TOTAL DRAWDOWN, IT=NO. OF TIME PERIODS, I=NO. OF PUMP WELLS
C  L= TOTAL NO. OF WELLS, KR= REVERSE OF TIME STEPS
C  R=WELL RADIUS IN FT, TRANS=TRANSMISSIVITY IN SQ FT/DAY
C  SP=WELL SPACING
      PI=22./7.
      KR= IT-IJ+1
      TOTDD=0.
C
      READ(3,2)I,L,IT,R,A,TIME,LENGTH,MODEL
C     WRITE(8,2)I,L,IT,R,A,TIME,LENGTH,MODEL
2     FORMAT(3I5,F5.2,F10.2,2A5,I5)
      IF(MODEL.EQ.1) THEN
         READ(3,4)QU,EP,HCMIN,HCMAX
C        WRITE(8,4)QU,EP,HCMIN,HCMAX
      ELSE
         READ(3,25)QU,EP,COVS,COVT,CL,F1
C        WRITE(8,55)QU,EP,COVS,COVT,CL,F1
      ENDIF
4     FORMAT(F10.2,F5.2,2F10.2)
25    FORMAT(F10.2,2F5.2,3F5.2)
55    FORMAT('LINE 2',F10.2,2F5.2,3F5.2)
C  READING LENGTH OF EACH SIDE AND NO. OF
C   WELLS ON A SIDE (2 * PUMP WELLS)
      DO 88 II=1,8
      READ(3,3)SL(II),NP(II)
C     WRITE(8,3)SL(II),NP(II)
3     FORMAT(F10.2,I5)
88    CONTINUE
C  READING GROUNDWATER TABLE AND GROUND SURFACE ELEVATIONS
C  FOR PUMPING WELLS HP(I,2) FROM FILE ______.DAT
C    *SOURCE GW TABLE ELEV. IS FIRST AFTER PUMP WELL DATA*
C
      DO 100 II=1,I
        READ(3,95)(HG(II,J),J=1,2)
```

```
C        WRITE(8,95)(HG(II,J),J=1,2)
100    CONTINUE
95     FORMAT(2F10.2)
C  READING GROUNDWATER TABLE ELEVATIONS FOR OBSERVATION WELLS
C  HO(L-I)-FROM FILE ______.DAT
C
       DO 201 II=1,L-I
        READ(3,96) HO(II)
C       WRITE(8,96) HO(II)
201    CONTINUE
C  READ THE SATURATED THICK. VALUES FOR EACH WELL FROM FILE ______.DAT
C   START W/SOURCE, THEN TO OBS WELL (X=A,Y=SL/2), THEN CCW
       DO 250 II=1,L
       READ(3,96) ST(II)
96     FORMAT(F10.2)
C      WRITE(8,96) ST(II)
250    CONTINUE
C READ THE SPACING OF WELLS ON EACH SIDE FROM TRANS.OUT SP=SPAC. ON
C  A REG SIDE; SP2=SPAC. ON IRREG.SIDE
       DO 9 II=1,8
       READ(9,1)SP(II)
C      WRITE(8,1)SP(II)
1      FORMAT(F10.2)
9      CONTINUE
C  READ ANISOTROPIC TRANS VALUES FOR PUMP WELLS FROM TRANS.OUT
       DO 99 IL=1,I
       READ(9,41) TRANS(IL)
C      WRITE(8,41) TRANS(IL)
41     FORMAT(D20.10)
99     CONTINUE
       LL= (L-1)/2
       PI=22./7.
C  READ THE FINAL PUMPING VALUES FROM ______.DAT
       DO 200 J=1,IT
       READ(3,86) Q(J)
86     FORMAT(F10.3)
       WRITE(8,87) Q(J),LENGTH,TIME
87     FORMAT('Q=',F10.3,' x 1000 CU.',1X,A4,'/',A4)
200    CONTINUE
C  READ THE INFLUENCE COEF. SUMS FOR THE PUMP WELL FROM KERNEL.OUT
       DO 301 IL= 1,I
       DO 301 IJ= 1,IT
       READ(2,97) SUMBP(IL,IJ)
301    CONTINUE
C  READ THE INFLUENCE COEF. SUMS FOR THE TARGET WELL FROM KERNEL.OUT
       READ(2,97)
       DO 400 IJ= 1,IT
       READ(2,97) PUMPSC(IJ)
C      WRITE(8,88), PUMPSC(IJ)
C8     FORMAT('PUMPSC IS', D15.4)
400    CONTINUE
C  READ THE INFLUENCE COEF. SUMS FOR ALL OBS WELLS FROM KERNEL.OUT
       DO 300 IL= 1,I
```

```
      DO 300 IJ= 1,IT
      READ(2,97) PUMPOB(IL,IJ)
97    FORMAT(D15.4)
C     WRITE(8,89) PUMPOB(IL,IJ)
C9    FORMAT('PUMPOB IS',D15.4)
300   CONTINUE
C     CALCULATE THE TOTAL DD AT THE TARGET WELL
      DO 805 IJ=1, IT
      KR=IT-IJ+1
      TOTDD= TOTDD + (PUMPSC(KR)*Q(IJ))
C     WRITE(8,11)TOTDD
C1    FORMAT('TOTDD FOR TARGET IS',F10.4)
805   CONTINUE
      ELEVS= HO(1)-TOTDD
C  WRITE THE TARGET ELEV IN FILE MODEL2.CAL
      WRITE(8,501) ELEVS,LENGTH
501   FORMAT(/,'TARGET ELEV IS',F10.4,1X,A4,/)
C  SET N SO THE CORRECT OBS WELL START ELEV. IS USED
      N= 2
C  CALCULATE THE FINAL ELEV. AT ALL OBS WELLS (ELEVOB)
C  SUMDIF IS THE SUM OF ELEV DIFFERENCES
      SUMDIF=0.
      DO 806 IL= 1,I
      TOTDD=0.
      DO 804 IJ= 1,IT
      KR=IT-IJ+1
      TOTDD= TOTDD + (PUMPOB(IL,KR)*Q(IJ))
804   CONTINUE
      ELEVOB(IL)= HO(N)-TOTDD
      SQDIFF= (ELEVS-ELEVOB(IL))**2
      SUMDIF=SUMDIF+SQDIFF
C  WRITE THE FINAL OBS WELL HEADS IN FILE MODEL2.CAL
      WRITE (8,502) IL, ELEVOB(IL),LENGTH
502   FORMAT('OBSER WELL',I4,2X,'ELEV. IS',F10.4,1X,A4)
      N=N+1
806   CONTINUE
      WRITE(8,503) SUMDIF,LENGTH
503   FORMAT(/,'SUM OF ELEV DIFFERENCES SQD. IS',F10.4,1X,A4,'**2')
C  CALCULATE PUMP WELL ELEVS. AND PUTTING ALL ELEVS. IN FILE MODEL4.DAT
C     IN THE READ FORMAT SO ANOTHER RUN CAN BE MADE WITH NEW ELEVATIONS
C     DO 1101 II=1,I
C     IF(II.EQ.I) GOTO 1099
C     ELPUMP(II)=(ELEVOB(II+1)+ELEVOB(II))/2
C     GOTO 1101
C099  ELPUMP(II)=(ELEVOB(II)+ELEVOB(1))/2
C101  CONTINUE
C     DO 1102 II=1,I
C     WRITE(9,1098)HG(II,1),ELPUMP(II)
C098  FORMAT(2F6.1)
C102  CONTINUE
C     WRITE(9,1097)ELEVS
C097  FORMAT(F6.1)
C     DO 1103 II=1,I
```

```
C103  WRITE(9,1097)ELEVOB(II)
C
C  CALCULATION OF STEADY STATE PUMPING AFTER PLUM IS STABILIZED
C
C  CALCULATE THE FINAL ELEVS AT PUMP WELLS
      DO 1806 IL=1,I
      TOTDD=0.
      DO 1804 IJ=1,IT
      KR= IT-IJ+1
      TOTDD= TOTDD + SUMBP(IL,KR)*Q(IJ)
1804  CONTINUE
      PUMPEL(IL)=HG(IL,2) -TOTDD
1806  CONTINUE
C  CALCULATE SS PUMPING Q=(2*PI*T/LN(RE/RW))(SW-SE) WHERE SW IS THE
C     AVG OF 1/2 DD FOR OBS WELLS ON ECH SIDE OF PUMP WELL
      DO 1906 IL=1,I
C     WRITE(8,1503)NP,NP2,SP,SP2,PI,TRANS(IL),HG(IL,2),R
C503  FORMAT(2I5,6F10.4)
C   CHANGE WELL SPACING FOR EACH SIDE OF THE OCTAGON
      MNP=NP(2)/2
      IF(IL.LE.MNP) SPAC=SP(2)
      LNP=NP(2)/2
      MNP=(NP(2)+NP(3))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(3)
      LNP=(NP(2)+NP(3))/2
      MNP=(NP(2)+NP(3)+NP(4))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(4)
      LNP=(NP(2)+NP(3)+NP(4))/2
      MNP=(NP(2)+NP(3)+NP(4)+NP(5))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(5)
      LNP=(NP(2)+NP(3)+NP(4)+NP(5))/2
      MNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(6)
      LNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6))/2
      MNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6)+NP(7))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(7)
      LNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6)+NP(7))/2
      MNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6)+NP(7)+NP(8))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(8)
      LNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6)+NP(7)+NP(8))/2
      MNP=(NP(2)+NP(3)+NP(4)+NP(5)+NP(6)+NP(7)+NP(8)+NP(1))/2
      IF(IL.GT.LNP.AND.IL.LE.MNP) SPAC=SP(1)
      IF(IL.EQ.I) HO(IL+2)=HO(2)
      IF(IL.EQ.I) ELEVOB(IL+1)=ELEVOB(1)
      QQ(IL)=(2*PI*TRANS(IL)/LOG(SPAC/(2.*R)))*(HG(IL,2)-PUMPEL(IL)
     !-(HO(IL+1)-ELEVOB(IL)+HO(IL+2)-ELEVOB(IL+1))/4.)
C  WRITE SS PUMPING IN FILE MODEL2.CAL
      WRITE(8,1502)IL,QQ(IL),LENGTH,TIME
1502  FORMAT(/,'PUMPING WELL',I3,1X,'STEADY STATE PUMPING IS',
     !F25.4,1X,'CU',1X,A4,'/',A4)
1906  CONTINUE
      CLOSE(3,ERR=1006, STATUS='KEEP')
      CLOSE(2,ERR=1007, STATUS='KEEP')
```

```
      CLOSE(8,ERR=1008, STATUS='KEEP')
      CLOSE(9,ERR=1010, STATUS='KEEP')
      GOTO 900
1003  PRINT 30
30    FORMAT('ERROR IN OPEN 5')
      GOTO 900
1004  PRINT 40
40    FORMAT('ERROR IN OPEN  2')
      GOTO 900
1005  PRINT 50
50    FORMAT('ERROR IN OPEN  8')
      GOTO 900
1009  PRINT 51
51    FORMAT('ERROR IN OPEN  9')
      GOTO 900
1006  PRINT 60
60    FORMAT('ERROR IN CLOSE 5')
      GOTO 900
1007  PRINT 70
70    FORMAT('ERROR IN CLOSE 2')
      GOTO 900
1008  PRINT 80
80    FORMAT('ERROR IN CLOSE 8')
      GOTO 900
1010  PRINT 81
81    FORMAT('ERROR IN CLOSE 9')
900   STOP
      END
```

Program 12 Data File MODEL2.DAT for use with MODEL2.FOR

```
  16   33    8  1.0  1086.0    DAY   FT    1                    CARD 1
400.      0.3       180.      270.  0.0                         CARD 2
900.          4
900.          4
900.          4
900.          4                                                  DATA SET 1
900.          4
900.          4
900.          4
900.          4
 120.0    96.
 120.0    98.
 120.0    100.
 120.0    102.0
 120.0    104.0
 120.0    106.0
 120.0    107.0
 120.0    107.0                                                  DATA SET 2
 120.0    106.0
 120.0    104.0
 120.0    102.
 120.0    100.
 120.0    98.
 120.0    96.
 120.0    95.
 120.0    95.
 101.0
  95.
  97.
  99.
 101.
 103.
 105.
 107.
 107.                                                            DATA SET 3
 107.
 105.
 103.
 101.
  99.
  97.
  95.
  95.
 50.
 50.
 50.                                                             DATA SET 4
 50.
 50.
```

50.	DATA SET 4 (cont.)
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
50.	
293.427	DATA SET 5
274.937	
259.482	
244.743	
177.996	
0.0	
0.0	
0.0	

Program 13 Data File SMODEL.DAT for use with SMODEL.FOR
(sample created by responses of Appendix IV)

```
16   33    8 1.00   1091.15 DAY  FEET    2                    CARD 1
 400.00 0.30 0.20 0.40 0.95 1.64  13500.36                     CARD 2
904.10    4
904.10    4
904.10    4
904.10    4                                                    DATA SET 1
904.10    4
904.10    4
904.10    4
904.10    4
120.00     96.90
120.00     98.31
120.00    100.01
120.00    102.00
120.00    103.69
120.00    105.10
120.00    105.80
120.00    105.80                                               DATA SET 2
120.00    105.10
120.00    103.69
120.00    102.00
120.00    100.01
120.00     98.31
120.00     96.90
120.00     96.20
120.00     96.20
101.00
 96.20
 97.61
 99.01
101.00
102.99
104.40
105.80
105.80                                                         DATA SET 3
105.80
104.40
102.99
101.00
 99.01
 97.61
 96.20
 96.20
 50.00
 50.00
 50.00                                                         DATA SET 4
 50.00
 50.00
```

50.00	DATA SET 4 (cont.)
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
50.00	
250.00	DATA SET 5
250.00	
250.00	
250.00	
250.00	
250.00	
250.00	

Program 14 XCON.FOR

```
C
C  PROGRAM TO ANALYZE DIFFERENT METHODS FOR CONTAINING A CONTAMINANT
PLUME
C
C   CHARAC IS USED TO REPRESENT THE STRING(1) ANSWERS OF THE USER
C   ROCK IS 1,2 OR 3 TO REPRESENT HOW MUCH ROCK IS IN THE SOIL
C   STRAT IS 1,2 OR 3 TO REPRESENT THE CONDITION OF THE INTERFACE
BETWEEN
C    THE AQUIFER AND THE BEDROCK
C   SOIL IS 1 TO 6 TO REPRESENT THE SOILTYPE. FROM THIS THE PRIOR MEANS
&
C    STAN. DEV. ARE COMPUTED FOR BAYSIAN ANALYSIS
C   CONF IS THE CONFIDENCE LEVEL THE USER GIVES HIS ANSWER. ONLY THE
C    SMALLEST VALUE IS STORED. COMPARES CONF(1) WITH CONF(2) AND PUTS
C    SMALLEST VALUE IN CONF(1)
C   TREL IS A RUNNING COUNT TO KEEP TRACK OF HOW MANY TIMES THE USER
SAYS
C    UNKNOWN OR HE DOESN'T UNDERSTAND ASSUMPTION THE SECOND TIME IT IS
C    GIVEN TO HIM. EACH REL REDUCES CONF BY 1%
C   N IS A COUNTER TO KEEP TRACK OF HOW MANY TIMES A USER DOESN'T
UNDERSTAND
C    A MODEL ASSUMPTION
C   NUMT IS THE NUMBER OF FIELD DATA FOR HYDRAULIC CONDUCTIVITY
C   NUMEP IS THE NUMBER OF FIELD DATA FOR EFFECTIVE POROSITY
C   TRANS(20) IS TO STORE THE HYDRAULIC COND. FIELD VALUES
C   EP(20) IS TO STORE THE EFF. PORO. FIELD VALUES
C   PRECIP IS THE ESTIMATE OF AVERAGE MONTHLY PRECIPITATION (IN/MONTH)
C   DRAIN IS A CLASSIFICATION OF THE TYPE OF DRAINAGE IN THE AREA (1 TO
7)
C   WT IS THE AVERAGE DEPTH TO THE BOTTOM OF AQUIFER (FT)
C   GRAD IS THE AVERAGE HYDRAULIC GRADIENT (0-.99)
C   SAT IS THE AVERAGE SATURATED THICKNESS OF AQUIFER (FT)
C   CHEM IS THE ANSWER AS TO WHETHER CERTAIN CHEMICALS ARE IN
CONTAMINANT
C   TIME IS THE NO. OF DAYS FROM PRESENT TO THE ESTIMATED TIME OF
CONTAINMENT
C     STRATEGY
C   EXTENT IS THE ESTIMATE OF THE MAXIMUM EXTENT OF THE PLUME FROM ITS
SOURCE
C   COEF IS A COEFFICIENT INPUT BY USER TO UPDATE 1986 CAPITAL COSTS TO
C     THE PRESENT
C   LWCF KEEPS TRACK OF WHICH DATA THE USER GAVE THE LOWEST VALUE TO
C   QUEST IS THE INDICATOR (1,2,3) OF WHAT THE USER HAS A QUESTION ABOUT
C   STABE IS THE NUMBER OF DAYS THE PUMPING STRATEGY HAS TO STABILIZE
PLUME
C   CHNGT INDICATES A CHANGE OF 1.SOIL TYPE 2.FIELD DATA OR 3.NONE FOR
HC
C   CHNGEP INDICATES A CHANGE OF 1.SOIL TYPE 2.FIELD DATA OR 3.NONE FOR
EP
C
      DIMENSION X(33),Y(33),Z(33),HP(33,2),HO(33),Q(20)
```

```
      DIMENSION SLFCT(6,4),CONF(2),TRANS(20),EP(20),REL(10)
      INTEGER ROCK,CONF,CF,REL,TREL,STRAT,SOIL,PRECIP,DRAIN,TIME,QUEST
      INTEGER STABE,CHNGT,CHNGEP
      INTEGER TPW,TW,PERIOD,RELIA
      REAL MAXLFT,MC
      CHARACTER*1 CHARAC,CHEM,PRINT,CHARAC2
      CHARACTER*4 FRAME
      CHARACTER*5 TFRAME,LENGTH
      OPEN(1,FILE='SMODEL.DAT',STATUS='NEW',ERR=1600)
      OPEN(UNIT=9,FILE='PRN')
C
C READING THE HYD. COND. AND EFF. PORO. UPPER & LOWER LIMITS FOR THE 6
C  SOIL TYPES.
C  THESE VALUES ARE READ IN THE ORDER OF THE SOIL TYPE TABLE; READING
ALL
C  LL HC FIRST (FT/D), THEN ALL UL HC, THEN ALL LL EP, THEN ALL UL EP.
C
      PI = 22./7.
      DATA SLFCT /.26,.16,.003,.0025,3.28E-6,.066,1873.,820,3.28,1.97,
     #.0013,52.5,.13,.16,4*.01,.4,.46,.39,.28,2*.46/
C     DO 2 I = 1,6
C     WRITE(*,1)(SLFCT(I,J),J=1,4)
C     FORMAT(4E15.4)
C     CONTINUE
C     PAUSE
      CONF(1) = 100
C     GOTO 5000
      WRITE(*,10)
10    FORMAT(////,T18,'EXPERT PROGRAM TO DETERMINE ECONOMIC METHOD'
     1,/,T21,'FOR CONTAINING A CONTAMINANT PLUME')
      WRITE(*,20)
20    FORMAT(///,T6,'This system will determine the best possible techni
     1que to contain',/,
     1' a contaminant plume based on input from you and your confidence
     1in',/,
     1' that input. There are three possible answers for any one questio
     1n.',/,T6,
     1'1. (W)hy; if you wish to know the reason a question is asked.',/,
     1T6,
     1'2. (U)nknown; if you do not know an answer and wish the program t
     1o',/,T10,'estimate an answer.',/,T6,
     1'3. (Y)es followed by the answer to the question and a confidence'
     1,/,T10,'level for your answer.',/,T6,
     1'ALL RESPONSES SHOULD BE IN CAPITAL LETTERS.',///////)
      PAUSE ' Hit ENTER when you are ready to continue.'
      WRITE(*,22)
22    FORMAT(/
     1///,T6,'This system analyzes three possible containment techniques
     1; slurry',/,
     1' wall, sheet piling and pumping. All three strategies are based o
     1n the',/,' assumptions that:',/,T10,
     1'1. The contaminant is from a source which forms an elliptically s
     1haped',/,T13,'plume.',/,T10,
```

```
     1'2. All containment techniques are configured in the shape of a re
     1gular',/,T13,
     1'octagon centered on the contaminant source.',/,T15,
     1'a. The pumping technique is based on wells located on all eight'
     1,/,T18,'sides of the octagon.',/,T15,
     1'b. The other two techniques are based on forming only the five'
     1,/,T18,'down-gradient sides of the octagon.',/////,T6,
     1'The following questions are intended to characterize the soil env
     1iroment.',/,' If you are ready to continue type CLS and hit ENTER
     1to clear the screen and',/,' hit ENTER again to begin the question
     1s. If at any time, you wish to quit in the middle of the program s
     1imply hit Ctrl C.',/)
      PAUSE
C
C ASKING QUESTION ABOUT SOIL HOMOGENEITY ASSUMPTION REL(1)=1
C
      N = 0
25    WRITE(*,30)
      N = N + 1
      IF(N.EQ.3) REL(1) = 1
      TREL = REL(1)
C      WRITE(*,28)TREL
28    FORMAT(I4)
30    FORMAT(///,T6,'Do you understand that the system assumes the soil
     1is homogeneous',/,' in the area of contamination when it estimates
     1 the size of the plume?',/,
     1' Answer (W)hy, (Y)es or (N)o.')
      READ(*,40) CHARAC
40    FORMAT(A1)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') WRITE(*,45)
45    FORMAT(//,T6,'Your answer does not correspond to one of the choice
     1s. Hit ENTER when',/,
     1' you are ready to give a response corresponding to one of the cho
     1ices.')
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') GOTO 25
      IF(CHARAC.EQ.'Y') GOTO 60
      WRITE(*,50)
50    FORMAT(//,T6,'This assumption is important in maintaining a unifor
     1m pollutant velocity.',/,
     1' If nonhomogenity exists, the pollutant will travel at varying ve
     1locities',/,
     1' depending upon where within the aquifer the pollutant is. This s
     1ituation',/,
     1' would make it impossible to predict plume movement.')
      GOTO 25
C
C  ASKING QUESTION ABOUT AMOUNT OF ROCK IN SOIL  REL(2)=1 LWCF=2
C
60    WRITE(*,70)
70    FORMAT(//,T6,'Do you have an estimate of rock in the soil? Answer
```

```
     1(W)hy, (U)nknown',/,' or (Y)es.',//,
     1T10,'1. None (0-10% by volume)',/,
     1T10,'2. Small (11-30%)',/,
     1T10,'3. Large (> 30%)',//,T10)
      READ(*,80)CHARAC
80    FORMAT(A1)
      IF(CHARAC.EQ.'Y')WRITE(*,75)
75    FORMAT(/,T6,' Give your answer (Type 1,2, or 3) and your confidenc
     1e (in per cent) in',/,' the answer. Separate each response with a
     1space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)ROCK,CONF(1)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE 'W') GOTO 60
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,85)ROCK,CONF(1)
85    FORMAT(//,T6,'You have input no. ',I2,' as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 60
      IF(CHARAC.EQ.'W') WRITE(*,90)
90    FORMAT(//,T6,'If there is a large volume of rock in the soil, shee
     1t piling is not',/,' a viable solution. Therefore, it would not be
     1 considered in the',/,' strategy economic comparison.')
      IF(CHARAC.EQ.'U') REL(2) =  1
      TREL = REL(2) + TREL
      LWCF = 2
      IF(CHARAC.EQ.'U') CONF(1) = 100
      IF(CHARAC.EQ.'U') WRITE(*,100)
100   FORMAT(//,T6,'Since unknown was given as the answer the model will
     1 assume sheet',/,' piling is a viable solution.')
      IF(CHARAC.EQ.'W')GOTO 60
C      WRITE(*,81)CHARAC,ROCK,CONF(1),REL(2),TREL,LWCF
81    FORMAT(T10,A1,I2,4I4)
C
C  ASKING QUESTION ABOUT STRATIFICATION AT SOIL-BEDROCK INTERFACE
C        REL(3)=1 LWCF=3
C
105   WRITE(*,110)
110   FORMAT(//,T6,'Do you know the condition of the boundary between th
     1e aquifer material',/,
     1' and the bedrock? Answer (W)hy, (U)nknown or (Y)es.'
     1,//,T10,'1. Very irregular (large irregular change in depth to bed
     1rock or',/,
     1T13,'bedrock is highly fractured)',/,
     1T10,'2. Slightly irregular (small regular change in depth to bedro
     1ck or',/,
     1T13,'bedrock has small fractures)',/,
     1T10,'3. Regular (little change in depth to bedrock or bedrock has
```

```
     1no',/,
     1T13,'fractures',//,T10)
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y')WRITE(*,115)
115   FORMAT(/,T6,' Give your answer (Type 1,2, or 3) and your confidenc
     1e (in per cent) in',/,' the answer. Separate each response with a
     1space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)STRAT,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE 'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'U'.AND.CHARAC.NE.'W') GOTO 105
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,125)STRAT,CONF(2)
125   FORMAT(//,T6,'You have input no. ',I2,' as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 105
      IF(CHARAC.EQ.'W')WRITE(*,130)
130   FORMAT(//,T6,'If the boundary between the aquifer material and the
     1 bedrock is very',/,
     1' irregular in shape or the bedrock has fractures in it there is a
     1 good',/,
     1' chance of groundwater leakage and it would be necessary to key t
     1he',/,
     1' slurry wall into the bedrock to provide an impermeable barrier.
     1This',/,
     1' causes the slurry wall to be cost prohibitive and not a viable s
     1olution.')
      IF(CHARAC.EQ.'U') REL(3) =  1
      TREL = TREL + REL(3)
      IF(CHARAC.EQ.'U') CONF(2) = 100
      IF(CHARAC.EQ.'U') WRITE(*,140)
140   FORMAT(//,T6,'Since unknown was given as the answer the model will
     1 assume a slurry ',/,' trench is a viable solution.')
      IF(CHARAC.EQ.'W')GOTO 105
      IF(CONF(2).LT.CONF(1)) LWCF = 3
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,141)CHARAC,ROCK,CONF(1),REL(3),TREL,LWCF
141   FORMAT(T10,A1,I2,4I4)
C
C  ASKING QUESTION ABOUT SOIL TYPE TO DETERMINE PRIOR MEAN AND SD FOR
BAYSIAN
C    ANALYSIS (USE LOG-NORMAL FOR TRANS & NORMAL FOR EFF. PORO.) LWCF=4
C
145   WRITE(*,150)
150   FORMAT(///,T6,'Do you know what soil type best describes the aquif
     1er material? An',/,
     1' answer must be given (U is unacceptable). Answer (W)hy or (Y)es.
     1',//,T15,'SOIL TYPE',T30,'% CLAY',T40,'% SAND',T50,'%SILT',/,
```

```
     1T10,'-----------------------------------------------',//,
     1T10,'1. Sand',T31,'<10%',T41,'>90%',T51,'>90%',/,
     1T10,'2. Sandy-loam',T31,'<20%',T41,'>85%',T50,'50-70%',/,
     1T10,'3. Sandy-clay',T30,'35-55%',T40,'60-85%',T50,'50-65%',/,
     1T10,'4. Silty-clay',T30,'40-60%',T40,'20-40%',T50,'40-60%',/,
     1T10,'5. Clay',T31,'>40%',T40,'30-75%',T51,'<60%',/,
     1T10,'6. Loam',T31,'5-25%',T40,'40-60%',T50,'75-95%',//,T10)
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y')WRITE(*,155)
155   FORMAT(/,T6,' Give your answer (Type 1-6) and your confidence (in
     1per cent) in',/,' the answer. Separate each responsc with a space.
     1',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)SOIL,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 145
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,165)SOIL,CONF(2)
165   FORMAT(//,T6,'You have input no. ',I2,' as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 145
      IF(CHARAC.EQ.'W')WRITE(*,170)
170   FORMAT(//,T6,'Characterizing the soil type allows the determinatio
     1n of a mean and',/,' standard deviation for hydraulic conductivity
     1 and effective porosity based on',/,
     1' past field data. This "prior" knowledge was obtained from severa
     1l sources',/,
     1' and will be used as the mean and standard deviation for these pa
     1rameters',/,' if no field data is available.')
      IF(CHARAC.EQ.'W')GOTO 145
      IF(CONF(2).LT.CONF(1)) LWCF = 4
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,171)CHARAC,SOIL,CONF(1),TREL,LWCF
171   FORMAT(T10,A1,I2,3I4)
C
C  CALCULATION OF THE PRIOR MEAN (XoT) AND STANDARD DEVIATION (VoT)
C    FOR HYDRAULIC CONDUCTIVITY
C
C      WRITE(*,172)SLFCT(SOIL,1),SLFCT(SOIL,2)
      Y1 = ALOG(SLFCT(SOIL,1))
      Y2 = ALOG(SLFCT(SOIL,2))
C      WRITE(*,172)SLFCT(SOIL,1),Y1,SLFCT(SOIL,2),Y2
172   FORMAT(4E15.2)
      XoT = (Y1 + Y2)/2.
      VoT = ABS(Y2 - XoT)/3.
C
C  CALCULATION OF THE PRIOR MEAN (XoEP) AND STANDARD DEVIATION (VoEP)
C    FOR EFFECTIVE POROSITY
```

```
C
      XoEP = (SLFCT(SOIL,3)+SLFCT(SOIL,4))/2.
      VoEP = (SLFCT(SOIL,4)-XoEP)/3.
C      WRITE(*,173)XoT,VoT,XoEP,VoEP
173   FORMAT(4E15.2)
C
C  IF CHNGT IS 1 IT MEANS THE SOIL TYPE ONLY WAS CHANGED AND THEIR
C     IS NO REASON TO GET FIELD DATA AGAIN
C
      IF(CHNGT.EQ.1) GOTO 240
C
C  ASKING FOR ANY FIELD DATA FOR HYDRAULIC CONDUCTIVITY REL(4)=3
C
175   WRITE(*,180)
180   FORMAT(///,T6,'Do you have any field data of hydraulic conductivit
     1y (ft/d)?',/,' Answer with (W)hy, (N)o or (Y)es.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y')WRITE(*,181)
181   FORMAT(/,T6,' How many field values do you have for hydraulic cond
     1uctivity.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*) NUMT
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') GOTO 175
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,185)NUMT
185   FORMAT(//,T6,'You have declared that you have ',I3,' hydraulic con
     1ductivity values.',/,
     1' Do you wish to change this? Only (Y)es will allow you to change
     1this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 175
      IF(CHARAC.EQ.'W')WRITE(*,190)
190   FORMAT(//,T6,' Field data is the most reliable information to use
     1to determine aquifer',/,
     1' parameters. If you have 4 or more values the "soil type" data is
     1 ignored and',/,
     1' the mean and standard error are calculated using only field data
     1. If there',/,
     1' are 1 - 3 values for a particular parameter the past data and pr
     1esent data are',/,
     1' combined using Bayesian theory to obtain a mean and standard err
     1or reflecting',/,' knowledge of both sets of data.')
      IF(CHARAC.EQ.'W')GOTO 175
      IF(CHARAC.EQ.'N') REL(4) = 3
      TREL = TREL + REL(4)
C
C  IF THERE IS NO FIELD DATA THE "SOIL TYPE" VALUES ARE USED
C
      IF(CHARAC.EQ.'N') THEN
         EET = XoT
```

```
          ESDT = VoT
C  CALCULATING THE ESTIMATED MEAN AND SD FOR THE ACTUAL VALUES OF H.C.
C    FROM THE MEAN AND SD FOR THE LN VALUES FOR H.C.(REF. JOHNSON &
KOTZ)
           ET = EXP(EET + (ESDT**2)/2)
           SDT = SQRT(EXP(ESDT**2 + 2*EET)*(EXP(ESDT**2)-1.))
          GOTO 1755
        ENDIF
C
C  READING THE FIELD VALUES
C
195     WRITE(*,200)
200     FORMAT(///,T6,'Enter all hydraulic conductivity values (ft/d). Typ
       1e all values on',/,' one line with a space between each value and
       1then press ENTER. Decimals',/,
       1' are accepted but not required.',//,T5)
        READ(*,*)(TRANS(I),I=1,NUMT)
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
        WRITE(*,205)(TRANS(I),I=1,NUMT)
205     FORMAT(//,T6,'You have input these hydraulic conductivity values:'
       1,//,2X,6E10.3)
        WRITE(*,206)
206     FORMAT(/,T6,'Do you wish to change any of these values? Only (Y)es
       1 will allow',/,' you to change this input.')
        READ(*,80)CHARAC2
        IF(CHARAC2.EQ.'Y') GOTO 195
C
C  CALCULATING THE POSTERIOR MEAN (ET) AND STANDARD DEVIATION (SDT)
C    FOR HYDRAULIC CONDUCTIVITY. IF THERE IS ONLY 1 FIELD VALUE THE
'LIKELIHOOD'
C    MEAN (XT) IS THE ONE VALUE AND THE 'LIKELIHOOD' STD ERROR (VT) IS
EQUAL
C    TO THE PRIOR VoT. IF THERE ARE >3 VALUES THE MEAN AND STD ERROR ARE
C    FOUND STRICTLY FROM THE FIELD DATA. IF 2 OR 3 VALUES BAYSIAN EQS.1
AND
C    2 ARE USED.
C
        IF(NUMT.EQ.1) THEN
           VT = VoT
           XT = ALOG(TRANS(1))
C           WRITE(*,221)VoT,VT,TRANS(1),XT,XoT
221        FORMAT(5F10.5)
C           PAUSE
           A = VoT
           B = A**(-2)
           C = XoT
           D = XT
           E = VT
           F = E**(-2)
C           WRITE(*,222)A,B,C
C           WRITE(*,222)D,E,F
222        FORMAT(3E15.4)
C           PAUSE
```

```
          EET = (1/(B + F)) * (B*C + F*D)
C          WRITE(*,222)ET
C          PAUSE
          ESDT = SQRT((B + F)**(-1))
C  CALCULATING THE ESTIMATED MEAN AND SD FOR THE ACTUAL VALUES OF H.C.
C    FROM THE MEAN AND SD FOR THE LN VALUES FOR H.C.(REF. JOHNSON &
KOTZ)
          ET = EXP(EET + (ESDT**2)/2)
          SDT = SQRT(EXP(ESDT**2 + 2*EET)*(EXP(ESDT**2)-1.))
C          WRITE(*,222)SDT
C          PAUSE
      ENDIF
      IF(NUMT.EQ.1) GOTO 240
C
C  DO LOOP TO GET THE SUM OF T (SUMT) AND THE SUM OF T**2 (SUMTSQ) TO
USE
C   IN THE STANDARD MEAN AND STAND ERROR FORMULAS
C
      SUMT = 0.
      SUMTSQ = 0.
      DO 230 I = 1,NUMT
         SUMT = SUMT + ALOG(TRANS(I))
         SUMTSQ = SUMTSQ + ALOG(TRANS(I))**2
230   CONTINUE
        XT = SUMT/NUMT
        VT = SQRT((ABS(NUMT*SUMTSQ-SUMT**2))/(NUMT*(NUMT-1)))
      IF(NUMT.GT.3) THEN
C  CALCULATING THE ESTIMATED MEAN AND SD FOR THE ACTUAL VALUES OF H.C.
C    FROM THE MEAN AND SD FOR THE LN VALUES FOR H.C.(REF. JOHNSON &
KOTZ)
          ET = EXP(XT + (VT**2)/2)
          SDT = SQRT(EXP(VT**2 + 2*XT)*(EXP(VT**2)-1.))
        GOTO 240
      ENDIF
          A = VoT
          B = A**(-2)
          C = XoT
          D = XT
      IF(VT.EQ.0.) THEN
         E = 0.
         F = 1.
         GOTO 235
      ENDIF
          E = VT
          F = E**(-2)
C235       WRITE(*,222)A,B,C
C          WRITE(*,222)D,E,F
C          PAUSE
235       EET = (1/(B + F)) * (B*C + F*D)
C          WRITE(*,222)ET
C          PAUSE
          ESDT = SQRT((B + F)**(-1))
C  CALCULATING THE ESTIMATED MEAN AND SD FOR THE ACTUAL VALUES OF H.C.
```

```
C    FROM THE MEAN AND SD FOR THE LN VALUES FOR H.C.(REF. JOHNSON &
KOTZ)
          ET = EXP(EET + (ESDT**2)/2)
          SDT = SQRT(EXP(ESDT**2 + 2*EET)*(EXP(ESDT**2)-1.))
C          WRITE(*,222)SDT
C          PAUSE
C
C  IF THE MEAN OF FIELD DATA HC IS FARTHER THAN 3 STANDARD DEVIATIONS
FROM
C   THE MEAN HC OF SOIL TYPE USER IS GIVEN CHANCE TO CHANGE SOIL TYPE OR
C   FIELD DATA
C
240    THRESDT = 3.* VoT
       DIFT = ABS(XT - XoT)
       IF(DIFT.LE.THRESDT) CHNGT = 0
       IF(DIFT.GT.THRESDT) WRITE(*,242)
242    FORMAT(///,T6,'The mean hydraulic conductivity for your field data
     1 is over 3 standard',/,' deviations from the mean of the soil type
     1 you have chosen. This is',/,' contradictory information. Would yo
     1u like to change your input of:',/,T10,'1. Soil type',/,T10,
     1'2. Field data',/,T10,'3. None',//,T6,'Answer 1, 2 or 3',2X)
       IF(DIFT.GT.THRESDT) READ(*,*)CHNGT
       IF(CHNGT.EQ.1) GOTO 145
       IF(CHNGT.EQ.2) GOTO 175
C      IF(CHNGEP.EQ.1) GOTO 270
C
C  ASKING FOR ANY FIELD DATA FOR EFFECTIVE POROSITY REL(5)=3
C
1755   WRITE(*,1805)
1805   FORMAT(///,T6,'Do you have any field data of effective porosity fo
     1r this aquifer?',/,' Answer with (W)hy, (N)o or (Y)es.')
       READ(*,80)CHARAC
       IF(CHARAC.EQ.'Y')WRITE(*,1815)
1815   FORMAT(/,T6,' How many field values do you have for effective poro
     1sity.',//,T10)
       IF(CHARAC.EQ.'Y') READ(*,*) NUMEP
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
       IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W')
     1WRITE(*,45)
       IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') PAUSE
       IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') GOTO 1755
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
       IF(CHARAC.EQ.'Y') WRITE(*,1855) NUMEP
1855   FORMAT(//,T6,'You have declared that you have ',I3,' effective por
     1osity values. Do',/,
     1' you wish to change this? Only (Y)es will allow you to change thi
     1s input.')
       IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
       IF(CHARAC2.EQ.'Y') GOTO 1755
       IF(CHARAC.EQ.'W')WRITE(*,190)
       IF(CHARAC.EQ.'W')GOTO 1755
       IF(CHARAC.EQ.'N') REL(5) = 3
```

```
      TREL = TREL + REL(5)
C
C  IF THERE IS NO FIELD DATA THE SOIL TYPE VALUES ARE USED
C
      IF(CHARAC.EQ.'N') THEN
          EEP = XoEP
          SDEP = VoEP
          GOTO 275
      ENDIF
C
C  READING THE FIELD VALUES FOR EFFECTIVE POROSITY
C
2085  WRITE(*,2105)
2105  FORMAT(///,T6,'Enter all effective porosity values (in decimal). T
     1type all values on',/,
     1' one line with a space between each value and then press ENTER.'
     1,//,T5)
      READ(*,*)(EP(I),I=1,NUMEP)
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      WRITE(*,2155)(EP(I),I=1,NUMEP)
2155  FORMAT(//,T6,'You have input these effective porosity values:',//
     1,2X,6E10.3)
      WRITE(*,2175)
2175  FORMAT(/,T6,'Do you wish to change any of these values? Only (Y)es
     1 will allow',/,' you to change this input.')
      READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 2085
C
C  CALCULATING THE POSTERIOR MEAN (EEP) AND STANDARD DEVIATION (SDEP)
FOR
C    EFFECTIVE POROSITY. IF THERE IS ONLY 1 FIELD VALUE THE 'LIKELIHOOD'
C    MEAN (XEP) IS THE ONE VALUE AND THE 'LIKELIHOOD' STD ERROR (VEP) IS
EQUAL
C    TO THE PRIOR VoEP. IF THERE ARE >3 VALUES THE MEAN AND STD ERROR
ARE
C    FOUND STRICTLY FROM THE FIELD DATA. IF 2 OR 3 VALUES BAYSIAN EQS.1
AND
C    2 ARE USED.
C
      IF(NUMEP.EQ.1) THEN
          VEP = VoEP
          XEP = EP(1)
C          WRITE(*,251)VoEP,VEP,EP(1),XEP,XoEP
251       FORMAT(5F10.5)
C          PAUSE
          A = VoEP
          B = A**(-2)
          C = XoEP
          D = XEP
          E = VEP
          F = E**(-2)
C          WRITE(*,252)A,B,C
C          WRITE(*,252)D,E,F
```

```
252         FORMAT(3E15.4)
C            PAUSE
            EEP = (1/(B + F)) * (B*C + F*D)
C            WRITE(*,252)EEP
C            PAUSE
            SDEP = SQRT((B + F)**(-1.))
C            WRITE(*,252)SDEP
C            PAUSE
      ENDIF
      IF(NUMEP.EQ.1) GOTO 275
C
C  DO LOOP TO GET THE SUM OF EP (SUMEP) AND THE SUM OF EP**2 (SUMEPSQ)
TO USE
C   IN THE STANDARD MEAN AND STAND ERROR FORMULAS
C
      SUMEP = 0.
      SUMEPSQ = 0.
      DO 260 I = 1,NUMEP
         SUMEP = SUMEP + EP(I)
         SUMEPSQ = SUMEPSQ + EP(I)**2.
260   CONTINUE
        XEP = SUMEP/NUMEP
        VEP = SQRT((ABS(NUMEP*SUMEPSQ-SUMEP**2))/(NUMEP*(NUMEP-1)))
      IF(NUMEP.GT.3) THEN
        EEP = XEP
        SDEP = VEP
        GOTO 275
      ENDIF
          A = VoEP
          B = A**(-2)
          C = XoEP
          D = XEP
          E = VEP
          F = E**(-2)
C          WRITE(*,222)A,B,C
C          WRITE(*,222)D,E,F
C          PAUSE
          EEP = (1/(B + F)) * (B*C + F*D)
C          WRITE(*,222)EEP
C          PAUSE
          SDEP = SQRT((B + F)**(-1.))
C          WRITE(*,222)SDEP
C          PAUSE
C
C  IF THE MEAN OF FIELD DATA EF PORO IS FARTHER THAN 3 STANDARD
DEVIATIONS FROM
C   THE MEAN EF PORO OF SOIL TYPE USER IS GIVEN CHANCE TO CHANGE SOIL
TYPE OR
C   FIELD DATA
C
C270   THRESDEP = 3.*VoEP
C      DIFEP = ABS(XEP-XoEP)
C      IF(DIFEP.LE.THRESDEP) CHNGEP = 0
```

```
C       IF(DIFEP.GT.THRESDEP) WRITE(*,272)
C272    FORMAT(///,T6,'The mean effective porosity for your field data is
C      lover 3 standard',/,' deviations from the mean of the soil type
you
C      1 have chosen. This is',/,' contradictory information. Would you
li
C      lke to change your input of:',/,T10,'1. Soil type',/,T10,
C      1'2. Field data',/,T10,'3. None',//,T6,'Answer 1, 2 or 3',2X)
C       IF(DIFEP.GT.THRESDEP) READ(*,*)CHNGEP
C       IF(CHNGEP.EQ.1) GOTO 145
C       IF(CHNGEP.EQ.2) GOTO 1755
C
C  STATING THE AQUIFER PARAMETER VALUES TO THE USER
C
275    WRITE(*,276)ET,SDT,EEP,SDEP
276    FORMAT(///,' Based on soil type, field or lab data or a combinatio
      1n of both:',//,
      1T3,'the mean hydraulic conductivity is ',F9.4,' ft/d',/,
      1T3,'with a standard error of ',F9.4,/,
      1T3,'the mean effective porosity is ',F7.2,/,
      1T3,'with a standard error of ',F7.2,//)
C
C  IF THE SOIL IS ROCKY, INTERFACE IS IRREGULAR, AND H.C.<.002
C  NONE OF THE STRATEGIES CAN BE USED
C
       IF(STRAT.EQ.1.AND.ROCK.EQ.3.AND.ET.LT.0.002) THEN
         PAUSE' According to your input none of the strategies can be
      1used. Hit ENTER to receive an explanation.'
         WRITE(*,277)
277      FORMAT(///////)
         WRITE(*,278)
278    FORMAT(///,T6,'According to your input none of the strategies are
      1viable solutions',/,' because:',//,
      1' 1. For slurry wall',/,
      1'       the aquifer-bedrock interface was very irregular',/,
      1' 2. For sheet piling',/,
      1'       the soil is too rocky and',/,
      1' 3. For pumping',/,
      1'       the mean hydraulic conductivity is below .002 ft/d.')
         GOTO 1280
       ENDIF
C       WRITE(*,277)XoT,XoEP,VoT,VoEP
C       WRITE(*,277)XT,XEP,VT,VEP,REL(4),TREL
C77    FORMAT(4F15.5,2I5)
       WRITE(*,280)
280    FORMAT(///,T25,'** Soil Characterization Complete **',///)
C
C  QUESTIONS TO CHARACTERIZE THE SITE ENVIROMENT
C
       WRITE(*,290)
290    FORMAT(T6,'The following questions are intended to characterize th
      1e site enviroment.',/,' All questions require an answer. (U)nknown
      1 is unacceptable. If you are ready',/,' to continue type CLS and h
```

```
     lit ENTER to clear the screen and hit ENTER again to',/,' begin the
     1 questions.',/)
      PAUSE
C
C  ASKING QUESTION ABOUT CONSTANT ENVIROMENT ASSUMPTION REL(6)=1
C
      N = 0
300   WRITE(*,310)
      N = N + 1
      IF(N.EQ.3) REL(6) =  1
      TREL = TREL + REL(6)
C      WRITE(*,28)TREL
310   FORMAT(///,T6,' Do you understand the system assumption that const
     1ant enviromental',/,' conditions exist (and no other remedial acti
     1on has been attempted) throughout',/,' the containment period? Ans
     1wer (W)hy, (Y)es or (N)o.')
      READ(*,320)CHARAC
320   FORMAT(A1)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') GOTO 300
      IF(CHARAC.EQ.'Y') GOTO 340
      WRITE(*,330)
330   FORMAT(//,T6,'This assumption is important because the model assum
     1es that the initial',/,
     1' gradient is at steady-state conditions.')
      GOTO 300
C
C  ASKING QUESTION ABOUT AVERAGE PRECIPITATION  LWCF=5
C
340   WRITE(*,350)
350   FORMAT(//,T6,'The following are acceptable estimates of average mo
     1nthly precipitation',/,
     1' (in/m) at the site during the entire pumping period. Can you est
     1imate the',/,
     1' average monthly precip. at your site for the time period that in
     1cludes the',/,
     1' optimal pumping and the steady pumping periods. Answer (W)hy or
     1(Y)es.'
     1,//,T10,'1. 0 - 2',/,T10,'2. 2 - 4',/,T10,'3. > 4',//,T10)
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y') WRITE(*,355)
355   FORMAT(/,T6,' Give your answer (Type 1,2, or 3) and your confidenc
     1e (in per cent) in',/,' the answer. Separate each response with a
     1space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)PRECIP,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 340
```

```
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,365)PRECIP,CONF(2)
365   FORMAT(//,T6,'You have input no. ',I2,' as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 340
      IF(CHARAC.EQ.'W')WRITE(*,370)
370   FORMAT(//,T6,'The amount of precipitation and how well this precip
     1itation drains off',/,
     1' the site can affect the contaminant movement within the aquifer.
     1 The safety',/,' factor used to determine plume extent will be lar
     1ger with increased',/,' precipitation and poor drainage.')
      IF(CHARAC.EQ.'W')GOTO 340
      IF(CONF(2).LT.CONF(1)) LWCF = 5
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C  SFP IS THE ADDED SAFETY FACTOR TO CALCULATE PLUME MOVEMENT BASED ON
C   LARGE PRECIP
      SFP = .02*(PRECIP-1)
C      WRITE(*,371)CHARAC,PRECIP,CONF(1),TREL,LWCF,SFP
371   FORMAT(T10,A1,I2,3I4,F6.2)
C
C  ASKING QUESTION ABOUT DRAINAGE AT THE SITE WLCF=6
C
380   WRITE(*,390)
390   FORMAT(////,T6,'Below are common descriptions of drainage classes.
     1Can you describe',/,
     1' drainage at the site? Answer (W)hy or (Y)es.'
     1,//,T6,'Drainage Class',T40,'Observable action'
     1,/,' ------------------------------------------------------------
     1------',//,
     1' 1.Very poorly drained',T33,'Water remains at or on the surface'
     1,/,T33,'most of the year',/,
     1' 2.Poorly drained',T33,'Water remains at or on the surface',/,T33
     1,'some of the year',/,
     1' 3.Somewhat poorly drained',T33,'Soils are wet for significant po
     1rtions',/,T33,'of the year',/,
     1' 4.Moderately well drained',T33,'Soils are seasonably wet (high s
     1pring',/,T33,'water table)',/,
     1' 5.Well drained',T33,'Water readily removed from the soil',//,
     1' 6.Somewhat excessively',T33,'Water is rapidly removed from the s
     1oil',/,T33,'(i.e. uniform drained sands)',/,
     1' 7.Excessively drained',T33,'Very rapid removal of water, little
     1or',/,T33,'no retention',//,T10)
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y')WRITE(*,400)
400   FORMAT(/,T6,' Give your answer (Type 1-7) and your confidence (in
     1per cent) in',/,' the answer. Separate each response with a space.
     1',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)DRAIN,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
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      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 380
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,405)DRAIN,CONF(2)
405   FORMAT(//,T6,'You have input no. ',I2,' as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 380
      IF(CHARAC.EQ.'W')WRITE(*,410)
410   FORMAT(//,T6,'The amount of precipitation and how well this precip
     1itation drains off',/,
     1' the site can affect the contaminant movement within the aquifer.
     1 The safety',/,' factor used to determine plume extent will be lar
     1ger with increased',/,' precipitation and poor drainage.')
      IF(CHARAC.EQ.'W')PAUSE'Hit ENTER when you are ready to continue'
      IF(CHARAC.EQ.'W')GOTO 380
      IF(CONF(2).LT.CONF(1)) LWCF = 6
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C  SFD IS THE ADDED SAFETY FACTOR TO CALCULATE PLUME MOVEMENT BASED ON
C   POOR DRAINAGE
      SFD = .03-(DRAIN-1)*.005
C      WRITE(*,411)CHARAC,DRAIN,CONF(1),TREL,LWCF,SFD
411   FORMAT(T10,A1,I2,3I4,F6.3)
C
C  ASKING QUESTION ABOUT AVERAGE DEPTH TO BOTTOM OF AQUIFER LWCF=7
C
420   WRITE(*,430)
430   FORMAT(///,T6,'Can you estimate the average depth (ft) to the base
     1 of the aquifer?',/,
     1' Answer (W)hy or (Y)es.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y')WRITE(*,440)
440   FORMAT(/,T6,' Give your answer and your confidence (in per cent) i
     1n the answer.',/,
     1' Separate each response with a space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)WT,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 420
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,445)WT,CONF(2)
445   FORMAT(//,T6,'You have input ',F7.2,' ft as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 420
      IF(CHARAC.EQ.'W')WRITE(*,450)
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450   FORMAT(//,T6,'Depth to the bottom of the aquifer affects the econo
     1mics of all three',/,' containment methods. The cost of constructi
     1on increases as depth increases.')
      IF(CHARAC.EQ.'W')GOTO 420
      IF(CONF(2).LT.CONF(1)) LWCF = 7
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,451)CHARAC,WT,CONF(1),TREL,LWCF
451   FORMAT(T10,A1,F10.5,I4,2I3)
C
C  ASKING QUESTION ABOUT AVERAGE SATURATED THICKNESS OF AQUIFER LWCF=8
C
453   WRITE(*,455)
455   FORMAT(///,T6,'Can you estimate the average saturated thickness (f
     1t) of the',/,
     1' aquifer? Answer (W)hy or (Y)es.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y') WRITE(*,456)
456   FORMAT(/,T6,' Give your answer and your confidence (in per cent) i
     1n the answer.',/,
     1' Separate each response with a space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)SAT,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 453
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,457)SAT,CONF(2)
457   FORMAT(//,T6,'You have input ',F7.2,' ft as your answer and ',I3,'
     1% as your confidence',/,' in that answer. Do you wish to change ei
     1ther one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 453
      IF(CHARAC.EQ.'W')WRITE(*,458)
458   FORMAT(//,T6,'Saturated thickness of the aquifer is used (along wi
     1th the average',/,' hydraulic conductivity) to determine the trans
     1missivity, which is the',/,' measure of potential for fluid moveme
     1nt within the aquifer.')
      IF(CHARAC.EQ.'W')GOTO 453
      IF(CONF(2).LT.CONF(1)) LWCF = 8
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,459)CHARAC,SAT,CONF(1),TREL,LWCF
459   FORMAT(T10,A1,F10.5,I4,3I3)
C
C  CALCULATE THE MAXIMUM PUMPING LIFT BASED ON A MAXIMUM DRAWDOWN OF
C    1/2 OF THE SATURATED THICKNESS
C
      MAXLFT = WT - 0.5*SAT
C
C  ASKING QUESTION ABOUT AVERAGE HYDRAULIC GRADIENT LWCF=9
C
460   WRITE(*,470)
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```
470   FORMAT(///,T6,'Can you estimate the average hydraulic gradient (0.
     10-0.99) of the',/,
     1' potentiometric surface of the aquifer in the direction of plume
     1movement?',/,
     1' Answer (W)hy or (Y)es.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y') WRITE(*,480)
480   FORMAT(/,T6,' Give your answer and your confidence (in per cent) i
     1n the answer.',/,
     1' Separate each response with a space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)GRAD,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 460
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,485)GRAD,CONF(2)
485   FORMAT(//,T6,'You have input ',F6.3,' as your answer and ',I3,
     1' % as your confidence',/,' in that answer. Do you wish to change
     1either one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 460
      IF(CHARAC.EQ.'W')WRITE(*,490)
490   FORMAT(//,T6,'The gradient will be used to calculate the Darcy vel
     1locity. The extent',/,' of the plume at the time the containment st
     1rategy is implemented can then',/,' be estimated.')
      IF(CHARAC.EQ.'W')GOTO 460
      IF(CONF(2).LT.CONF(1)) LWCF = 9
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,451)CHARAC,GRAD,CONF(1),TREL,LWCF
      WRITE(*,500)
500   FORMAT(///,T25,'** Site Characterization Complete **',///)
C
C  QUESTIONS TO CHARACTERIZE THE CONTAMINANT
C
      WRITE(*,505)
505   FORMAT(T6,'The following questions are intended to characterize th
     1e contaminant.',/,
     1' All questions require an answer. (U)nknown is unacceptable. If y
     1ou are ready',/,
     1' to continue type CLS and hit ENTER to clear the screen and hit E
     1NTER again to',/,' begin the questions.',/)
      PAUSE
C
C  ASKING QUESTION ABOUT ADVECTION ASSUMPTION  REL(7)=1
C
      N = 0
510   WRITE(*,520)
      N = N + 1
      IF(N.EQ.3) REL(7) =  1
      TREL = TREL + REL(7)
```

```
C       WRITE(*,28)TREL
520   FORMAT(///,T6,' Do you understand the system assumption that water
     1 is the contaminant',/,' carrier and that advection is the major m
     1echanism of contaminant movement?',/,' Answer (W)hy, (Y)es or (N)o
     1.')
      READ(*,530)CHARAC
530   FORMAT(A1)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'N'.AND.CHARAC.NE.'W') GOTO 510
      IF(CHARAC.EQ.'Y') GOTO 550
      WRITE(*,540)
540   FORMAT(//,T6,'These are assumptions that greatly simplify the pred
     1liction of plume',/,' movement. A more sophisticated model is neede
     1d for mechanical dispersion',/,' or molecular diffusion are also me
     1chanisms of contaminant transport. The',/,
     1' safety factor used in the calculation of plume extent provides f
     1or enough',/,
     1' margin to account for dispersion.')
      GOTO 510
C
C  ASKING QUESTION ABOUT CERTAIN CHEMICALS IN CONTAMINANT  LWCF=10
C
550   WRITE(*,560)
560   FORMAT(///,T6,'Does the contaminant contain any of the following c
     1ompounds? Answer',/,' (W)hy, (N)o or (Y)es.'
     1,//,T5,'Alcohol',T25,'Sulfuric acid',T45,
     1'Calcium hydroxide',/,T5,'Hydrochloric acid',T25,
     1'Sodium hydroxide',T45,'Brine (sp. gravity > 1.2)',/,T5,
     1'Aldehydes',T25,'Ketones',T45,'Hydrocarbons (aliphatic and ',/,
     1T60,'aromatic)',/,T5,
     1'Heterocyclics',T25,'Organic acids',T45,'Acid chlorides',/,T5,
     1'Phenols',T25,'Glycols'
     1,//,T10)
      READ(*,80)CHEM
      IF(CHEM.EQ.'Y'.OR.CHEM.EQ.'N') WRITE(*,565)
565   FORMAT(//,T6,'What is your confidence (in per cent) in this answer
     1?',/,T10)
      IF(CHEM.EQ.'Y'.OR.CHEM.EQ.'N') READ(*,*) CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHEM.NE.'Y'.AND.CHEM.NE.'N'.AND.CHEM.NE.'W') WRITE(*,45)
      IF(CHEM.NE.'Y'.AND.CHEM.NE.'N'.AND.CHEM.NE.'W') PAUSE
      IF(CHEM.NE.'Y'.AND.CHEM.NE.'N'.AND.CHEM.NE.'W') GOTO 550
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHEM.EQ.'Y'.OR.CHEM.EQ.'N') WRITE(*,575)CHEM,CONF(2)
575   FORMAT(//,T6,'You have input ',A1,' as your answer and ',I3,
     1' % as your confidence',/,' in that answer. Do you wish to change
     1either one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHEM.EQ.'Y'.OR.CHEM.EQ.'N') READ(*,80)CHARAC2
```

```
      IF(CHARAC2.EQ.'Y') GOTO 550
      IF(CHEM.EQ.'W') WRITE(*,580)
580   FORMAT(//,T6,'These compounds could increase the permeability of a
     1 bentonite slurry',/,
     1' wall by as much as 10 times. Therefore, a slurry wall is not a v
     1iable solution',/,
     1' if any of these compounds are present in the contaminant.')
      IF(CHEM.EQ.'W') GOTO 550
      IF(CONF(2).LT.CONF(1)) LWCF = 10
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,581)CHEM,CONF(1),TREL,LWCF
581   FORMAT(T10,A1,I4,2I4)
C
C  IF THE SOIL IS ROCKY, CERTAIN CHEMICALS ARE PRESENT, AND H.C.<.002
C  NONE OF THE STRATEGIES CAN BE USED
C
      IF(CHEM.EQ.'Y'.AND.ROCK.EQ.3.AND.ET.LT.0.002) THEN
        PAUSE' According to your input none of the strategies can be
     1used. Hit ENTER to receive an explanation.'
        WRITE(*,582)
582     FORMAT(////////)
        WRITE(*,585)
585   FORMAT(///,T6,'According to your input none of the strategies are
     1viable solutions',/,' because:',//,
     1' 1. For slurry wall',/,
     1'       there were chemicals in the contaminant that would',/,
     1'            increase the permeability of the wall',/,
     1' 2. For sheet piling',/,
     1'       the soil is too rocky and',/,
     1' 3. For pumping',/,
     1'       the mean hydraulic conductivity is below .002 ft/d.')
        GOTO 1280
      ENDIF
C
C  ASKING FOR AN ESTIMATE OF THE TIME UNTIL CONTAINMENT STRATEGY IS
IMPLEMENTED
C      LWCF=11
C
590   WRITE(*,600)
600   FORMAT(///,T6,'Can you estimate the length of time (days) from the
     1 present until',/,
     1' a containment strategy can be implemented? Answer (W)hy or (Y)es
     1.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y') WRITE(*,610)
610   FORMAT(/,T6,' Give your answer and your confidence (in per cent) i
     1n the answer.',/,
     1' Separate each response with a space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)TIME,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
```

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      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 590
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,615)TIME,CONF(2)
615   FORMAT(//,T6,'You have input ',I3,' days as your answer and ',I3,
     1' % as your confidence',/,' in that answer. Do you wish to change
     1either one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 590
      IF(CHARAC.EQ.'W')WRITE(*,620)
620   FORMAT(//,T6,'The size of the octagonal configuration, which is us
     1ed by all 3',/,
     1' possible techniques is sized based on the estimated extent of th
     1e contaminant',/,
     1' plume at the time of containment strategy implementation. This e
     1stimate',/,
     1' is based on:',/,T6,
     1'1. the present extent of the plume and',/,T6,
     1'2. the estimated distance the plume will move from the present ti
     1me',/,T9,
     1'until the strategy is implemented.This estimated plume movement',
     1/,T9,
     1'is based on Darcy velocity and estimated time until containment',
     1/,T9,
     1'strategy is implemented.')
      IF(CHARAC.EQ.'W')GOTO 590
      IF(CONF(2).LT.CONF(1)) LWCF = 11
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,621)CHARAC,TIME,CONF(1),TREL,LWCF
621   FORMAT(T10,A1,4I4)
C
C  ASKING FOR THE PRESENT FURTHEST EXTENT OF THE CONTAMINANT PLUME
C        LWCF=12
C
630   WRITE(*,640)
640   FORMAT(///,T6,'Can you estimate the present detectable maximum ext
     1ent of the',/,
     1' contaminant plume (ft) from its source point? Answer with (W)hy
     1or (Y)es.')
      READ(*,80)CHARAC
      IF(CHARAC.EQ.'Y') WRITE(*,650)
650   FORMAT(/,T6,' Give your answer and your confidence (in per cent) i
     1n the answer.',/,
     1' Separate each response with a space.',//,T10)
      IF(CHARAC.EQ.'Y') READ(*,*)EXTENT,CONF(2)
C  IF THE USER DOES NOT GIVE A CORRECT ANS. HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') WRITE(*,45)
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') PAUSE
      IF(CHARAC.NE.'Y'.AND.CHARAC.NE.'W') GOTO 630
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      IF(CHARAC.EQ.'Y') WRITE(*,655)EXTENT,CONF(2)
655   FORMAT(//,T6,'You have input ',F5.1,' ft. as your answer and ',I3,
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     1' % as your confidence',/,' in that answer. Do you wish to change
     1either one of these values?',/,
     1' Only (Y)es will allow you to change this input.')
      IF(CHARAC.EQ.'Y') READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 630
      IF(CHARAC.EQ.'W')WRITE(*,620)
      IF(CHARAC.EQ.'W')GOTO 630
      IF(CONF(2).LT.CONF(1)) LWCF = 12
      IF(CONF(2).LT.CONF(1)) CONF(1) = CONF(2)
C      WRITE(*,661)CHARAC,EXTENT,CONF(1),TREL,LWCF
661   FORMAT(T10,A1,F5.1,3I4)
      WRITE(*,670)
670   FORMAT(///,T21,'** Contaminant Characterization Complete **',///)
      PAUSE' When you are ready to continue hit ENTER'
C
C  ASKING FOR THE MAXIMUM PUMPING FOR EACH PUMP BASED ON A MAXIMUM LIFT
C   OF 1/2 OF THE SATURATED THICKNESS
C
671   WRITE(*,672) MAXLFT
672   FORMAT(///,T6,'What is the upper limit on pumping (cu.ft./pump/day
     1) for a maximum lift',/,
     1' of',F10.2,' ft. based on available equipment? This will be used
     1to estimate',/,
     1' the operating costs of the pumps for preliminary economic compar
     1ison between',/,
     1' the systems under consideration.',//,
     1' W or Y are not necessary. Just input an answer.',//,T10)
      READ(*,*) QX
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      WRITE(*,673) QX
673   FORMAT(//,T6,'You have input ',F10.2,' cu.ft./pump/day as your ans
     1wer. Do you wish to',/,
     1' change this input? Only (Y)es will allow you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 671
C
C  CALCULATING THE MEAN TRANSMISSIVITY (TR); EQUALS MEAN HYDR.COND.
TIMES
C    SATURATED THICKNESS
C  STANDARD ERROR OF TRANSMISSIVITY = STANDARD ERROR OF HYDR. COND. (IF
SAT.
C    THICKNESS IS CONSTANT)
C
       TR = ET*SAT
       SDTR = SDT
C
C  DETERMINATION OF SAFETY FACTOR FOR USE IN PLUME MOVEMENT CALCULATION
C   IT IS NEVER GREATER THAN 2.
C
C  COVT IS THE COEFFICIENT OF VARIATION FOR TRANSMISSIVITY
C      COVT = SDT/LN(MEAN)
C
      IF(SDT.EQ.0.) THEN
```

```
        COVT = 0.
        GOTO 675
      ENDIF
      COVT = ABS(SDTR/ALOG(TR))
675   SFPD = SFP + SFD
      SF = 1. + COVT +SFPD
      IF(SF.GT.2.) SF = 2.
C
C  COVEP IS THE COEFFICIENT OF VARIATION FOR EFFECTIVE POROSITY
C       COVEP = SDEP/MEANEP
C
      IF(SDEP.EQ.0.) THEN
        COVEP = 0.
        GOTO 678
      ENDIF
      COVEP = ABS(SDEP/EEP)
C
C  EXPLAINING TO THE USER HOW THE S.F. FOR PLUME MOVEMENT WAS DETERMINED
C
678   WRITE(*,680)
680   FORMAT(////,T6,'A safety factor is used to estimate the future ext
     1ent of the contaminant',/,
     1' plume to ensure that the proposed octagonal containment strategy
     1 is outside the',/,
     1' limits of the plume at the time of strategy implementation. Afte
     1r many',/,
     1' simulation runs of hypothetical contamination problems it has be
     1en determined',/,
     1' that this safety factor is most influenced by:',//,T6,
     1'1. The uncertainty of the transmissivity value used;',/,
     1T9,'this uncertainty is measured by the coefficient of variation w
     1hich is',/,T9,
     1'the standard error divided by the mean. This is based on a log-no
     1rmal',/,T9,
     1'distribution for hydraulic conductivity.',/,T6,
     1'2. The amount of precipitation in the contaminated area.',/,T6,
     1'3. The drainage in the contaminated area.',/,T6,
     1'4. A dispersivity value of 100 ft.',//,T6,
     1'However, it was determined that any safety factor greater than 2
     1serves',/,
     1' no purpose. Therefore, the largest safety factor used is 2.',//
     1//)
      PAUSE' When you are ready to continue hit ENTER'
      WRITE(*,685)SF,COVT,SFPD
685   FORMAT(///////,T6,'The safety factor that will be used to estimate
     1 future plume extent is ',/,1X,F5.3,
     1'. This safety factor is based on an increase of ',F6.3, ' due to
     1the',/,
     1' transmissivity coefficient of variation and an additional increa
     1se of ',F4.3,/,' due to your input of:')
      IF(PRECIP.EQ.1)WRITE(*,690)
690   FORMAT(/,T6,'1. 0-2 in/month of precipitation and')
      IF(PRECIP.EQ.2)WRITE(*,700)
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700   FORMAT(/,T6,'1. 2-4 in/month of precipitation and')
      IF(PRECIP.EQ.3)WRITE(*,710)
710   FORMAT(/,T6,'1. > 4 in/month of precipitation and')
      IF(DRAIN.EQ.1)WRITE(*,720)
720   FORMAT(/,T6,'2. Area is very poorly drained.',/)
      IF(DRAIN.EQ.2)WRITE(*,730)
730   FORMAT(/,T6,'2. Area is poorly drained.',/)
      IF(DRAIN.EQ.3)WRITE(*,740)
740   FORMAT(/,T6,'2. Area is somewhat poorly drained.',/)
      IF(DRAIN.EQ.4)WRITE(*,750)
750   FORMAT(/,T6,'2. Area is moderately well drained.',/)
      IF(DRAIN.EQ.5)WRITE(*,760)
760   FORMAT(/,T6,'2. Area well drained.',/)
      IF(DRAIN.EQ.6)WRITE(*,770)
770   FORMAT(/,T6,'2. Area is somewhat excessively drained.',/)
      IF(DRAIN.EQ.7)WRITE(*,780)
780   FORMAT(/,T6,'2. Area is excessively drained.',/)
      WRITE(*,785)
785   FORMAT(////)
      PAUSE' When you are ready to continue press ENTER'
C
C  ADJUSTING THE PRESENT EXTENT OF PLUME INPUT BY USER (EXTENT) BY THE
C   SAFETY FACTOR
C
      PEXTENT = EXTENT*SF
C
C  CALCULATING THE ESTIMATED FUTURE EXTENT OF THE PLUME AT TIME OF
CONTAINMENT
C   STRATEGY IMPLEMENTATION
C
      FEXTENT = PEXTENT + ((ET*GRAD*TIME)/EEP)*SF
      WRITE(*,790)PEXTENT,FEXTENT,ET,GRAD,EEP,TIME,SF
790   FORMAT(/////,T6,
     1             'The system estimate for the present extent of the co
     1ntaminant',/,' plume is ',F8.3,' ft.',/,T6,'The system estimate fo
     1r the future extent of the plume at containment',/,' implementatio
     1n is ',F8.3,' ft. This is based on:',//,
     1T6,'1. A hydraulic conductivity of ',F8.3,' ft/d',/,
     1T6,'2. A hydraulic gradient of ',F6.4,/,
     1T6,'3. An effective porosity of ',F5.3,/,
     1T6,'4. Time to containment implementation of ',I3,' days and',/,
     1T6,'5. A safety factor of ',F5.3,//////)
      PAUSE' When you are ready to continue press ENTER'
C
C  SIZING A REGULAR OCTAGON BASED ON THE FUTURE EXTENT OF THE PLUME
C   SL IS THE LENGTH OF EACH SIDE OF THE OCTAGON
C
      SL = FEXTENT/(0.5 + COS(2*PI/8.))
      WRITE(*,800)FEXTENT,SL
800   FORMAT(//////,T6,
     1             'Based on a predicted plume extent of ',F8.2,' ft. e
     1ach side of the',/,' regular octagon will be ',F8.2,' ft. long. Th
     1e capital cost estimate',/,' for the pumping scheme will be based
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     1on a well spacing of one-fourth of the',/,' side length (2 pump we
     1lls per side located at the 1/4 and 3/4 points), well ',/,
     1' holes drilled are 24" in diameter and fully penetrate the aquife
     1r and a',/,
     1' 1986 purchase price of $1500/pump.',//,T3,
     1'The program will now calculate the capital costs for the three co
     1ntainment',/,T6,' schemes.',/////////)
      PAUSE ' When you are ready to continue hit ENTER'
C
C   ECONOMIC ANALYSIS OF THE THREE CONTAINMENT STRATEGIES
C
C   INPUT OF COEFFICIENT TO UPDATE 1986 CAPITAL COSTS TO PRESENT
C
805   WRITE(*,810)
810   FORMAT(//,T6,'The unit costs for the economic comparison are based
     1 on 1986 prices.',/,' Enter a coefficient to update these costs (E
     1nter 1.00 if 1986 costs are',/,' acceptable).',//,T10)
      READ(*,*)COEF
      WRITE(*,825)COEF
825   FORMAT(//,T6,'You have input ',F4.2,' as the coefficient to update
     1 the 1986 capital',/,' costs. Do you wish to change this? Only (Y)
     1es will allow you to change this',/,' input.')
      READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 805
C
C  CALCULATION OF COST FOR SLURRY WALL ($50,000 EQUIPMENT MOBILIZATION
AND
C   $67/CU.YD.) DEPENDENT ON DEPTH TO BOTTOM OF AQUIFER (WT) AND LENGTH
OF
C   OCTAGON SIDE (SL) ASSUMED 3 FT WIDE & ONLY 5 SIDES OF OCTAGON
C   NOT CALCULATED IF AQUIFER-BEDROCK INTERFACE IS BAD (STRAT=1)
C   OR CERTAIN CHEMICALS IN CONTAMINANT (CHEM='Y')
C
      SWCOST = 1.E15
      IF(STRAT.EQ.1.OR.CHEM.EQ.'Y') GOTO 830
      SWCOST = 50000. + 67.*SL*5*3*WT
C
C  CALCULATION OF COST FOR SHEET PILING ($1500/TON FOR MATERIAL AND
$250/TON
C   FOR INSTALLATION) BASED ON DEPTH TO BOTTOM OF AQUIFER (WT), LENGTH
OF
C   OCTAGON SIDE (SL) AND WEIGHT OF 12 LBS/SQ.FT. ASSUMED OVERLAP OF 10%
C   & ONLY 5 SIDES USED
C   NOT CALCULATED IF VERY ROCKY SOIL (ROCK=3)
C
830   SPCOST = 1.E15
      IF(ROCK.EQ.3) GOTO 840
      SPCOST = (1500. + 250.)*(WT*SL*5*1.1*12.)/2000.
C
C  CALCULATION OF COST FOR WELL PUMPING ($3/IN. DIAMETER/FT. OF DEPTH,
PLUS
C   CASING AT $15/FT PLUS PUMPS AT $1500 EACH. ASSUMES 2 PUMPS/SIDE, AND
ARE
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C   DRILLED TO BOTTOM OF AQUIFER WITH 24" DIA. HOLE PLUS $50,000 EQUIP.
SET-UP.
C   IF MEAN TRANSMISSIVITY (ET) IS LESS THAN .002 THIS IS NOT
CALCULATED.
C   WELLS ARE PUT ON ALL 8 SIDES AND NEED 8" HEADER ALL THE WAY AROUND
C   $75,000 FOR A 50 YD x 30 YD x 10 YD SETTLING POND
C
840   PWCOST = 1.E15
      IF(ET.LT.0.002)GOTO 850
      PWCOST = 3.*(24.*WT*2*8) + 15.*WT*2*8 + 1500.*2*8+50000.+8*SL*55.
C
C  CALCULATION OF THE CONFIDENCE FACTOR. IT IS EQUAL TO THE LOWEST
FACTOR
C   GIVEN BY THE USER, CONF(1), MINUS ANY UNKNOWNS, ASSUMPTIONS NOT
UNDERSTOOD
C   OR NO FIELD DATA (MEASURED WITH TREL)
C
850   CF = CONF(1) - TREL
C      WRITE(*,852)SWCOST,SPCOST,PWCOST,CF
852   FORMAT(3E10.2,I5)
C
C  EXPLAINING TO THE USER THE RECOMMENDED STRATEGY AND ITS CONFIDENCE IN
THAT
C   STRATEGY
C
      IF(SWCOST.EQ.1.E15)GOTO 865
      IF(SWCOST.LT.SPCOST.AND.SWCOST.LT.PWCOST)WRITE(*,860)CF
860   FORMAT(///,T6,'The system recommends a slurry wall containment str
     1ategy. Its confidence',/,' in this recommendation is ',I3,'%.')
865   IF(SPCOST.EQ.1.E15)GOTO 875
      IF(SPCOST.LT.SWCOST.AND.SPCOST.LT.PWCOST)WRITE(*,870)CF
870   FORMAT(///,T6,'The system recommends a sheet piling containment st
     1rategy. Its confidence',/,' in this recommendation is ',I3,'%.')
875   IF(PWCOST.EQ.1.E15)GOTO 885
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST)WRITE(*,880)CF
880   FORMAT(///,T6,'The system recommends a pumping containment strateg
     1y. Its confidence',/,' in this recommendation is ',I3,'%.')
C
C  ASK USER IF HE HAS ANY QUESTIONS ABOUT RECOMMENDATION OR CONFIDENCE
C
885   WRITE(*,890)
890   FORMAT(//,T6,'Do you have any questions about:',//,
     1T10,'1. Recommendation',/,
     1T10,'2. Confidence value',/,
     1T10,'3. None',//,
     1T6,'Indicate by number.',//,T10)
      READ(*,900)QUEST
900   FORMAT(I2)
C
C  EXPLAINING THE RECOMMENDATION TO THE USER
C
      WRITE(*,905)
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905   FORMAT(//////)
      IF(QUEST.NE.1)GOTO 1050
      IF(SPCOST.EQ.1.E15)WRITE(*,910)
910   FORMAT(//,T3,'Sheet piling is not a viable alternative because the
     1 soil is too rocky.')
      IF(PWCOST.EQ.1.E15)WRITE(*,920)
920   FORMAT(//,T3,'Pumping is not a viable alternative because the mean
     1 hydraulic',/,' conductivity is less than .002 ft/d.')
      IF(SWCOST.EQ.1.E15)WRITE(*,930)
930   FORMAT(//,T3,'A slurry wall is not a viable alternative because:')
      IF(STRAT.EQ.1) WRITE(*,932)
932   FORMAT(/,T6,'The aquifer-bedrock interface is too irregular.')
      IF(CHEM.EQ.'Y') WRITE(*,934)
934   FORMAT(/,T6,'Certain chemicals are in the contaminant that increas
     1e the',/,T8,'permeability of a bentonite slurry wall.')
C
C  EXPLAINING THAT A SLURRY WALL IS THE BEST ALTERNATIVE
C
      IF(SWCOST.GT.SPCOST.OR.SWCOST.GT.PWCOST) GOTO 970
      IF(SWCOST.LT.SPCOST.AND.SWCOST.LT.PWCOST)WRITE(*,940)SWCOST
940   FORMAT(//,T6,'The slurry wall capital cost was the smallest of the
     1 techniques',/,' considered. The costs were:',/,
     1T10,'Slurry wall',T30,F15.2)
      IF(SPCOST.NE.1.E15)WRITE(*,950)SPCOST
950   FORMAT(/,T10,'Sheet piling',T30,F15.2)
      IF(PWCOST.NE.1.E15)WRITE(*,960)PWCOST
960   FORMAT(/,T10,'Pumping',T30,F15.2)
C
C  EXPLAINING THAT SHEET PILING IS THE BEST ALTERNATIVE
C
970   IF(SPCOST.GT.SWCOST.OR.SPCOST.GT.PWCOST) GOTO 1010
      IF(SPCOST.LT.SWCOST.AND.SPCOST.LT.PWCOST)WRITE(*,980)SPCOST
980   FORMAT(//,T6,'The sheet piling capital cost was the smallest of th
     1e techniques',/,' considered. The costs were:',/,
     1T10,'Sheet piling',T30,F15.2)
      IF(SWCOST.NE.1.E15)WRITE(*,990)SWCOST
990   FORMAT(/,T10,'Slurry wall',T30,F15.2)
      IF(PWCOST.NE.1.E15)WRITE(*,1000)PWCOST
1000  FORMAT(/,T10,'Pumping',T30,F15.2)
C
C  SETTING UP THE OPERATING (C) AND MAINTENANCE (CC) UNIT COSTS
C    OPERATING IS IN CU.FT./FT. AND MAIN. IS IN CU.FT.
C
      OC = .000004132
      MC = .00003788
C
C  EXPLAINING THAT PUMPING IS THE BEST ALTERNATIVE
C
1010  IF(PWCOST.GT.SWCOST.OR.PWCOST.GT.SPCOST) GOTO 1050
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST)WRITE(*,1020)PWCOST
1020  FORMAT(//,T6,'The pumping capital cost was the smallest of the tec
     1hniques',/,' considered. The costs were:',/,
     1T10,'Pumping',T30,F15.2)
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      IF(SPCOST.NE.1.E15) THEN
        WRITE(*,1030)SWCOST
1030    FORMAT(T10,'Slurry wall',T30,F15.2)
C
C  CALCULATING OPERATING TIME (TT) BEFORE PUMPING IS NOT MOST ECONOMIC
C   BASED ON MAX. PUMPING (QX),MAX. LIFT (MAXLFT),NEW PUMPS EVERY 10 YRS
C   MAINTENANCE TRIPLES EVERY 10 YRS & UTILITIES ARE 1.5 TIMES EVERY 10
YRS.
C
        TT = ((SPCOST-PWCOST-16*5*1500.)/(7.6*4.132E-6*MAXLFT*QX*16.
     1      + 2.*3.788E-5*QX*16.))/30.4
      ENDIF
      IF(SPCOST.NE.1.E15)WRITE(*,1040)SPCOST
1040  FORMAT(T10,'Sheet Piling',T30,F15.2)
      IF(SPCOST.EQ.1.E15.AND.SWCOST.NE.1.E15)
     1  TT = ((SWCOST-PWCOST-16*5*1500.)/(7.6*4.132E-6*MAXLFT*QX*16.
     1      + 2.*3.788E-5*QX*16.))/30.4
C  PUTTING TT IN UNITS OF YEARS
      TT = TT/12.
      IF(TT.GT.50.) THEN
C        WRITE(*,1031) TT
        TT = 50.
      ENDIF
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST) WRITE(*,1045) TT
1045  FORMAT(/,T6,'However, it should be kept in mind that operation and
     1 maintenance',/,' (O&M) costs were not considered in this capital
     1cost comparison.',/,
     1' If the pumping technique is to be utilized for a long period of
     1time the',/,
     1' O&M costs for pumping become a major part of the economic analys
     1is and',/,
     1' need to be considered.',/,T6,
     1'Therefore, based on these assumptions:',/,
     1T10,'1. Pumping at all wells is at the upper limit specified.',/,
     1T10,'2. Pumping lift is the maximum allowed (1/2 of the saturated'
     1,/,T13,'thickness).',/,
     1T10,'3. Pumps are replaced every 10 years.',/,
     1T10,'4. Operating costs are $4.13 per 1 million cu.ft/ft and incre
     1ase',/,T13,'by 1.5 times every 10 yrs.',/,
     1T10,'5. Maintenance costs are $38 per 1 million cu.ft and triple o
     1ver',/,T13,'a 10 yr. period.',/,
     1T6,'If the pumping strategy exceeds ',E8.2,' years,',/,T6,
     1' the next least capital cost technique is the most economic.')
C       WRITE(*,1031) TT,OC,MAXLFT,QX,MC
1031  FORMAT(5E12.2)
      PAUSE' If you are ready to continue hit ENTER'
C
C EXPLAINING THE CONFIDENCE VALUE TO THE USER
C
1050      IF(QUEST.NE.2)GOTO 1240
C
C  CF FIRST BASED ON THE LOWEST CF,CONF(1), GIVEN BY THE USER
C
```

```
      WRITE(*,905)
      WRITE(*,1060)CF
1060  FORMAT(//,T6,'The system confidence of ',I3,'% is based on:',/)
      IF(LWCF.EQ.2)WRITE(*,1070)CONF(1)
1070  FORMAT(T10,'The user confidence of ',I3,'% in the amount of rock i
     1n the soil.',/)
      IF(LWCF.EQ.3)WRITE(*,1080)CONF(1)
1080  FORMAT(T10,'The user confidence of ',I3,'% in amount of irregulari
     1ty in the',/,' aquifer-bedrock interface.',/)
      IF(LWCF.EQ.4)WRITE(*,1090)CONF(1)
1090  FORMAT(T10,'The user confidence of ',I3,'% in the soil type.',/)
      IF(LWCF.EQ.5)WRITE(*,1100)CONF(1)
1100  FORMAT(T10,'The user confidence of ',I3,'% in the average precipit
     1ation.',/)
      IF(LWCF.EQ.6)WRITE(*,1110)CONF(1)
1110  FORMAT(T10,'The user confidence of ',I3,'% in the drainage at the
     1site.',/)
      IF(LWCF.EQ.7)WRITE(*,1120)CONF(1)
1120  FORMAT(T10,'The user confidence of ',I3,'% in the average depth to
     1 the',/,' bottom of the aquifer.',/)
      IF(LWCF.EQ.8)WRITE(*,1130)CONF(1)
1130  FORMAT(T10,'The user confidence of ',I3,'% in the average saturate
     1d',/,' thickness of the aquifer.',/)
      IF(LWCF.EQ.9)WRITE(*,1140)CONF(1)
1140  FORMAT(T10,'The user confidence of ',I3,'% in the hydraulic gradie
     1nt.',/)
      IF(LWCF.EQ.10)WRITE(*,1150)CONF(1)
1150  FORMAT(T10,'The user confidence of ',I3,'% in certain chemicals (t
     1hat affect',/,' the permeability of a slurry wall) being in the co
     1ntaminant.',/)
      IF(LWCF.EQ.11)WRITE(*,1160)CONF(1)
1160  FORMAT(T10,'The user confidence of ',I3,'% in the time until conta
     1inment',/,' strategy is implemented.',/)
      IF(LWCF.EQ.12)WRITE(*,1170)CONF(1)
1170  FORMAT(T10,'The user confidence of ',I3,'% in the present furthest
     1 extent',/,' of the plume.',/)
C
C  SUBTRACTING FROM LWCF ANY "UNKNOWNS" OR NOT SAYING YES TO A
ASSUMPTION
C   TWO TIMES OR MORE
C
      IF(CF.LT.CONF(1)) WRITE(*,1175)
1175  FORMAT(T10,'In addition, the confidence factor was further reduced
     1 because:',/)
      IF(REL(1).EQ.1)WRITE(*,1180)
1180  FORMAT(T12,'The user did not understand the soil homogeniety assum
     1ption.',/)
      IF(REL(2).EQ.1)WRITE(*,1190)
1190  FORMAT(T12,'The user was uncertain about the amount of rock in the
     1 soil.',/)
      IF(REL(3).EQ.1)WRITE(*,1200)
1200  FORMAT(T12,'The user was uncertain about the amount of irregularit
     1y in the',/,T14,' aquifer-bedrock interface.',/)
```

```
      IF(REL(4).EQ.3)WRITE(*,1210)
1210  FORMAT(T12,'The user supplied no hydraulic conductivity field data
     1.',/)
      IF(REL(5).EQ.3)WRITE(*,1215)
1215  FORMAT(T12,'The user supplied no effective porosity field data.'
     1,/)
      IF(REL(6).EQ.1)WRITE(*,1220)
1220  FORMAT(T12,'The user did not understand the constant enviroment as
     1sumption.',/)
      IF(REL(7).EQ.1)WRITE(*,1230)
1230  FORMAT(T12,'The user did not understand the advection assumption
     1.',/)
      WRITE(*,1235)
1235  FORMAT(/)
      PAUSE ' If you are ready to continue hit ENTER'
1240  IF(QUEST.NE.3)GOTO 885
      IF(PWCOST.GT.SWCOST.OR.PWCOST.GT.SPCOST) GOTO 1280
C
C SHOWING USER THE PROGRAM'S SUGGESTED INPUT TO OPTIMIZATION MODEL AND
C  ASKING IF HE WANTS A PRINTOUT
C
      IF(PWCOST.EQ.E+15)GOTO 1280
C
C  ASKING THE USER HOW LONG THE PUMPING STRATEGY HAS TO STABILIZE THE
PLUME
C
      WRITE(*,1242)
1242  FORMAT(//////////)
1244  IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST) WRITE(*,1245)
1245  FORMAT(//,T6,'How much time (days) should be allowed to stabilize
     1the plume',/,
     1' once the pumping strategy is begun (assuming pumping is begun im
     1mediately',/,
     1' following installation of the wells? (Y)es is unnecessary. Just
     1input',/,
     1' a value.',//,T10)
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST) READ(*,*)STABE
C  SHOWS USER HIS INPUT AND ALLOWS HIM TO CHANGE THE INPUT
      WRITE(*,905)
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST) WRITE(*,1248)STABE
1248  FORMAT(//,T6,'You have input ',I4,' days as your answer.Do you wis
     1h to change this?',/,' Only (Y)es will allow you to change this in
     1put.')
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST) READ(*,80)CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1244
      WRITE(*,1295)
1295  FORMAT(/////,T6,'The user can now run either the deterministic ver
     1sion or the stochastic',/,
     1' version of the optimization program. If field or lab data is ple
     1ntiful for',/,
     1' this aquifer then it is recommended that the deterministic versi
     1on be run',/,
     1' because it develops optimal pumping values that are more predict
```

```
     lable for the',/,
     1' situation. The deterministic version is run by developing an inp
     lut file,',/,
     1' MODEL2.DAT, as described in Section VI and Appendix V.',/,T6,
     1'The stochastic version is normally run if field or lab data is sc
     larce.',/,
     1' Because of the uncertainty involved in the data and the required
     1 reliability',/,
     1' in the solution, the optimal pumping allowed within the constrai
     lnts is less',/,
     1' and the resulting heads at all wells are at higher estimated ele
     lvations as',/,
     1' compared to the deterministic version. The input file and runnin
     lg of the',/,
     1' stochastic version are also described in Section VI and Appendix V.
     1 In many',/,
     1' cases it is advantageous to run both deterministic and stochasti
     lc versions',/,
     1' and compare the results.',////,
     1' When you are ready to continue hit ENTER.')
      PAUSE
      WRITE(*,905)
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST)
     1WRITE(*,1250)TR,COVT,EEP,COVEP,SL,STABE,STABE
1250  FORMAT(/,T6,'If you wish to create the input file, SMODEL.DAT, for
     1 the stochastic',/,
     1' version the suggested input to the optimization program is:',//,
     1T10,'Transmissivity',/,
     1T15,'mean ',F10.3,' ft.sqd./d',/,
     1T15,'coefficient of variation ',F8.3,//,
     1T10,'Effective porosity',/,
     1T15,'mean ',F5.3,/,
     1T15,'coefficient of variation ',F5.3,//,
     1T15,'Octagon side length ',F7.2,' ft.',//,
     1T15,'Time period to stabilize plume ',I4,' days',//,
     1T15,'Well spacing - 1/2, 1/4, 1/8 of side length',/,
     1T17,'Due to memory limitations, the stochastic optimization',/,
     1T17,'model cannot compute a strategy if wells are spaced at 1/8',/
     1,T17,'of the side length. However, the pump spacing should never',
     1/,T17,'exceed the "effective radius of influence" of the pump',/,
     1T17,'for the',I4,' day time period specified',/,
     1T6,'Would you like a hard copy of this information? (Make sure you
     1r printer',/,' is turned on.) Answer (Y)es or (N)o.')
      IF(PWCOST.LT.SWCOST.AND.PWCOST.LT.SPCOST)
     1READ(*,1260)PRINT
1260  FORMAT(A1)
C
C  PRODUCING A PRINTOUT OF WHAT SHOULD BE INPUT TO OPTIMIZATION MODEL
C
      IF(PRINT.EQ.'Y')WRITE(9,1270)TR,COVT,EEP,COVEP,SL,STABE,STABE
1270  FORMAT(/,T6,'Input to the optimization program should be:',//,
     1T10,'Transmissivity',/,
     1T15,'mean ',F10.3,' ft.sqd./d',/,
```

```
     1T15,'coefficient of variation ',F8.3,//,
     1T10,'Effective porosity',/,
     1T15,'mean ',F5.3,/,
     1T15,'coefficient of variation ',F5.3,//,
     1T15,'Octagon side length ',F7.2,' ft.',//,
     1T15,'Time period to stabilize plume ',I4,' days',//,
     1T15,'Pump spacing - 1/2, 1/4, 1/8 of side length',/,
     1T17,'Due to memory limitations, the stochastic optimization',/,
     1T17,'model can not handle the wells spaced at 1/8 of the',/,
     1T17,'side length.',//,
     1T17,'However, the pump spacing should never exceed the',/,
     1T17,'"effective radius of influence" of the pump for',/,
     1T17,'the ',I4,' day time period specified.')
C
C  REMAINDER OF PROGRAM INPUTS DATA INTO SMODEL.DAT FILE FOR RUNNING OF
C     STOCHASTIC OPTIMIZATION - SMODEL.FOR
C
C NP IS THE NUMBER OF TOTAL WELLS ON A SIDE
      NP = 4
C TPW IS THE TOTAL NUMBER OF PUMPING WELLS
      TPW = 16
C TW IS THE TOTAL NUMBER OF WELLS (EQUALS 2*TPW + 1)
      TW = 33
C R IS THE RADIUS OF THE PUMPING WELL
      R = 1.0
C MODEL = 2 SAYS TO RUN THE STOCHASTIC VERSION
      MODEL = 2
C LENGTH SHOWS THE USER WHAT UNIT OF LENGTH IS BEING USED
      LENGTH = 'FEET'
1299  WRITE(*,1300)
1300  FORMAT(////////,T6,'This program has the ability to develop an inpu
     1t file, SMODEL.DAT,',/,
     1' for use with the stochastic version of the optimization model. T
     1his',/,
     1' input is based on the mean and coefficient of variation for tran
     1smissivity',/,
     1' and effective porosity calculated previously. The well configura
     1tion is',/,
     1' based on 1 ft. radius pump wells located at the 1/4 and 3/4 poin
     1ts of each',/,
     1' side of the octagon. The user will input an average ground slope
     1 and direction',/,
     1' of that slope. The program assumes the hydraulic gradient to be
     1symmetrical',/,
     1' to the x-axis of the octagon and that the saturated thickness is
     1 constant.',/////////,
     1T6,'Do you wish the program to develop this input file for you? An
     1swer',/,' (Y)es or (N)o.')
      READ(*,80)CHARAC
C  IF USER DOES NOT RETURN A CORRECT ANSWER HE IS RETURNED TO THE
QUESTION
      IF(CHARAC.NE.'N'.AND.CHARAC.NE.'Y') THEN
      WRITE(*,45)
```

```
      PAUSE
      GOTO 1299
      ENDIF
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      IF(CHARAC.EQ.'Y') THEN
      WRITE(*,1305)
1305  FORMAT(//,T6,'You have asked the program to develop a data file to
     1 be used with the',/,' stochastic optimization model. Do you wish
     1to change this input? Only (Y)es',/,
     1' will allow you to make a change.')
      READ(*,80) CHARAC2
      ENDIF
      IF(CHARAC2.EQ.'Y') GOTO 1299
C  IF USER ANSWERS NO HE IS TAKEN TO THE END OF THE PROGRAM
      IF(CHARAC.EQ.'N') GOTO 1280
C
C  ASKING FOR UNITS AND NUMBER OF TIME PERIODS ALLOWED FOR STABILIZATION
C   OF PLUME
C
1308  WRITE(*,1310)
1310  FORMAT(//,T6,'A maximum of 10 "time periods" is allowed in the opt
     1imization program',/,
     1' for the pumping strategy to stabilize the plume. Select the unit
     1s you wish',/,
     1' to use for each time period (1,2 or 3).',//,
     1T10,'1. Day',/,T10,'2. Week',/,T10,'3. Month',//,T10)
      READ(*,*) PERIOD
      IF(PERIOD.EQ.1) THEN
      TFRAME = 'DAY'
      FRAME = 'DAY'
C  DIVIDE U.L. ON PUMPING BY 1000 BECAUSE INFLUENCE COEFFICIENTS ARE FOR
C    1000 UNITS
      QU = QX/1000.
      ENDIF
      IF(PERIOD.EQ.2) THEN
         TFRAME = 'WEEK'
         FRAME = 'WEEK'
C  PUTTING TRANSMISSIVITY IN THE CORRECT UNITS
         ET = ET*7
C PUTTING PUMPING IN CORRECT UNITS & DIVIDE BY 1000 BECAUSE INFLUENCE
C   COEFFICIENTS FOR MODEL ARE FOR UNITS OF 1000.
         QU = QX*7/1000.
      ENDIF
      IF(PERIOD.EQ.3) THEN
         TFRAME = 'MONTH'
         FRAME ='MNTH'
C  PUTTING TRANSMISSIVITY IN THE CORRECT UNITS
         ET = ET*30.4
C PUTTING PUMPING IN CORRECT UNITS & DIVIDE BY 1000 BECAUSE INFLUENCE
C   COEFFICIENTS FOR MODEL ARE FOR UNITS OF 1000.
         QU = QX*30.4/1000.
      ENDIF
      WRITE(*,1320) TFRAME
```

```
1320  FORMAT(//,T6,'How many ',A6,'(s) will you allow for the pumping st
     1rategy to stabilize',/,
     1' movement of the plume once the wells are in place and functionin
     1g?',//,T10)
      READ(*,*) IT
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      WRITE(*,1330) IT,TFRAME
1330  FORMAT(//,T6,'You have input ',I3,1X,A6,'(s) as your answer. Do yo
     1u wish to change',/,
     1' this input? Only (Y)es will allow you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1308
C
C  ASKING FOR THE REQUIRED RELIABILITY
C
1355  WRITE(*,1360)
1360  FORMAT(//,T6,'How confident do you want to be in the final heads a
     1t the observation',/,
     1' wells and the drawdowns at the pumping wells that are generated
     1by the',/,
     1' optimization program (This is referred to as a reliability level
     1)?',/,
     1' A reliability of 50% is equivalent to running the deterministic
     1version',/,
     1' using the mean values of hydraulic conductivity and effective po
     1rosity.',/,
     1' Answer 1,2,3,4 or 5',//,
     1T10,'1. 99%',/,
     1T10,'2. 95%',/,
     1T10,'3. 90%',/,
     1T10,'4. 85%',/,
     1T10,'5. 80%',/,
     1T10,'6. 50%',//,T10)
      READ(*,*)RELIA
      IF(RELIA.EQ.1) THEN
        CL = .99
        F1 = 2.33
      ENDIF
      IF(RELIA.EQ.2) THEN
        CL = .95
        F1 = 1.64
      ENDIF
      IF(RELIA.EQ.3) THEN
        CL = .90
        F1 = 1.28
      ENDIF
      IF(RELIA.EQ.4) THEN
        CL = .85
        F1 = 1.04
      ENDIF
      IF(RELIA.EQ.5) THEN
        CL = .80
        F1 = 0.84
```

```
      ENDIF
      IF(RELIA.EQ.6) THEN
        CL = .50
        F1 = 0.00
      ENDIF
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      WRITE(*,1370) CL
1370  FORMAT(//,T6,'You have input ',F4.2,' as the required confidence l
     level for the',/,
     1' optimization program. Do you wish to change this input? Only (Y)
     les will',/,' allow you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1355
C
C  ASKING FOR THE AVERAGE SLOPE OF THE LAND AND THE ANGLE (CCW) IT MAKES
C   WITH THE X-AXIS
C
1375  WRITE(*,1380)
1380  FORMAT(//,T6,'Input the average ground slope (ft/ft) in the area o
     1f contamination',/,
     1' and the counter clockwise angle (degrees) from the positive x-ax
     1is to',/,
     1' a line in the direction of the DOWNWARD slope. The positive x-ax
     1is is in',/,
     1' the direction of the downward hydraulic gradient and the octagon
     1 of wells',/,
     1' is symmetrical with respect to it. Separate the two values with
     1a space.',//,T10)
      READ(*,*)SLOPE,ANGLE
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      WRITE(*,1390) SLOPE,ANGLE
1390  FORMAT(//,T6,'You have input ',F6.4,' as the average slope of the
     1ground and ',F5.1,/,
     1' degrees as the angle the downward slope makes with the direction
     1 of the',/,
     1' hydraulic gradient (the x-axis). Do you wish to change this inpu
     1t? Only',/,
     1' (Y)es will allow you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1375
C
C  CONVERTS THE ANGLE IN DEGREES TO RADIANS
C
      RAD = (ANGLE/360.)*2*PI
C
C ASKS FOR THE GROUND ELEVATION (ZO) AND THE POTENTIOMETRIC SURFACE
ELEVATION
C  [HO(1)] AT THE CONTAMINANT SOURCE
C
1395  WRITE(*,1400)
1400  FORMAT(//,T6,'Input the ground elevation (ft) and the potentiometr
     1ic surface elevation',/,
     1' (ft) at the contaminant source. Separate the two values with a s
```

```
     1pace.',//T10)
      READ(*,*)ZO,HO(1)
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
      WRITE(*,1410) ZO,HO(1)
1410  FORMAT(//,T6,'You have input ',F7.2,' as the ground elevation and
     1',F7.2,' as the',/,
     1' potentiometric surface elevati  at the contaminant source. Do y
     1ou wish to',/,
     1' change this input? Only (Y)es will allow you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1395
C
C  CALCULATION OF COORDINATES OF ALL WELLS (OBS & PUMP) STARTING WITH
C  SOURCE WELL AND THEN TO WELL (A,SL/2) AND THEN CCW
C
      X(1)= 0.
      Y(1)= 0.
C     WRITE(6,13)X(1),Y(1)
      X(2)= FEXTENT
      Y(2)= SL/2.
C     WRITE(6,13)X(2),Y(2)
13    FORMAT(2F10.2)
      DO 1420 II=3,6
      X(II)=X(II-1)-(SL/4.)*SIN(PI/4.)
      Y(II)=Y(II-1)+(SL/4.)*COS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1420  CONTINUE
      DO 1430 II=7,10
      X(II)=X(II-1)-(SL/4.)
      Y(II)=Y(II-1)
C     WRITE(6,13)X(II),Y(II)
1430  CONTINUE
      DO 1440 II=11,14
      X(II)=X(II-1)-(SL/4.)*SIN(PI/4.)
      Y(II)=Y(II-1)-(SL/4.)*COS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1440  CONTINUE
      DO 1450 II=15,18
      X(II)= X(II-1)
      Y(II)=Y(II-1)-(SL/4.)
C     WRITE(6,13)X(II),Y(II)
1450  CONTINUE
      DO 1460 II=19,22
      X(II)=X(II-1)+(SL/4.)*SIN(PI/4.)
      Y(II)=Y(II-1)-(SL/4.)*COS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1460  CONTINUE
      DO 1470 II=23,26
      X(II)=X(II-1)+(SL/4.)
      Y(II)=Y(II-1)
C     WRITE(6,13)X(II),Y(II)
1470  CONTINUE
      DO 1480 II=27,30
```

```
      X(II)=X(II-1)+(SL/4.)*SIN(PI/4.)
      Y(II)=Y(II-1)+(SL/4.)*COS(PI/4.)
C     WRITE(6,13)X(II),Y(II)
1480  CONTINUE
      DO 1490 II=31,33
      X(II)= X(II-1)
      Y(II)=Y(II-1)+(SL/4.)
C     WRITE(6,13)X(II),Y(II)
1490  CONTINUE
C
C  CALCULATING THE PUMPING WELLS GROUND ELEVATION HP(I,1) AND THE
C   POTENTIOMETRIC SURFACE ELEVATION HP(I,2)
C
      DO 1500 I = 3,33,2
        HP(I,1) = ZO + (Y(I)*((-SLOPE)*SIN(RAD))
     1               + X(I)*((-SLOPE)*COS(RAD)))
        HP(I,2) = HO(1) - X(I)*GRAD
1500  CONTINUE
C
C  CALCULATING THE POTENTIOMETRIC SURFACE ELEVATION (HO(I) AT
OBSERVATION
C   WELLS
C
      DO 1510 I = 2,32,2
        HO(I) = HO(1) - X(I)*GRAD
1510  CONTINUE
C
C  ASKING FOR INITIAL PUMPING VALUES (CU.FT./TIME PERIOD) FOR EACH TIME
C   PERIOD
C
1515  WRITE(*,1520) TFRAME,TFRAME,IT
1520  FORMAT(//,T6,'As described in Volume I, one must usually run the s
     1tochastic model',/,
     1' several times to assure validity of results. This iterative proc
     1ess is',/,
     1' performed until assumed pumping values input into the model are
     1within',/,
     1' about 5% of the optimal values subsequently computed by the mode
     1l.',/,T6,
     1'You are now ready to input assumed pumping values for SMODEL.DAT
     1in',/,
     1' cu.ft./',A6,'/pump. If this data is for the first optimization,
     1 simply',/,
     1' guess values for each',A6,'. For all others use the optimal valu
     1es',/,
     1' from the previous optimization as assumed values.',//,
     1' Input ',I3,' pumping values with a space between each value (onl
     1y 5',/,
     1' values per line, then hit return). These values must be less tha
     1n the',/,
     1' upper limit on pumping input previously.',//)
      READ(*,*)(Q(I),I=1,IT)
C SHOWS THE USER HIS INPUT AND ALLOWS HIM TO CHANGE IT
```

```
1525  WRITE(*,1530)
1530  FORMAT(//,T6,'You have input the following initial pumping values:
     1',//)
      WRITE(*,1535)(Q(I),I=1,IT)
1535  FORMAT(1X,10E7.2)
      WRITE(*,1540)
1540  FORMAT(//,' Do you wish to change this input? Only (Y)es will allo
     1w you to change this.')
      READ(*,80) CHARAC2
      IF(CHARAC2.EQ.'Y') GOTO 1515
C
C OUTPUTING THE DATA INTO FILE SMODEL.DAT
C
      WRITE(1,1550)TPW,TW,IT,R,FEXTENT,FRAME,LENGTH,MODEL
1550  FORMAT(3I5,F5.2,F10.2,1X,A4,1X,A4,I5)
      WRITE(1,1560)QU,EEP,COVEP,COVT,CL,F1,TR
1560  FORMAT(F10.2,F5.2,4F5.2,F10.2)
      DO 1580 I = 1,8
       WRITE(1,1570)SL,NP
1570   FORMAT(F10.2,I5)
1580  CONTINUE
      DO 1610 I = 3,33,2
       WRITE(1,1620)(HP(I,J),J=1,2)
1620   FORMAT(2F10.2)
1610  CONTINUE
      WRITE(1,1620)HO(1)
      DO 1630 I = 2,32,2
       WRITE(1,1620)HO(I)
1630  CONTINUE
      DO 1640 I = 1,33
       WRITE(1,1620)SAT
1640  CONTINUE
C  QE ARE THE ESTIMATED INITIAL VALUES. DIVIDE BY 1000 BECAUSE INFLUENCE
C   COEFS. ARE FOR 1000 UNITS
      DO 1650 I = 1,IT
       QE = Q(IT)/1000.
       WRITE(1,1620) QE
1650  CONTINUE
C
C INFORMING THE USER INPUT FILE SMODEL.DAT HAS BEEN CREATED
C
      WRITE(*,1680)
1680  FORMAT(//,T6,'The input data file, SMODEL.DAT, has been created fo
     1r running the',/,
     1' stochastic version of the optimization program. Follow the detai
     1led',/,
     1' instructions in              Section VI to run the program.
      CLOSE(1, ERR=1660, STATUS='KEEP')
      GOTO 1280
1600  WRITE(*,1602)
1602  FORMAT(' ERROR IN OPEN 7')
      GOTO 1280
1660  WRITE(*,1670)
```

```
1670  FORMAT(' ERROR IN CLOSE 7')
1280  WRITE(*,1290)
1290  FORMAT(//,T6,'This program is complete. We hope it has been an aid
     1 in',/,
     1' analyzing your contamination problem. If you had the program dev
     1elop',/,
     1' input file SMODEL.DAT then you can run the stochastic version of
     1 the',/,
     1' optimization model by typing FORT SMODEL BOB2 NO (or YES).')
      STOP
      END
```