STRENGTH DESIGN OF REINFORCED CONCRETE HYDRAULIC STRUCTURES

Report 8

DESIGN OF BURIED CIRCULAR CONDUITS - FLEXURE AND SHEAR

by

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The contents of this report are not to be used for advertising, publication, or promotional purposes. Citation of trade names does not constitute an official endorsement or approval of the use of such commercial products.
A strength design (SD) procedure for buried reinforced concrete circular conduits is presented. The procedure is a combination of results produced through recent work sponsored by the Corps of Engineers and recent studies sponsored by the American Concrete Pipe Association. The SD procedure accounts for both flexural strength and shear in buried circular conduits subjected to the uniform loading conditions of Engineering Manual 1110-2-2902. Expressions are included that account for the effects of concrete compressive strength and radial tension. A criterion for limiting the flexural reinforcement ratio to prevent unacceptable crack width is also included.

This organized presentation of the procedure will enable designers to evaluate further the application of recent research to real problems, with the goal of improving the Corps' SD methods.
PREFACE

This study was conducted during the period September 1987 through September 1988 by the US Army Engineer Waterways Experiment Station (WES) under the sponsorship of Headquarters, US Army Corps of Engineers (HQUSACE). The Technical Monitor was Dr. Tony Liu, HQUSACE.

This work was conducted under the supervision of Messrs. Bryant Mather, Chief, Structures Laboratory (SL); and James T. Ballard, Assistant Chief, SL; and Dr. Jimmy P. Balsara, Chief, Structural Mechanics Division (SMD), SL. Dr. Robert L. Hall, SMD, monitored this study. Mr. Stanley C. Woodson, SMD, performed the study and prepared this report.

Acting Commander and Director of WES during preparation of this report was LTC Jack R. Stephens, EN. Technical Director was Dr. Robert W. Whalin.
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CONVERSION FACTORS, NON-SI TO SI (METRIC)
UNITS OF MEASUREMENT

Non-SI units of measurement used in this report can be converted to SI (metric)
units as follows:

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STRENGTH DESIGN OF REINFORCED CONCRETE HYDRAULIC STRUCTURES

DESIGN OF BURIED CIRCULAR CONDUITS - FLEXURE AND SHEAR

PART I: INTRODUCTION

Background

1. The development of a strength design (SD) methodology for the Corps' reinforced concrete hydraulic structures (RCHS) was initiated at the U.S. Army Engineer Waterways Experiment Station (WES) in 1978 (Liu, 1980). The objective of the first phase of the study was to develop general SD criteria to yield designs equivalent to those designed by the working-stress method for RCHS. The primary reason for the Corps' interest in developing SD criteria is the adoption of the SD approach by the structural engineering profession. Engineering schools are emphasizing the SD approach, and the research community will be using only the SD method. Also, the development of a SD approach may result in the design of more economical structures.

2. Recent phases of the study to develop SD criteria for RCHS concerned the development of methodology for the design of particular RCHS and the publication of design aids. Five WES technical reports (Chiarito and Mlakar, 1986; Wright and Chiarito, 1987; Gerstle, 1987; Gerstle, 1988; and Sandhu and Chen, 1988) describe investigations related to the development of a SD procedure for buried reinforced concrete conduits. Both experimental and analytical studies were conducted. Except for the work by Sandhu and Chen, only circular conduits were studied. This report also deals with the SD approach for buried circular reinforced concrete conduits. However, a study conducted by Professor Gerstle at the University of Colorado on SD of buried reinforced concrete conduits of general shapes (oblong and horseshoe) is being
reported concurrently as Report 9.

3. The Engineer Manual (EM) 1110-2-2902 (Headquarters, Department of the Army, 1969) is the current EM giving guidance for the design of the Corps' conduits, culverts, and pipes. EM 1110-2-2902 follows the working stress design approach and discusses the distribution of loads on buried conduits. In general, the EM loading criteria results in uniformly distributed vertical and horizontal pressures with the horizontal pressure being some fraction of the vertical pressure as shown in Figure 1. This type of loading distribution will be referred to as the "EM loading" throughout this report. The Engineer Technical Letter (ETL) 1110-2-312 (Headquarters, Department of the Army, 1988) is a recent, although general, document providing guidance for the design of RCHS by the SD method. The ETL calls for the design of RCHS by the SD method in accordance with the American Concrete Institute (ACI) Building Code (American Concrete Institute, 1983) with some modifications.

4. Report 4 (Chiarito and Mlakar, 1986) of this series of investigations on SD of RCHS is a study of the effect of initial curvature on the moment-thrust characteristics of reinforced concrete circular sections. When compared to straight beam behavior, initial curvature has no significant effect for the range of conduit section variables used by the Corps. Report 5 (Wright and Chiarito, 1987) is a description of an experimental study on the behavior of model reinforced concrete circular conduits conducted for observation of failure modes. In the model experiments, load was applied through a system of hydraulic rams positioned radially about a circular conduit test specimen. The model tests were compared to nonlinear finite element analyses in Report 6 (Gerstle, 1987). In general, the analyses underestimated the deflections and overestimated the strength of the model rings. The ring tests were studied
Figure 1. Uniformly Distributed Loads Simulating Buried Pipe Conditions
further as discussed in Report 7 (Gerstle, 1988) with analyses of the flexural strength of the specimens according to elastic and plastic theories. A SD procedure combining ultimate flexural section strength with elastic structure analysis for circular conduits under the EM loading is also presented in Report 7.

Objective

5. The objective of this study was to present a SD procedure for buried circular reinforced concrete conduits based on recent research, accounting for both flexure and shear. With the goal of improving the Corps' SD methods, an organized presentation of the procedure will enable further discussion and consideration of the recent research conducted by others.

Scope

6. The SD procedure presented in this report is a combination of results produced through the work of Gerstle (1988) and several studies sponsored by the American Concrete Pipe Association (ACPA) that will be discussed. The ACPA studies have resulted in design criteria for the effects of radial tension and diagonal tension (shear) in buried reinforced concrete conduits. The procedure presented in this report combines the SD procedure given in Report 7 (Gerstle, 1988) for flexural strength with the radial tension and shear criteria developed by ACPA. The procedure uses the EM loading distribution which does not account for soil-structure interaction, but is conservative. A discussion of design criteria given in EM 1110-2-2902 and ETL 1110-2-312 is included.
PART II: DISCUSSION OF CURRENT DESIGN GUIDANCE

Load Distribution

7. EM 1110-2-2902 discusses the structural design and construction of conduits, culverts, and pipes through embankments and pressure conduits for interior drainage of local flood protection projects. The structural design criteria are of a general nature, but considerable detail is given for the determination of loading distributions. The EM also gives some guidance concerning conduit shape. It states that for fills of moderate height, circular or rectangular openings will frequently be the most practicable shape because of the speed and economy obtainable in design and construction. The EM states that circular shapes are more adaptable to changes in loadings and stresses which may be caused by unequal fill or foundation settlement.

8. EM 1110-2-2902 gives guidance for computing loads on buried conduits due to groundwater and surcharge water, concentrated live loads, and loads due to backfill. The EM approach to loading converts each of these types of loads to a vertical and horizontal pressure of the form shown in Figure 2K, which was taken from the EM. Notes 5 and 6 of Figure 2 give guidance for the approximation of the water pressure and the lateral soil pressure against the conduit as an uniformly distributed load as in Figure 1.

9. The EM loading is considered to be a conservative approximation of the real loading applied to a buried conduit. The EM loading does not account for the interaction between the soil and the structure. A computer-aided direct design procedure for buried pipe was recently developed through research sponsored by ACPA (Heger, Liepins, and Selig, 1985). The computer program called SPIDA (Soil-Pipe Interaction Design and Analysis) incorporates multiple finite-element analyses of the pipe-soil system as it changes in the

* A table of factors for converting non-SI units of measurement to SI (metric) units is presented on page 3.
1. Loadings shown or noted on this plate are typical conditions and each designer should modify them as necessary to match the local conditions anticipated.

2. $H'$ represents a height of fill equivalent to the total weight of all dead and live loads effectively supported by the conduit, including the weight of soil and water and the weight of any loads transmitted thru the soil to the conduit except impact. When part or all of the soil above the point in question is saturated or submerged, the weight and pressure of water may be treated separately. The weights and pressures due to soil then are based on its weight reduced by 62.5 lbs/c.ft. Impact is not usually considered where backfill above the top of the conduit exceeds three feet.

3. Foundation reactions are usually assumed to be uniformly distributed across the full pipe width of cast-in-place conduits and uniformly across the bedding width of precast conduits corresponding to the conditions to be shown and specified in the construction plans and specifications.

4. $h$ = the head of water (in feet) above the point in question, $w$ = the unit weight of water expressed as 62.5 lbs/c.f. $w_d$ = natural drained weight of soil. $w_s$ = buoyant (submerged) weight of soil. $w_{sat}$ = saturated weight of soil.

5. When the head of water above the crown exceeds about 20, the water pressure against the conduit, internally or externally, may be assumed as uniform and equal to the head at the conduit's horizontal axis. However, uplift should normally be based on head at bottom of conduit for examining flotation possibilities.

6. When $H'$ exceeds about twice the height of conduit, the horizontal pressure of soil against the conduit may be considered as uniform and equal to the soil pressure at the conduit's horizontal axis.

7. Effective weights of water and soil are weight scales and do not coincide with height scales.

Figure 2. Loading Diagram of EM 1110-2-2902
construction sequence of installation. It accounts for soil stiffness that varies as backfill is placed around and over the pipe and pipe stiffness that varies for each assumed construction layer with the state of cracking in the pipe. The SPIDA analysis determines the total field load acting on the pipe, the pressure distribution in the earth at its interface with the pipe, and the resulting moments, thrusts, and shears in the pipe. A thorough study of SPIDA and its comparison with the EM loading is needed before its acceptance by the Corps is recommended. However, results of supporting research on radial tension and diagonal tension used in the development of SPIDA has been incorporated into Section 17 of the American Association of State Highway and Transportation Officials (AASHTO) "Standard Specifications for Highway Bridges" (1983) and are discussed later in this report.

**Design for Flexure and Shear**

10. EM 1110-2-2902 follows the working-stress approach for the design of cast-in-place reinforced concrete conduits. The EM also calls for an analysis of critical sections by the SD method based on a minimum load factor of 1.8. A suggested procedure for the SD analysis is given in the EM. The SD procedure is only an analytical verification of the section already designed by the working-stress method; it is not a design procedure.

11. EM 1110-2-2902 calls for the design of precast conduits or pipe using bedding condition load factors and the D-load. In this procedure, the load factor is the ratio of the supporting strength of a conduit installed for a given bedding condition to the supporting strength of the same conduit when tested by the three-edge bearing method. The D-load is the three-edge bearing test load in pounds per foot of pipe length divided by the pipe's inside
diameter. The limiting D-load for a pipe is that which produces a 0.01-inch wide crack in the reinforced concrete pipe. For a buried pipe, the required D-load is computed using the estimated vertical loading, the load factor, and a safety factor of 2.0. A pipe design with a D-load value greater than or equal to the required D-load is then selected.

12. Heger and McGrath (1982a) discuss the use of the D-load method and state that it does not always provide an accurate basis for determining the in-ground strength of a buried concrete pipe. For example, a pipe whose ultimate strength under three-edge bearing conditions is governed by diagonal tension may have its ultimate strength in the ground governed by flexure or radial tension due to the differences in the relationship of moment, thrust, and shear between in-ground load conditions and three-edge bearing conditions. Also, a pipe in the ground may have significant thrusts at all cross sections which govern design requirements, while thrust is zero at the critical crown and invert sections under three-edge bearing loading.

13. ETL 1110-2-312 does not discuss the design of buried conduits in great detail but does state that the reinforcement ratio for conduits or culverts designed in accordance with EM 1110-2-2902 shall not exceed 0.375$P_b$ unless it can be shown that deflections will not be excessive. The ETL also limits the design yield strengths of reinforcement to 40,000 psi and 48,000 psi for Grade 40 and 60 steels, respectively. Requirements are also given for load factors on dead loads and live loads to be used in SD of RCHS.

14. Shear and diagonal tension requirements of EM 1110-2-2902 are met when the principal diagonal tension at points of maximum shear does not exceed $2[f'_{c}]^{1/2}$. ETL 1110-2-312 is based on the SD approach and gives more detail on shear criteria. The ETL calls for the computation of the shear strength
provided by concrete, $V_c$, in accordance with the American Concrete Institute (ACI) Building Code (ACI, 1983) with modified expressions for some members. Two expressions are given for $V_c$ as modifications to ACI. One expression (Equation 1) is for deep straight members with rigid, continuous joints and subjected to uniformly distributed loads.

$$V_c = \{(11.5 - 1_{n/d})(f_c')^{1/2}[1 + \left(\frac{N_u}{A_g}\right)/5(f_c')^{1/2}]^{1/2}\}bd \quad (1)$$

ETL 1110-2-312 restricts Equation 1 to the design of members with a clear span-to-effective depth ratio ($l_n/d$) less than 9. Equation 1 was empirically derived from a series of tests on thick walled reinforced concrete conduits (Ruzicka, Gamble, and Mohraz, 1976) where it was considered to be valid for $l_n/d$ ratios between 2 and 6.

15. The second expression for $V_c$ (Equation 2) is given for uniformly loaded curved cast-in-place members with $R/d > 2.5$, where $R$ is the radius of curvature to the centerline of the member and $d$ is the effective depth of the member.

$$V_c = \{(4(f_c')^{1/2}1 + \left(\frac{N_u}{A_g}\right)/4(f_c')^{1/2}]^{1/2}bd \quad (2)$$

The shear strength defined by Equation 2 is limited to $10(f_c')^{1/2}bd$.

**Summary**

16. The current EM used by the Corps for the design of buried conduits uses the working-stress method. A more recent ETL follows the SD approach but is primarily based on the ACI Building Code with some modifications rather than a SD procedure developed directly for buried conduits. Research has been
conducted to improve loading definitions on buried conduits, but a thorough
study to determine the conservatism of the EM loading by comparison with the
finite-element loading scheme is needed. State-of-the-art radial tension and
shear design criteria developed by ACPA specifically for buried conduits will
be discussed further. A SD procedure for flexure developed specifically for
buried circular conduits will also be presented in the following sections.
PART III: DESIGN FOR FLEXURE

Gerstle Strength Envelope Procedure

17. Gerstle (1988) studied methods of flexural-strength prediction based on both elastic and plastic approaches for the loading of the WES model ring tests as well as the EM loading for buried reinforced concrete conduits. Gerstle found that both plastic and elastic analyses of circular sections under distributed loads are practical and convenient tools for strength determination, but that elastic analysis is even simpler than plastic analysis for the EM loading and the radial loading of the WES tests. The working-stress approach underestimated the strength of the model rings by a wide margin. Gerstle recommended the use of the strength method, which combines ultimate section strength with elastic structure analysis, for pipe design. Some reasons for the recommendation include: (1) It follows well-known concepts; (2) It is convenient; (3) It produces results slightly on the conservative side of fully plastic analysis; and (4) It provides a basis for future inclusion of soil-structure interaction.

18. Since Gerstle's report includes a thorough discussion of plastic and elastic theory for the radial loading of the WES tests and for the EM loading, the recommended SD procedure for circular conduits under the EM loading will be presented for simplicity. The application of the procedure is demonstrated in Part V. The reader is referred to Report 7 (Gerstle, 1988) for the derivation and in-depth discussion of this procedure.

19. For the uniformly distributed EM loading, the internal forces along the ring are found by statics to be:
where

\[ N = wR[(1 + k) + (1 - k)\cos 2\theta]/2 \]  
\[ V = wR[(1 - k)\sin 2\theta]/2 \]  
\[ M = wR^2[(1 - k)\cos 2\theta]/4 \]

N = axial thrust, kips  
V = shear force, kips  
M = bending moment, inch-kips  
w = vertical load, kips/square inch  
k = ratio of horizontal to vertical pressure  
R = radius of curvature to centerline of conduit  
\[ \theta \] = angle to point of interest as defined in Figure 3

RCHS should be designed to have the strength required to resist factored loads based on the factors defined in ETL 1110-2-312. The factored load \( w_u \) will be defined by the EM loading multiplied by the load factors given in the ETL.

20. Critical sections will be either at the springing or at the crown section, depending on loading and conduit characteristics. For a given combination of axial load and moment, the structure strength is defined as that at which the capacity of either the springing or the crown section is reached. In keeping with strength-design philosophy, no force redistribution is considered.

21. The development of dimensionless strength curves (moment-thrust) is needed for the design procedure. Structural engineers are familiar with the development and use of similar strength curves provided by ACI (1985) for the SD of reinforced concrete columns. Gerstle (1988) found that the choice of the crushing strain of concrete used in the development of the strength curves has only a minor influence on the ultimate section strength and recommends a value of 0.003 in/in. Radial lines defining the load paths at the springing and the crown for a circular conduit under the EM loading have slopes computed by Equations 6 and 7.
Figure 3. Definition of Section
\[ e_{sp} = R(1 - k)/4 \]  
\[ e_{cr} = R(1 - k)/4k \]

where \(e_{sp}\) and \(e_{cr}\) are values of eccentricity at the springing and crown, respectively.

22. With the EM loading and non-dimensional section strength envelopes known, equilibrium of axial force and moment at the critical crown and springing sections can be represented by radial lines of slope \(1/e_{cr}\) and \(1/e_{sp}\), respectively. The strength envelopes are non-dimensional in terms of the dimensionless axial force \(n = N/f_c'bh\), the dimensionless moment \(m = M/f_c'bh^2\), the reinforcing index \(\omega = A_s f_y/bh f_c'\), and the cross-section parameter \(Y = (h - 2d')/h\). For the EM loading, the non-dimensional terms for axial force and moment are given in Equations 8 - 10.

\[ n_{sp} = \omega R/f_c'bh \]  
\[ n_{cr} = k\omega R/f_c'bh \]  
\[ m_{sp} = m_{cr} = (1 - k)\omega R^2/4f_c'bh^2 \]

The slopes of the non-dimensional radial eccentricity lines are as given in Equations 11 and 12 for springing and crown, respectively.

\[ h/e_{sp} = 4h/(1 - k)R \]  
\[ h/e_{cr} = 4kh/(1 - k)R \]

23. As demonstrated in the examples of Part V of this report, the intersection of two straight lines, one radial and one horizontal, with the strength envelope provides the solution. Two failure modes are possible:

Either the eccentricity line for the springing section intersects the strength curve first, indicating compression failure at the springing, or the eccentricity line for the crown section intersects the strength envelope first, indicating tension failure at the crown.
Limits on Flexural Reinforcement

24. The required area of reinforcement determined from the flexural design procedure should be checked against a minimum allowable value prior to an evaluation of shear strength since the shear strength analysis procedure to be presented in Part IV uses the flexural-reinforcement ratio. Gerstle (1988) does not discuss minimum and maximum limits on the area of flexural reinforcement. The criteria given by Heger and McGrath (1982a) for the minimum area of reinforcement are given by Equation 13 for the inside face and Equation 14 for the outside face.

\[ \text{min. } A_{si} = 0.002bh \]  
\[ \text{min. } A_{so} = 0.0015bh \]  

25. The pipe wall section and concrete strength used in the flexural design procedure should also be checked to ensure that there is adequate compressive strength to develop the ultimate bending moment. This is merely a check to insure ductile behavior and is consistent with the ACI criteria limiting the reinforcement ratio to 0.75\(P_b\). AASHTO (1983) presents the Heger-McGrath equations for performing this check in a form that is slightly different from the Heger-McGrath form of the equations. The Heger-McGrath formulation is recommended since it isolates the reinforcement area term on the left-hand side. The expression is given as:

\[ A_{sc} = [55,000bBf_c \cdot \psi d/f_y(87000 + f_y)] - (0.75Nu/f_y) \]  

where:

- \(A_{sc}\) = the maximum allowable area of flexural reinforcement based on concrete compressive strength
- \(B\) = 0.85 - 0.05(f_c' - 4000)/1000 \(0.65 < B < 0.85\)
- \(\psi\) = capacity reduction factor: AASHTO recommends 1.0 for flexure and 0.9 for shear; Heger and McGrath suggest values from 0.9 to 0.95.
Heger and McGrath allow an increase in $A_{sc}$ up to $0.75A_s$ (where $A_s$ is the compression reinforcement area) if ties are provided connecting the inner and outer cages. The maximum allowable spacing of the ties is the least of the following:

a. wall thickness, $h$

b. 16 times the diameter of the compression steel bar
c. 48 times the diameter of a tie

26. From Equation 15 it is obvious that the limiting value, $A_{sc}$, is a minimum when $N_u$ takes on its maximum value. For the EM loading of a buried circular conduit, $N_u$ is a maximum when $\theta = 0$ degrees in Equation 3 with $w = w_u$. The result is the following expression:

$$\max N_u = w_u R$$

(16)

27. Heger and McGrath state that if the area of reinforcement is found to be greater than $A_{sc}$, cross ties will generally also be required in order to meet diagonal-tension and radial-tension limits. If $A_s$ is greater than the limit, $A_{sc}$, computed using Equation 15, or if the limit cannot be satisfied by an increase due to the use of ties (if desirable), then a new section should be selected and Gerstle's flexure design procedure performed again.

28. Crack width in the reinforced concrete conduit must be limited in order to protect the reinforcement from corrosion, and perhaps to limit infiltration or exfiltration of fluids. Heger and McGrath (1982a) introduce a crack control factor, $F_{cr}$, which indicates an average maximum crack width of 0.01 inch when it has a value of 1.0. The computation of $F_{cr}$ is based on semi-empirical coefficients associated with reinforcement areas needed to control maximum crack width under service loads. The coefficients were derived from statistical analyses of many 3-edge bearing tests for load corresponding to
the formation of the first 0.01-inch wide crack. A data base does not exist for crack widths other than 0.01 inch. Therefore, values of Fcr above or below 1.0 only indicate an increase or decrease in expected crack width.

Equation 17 (given below) for determining Fcr should not be used outside of the range of Fcr = 0.7 to 1.5. Also, Heger and McGrath suggest that the procedure not be applied to pipe having more than about 1.5 inches of concrete cover thickness until additional test data are available. The AASHTO presentation of the procedure is recommended due to its simplicity and is presented below. If an increase in the area of reinforcement is required by Equation 17, the maximum allowable area as limited by the section's concrete compressive strength must be checked for the increased As.

\[ F_{cr} = B_1 \left[ M_s + N_s (d - h/2)/j \right] - C_1 b h^2 (f'_c)^{1/2} / 30,000 d A_s \]  

(17)

where:

- \( M_s \) = bending moment, service load, psi
- \( N_s \) = thrust (positive when compressive), service load, psi
- \( j \) = approximately \( 0.74 + 0.1e/d \)
- \( i \) = \( 1/(1 - jd/e) \)
- \( e \) = \( (M/N) + d - (h/2) \)
- \( M \) = moment acting on cross section of width, b, service load conditions (always +), psi
- \( N \) = thrust acting on cross section of width, b, service load conditions (always +), psi

\( (e/d)_{min} = 1.15 \)

- \( S \) = spacing of circumferential reinforcement in inches
- \( t_b \) = clear cover over reinforcement in inches
- \( h \) = wall thickness of pipe in inches
- \( B_1 \) and \( C_1 \) = crack control coefficients dependent on type of reinforcement used as follows:

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<th>Type of Reinforcement</th>
<th>( B_1 )</th>
<th>( C_1 )</th>
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<td>1. Smooth wire or plain bars</td>
<td>((0.5t_b^2s/n)^{1/3})</td>
<td>1.0</td>
</tr>
<tr>
<td>2. Welded smooth wire fabric, 8-inch maximum spacing of longitudinals</td>
<td>1.0</td>
<td>1.5</td>
</tr>
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3. Welded deformed wire fabric, deformed wire, deformed bars or any reinforcement with stirrups anchored thereto

\[(0.5t_b^2s/n)^{1/3} \leq 1.9\]

Notes:

a. Use \(n = 1\) when the inner and the outer cages are each a single layer. Use \(n = 2\) when the inner and the outer cages are each made up of multiple layers.

b. For type 2 reinforcement having \(t^2s/n > 3.0\), also check \(F_{cr}\) using coefficients \(B_1\) and \(C_1\) for type 3 reinforcement, and use larger value for \(F_{cr}\).

c. When \(F_{cr} = 1.0\), the reinforcement area, \(A_s\), will produce and average maximum crack width of 0.01 inch. For \(F_{cr}\) values less than 1.0, the probability of a 0.01-inch crack is reduced, and for larger values, cracks greater than 0.01 inch may occur.

d. Higher values for \(C_1\) may be used if substantiated by test data and approved by the Engineer.

29. It is not obvious from Equation 17 where the controlling section for crack-width control is located. \(F_{cr}\) is a function of the service load bending moment, \(M_s\), and the axial thrust, \(N_s\). The service-load moment and thrust values may be obtained from Equations 3 and 5, respectively. Values of \(M_s\) and \(N_s\) computed for \(\theta\) varying from 0 to 180 degrees may be used in Equation 17.

30. A final check on the flexural-reinforcement design is for the prevention of premature radial-tension failure. Radial-tension failure is characterized by the formation of a circumferential crack along the line of inner tension reinforcement and straightening of the curved reinforcement. The radial-tension analysis/design procedure is presented in Part V with the shear-investigation procedure since both may require the use of stirrups and, expressions are given for the combined effects.
Summary of Procedure

31. The following summarizes the steps required for the design for flexure.

1. Select trial cross-sectional parameters h, b, d, d', R, and Y.

2. Compute $n_{sp}$, $n_{cr}$, $h/e_{sp}$, and $h/e_{cr}$ from Equations 8, 9, 11, and 12, respectively.

3. Enter appropriate graph of interaction diagrams with values computed in Step 2 and obtain required $\omega$.

4. Compute $p$ and $A_s$ due to $\omega$: $P_{total} = \omega f'_c / f_y$ and $A_{s \, total} = pbh$.

5. Select inner and outer reinforcement to satisfy $A_s$ required.

6. Check limits on $A_{sI}$ and $A_{sO}$ by Equations 13 and 14.

7. Check limit ($A_{sc}$) on reinforcement imposed by concrete compression (Equation 15). If $A_{sc}$ is too small, it may be increased by $0.75 A_s$ if ties are included, or increase section thickness or the concrete compressive strength.

8. Check limits on crack width ($F_{cr}$) by Equation 17 if concrete cover does not exceed 1.5 inches; otherwise, cracking is controlled by the limits ETL 1110-2-312 such that $p < 0.375 p_b$ and the reinforcement spacing is less than 12 and 18 inches for Grades 60 and 40 steel, respectively. Use Equations 3 and 5 to compute thrust and moment values needed for Equation 17.
PART IV: DESIGN FOR SHEAR

32. Heger and McGrath (1982a) present a procedure for the design of buried conduits to resist shear and radial-tension stresses based on research sponsored by the American Concrete Pipe Association (ACPA). As mentioned in Part II, this procedure is part of the SPIDA computer program which accounts for soil-structure interaction in the design of buried pipe. Also, a simplified version of the Heger-McGrath procedure is presented in Section 17 of the AASHTO bridge specification (1984).

33. Heger and McGrath's criteria are based on three-edge bearing tests which produces loading conditions different from loading conditions on pipe in the ground. Gerstle (1988) applied part of the Heger-McGrath procedure to the WES ring sections for comparison with data. Gerstle concluded that the Heger-McGrath procedure is very conservative for shear-strength determination of the WES ring sections, but is satisfactory for rings experiencing the three-edge bearing loading conditions. The procedure predicts shear failure for all but two of the WES ring sections, although none of the rings failed in shear. The Heger-McGrath procedure is based on the assumption that the critical section for diagonal tension occurs where $\frac{M_u}{V_uOd} = 3.0$. As discussed by Heger and McGrath (1982a), this assumption is based on data from tests on beams and frames. It is not clear that this assumption is valid for circular conduits under distributed loading.

34. The development of shear criteria for reinforced concrete members has always been recognized as a difficult problem. The ACI shear-strength criteria for structural members are empirically based on an extensive number of beam tests and were formulated with conservatism. For buried circular conduits, there are less data and a greater lack of understanding of the shear.
resistance than for beams. The ACPA-sponsored work is the most comprehensive research program conducted to date on the shear strength of circular conduits. The Heger-McGrath procedure is also the first method published for the evaluation of the radial-tension strength at sections of governing moment with tension on the inside of the curved surfaces.

35. Heger and McGrath (1982a) discuss the applicability of their design method and compare it to other current design procedures. The design method was developed primarily for application to buried pipe and box sections subject to external loads. The design method is also applicable to the general class of under-reinforced concrete straight and curved slabs, beams, and members subject to combined axial force and bending. Heger and McGrath state that the provisions suggested for radial tension and diagonal tension are applicable to the type of structures covered in both the AASHTO Bridge Specification and the ACI Building Code. Of course, their discussion being summarized here was written prior to AASHTO's acceptance of the procedure. Heger and McGrath believe that the method provides significant improvements and possible economies in the design of both curved and straight members because the criteria for shear strength and crack control for slab-like members are more accurate than those given in other design procedures which have no criteria for the effects of curvature in thin flexural members. Heger and McGrath conclude from comparison of their procedure with others (particularly the former AASHTO criteria and ACI) that the other procedures are over-simplified and may seriously over-estimate shear strength for certain members with low reinforcement ratios subject to concentrated loads that cause $M/Vd$ ratios of 3 or more. The comparison indicates that the other procedures
underestimate the diagonal tension strength of uniformly loaded beams, frames, slabs, buried pipe, and other members with low M/Vd ratios.

Radial Tension

36. Heger and McGrath (1982a) give a detailed procedure for determining the maximum allowable area of flexural reinforcement as limited by radial tension when stirrups are not present. The procedure is based on a limited number of tests on portions of 84-inch diameter pipe sections (Heger and McGrath, 1982b), but reflects the state-of-the-art in the consideration of radial tension effects. The procedure is as follows:

\[
R_{rt} = \frac{t_{ru}}{t_{rc}}
\]

(18)

where:

\[
t_{ru} = \frac{(M_u - 0.45N_u d)}{bdr_s}
\]

\[
t_{rc} = 1.2(f_c')^{1/2}
\]

for circular pipe, \( r_s = 0.5(D_i + 2t_b) \)

37. Heger and McGrath also give Equation 19 as a close approximation of \( R_{rt} \).

\[
R_{rt} = \frac{A_{sfy}}{16r_s(f_c')^{1/2}}
\]

(19)

AASHTO uses a form of Equation 19 with \( R_{rt} \) taken to be 1.0, allowing the computation of a corresponding maximum allowable area of flexural steel. The computation of Equation 18 is recommended for greater accuracy.

38. The value of \( R_{rt} \) in Equation 18 is a function of \( M_u \) and \( N_u \), and it is obvious that the greatest value of \( R_{rt} \) will result when the difference between \( M_u \) and \( N_u \) is the greatest with \( M_u > N_u \). This occurs at the springing (\( \theta = 0 \) degrees) for the EM loading. Therefore, the designer should first compute \( R_{rt} \) for \( \theta = 0 \) degrees. If \( R_{rt} \) for \( \theta = 0 \) degrees is less than 1.0, no further evaluations of \( R_{rt} \) are required. If \( R_{rt} \) is computed to be greater than 1.0 by Equation 18, ties must be used to resist radial tension forces. The equation
for computing the required area of radial tension stirrups given in AASHTO is
the same as that of Heger and McGrath (1982a) and is given by:

\[ A_{vr} = 1.1s(M_u - 0.45N_u \theta d)/f_{vy}r_d \theta d \]  

(20)

where:

- \( A_{vr} \) = required area of stirrup reinforcement for radial tension
- \( s \) = circumferential spacing of stirrups \( (s_{\text{max}} = 0.750d) \)
- \( f_{vy} \) = maximum allowable strength of stirrup material \( (f_{\text{max}} = f_y) \)

### Diagonal Tension

39. The application of the Heger-McGrath procedure requires the knowledge of
the critical section \( (M_u/V_u \theta d = 3.0) \) location. \( M_u \) and \( V_u \) are determined from
Equations 4 and 5 using the factored load, \( w_u \). The \( \theta \) term is the capacity
reduction factor used to account for variations in construction with values of
0.90 to 0.95 suggested by Heger and McGrath and 0.90 required by AASHTO
(1984). Part V gives an example of locating the critical section. Appendix C
of Report 7 (Gerstle, 1988) gives expressions for determining the location of
the section where \( M_u/V_u \theta d = 3.0 \) for the EM loading conditions. From Equations
4 and 5:

\[ M/V = R(\cos2\theta)/2(\sin2\theta) = (R/2)(\cot2\theta) \]  

(21)

When \( M_u/V_u = 30d \), we have:

\[ \cot2\theta = 6\theta/R \]  

(22)

Since \( \theta = (h - 2d')/h \) and \( d = h - d' \), then \( d = h( + 1)/2 \) and:

\[ \cot2\theta = 6\theta/R = 3(1 + )\theta/(R/h) \]  

(23)

Therefore, \( \theta \) for the critical section may be computed for a known \( \theta \).

40. Once the location of the critical section is known, the ultimate shear
force \( (V_u) \) at the critical section may be computed from Equation 4 using the
factored load, \( w_u \). \( V_u \) is then compared to what Heger and McGrath term the "basic" shear strength, \( V_b \), defined by Equation 24.

\[
V_b = b\delta F_{vp}(4.4 + 250p)F_d(f_c')^{1/2}/(M_u/V_u\delta) + 1)F_cF_n
\]  

where:

- \( b \) = width of section (usually taken as 12 inches)
- \( \delta \) = capacity reduction factor (0.90)
- \( d \) = distance from compression face to centroid of tension reinforcement
- \( F_{vp} \) = factor for process of local materials that affect shear strength
- \( p \) = ratio of reinforcement area to concrete area
- \( F_d \) = factor for crack depth effect
- \( F_c \) = factor for effect of curvature on shear strength
- \( F_n \) = factor for effect of thrust on shear strength

Equation 24 reduces to Equation 25 when \( M_u/V_u\delta = 3.0 \).

\[
V_b = b\delta F_{vp}(f_c')^{1/2}(1.1 + 63p)F_d/F_cF_n
\]  

Heger and McGrath (1982a) report that \( F_{vp} \) can be 10 to 15 percent greater than 1.0 when inner reinforcement is comprised of multiple cages of welded wire fabric or with pipe manufacturing processes which densify the concrete or use certain angular and strong coarse aggregates in the concrete. The AASHTO specification presents Equation 25 and specifies that \( F_{vp} = 1.0 \) unless a higher value is substantiated by test data and approved by the Engineer. The depth factor, \( F_d \), provides an increase in shear strength as cross sections become thinner. Test results, as discussed by Heger and Gillespie (1967), show an increase in diagonal-tension strength which is cut off at a maximum \( F_d = 1.25 \) until more research is available. \( F_d \) is defined by Equation 26.

\[
F_d = 0.8 + 1.6/\delta
\]  

The curvature factor, \( F_c \), modifies the shear strength of a curved member to account for the increased or decreased nominal shear stress in a curved member compared to the shear stress resulting from the same force in a
straight member. Heger and McGrath explain that when bending produces tension on the inside of the curved member, compressive thrust reduces as bending increases, resulting in an increase in nominal shear stress for a given shear force. When bending produces tension on the outside of the curved member, compressive thrust increases with bending moment, resulting in a decrease in nominal shear stress for a given shear force. \( F_c \) is defined by Equation 27.

\[
F_c = 1 \pm \left( \frac{d}{2r} \right) (\text{tension on the inside of the pipe})
- \left( \text{tension on the outside of the pipe} \right) \\
\]

where \( r \) = radius to centerline of pipe wall

43. \( F_n \), the thrust factor, provides for the increase in shear strength that occurs when a compressive thrust force, \( N_u \), acts on a section in combination with a shear force \( V_u \) and a moment \( M_u \). The factor also accounts for the reduction in shear strength occurring when a tensile thrust acts in combination with \( V_u \) and \( M_u \). \( F_n \) is defined by Heger and McGrath as follows:

\[
F_n = 0.5 - \left( \frac{N_u}{6V_u} \right) + \left[ 0.25 + \left( \frac{N_u}{V_u} \right)^2 \right]^{1/2} \\
\]

\( N_u \) is positive for compression and negative for tension.

44. Heger and McGrath give approximations for \( F_n \) as follows:

\[
F_n = 1.0 - 0.12\frac{N_u}{V_u} \quad 0 < \frac{N_u}{V_u} < 2.1 \\
F_u = 0.82 - 0.003\frac{N_u}{V_u} \quad 2.1 \leq \frac{N_u}{V_u} < 4.0 \\
F_u = 0.7 \quad 4.0 \leq \frac{N_u}{V_u} < \infty \\
F_u = 1.0 - 0.24\frac{N_u}{V_u} \quad -2.0 \leq \frac{N_u}{V_u} < 0 \\
\]

The AASHTO specification gives only the expression of Equation 29a. Equation 28 should be used if this procedure is incorporated into a computer program, otherwise the use of Equations 29a through 29d is recommended.

45. The maximum value allowed for \( f'_c \) is 7000 psi since Equation 24 was
formulated from test data with $f'_c$ below that value. The reinforcement ratio $p = A_s/bOd$ is limited to 0.02. If the actual value of $p$ is greater than 0.02, then 0.02 should be used in Equations 24 and 25.

46. If the applied shear force, $V_u$, at the critical section is greater than the computed basic shear strength, three options may be considered for increasing the shear strength of the section. The options, as presented by Heger and McGrath (1982a) are as follows:

a. Increase $A_s$ to the following value and re-compute $V_b$:

$$A_s = \left[ 0.016 F_c F_n V_u / F_d (f'_c)^{1/2} \right] - 0.0175 bOd$$  \hspace{1cm} (30)

If $V_u > V_b$, it is probably not economical to increase the diagonal tension strength by this method. All previously discussed limits on $A_s$ must be considered.

b. Increase the concrete compressive strength to the following value and re-compute $V_b$:

$$f'_c = 0.91 V_u F_c F_n / bOd (1.0 + 57p) F_d$$  \hspace{1cm} (31)

$$r = 0.5 (D_i + 2t_b) \text{ for circular pipe}$$

c. Provide stirrups. The circumferential spacing of stirrups should not exceed $0.750d$. The required area of stirrups per cross-sectional width, $b$, is given as:

$$A_{Vs} = (1.1s/f_v 0d) [V_u F_c - V_c] + A_{Vr}$$  \hspace{1cm} (32)

where:

$A_{Vr}$ is given by Equation 20.

$V_c = V_b$ as defined by Equation 24.

$V_{cmax} = 2Od (f'_c)^{1/2}$

Equation 32 includes the effect of radial tension; therefore, $A_{Vr}$ and $A_{Vs}$ should not be added to give a total required area of stirrups. If stirrups
are not needed to resist diagonal tension forces, but are needed for radial
tension, only the area $A_{v_f}$ is required.

**Extent of Stirrups**

47. For the case where stirrups are required for radial tension only, stirrups should be provided at all sections where $R_{rt}$ as computed by Equation 18 is greater than 1.0. If stirrups are required at the critical section due to diagonal tension, the extent of the structure that requires stirrups may be determined by comparing $V_u$ to the basic shear strength computed from Equation 24 at various sections. Stirrups will be required where $V_u$ is greater than $V_b$. Equation 24 must be used for this procedure since $V_b$ varies with $M_u$ and $V_u$, which vary within the structure. The AASHTO criteria do not give guidance for determining the extent of stirrup placement. As mentioned before, AASHTO only presents the form of the expression for $V_b$ that results from letting $M_u/V_uOd = 3.0$. Therefore, it is not obvious that $V_b$ may be computed at other locations. Apparently, the AASHTO procedure requires the placement of stirrups throughout the structure at a spacing and with an area corresponding to that required at the critical section. For economy, the required extent of stirrup placement should be determined using Equation 24. The designer should give consideration to the practicality of construction when determining the extent of stirrup placement.

**Summary of Procedure**

48. The following summarizes the steps required in the design for shear and radial tension.

1. Check maximum allowable area of flexural reinforcement as limited by radial tension when stirrups are not present using Equation 18. If $R_{rt}$ for $\theta = 0$ degrees is less than 1.0, radial tension is not a problem; otherwise, ties must be used to resist radial-tension forces. The required area of radial-tension stirrups is computed by Equation 20.
2. For diagonal-tension evaluation, locate the critical section by Equation 22 or 23. Compute $V_u$ at the critical section by Equation 4.

3. Compare $V_u$ to $V_b$ computed by Equation 25. If $V_u > V_b$, provide stirrups or increase $A_s$ or $f_c$ to values determined by Equations 30 and 31, respectively. For stirrups, compute the required area by Equation 32. If stirrups are needed for radial tension, but not diagonal tension, then only the area $A_{vr}$ in Equation 32 is required.

4. If stirrups are needed only for radial tension, provide them at all sections where $R_{rb}$ of Equation 18 is greater than 1.0. If stirrups are required at the critical section for diagonal tension, determine the extent of the structure requiring stirrups by comparing $V_u$ to $V_b$ (Equation 24) at all locations.
PART V: EXAMPLE APPLICATIONS OF DESIGN PROCEDURE

Example 1. Stirrups Required

49. The following is an example of the design of a buried circular conduit using the flexure-shear design procedure discussed in this report. Although the entire procedure is demonstrated, more detail is given to the criteria established by Heger and McGrath, since Gerstle (1988) demonstrated the flexure portion of the procedure. In order to maximize the use of previous work, the example will begin with the problem definition of Gerstle’s example given in Part V of Report 7 and expand it to include the evaluation of allowable limits on the area of flexural reinforcement and the design for shear and radial-tension effects.

50. A 72-inch inside diameter conduit with a wall thickness of 7 inches for the circular conduit is to be designed for the factored loads shown in Figure 4. Other specified parameters include: concrete cover = 1 inch; $f'_c = 4.8$ ksi; and $f_y = 88$ ksi. The area of flexure reinforcement and, if needed, the area of and location of stirrups for diagonal-tension and radial-tension effects are required.

![Diagram of conduit with loads](image)

- Factored vertical distributed loading, $w_u = 24$ kips/ft$^2$
- Factored lateral distributed loading, $k_{wu} = 12$ kips/ft$^2$

Figure 4. Example 1 Problem Definition
51. Gerstle generated strength envelopes for equal inside and outside reinforcing for his example. The same envelopes will be used herein. As Gerstle stated, the incorporation of this design procedure into a computer program which generates strength envelopes for unequal inside and outside reinforcement will aid in the design of economical pipe structures.

**Flexure**

\[
R = \left( \frac{D_i + h}{2} \right) - \frac{(72 + 7)}{2} = 39.5 \text{ in}
\]

\[
R/h = 39.5/7 = 5.64
\]

The cross-section parameter, \( \gamma = \frac{(h - 2d')}{2} \) is approximately equal to: \([7 - 2(1)]/7 = 0.71; \) use 0.7.

\( K = \frac{\text{lateral uniform load}}{\text{vertical uniform load}} = 12 \text{ ksf}/24 \text{ ksf} = 0.5 \)

It is convenient to let \( b = 12 \text{ in.} \)

Units: inches per foot length of pipe.

\[
\omega_u = [24 \text{ ksf (1 ft/12 in)}] \times 1 \text{ ft} = 2 \text{ k/in}
\]

\[
n_{sp} = \omega_u R/f_c' bh = \frac{(2 \text{ k/in} \times 39.5 \text{ in})}{(4.8 \text{ k/in}^2 \times 12 \text{ in} \times 7 \text{ in})} = 0.196
\]

\[
n_{cr} = K \omega_u R/f_c' bh = 0.5n_{sp} = 0.098
\]

\[
h/e_{sp} = \frac{4}{(1 - K)}[1/(R/h)] = \frac{4}{(1 - 0.5)}(1/5.64) = 1.418
\]

\[
h/e_{cr} = \frac{4K}{(1 - K)}[1/(R/h)] = \frac{4(0.5)}{(1 - 0.5)}(1/5.64) = 0.709
\]

Enter graph (Figure 5) for \( \gamma = 0.7: \) Springing lines intersect at \( \omega_{sp} = 0.18, \) and crown lines intersect at \( \omega_{cr} = 0.27. \)

52. The crown section intersects the strength envelope first. Therefore, the crown section controls with a value of 0.27 required for \( \omega \) as determined from Figure 5. The required flexural reinforcement is determined as:

\[
P_{\text{total required}} = \frac{\omega f_c'}{f_y} = 0.27(4.8 \text{ ksi}/88 \text{ ksi}) = 0.0147
\]

\[
A_s(\text{total required}) = pbh = 0.0147(12 \text{ in})(7 \text{ in}) = 1.24 \text{ in}^2
\]
53. For equal inner and outer cages: $A_{s1} = A_{s0} = 1.24 \text{ in}^2/2 = 0.62 \text{ in}^2/\text{ft. continuous ring reinforcement}$

A possible choice of reinforcement is two #5 deformed bars:

$$A_{s1} = A_{s0} = 2(0.31 \text{ in}^2) = 0.62 \text{ in}^2.$$  ok

#5 bar implies $d = 7 \text{ in} - [1 + (1/2)(5/8)] = 5.7 \text{ in}$

**Limits for Flexural Reinforcement**

**Limits for Minimum Reinforcement:**

Heger and McGrath:

$$\min A_{s1} = 0.002bh$$

$$= 0.002(12)(7) = 0.168 \text{ in}^2$$

$$0.168 < 0.62 \text{ in}^2$$  ok

$$\min A_{s0} = 0.0015bh$$

$$= 0.0015(12)(7) = 0.126 \text{ in}^2$$

$$0.126 < 0.62 \text{ in}^2$$  ok

**Limits due to concrete compression:**

$$A_{sc} = [55,000bBf_c'\varnothing d/f_y(87000 + f_y)] - (0.75N_u/f_y)$$  Eq. 15

$$B = 0.85 - 0.05(4800 - 4000)/1000 = 0.81$$

$$0.65 \leq B \leq 0.85$$  ok

$$N_u = w_uR$$

$$= 2 \text{ k/in}(39.5 \text{ in})$$

$$= 79 \text{ kips} = 79,000 \text{ lbs}$$

$$\varnothing = 1.0 \text{ (flexure)}$$

$$d = 5.7 \text{ in}$$

$$A_{sc} = ([55,000(12)(0.81)(4800)(1.0)(5.7)]/88000(87000 + 88000)) -$$

$$[0.75(79000)/88000]$$

$$A_{sc} = 0.276 \text{ in}^2 < A_{s1} = A_{s0} = 0.62$$  modifications required

If ties are provided, $A_{sc}$ may be increased by $0.75A_s$:

$$0.75(0.62) = 0.47$$
Now, \( A_{sc} = 0.276 + 0.47 - 0.75 \text{ in}^2 > 0.62 \) \text{ ok}

54. The concrete compression limit is met when ties are provided. Since ties may be needed for shear or radial tension, the procedure should proceed on that assumption. If ties are later found to not be needed for shear or radial tension, then it may be more economical to increase the section thickness or the concrete compressive strength rather than provide ties to satisfy this requirement.

**Limits due to Crack Width:**

\[
F_{cr} = \left( [M_s + N_s(d - h/2)/ij] - [C_1 h^2 (f_c')^{1/2}] B_1 / 30,000 d A_s \right) \text{ Eq. 17}
\]

for deformed bars:

- \( B_1 = (0.5 t_b^2 s/n)^{1/3} \)
- \( A_s = 0.62 \)
- \( d = 5.7 \)

\[
F_{cr} = \left( [M_s + N_s(5.7 - 7/2)/ij] - [1.9(12)(7^2)(4800)^{1/2}] 1.44 / 30,000(5.7)(.62) \right) \left[ 1/[1 - (jd/e)] - 1/[1 - (5.7j/e)] \right] \]

\[
e = M/N + d - h/2 \quad M \text{ and } N \text{ always positive.}
\]

Assuming load factors from ETL 1110-2-312 of 1.9(Dead Load + Live Load), the service loads for this example are:

- \((24 \text{ k/ft}^2)/1.9 = 12.63 \text{ k/ft}^2 \) vertical
- \((12 \text{ k/ft}^2)/1.9 = 6.32 \text{ k/ft}^2 \) lateral

Similar to the determination of \( w_u \):

- \( w_s = [12.63 \text{ k/ft}^2(1 \text{ ft}/12 \text{ in})](1 \text{ ft}) = 1.05 \text{ k/in} \)

\( M_s, N_s, M, \) and \( N \) may be computed from Equations 3 and 5 with \( w = w_s \).
Table 1. $F_{cr}$ Analysis

<table>
<thead>
<tr>
<th>$\Theta$ (degrees)</th>
<th>$N_s$ (lbs)</th>
<th>$M_s$ (in-lbs)</th>
<th>$e$</th>
<th>$j$</th>
<th>$i$</th>
<th>$E_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>41,570</td>
<td>204,780</td>
<td>7.13</td>
<td>0.87</td>
<td>3.28</td>
<td>0.36</td>
</tr>
<tr>
<td>45</td>
<td>31,180</td>
<td>0</td>
<td>6.56*</td>
<td>0.86</td>
<td>3.89</td>
<td>-0.77</td>
</tr>
<tr>
<td>90</td>
<td>20,790</td>
<td>-204,780</td>
<td>12.05</td>
<td>0.95</td>
<td>1.82</td>
<td>-2.30</td>
</tr>
<tr>
<td>135</td>
<td>31,180</td>
<td>0</td>
<td>6.56*</td>
<td>0.86</td>
<td>3.89</td>
<td>-0.77</td>
</tr>
<tr>
<td>180</td>
<td>41,570</td>
<td>204,780</td>
<td>7.13</td>
<td>0.87</td>
<td>3.28</td>
<td>0.36</td>
</tr>
</tbody>
</table>

*minimum e/d = 1.15 implies min. e = 6.56

$F_{cr}$ remains significantly less than 1.0. The probability of a 0.01-inch crack is very small.

Radial Tension

$$R_{rt} = \frac{t_{ru}}{t_{rc}}$$  \hspace{1cm} \text{Eq. 18}

$$t_{rc} = 1.2(4800)^{1/2} = 83.14$$

$$r_s = 0.5(72 + 2(1)) = 37 \text{ in}$$

Table 2. Radial Tension Analysis

<table>
<thead>
<tr>
<th>$\Theta$ (degrees)</th>
<th>$M_y$ (in-lbs)</th>
<th>$N_u$ (lbs)</th>
<th>$E_{ru}$</th>
<th>$R_{rt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>390,060</td>
<td>79,000</td>
<td>74.06</td>
<td>0.89</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>59,250</td>
<td>-60.05</td>
<td>-0.72</td>
</tr>
<tr>
<td>90</td>
<td>-390,060</td>
<td>39,500</td>
<td>-194.16</td>
<td>-2.34</td>
</tr>
<tr>
<td>135</td>
<td>0</td>
<td>59,250</td>
<td>-60.05</td>
<td>-0.72</td>
</tr>
<tr>
<td>180</td>
<td>390,060</td>
<td>79,000</td>
<td>74.06</td>
<td>0.89</td>
</tr>
</tbody>
</table>

55. As mentioned in the procedure given in Part IV, the value of $R_{rt}$ is greatest when $\Theta = 0$ or 180 degrees. For this example, $R_{rt}$ equals 0.89 or less throughout the structure. Since 0.89 < 1.0, radial tension is not a problem.
Compare this value to that of the approximate solution of $R_{rt}$ developed by Heger and McGrath and used by AASHTO:

$$R_{rt} = \frac{A_{sf}f_y}{[16r_s(f'_c)]^{1/2}} \quad \text{Eq. 19}$$

$$R_{rt} = 0.62(88,000)/[16(37)(4800)^{1/2}] = 1.33$$

The value of $f_y$ plays a greater role in Equation 19 than in Equation 18. The value of $f_y = 88,000$ psi is greater than that of most ductile steels. A $f_y$ of 65,000 yields an $R_{rt}$ value of 0.98 in Equation 19. The more accurate computation given by Equation 18 is recommended.

**Diagonal Tension**

Locate the assumed critical section ($M_u/V_uO_d = 3.0$):

$$\gamma = 0.7 \quad \beta = 0.9$$

$$\cot 2\theta = \frac{60d}{R} \quad \text{Eq. 22}$$

or

$$\cot 2\theta = \frac{30(1 + \gamma)}{(R/h)} \quad \text{Eq. 23}$$

From Equation 22:

$$\cot 2\theta = \frac{[6(.9)(5.7 \text{ in})]/39.5 \text{ in}} = 0.78$$

$$\theta = 52.05 \text{ degrees}$$

$$\alpha = 26.0 \text{ degrees}$$

Determine $M_u$ and $V_u$:

$w_u = 2 \text{ k/in}$

$$M_u = [(2 \text{ k/in})(39.5 \text{ in})^2/4](1 - .5)(\cos 52.05) = 239.88 \text{ in-k}$$

$$V_u = [2(39.5)/2](1 - .5)(\sin 52.05) = 15.57 \text{ k}$$

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Basic Shear Strength:

\[ V_b = b\theta d F_{vp}(f_c')^{1/2}(1.1 + 63p)F_d/(F_cF_n) \]  
\[ F_{vp} = 1.0 \]  
\[ F_d = 0.8 + 1.6/0.9(5.7) = 1.11 \]  
\[ F_c = 1 + 0.9(5.7)/2(39.5) = 1.07 \]  
\[ N_u = [2(39.5)/2][(1 + .5) + (1 - .5)(\cos 52.05)] = 71.40 \] k  
\[ N_u/V_u = 4.59, \text{ therefore:} \]  
\[ F_n = 0.7 \]  
\[ P = P_{\text{total}}/2 = 0.0147/2 = 0.0074 \]  
\[ V_b = (12)(0.9)(5.7)(1.0)(4800)^{1/2}[1.1 + 63(0.0074)]/(1.11)/(1.07)(0.7) \]  
\[ V_b = 9899 \text{ lb} = 9.90 \text{ k} \]

Compare \( V_b \) to \( V_u = 15.57 \text{ k} \): \( V_u \) is much greater than \( V_b \).

Therefore, increases in \( A_s \) or \( f_c' \) to increase \( V_b \) are not practical.

Provide stirrups.

\[ A_{vs} = (l.1s/f_o\theta d)[V_uF_c - V_c] + A_{vr} \]  
\[ V_c = V_b \]

But, \( V_c \max = 2b\theta d(f_c')^{1/2} = 8.53 \text{ k} \).

Try maximum allowable spacing: \( s_{\text{max}} = 0.75\theta d = 3.85 \text{ in} \):

From Equation 20:

\[ A_{vr} = [1.1(3.85)/(65000)(40.81)(0.9)(5.7)][239,880 - 0.45(71,400)(0.9)(5.7)] \]

\[ r_s = D_i/2 + t_b + d_b/2 = 40.81 \]

\[ A_{vr} = 0.0234 \]
Therefore:

\[
A_{VS} = [1.1(3.85)/(65000)(.9)(5.7)][15,570(1.07) - 8,530] + .0234
\]

\[
A_{VS} = 0.103 + 0.0234 = 0.13 \text{ in}^2
\]

\(A_{VS}\) is the area of stirrup required per width \(b\). The flexural design resulted in two No. 5 bars per width \(b\) to resist bending stresses.

The area of a D7 deformed wire = 0.07 \(\text{in}^2\). Therefore, use D7 deformed wire stirrups (one per bar of flexural reinforcement) at a circumferential spacing of 3.85 in.

**Extent of Stirrups**

Determine the extent of stirrups using Equation 24.

Recall that the critical section is at \(\theta = 26.0\) degrees.

minimum \(F_n = 0.7\)

<table>
<thead>
<tr>
<th>(\theta) degrees</th>
<th>(M_u) in-lb</th>
<th>(V_u) lb</th>
<th>(N_u) lb</th>
<th>(F_B)</th>
<th>(F_C)</th>
<th>(V_b) lb</th>
<th>Need Stirrups?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>366,539</td>
<td>6,755</td>
<td>77,809</td>
<td>0.70</td>
<td>1.07</td>
<td>3,412</td>
<td>yes</td>
</tr>
<tr>
<td>45</td>
<td>0</td>
<td>19,750</td>
<td>59,250</td>
<td>0.73</td>
<td>1.07</td>
<td>37,880</td>
<td>no</td>
</tr>
<tr>
<td>80</td>
<td>-366,539</td>
<td>6,755</td>
<td>40,691</td>
<td>0.70</td>
<td>0.74</td>
<td>4,934</td>
<td>yes</td>
</tr>
<tr>
<td>50</td>
<td>-67,734</td>
<td>19,450</td>
<td>55,820</td>
<td>0.73</td>
<td>0.74</td>
<td>32,626</td>
<td>no</td>
</tr>
<tr>
<td>65</td>
<td>-250,727</td>
<td>15,129</td>
<td>46,555</td>
<td>0.73</td>
<td>0.74</td>
<td>12,947</td>
<td>yes</td>
</tr>
<tr>
<td>60</td>
<td>-195,031</td>
<td>17,104</td>
<td>49,375</td>
<td>0.73</td>
<td>0.74</td>
<td>16,996</td>
<td>yes</td>
</tr>
<tr>
<td>55</td>
<td>-133,409</td>
<td>18,559</td>
<td>52,495</td>
<td>0.74</td>
<td>0.74</td>
<td>22,502</td>
<td>no</td>
</tr>
<tr>
<td>30</td>
<td>195,031</td>
<td>17,104</td>
<td>69,125</td>
<td>0.70</td>
<td>1.07</td>
<td>12,258</td>
<td>yes</td>
</tr>
<tr>
<td>35</td>
<td>133,409</td>
<td>18,559</td>
<td>66,005</td>
<td>0.71</td>
<td>1.07</td>
<td>16,220</td>
<td>yes</td>
</tr>
<tr>
<td>37</td>
<td>107,516</td>
<td>18,985</td>
<td>64,694</td>
<td>0.72</td>
<td>1.07</td>
<td>18,255</td>
<td>yes</td>
</tr>
<tr>
<td>57</td>
<td>-158,653</td>
<td>18,043</td>
<td>51,217</td>
<td>0.73</td>
<td>0.74</td>
<td>20,181</td>
<td>no</td>
</tr>
</tbody>
</table>

57. Stirrups are not needed in the region for \(\theta\) from about 37 to 57 degrees in order to resist diagonal tension. This region exists in each quadrant of the circular conduit due to symmetry. The percentage of the structure not needing stirrups to resist diagonal tension stresses is about 22 percent.
Although a portion of the structure does not require shear stirrups, consideration must be given to the requirements based on the maximum allowable area of flexural reinforcement as determined by the compression strength of the section. Earlier in this example it was concluded that if stirrups are used to tie the inner and outer cages of reinforcement together, then the compression strength of the section will be adequate. Only the section where $N_u$ is maximum was checked for compression strength earlier. Table 4 shows values of $A_{sc}$ in the region where shear stirrups are not required.

<table>
<thead>
<tr>
<th>$\theta$ degrees</th>
<th>$A_{sc}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.502</td>
</tr>
<tr>
<td>50</td>
<td>0.474</td>
</tr>
<tr>
<td>35</td>
<td>0.387</td>
</tr>
</tbody>
</table>

It is obvious that $A_{sc}$ is less than the flexural reinforcement area in this region. Actually, this was already known since the reinforcement is continuous and the limiting value was computed by Equation 15. Since the use of ties was found to remedy this problem at the location of maximum $N_u$, it will also satisfy the requirements of this region. Therefore, stirrups are required throughout the structure. If it is desirable (practical), the spacing of the stirrups in the region for $\theta = 37$ to 57 degrees may be increased to 7 inches based on the criteria for maximum spacing given in paragraph 25 of this report.
Example 2. Stirrups Not Required

59. The previous example demonstrated the design of a conduit requiring stirrups, in order to demonstrate the entire design procedure. The following example applies the radial and diagonal tension criteria to a more realistic problem (Figure 6) taken from Liu (1980).

Figure 6. Example 2 Problem Definition
Given (per foot length of conduit):

\[ b = 12'' \], \[ h = 48'' \], \[ R = 132'' \], \[ f_c' = 4 \text{ ksi} \], \[ f_y = 40 \text{ ksi} \]
\[ A_{so} = 1.27 \text{ in}^2 \], \[ A_{sl} = 1.27 \text{ in}^2 \] except at crown (2.54 in²)

**Limits for Flexural Reinforcement**

**Limits for Minimum Reinforcement:**

Heger and McGrath:

\[ \text{min } A_{si} = 0.002bh \quad \text{Eq. 13} \]
\[ = 0.002(12)(48) = 1.15 \text{ in}^2 \]
\[ 1.15 < 1.27 \text{ ok} \]

\[ \text{min } A_{so} = 0.0015bh \quad \text{Eq. 14} \]
\[ = 0.86 \text{ in}^2 \]
\[ 0.86 < 1.27 \text{ ok} \]

**Limits due to Concrete Compression:**

Using Equation 15:

\[ A_{sc} = \left( \frac{[55,000(12)(0.85)(4000)(1.0)(43.37)]}{40000(87000 + 40000)} \right) = 15.5 \text{ in}^2 > A_{si} = A_{so} = 1.27 \text{ in}^2 \text{ ok} \]

where: \[ N_u = w_uR = (17.55 \text{ k/ft}^2)(1 \text{ ft})(1 \text{ ft/12 in})(132 \text{ in}) = 193.05 \text{ k} \]
\[ d = 43.37 \text{ in} \]

**Limits due to Crack Width:**

60. Since the reinforcement concrete cover (4.0 in) is greater than 1.5 inches, Equation 17 does not apply for an evaluation of crack width.

Therefore, the limits of ETL 1110-2-312 apply such that \( p < 0.375 \) \( p_b \) and reinforcing spacing is less than 18 inches for Grade 40 steel and 12 inches of Grade 60 steel. \text{ ok}

**Radial Tension**

\[ R_{rt} = \frac{t_{ru}}{t_{rc}} \quad \text{Eq. 18} \]
\[ t_{rc} = 1.2(4000)^{1/2} = 75.89 \]
\[ r_s = 0.5(108 + 2(4)) = 112 \text{ in} \]
\[ w_u = 1.46 \text{ k/in} \quad K = 0.33 \]

Use Equations 3 and 5 for moments and thrusts.
Table 4. Radial Tension Analysis

<table>
<thead>
<tr>
<th>$\theta$ degrees</th>
<th>$N_u$ lbs</th>
<th>$M_u$ in-lbs</th>
<th>$t_{ru}$</th>
<th>$R_{rt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>192,720</td>
<td>4,239,861</td>
<td>30.63</td>
<td>0.40</td>
</tr>
<tr>
<td>45</td>
<td>128,477</td>
<td>0</td>
<td>8.25</td>
<td>0.11</td>
</tr>
<tr>
<td>90</td>
<td>64,234</td>
<td>-4,239,861</td>
<td>-94.25</td>
<td>-1.24</td>
</tr>
</tbody>
</table>

$R_{rt}$ is always greatest when $\theta = 0$ degrees. Since $R_{rt}$ equals 0.40 or less throughout the structure, and $0.40 < 1.0$, radial tension is not a problem.

Diagonal Tension

Locate the assumed critical section ($M_u/V_u \theta d = 3.0$):

$$\gamma = \frac{(h-2d')}{h} \quad \text{Eq. 22}$$

$$\cot 2\theta = \frac{6d}{R}$$

$$\cot 2\theta = \frac{6(0.9)(43.37)}{132} = 1.77$$

$2\theta = 29.41$ degrees

$\theta = 14.7$ degrees

$w_u = 1.46$ k/in

$$M_u = \frac{[(1.46 \text{ k/in})(132 \text{ in})^2/4](1 - 0.33)(\cos 29.41)}{1 - 0.33}(\cos 29.41)$$

$= 3,693.6$ in-k

$$V_u = \frac{[(1.46 \text{ k/in})(132)/2](1 - 0.33)(\sin 29.41)}{1 - 0.33}(\sin 29.41)$$

$= 31.55$ k

$$N_u = \frac{[(1.46 \text{ k/in})(132)/2][(1 + .33) + (1 - .33)(\cos 29.41)]}{1 - 0.33}(\cos 29.41)$$

$= 184.44$ k
Basic Shear Strength:

Using Equation 25 for $V_b$:

$$F_{vp} = 1.0$$

$$F_d = 0.8 + 1.6/(0.9)(43.37) = 0.84$$

$$F_c = 1 + 0.9(43.37)/2(132) = 1.15$$

$$N_u/V_u = 5.85, \text{ therefore } F_n = 0.7$$

$$p = 1.27/12(43.37) = 0.0024 \text{ at } \theta = 14.7 \text{ degrees}$$

$$V_b = 12(.9)(43.37)(1.0)(4000)^{1/2}[1.1 + 63(.0024)(.84)/[(1.15)(.7)]$$

$$= 38,677 \text{ lb } = 38.7 \text{ k}$$

Compare $V_b$ to $V_u = 31.6 \text{ k} : \ V_b > V_u$, stirrups not needed.
PART VI: SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary

61. A strength-design procedure accounting for both flexure and shear in buried reinforced concrete circular conduits subjected to the EM loading is presented. The procedure is based on previous research by Gerstle (1988) on design for flexure and on research by Heger and McGrath (1982a) on design for shear. The design procedure includes expressions to account for the effects of the section's concrete compression strength and radial tension on the strength of the section. Criteria for limiting the flexural reinforcement ratio to prevent unacceptable crack widths are also included. Detailed examples of the new procedure are presented.

Conclusions

62. Designs for buried circular conduits developed by this SD procedure will account for flexure, shear, radial tension, and crack-width control. Although the procedure is conservative, the degree of conservatism is not known due to the engineering difficulties in defining shear strength of and loading on buried conduits.

63. The design procedure is slightly tedious, but easy to execute. The flexure design portion will be very easy to perform by hand if strength envelope design aids are developed. The entire design procedure may be programmed for personal computers without great difficulty.

64. Section 17 of the AASHTO Bridge Specification incorporates a simplified version of the Heger-McGrath shear and radial tension procedure. The procedure presented in this report combines the simplicity of the AASHTO procedure with the accuracy of the Heger-McGrath procedure where possible. The consideration of the EM loading allows the formulation of expressions that
give bounding values for some of the parameters involved, thereby reducing the computational effort.

**Recommendations**

65. The procedure presented in this report is recommended for consideration as part of the strength design for buried reinforced circular conduits. The development of both strength-envelope design charts and a personal-computer program of the entire procedure will result in a practical approach to the design of these structures.

66. The procedure should be extended to other shapes of buried conduits. As recognized by Gerstle (1988), the Heger-McGrath shear criteria have some inadequacies, but are considered to be a conservative approach. Additional research on the shear strength of structures of this type may result in the development of a less conservative procedure resulting in more economical designs of conduits. Also, the EM loading condition is conservative. A study comparing the EM loading with current finite-element computer programs such as the one mentioned in this report (SPIDA) may lead to an estimate of the degree of conservatism of the EM loading and result in an improved procedure for the determination of loading conditions.
REFERENCES


APPENDIX

NOTATION
$A_g$  Gross area of section, square inches

b  Width of section, inches

d  Depth from compression face of section to the centroid of the tension reinforcement (effective depth), inches

$D_i$  Inside diameter of circular pipe, inches

$f_y$  Yield strength of reinforcement, psi

$f_c'$  Compressive strength of concrete, psi

h  Wall thickness, inches

$N_u$  Axial thrust due to factored loads, lbs

p  Tension reinforcement ratio

$p_b$  Balanced reinforcement ratio

$t_b$  Concrete cover on reinforcement, inches