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22a. NAME OF RESPONSIBLE INDIVIDUAL: Dr. Arje Nachman

22b. TELEPHONE (Include Area Code): (202) 767-4939

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Numerical Solution of Ill Posed Problems in Partial Differential Equations

(October 1, 1984 - September 30, 1987)

by

Howard A. Levine
Department of Mathematics
Iowa State University
Ames, Iowa 50011

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I. Summary

This project is concerned with several questions concerning the existence, uniqueness, continuous data dependence and numerical computation of solutions of various ill posed problems in partial differential equations.

Several problems involving reaction diffusion equations with and without convection terms present were studied. In the latter case the ability of finite element approximate solutions to reproduce the continuous time dynamics was investigated. In the former case, a convective diffusion equation with a semilinear source in the boundary condition was analyzed. A fairly complete picture of the dynamics was obtained. With the source term in the equation, computations revealed a rich structure which has been partially analyzed theoretically.

Several problems for reaction diffusion equations in unbounded regimes were also investigated. It was shown that under certain conditions in the rate law all nonzero solutions blow up in finite time, while for other conditions in the rate law, solutions damp out.

It was shown that a potential well theory is possible for certain hyperbolic problems in which a nonlinear boundary condition is prescribed and not possible in certain cases when the forcing term in the differential equation is singular.

Numerical experiments performed on the wave equation with a singular forcing term have down that when quenching occurs, the time and exact derivatives blow up in finite time. The nature of the blowup was studied computationally.

An investigation was begun into the study of the existence of nonconstant (in time) time periodic solutions of semilinear wave equations in exterior domains (breathers). Necessary and sufficient conditions for the existence of such solutions were given.

Continuous data dependence results were established for certain classes of initial value problems.

Some seismic inverse problems were studied both analytically and numerically with a view toward recovering either the elastic coefficients or any source terms for problems in a layered half space. Such problems, being ill posed, require careful analysis from both points of view. Some new results for coefficient determination problems were obtained, which provide algorithms for computational purposes.

In the study of certain eigenvalue problems for the Laplacian, several interesting, geometry dependent, inequalities between Dirichlet and Neumann eigenvalues were obtained.

Numerical solution of a modified pendant drop equation demonstrated the existence of positive radial solutions of the equation in all of space.

II. Research Objectives

A. Reaction-Diffusion Equations. Here we are concerned with several types of problems involving reaction diffusion equations. In some of these problems, convection terms are present in the equation as for example in Burger's equation with a power law nonlinearity. In other problems, the underlying geometry is some unbounded region such as a cone or the exterior of a sphere. In still other problems, the reaction term is singular as arises in so called quenching phenomena in semiconductor theory. Still other problems arise in forced flows through porous media. In some problems the reaction terms enter through the boundary condition as for example in problems in penetrative convection. In others, the reaction terms are nonlocal appearing in the boundary condition or in the governing equation or perhaps both. We are studying these problems from the point of view of examining the long time behavior of their solutions.

B Semilinear Wave Equations. In their paper on quenching for solutions of wave equations with a singular forcing term, Levine and Chang showed that if $\epsilon > 0$ was large then solutions of the initial boundary value problem for the equation

$$u_{tt} = u_{xx} + \epsilon/(1-u)$$

with zero boundary and initial data reached one in finite time (quenched). We are concerned with the behavior of the derivatives at quenching time. Specifically, we want to know which derivatives in the equation blow up and the nature of the growth.

We consider, in several space dimensions, and initial-boundary value problem for the wave equation with a singular nonlinearity:

$$u_{tt} = \Delta_n u + \epsilon (1-u)^{-\beta}, \ (\beta > 0, \ \epsilon > 0).$$

we seek conditions on ϵ and the initial data for which the solution remains less than one (pointwise) for all time. The problem is complicated by the lack of an embedding inequality from H_0^1 into L^∞ for more than one space dimension.

We have been interested in the questions of existence and qualitative behavior of time-periodic solutions of the nonlinear wave equation

$$u_{tt} - \Delta_n u + g(u) = f,$$

in which u and f are real-valued functions defined on $R^n \times R$ $(n \geq 1)$ and $g:R \to R$. The function f is assumed to be T-periodic in time, radially symmetric in space and square-integrable over $(0,T)\times R$. A solution u should have the same properties. If $g(u) = m\,u + \sin(u)$ then the equation is the forced sine-Gordon equation with mass term.

A third problem of importance is the initial boundary value problem for the wave equation with a large oscillating nonlinearity, for example,

$$u_{tt} = u_{xx} + u^3 \cos u.$$

when the factor of cos(u) is replaced by either one or minus one, the dynamical problem is well studied and reasonably well (although not completely) understood. In this case, however, there is no general theory and many surprises as, for example, the existence of infinitely many positive stationary solutions. We want to investigate (numerically) the stability of these ground states.

In 1974, L. E. Payne and D. H. Sattinger developed a potential well theory and a corresponding existence - nonexistence theorem for initial-boundary value problems of the form

$$u_{tt} = \Delta_n u + f(u)$$

where (i) f is either a convex point function or (ii) convex for $u > 0$, concave for $u < 0$ and grows like $|u|^p$ for $p > 1$ at infinity. The arguments in the second case were seriously flawed. We consider the problem in this case, as well as the question of developing analogous results for a similar problem when the equation is linear but when $\partial u/\partial n = f(u)$ on a portion of the boundary of the spatial domain. We seek similar results for parabolic problems.

C. Inverse Problems. The general seismic inverse problem is that of determining the functions which characterize an elastic or acoustic medium, using measurements which can be made at the surface of the medium. Many versions of this problem may be formulated, depending on the geometry, nature of sources, nature of measurements, and on the a priori assumptions made on the medium. Typically such problems are ill-posed, and in practice data are noisy and band-limited. Thus one is interested not only in mathematical existence and uniqueness questions, but also in developing strategies for extracting the maximum amount of useful information out of the available data.

In our research we investigate the mathematical structure of various model problems, with special attention to constructive solution techniques which might be suggested by the analysis. Whenever feasible, we try to complement the theoretical analysis with numerical calculations. While much current research deals with somewhat idealized model problems, we hope eventually to be able to handle more realistic situations.

D. Evolutionary Equations. We have been looking at the question of finding continuous data dependence results for the Cauchy problem for evolutionary equations of the form

$$\frac{d^n u}{dt^n} = Mu$$

where $n = 1,2,3$, and M is an unbounded operator on a Hilbert space valued functions $u(t)$. This problem is always ill posed if $n > 3$. However, in special cases, Levine, Murray, Payne and others have found weak (logarithmic) continuous data dependence results. We wish to obtain Hölder continuous data dependence results for such problems (which are known to hold when $n = 1,2$ even when the Cauchy problem is not well posed).

E. Eigenvalue Problems. For bounded domains in R^N, Levine and Weinberger looked for sufficient conditions on the boundary curvatures to insure that $\mu_{k+R} < \lambda_k$ for $k = 1,2,\ldots$, where R is a fixed integer in $[1,N]$ and μ_i. λ_i denote the Neumann and Dirichlet eigenvalues for the Laplacian. Such results would extend an early result of Payne who showed that $\mu_{k+2} < \lambda_k$ if $N = 2$ and the domain was convex.

F. Numerical Solution of Nonlinear Partial Differential Equations. We seek positive global solutions of a modified pendant drop dquation in R^n:

$$\operatorname{div}\left(\frac{\operatorname{grad} u}{\left(1 + |\operatorname{grad} u|^2\right)^{\frac{1}{2}}}\right) + \lambda u^q - u = 0 \quad (\lambda > 0,\ q > 1).$$

in Rn, with the "boundary condition" $u(x) \to 0$ as $|x| \to \infty$. The question of the existence of such solutions was raised by J. Serrin.

III. Status of Rearch

A. Reaction Diffusion Equations.

i) Sacks has investigated decay rates for solutions of quasilinear parabolic problems of the form

$$u_t = \Delta\phi(u) - f(u) \qquad x \in \Omega \qquad t > 0$$

$$u(x,t) = 0 \qquad x \in 0\Omega \qquad t > 0$$

$$u(x,0) = u_0(x) \qquad x \in \Omega$$

Where $\Omega \subset \mathbf{R}^N$ is bounded. Sufficient conditions (and sometimes necessary conditions) on the nonlinearities ϕ and f are given so that an estimate of the form $||u(\cdot,t)||_{L^\infty(\Omega)} \leq \eta(t)$ holds where η is independent of u_0. Exact computation of the form of $\eta(t)$ is possible in many cases.

ii) Benilan, Crandall and Sacks have proved results about the continuous dependence of the solutions on the nonlinearities for the elliptic problem

$$\beta(u) - \Delta u = f \qquad x \in \Omega$$

$$-\frac{\partial u}{\partial n} = \gamma(u) \qquad x \in \partial\Omega$$

where $\Omega \subset \mathbf{R}^N$ is bounded, $f \in L^1(\Omega)$ and β, γ are maximal monotone graphs. These results imply corresponding ones for the related parabolic problem

$$v_t = \Delta\phi(u) \qquad x \in \Omega \qquad t > 0$$

$$-\frac{\partial}{\partial n}\phi(u) = \gamma(\phi(u)) \qquad x \in \partial\Omega \qquad t > 0$$

$$v(x,0) = v_0(x) \qquad x \in \Omega$$

where $\phi = \beta^{-1}$.

iii) Papers by Chen, Levine and Sacks and Levine, Payne, Sacks and Straughan considered the reaction-diffusion-convection problem

$$u_t = u_{xx} + \epsilon(g(u))_x + f(u) \qquad 0 < x < L \qquad t > 0$$

$$\text{(A)} \qquad u(0,t) = u(L,t) = 0$$

$$u(x,0) = u_0(x)$$

under various conditions on the nonlinearities f and g. Detailed analysis of the structure of the set of nonnegative steady state solutions was carried carried out, and this information was then used to describe the possible large time behavior for the solution $u(x,t)$.

iv) Related to the above problem is the case of a nonlinear boundary condition. Here Levine considered problems of the form

$$
\text{(B)} \qquad \begin{aligned}
&u_t = u_{xx} + \varepsilon\big(g(u)\big)_x \qquad 0 < x < L, \qquad T > 0,\\
&u(0,t) = 0\\
&u_x(1,t) = f\big(u(1,t)\big)\\
&u(x,0) = u_0(x) > 0.
\end{aligned}
$$

Such problems are not amenable to the Hopf-Cole transformation. When $\varepsilon = 0$, a fairly complete analysis of the time dependent problem was given by Levine and Smith who modified (and corrected) the potential well arguments of Payne and Sattinger to obtain a global existence-nonexistence result (which also holds in more than one space dimension). However, potential well theory cannot be applied when convection terms are present in the governing equation. Moreover, it has been found that when $\varepsilon = 0$, the results found for iii) (A) and for (B) are closely related (even in several dimensions). By examining (B), therefore, he hoped to learn more about (A). He found that (B), too, has a rich structure and that he could obtain very precise existence and stability-instability results which not only helped him to better understand (A) but were also obtainable without recourse to the computer.

v) Khalsa analyzed the ability of semidiscrete finite element approximations, with numerical integration, to reproduce qualitative dynamics

$$
(*) \qquad u_t = u_{xx} + f(u), \quad f(u) = -u(u-b)(u-1), \quad 0 < b < 1/2,
$$

in the region $\{|x| < L, \ t > 0\}$, recently analyzed by Conley and Smoller. For the semidiscrete approximations he established the asymptotic, as $t \to \infty$, optimal order convergence and error estimates that hold uniformly on the infinite time interval $[t_0, \infty)$, $0 < t_0$, for nonsmooth or incompatible initial data. Also approximation of the "spontaneous bifurcation" (with L as a parameter) for the steady-state problem has been analyzed.

For the problem

$$
(**) \qquad u_t = \Delta u + f(u) \text{ in } \Omega \times (0,\infty), \quad \Omega \subset R^d, \quad d \leq 3,
$$

he established that the error bound for the semidiscrete approximations consists of two terms. The first term has the power of h less than optimal, due to lack of smoothness of initial data, and decays exponentially in time, the second term is of optimal order in h and does not depend on time.

He has also analyzed the effect of numerical integration in finite element solution of some nonlinear problems.

vi) Bandle and Levine have shown that for a class of reaction-diffusion equations of the form

$$u_t = \Delta_n u + u^p$$

in a sector (cone in R^n) there are no global solutions for arbitrarily small initial values if p is small relative to the (solid) angle opening and that there are such solutions if p is large relative to the angle opening.

vii) Levine has examined the initial-boundary value problem for

$$u_t = u_{xx} + \epsilon (1-u)^{-\beta}$$

from the point of view of dynamical systems. He has obtained all of the stationary solutions and shown that there are initial values as close as one pleases to unstable stationary solutions for which the corresponding solution quenches (rather than simply stays outside of some neighborhood of the stationary solution).

viii) Levine has begun to look at problems in which the reaction terms contain nonlocal nonlinearities. The anlysis of the stationary solutions as well as there stability is complicated by the fact that one has to develop comparison theorems for these problems since the standard results do not apply to them.

B. Semilinear Wave Equations.

i) We are concerned with the behavior of the derivatives of the solution of

$$u_{tt} = u_{xx} + \epsilon/(1-u)$$

when the solution reaches one in a finite time. The derivatives of the solution of the problem are studied numerically near the time when u reaches one (this time will be called T*). The significant numerical observations are:

1) $u_t(0.5,t)$ behaves like $-\{\ln[\sqrt{2}\,(T^* - t)]\}^{\beta}$ with beta positive as $t \to T^*$.

2) $u_x(0.5,t)$ is bounded as $t \to T^*$.

3) u_{tt} blows up more quickly than $1/(T^* - t)$.

4) u_{xx} blows up like $\ln(T^* - t)$ in the sense that the limit of the ratio of these quantities is a nonzero constant.

5) It was demonstrated numerically by Chang and Levine that there was a critical ϵ, say ϵ^*, such that for $\epsilon > \epsilon^*$ the solution quenches, while if the reverse inequality holds, there is no quenching. We show (numerically) that the quenching time T^* behaves like $-\ln(\epsilon - \epsilon^*)$.

Smith examined the first initial-boundary value problem for

$$u_{tt} = \Delta_N u + \epsilon/(1-u)^{\beta} \qquad (\epsilon > 0)$$

in $\Omega \times [0,T)$ where $\Omega \subset R^N$. He showed that for $N \geq 2$, there was no potential well theory possible and that for $N = 2,3$ it was possible to solve the problem locally. He also carried out computations that indicate that there is a critical ϵ, ϵ^*, (depending on the initial values) such that for ϵ's larger than ϵ^*, the solutions quench, while for ϵ's smaller than ϵ^*, they exist globally. The computation was carried out in an N-Ball for several values of N.

ii) For the equation

$$u_{tt} - u_{rr} - \big((n-1)/r\big)u_r + g(u) = 0, \quad r > 1, \quad 0 < t < T$$

$$u(r,t) = u(r,\ t+T),$$

Levine has shown that if $g \in C^2(R^1)$, $g(0) = 0$,

(a) $$\int_R^{\infty} \int_0^T |u|\, dr\, dt < \infty$$

(b) $$\lim_{r \to +\infty} r^{2(n-1)} \int_0^T \big(u_r^2 + u_t^2\big)dt = 0$$

(c) $$\lim_{r \to \infty} r^{n-1}\, ||u(r,\cdot)||_{\infty} = 0$$

and $g'(0) < (2\pi/T)^2$, then u does not depend upon t. In the case that $n = 3$, $r^{n-1} = r^2$ may be replaced by the factor $r^{1+\beta}$ for some β, $0 < \beta < 1$. The arguments used can be modified to obtain related results for the equations of semilinear and nonlinear elasticity. For example, for

$$u_{tt} - u_{xx} - \frac{\partial}{\partial x}\left(\phi(u_x)\right) = 0,$$

with the periodic constraint,

$$u(x,t) = u(x+L,\ t),$$

one can show that if $\phi'(0) + 1 > 0$,

$$\lim_{t \to \infty} \int_0^L \left(u_x^2 + u_t^2\right)dt = \lim_{t \to \infty} \left|\left|u_x(x,t)\right|\right|_\infty = 0$$

then u must be constant. If $\phi'(0) + 1 < 0$, the result is false.

Prior to Smiley's work the only known results on the problem of time-periodic solutions of the nonlinear wave equation

$$u_{tt} - \Delta_n u + g(u) = f$$

assumed the spatial dimension to be one and the equation to be homogeneous. In this case $g(u) = mu$ and f is replaced by the zero function. The work of J. M. Coron, in 1982, established a link between the period T and the strength of the nonlinearity as measured by $g'(0)$. He showed that any T-periodic solution, of class C and decaying to zero as $x \to \infty$, would in fact be independent of time whenever $g'(0) < (2\pi/T)^2$. The above work of Levine extended Coron's result to problems in several dimensions. In what was apparently the only result showing existence of solutions, A. Weinstein showed in 1985 that there were indeed time-dependent solutions on the half-line when $g'(0) > (2\pi/T)^2$, and that these solutions were localized in space in the sense that they had exponential decay as $x \to \pm\infty$.

Smiley's work was initially focused on the linearized problem in which $g(u) = mu$ with m real. In addition he assumed radial symmetry in space. When the spatial dimension is either 1 or 3 he was able to completely characterize the set of solutions u for all forcing terms f having exponential decay as $r \to \infty$. It was also shown that all solutions inherited the property of exponential decay. Building on this analysis he was then able to establish the existence of solutions for the nonlinear problem in both dimensions 1 and 3. All solutions were shown to have exponential decay as $r \to \infty$, so they are localized in space. When f is identically zero it is shown that there are time-dependent solutions whenever $g'(0) > (2\pi/T)^2$.

These solutions have been referred to as breathers by several authors since the spatial profile exhibits an oscillation suggestive of breathing.

In addition to the theoretical work he developed Pascal programs which can be used to obtain approximate solutions, and to study the changes in the solution set as parameters in the problem are varied. In particular the programs can be used to numerically describe the bifurcation that takes place in the solution set as $g'(0)$ crosses one of the values $(2n\pi/T)^2$.

iii) Alexander is investigating the structure of solutions of

$$u_{tt} = u_{xx} + u^3 \cos u,$$

with, say, Dirichlet boundary conditions on $[0,1]$ in x.

In an attempt to build a picture of the time-dependent behavior by hanging it on the skeleton of the steady-state solutions, he examines the following BVP:

$$(**) \qquad u_{xx} + u^3 \cos u = 0, \qquad u(0) = u(1) = 0.$$

This is a plane autonomous conservative problem. The phase plane exhibits alternating saddles and centers along the u-axis, with a degenerate singular point at the origin.

In order to study (**) one considers the related initial value problem:

$$u_{xx} + u^3 \cos u = 0, \qquad u(0) = 0, \quad u_x(0) = s$$

Define the arrival time T to be the minimum positive x such that

$$u_x(T) = 0.$$

Then T is a function of s.

The initial value ODE problem has a conserved energy; one defines the segment between two consecutive saddles in the phase plane as a "window."

Numerical computations indicate that:

1. Within each window, arrival time T is a convex function of energy;

2. In the k'th window, the minimum arrival time is

$$0.09\, k^{-1/2} + o\,(k^{-1/2}),$$

the k'th window being the energy space between the saddle at $(4k-3)\,\pi/2$ and the saddle at $(4k+1)\,\pi/2$. This minimum arrival time is achieved by the solution having about 15/16ths of the maximum energy in the window.

3. The above implies that there are two initial conditions within each energy window yielding arrival times $T=1/2$. These give solutions of the BVP (**) above on $[0,1]$.

4. He has computed the solutions of the BVP (**) above for the first two windows. In the second window they already lie very close to the stable manifolds of the saddles that bound the window.

5. The linearized problem

$$v_{xx} + f'(u(x))\, v + m\, v, \qquad v(0) = v(1) = 0$$

$[f(u) = u^3 \cos u]$, linearized, that is, about solutions u of the BVP, appears to have spectrum that is entirely negative, or negative except for one positive eigenvalue m.

iv) In a pair of papers Levine and Smith investigated the following problems ($\Omega \subset R^N$, bounded):

A.
$$\begin{aligned} u_t &= \Delta u && \text{in } \Omega \times [0,T) \\ u &= 0 && \text{on } \textstyle\sum_1 \times [0,T) \\ \frac{\partial u}{\partial n} &= f(u) && \text{on } \textstyle\sum_2 \times [0,T) \\ u(x,0) &= u_0(x) && \text{on } \Omega \times \{0\} \end{aligned}$$

and

B.
$$\begin{aligned} u_{tt} &= \Delta u && \text{in } \Omega \times [0,T) \\ u &= 0 && \text{on } \textstyle\sum_1 \times [0,T) \\ \frac{\partial u}{\partial n} &= f(u) && \text{on } \textstyle\sum_2 \times [0,T) \\ u(x,0) &= U_0(x) && \text{on } \Omega \times \{0\} \\ u_t(x,0) &= V_0(x) && \end{aligned}$$

where $\sum_1 \cup \sum_2 = \partial\Omega$, $\sum_1 \cap \sum_2 = \phi$, and where $\frac{\partial}{\partial n}$ denotes the outer normal derivative. For suitable nonlinearities, f, a potential well theory is developed.

Each problem has a global weak solution provided the data lies in the potential well and the total initial energy is small. The global solution is obtained as an expansion in normal modes in terms of the Helmholtz eigenfunctions and the eigenfunctions for a modified Steklov problem. The proof of global existence is valid for all potential wells of positive depth and all dimensions $n > 1$, and can be used to generalize Sattinger's theory.

Solutions of these problems which start in a region exterior to the potential well with sufficiently small total initial energy can only exist for a finite time. The proof of this fact requires stronger hypotheses on f and corrects a key lemma of Payne and Sattinger.

C. Inverse problems

i) Sacks and Santosa consider the one dimensional velocity inversion problem: find $c(z)$ for $z > 0$ given $c(0)$ and $u(0,t)$ where $u(x,t)$ satisfies

$$u_{tt} = c^2(z)u_{zz} \qquad z > 0$$

$$u_x(0,t) = -\delta(t)$$

$$u(x,t) = 0 \qquad t < 0$$

where δ is the Dirac delta function. The solution to this problem is known to be unique under suitable regularity assumptions. In this paper an iterative solution method of quasi-Newton type is derived and analyzed, and numerical examples are given. Related least square problems are also studied.

ii) Sacks and Symes study the following inverse problem for a one dimensional elastic medium: Find $\rho(x_3)$, $\lambda(x_3)$ and $\mu(x_3)$ for $x_3 > 0$ given $\rho(0)$, $\lambda(0)$, $\mu(0)$ and $u_3(x_1,x_2,0,t)$ where $\vec{u} = (u_1,u_2,u_3) = \vec{u}(x_1,x_2,x_3,t)$ satisfies

$$\rho u_{itt} = \frac{\partial}{\partial x_j} \tau_{ij}(\vec{u}) \qquad i = 1,2,3 \qquad x_3 > 0$$

$$\tau_{13} = \tau_{23} = 0 \qquad x_3 = 0$$

$$\tau_{33} = \delta(x_1)\,\delta(x_2)\,\delta(t) \qquad x_3 = 0$$

where τ is the stress tensor, $\tau_{ij}(\vec{u}) = \lambda(\vec{\nabla}\cdot\vec{u})\,\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$. Continuous dependence estimates and a constructive approximate solution method for the linearized inverse problem are given, and a numerical example is shown. There is also heuristic discuss of iterative solution for the nonlinear problem.

iii) Bube, Lailly, Sacks, Santosa and Symes have considered the problem of simultaneous recovery of a velocity profile and the source wave form in the following problem: Find $c(z)$ for $z > 0$ and $f(t)$ for $t > 0$ given $c(0)$ and $u(x,0,t)$ where $u(x,z,t)$ satisfies

$$u_{tt} = c^2(z)\,(u_{xx} + u_{zz}) \qquad t > 0 \qquad z > 0$$

$$u_z(x,0,t) = f(t)\,\delta(x)$$

$$u(x,z,0) = 0 \qquad t < 0$$

Continuous dependence estimates and an constructive approximate solution method are given for the linearized inverse problem, and some numerical examples are given.

iv) Sacks has considered the inverse problem for a two dimensional acoustic medium. Find $\rho(x,z)$, $c(x,z)$ for $z > 0$, $-\infty < x < \infty$ given $\rho(x,0)$, $c(x,0)$ and $u(x,y,0,t)$ where $u(x,y,z,t)$ satisfies

$$\frac{1}{\rho c^2} u_{tt} = \nabla\cdot(\frac{1}{\rho} \nabla u) \qquad z > 0$$

$$u_z(x,y,0,t) = -\delta(y)\ \delta(t)$$

$$u(x,y,z,t) = 0 \qquad t < 0$$

The assumption of weak inhomogeneity is made (i.e. ρ and c are close to constant) and the corresponding linearized inverse problem is analyzed in some detail, with special attention to continuous dependence questions. It is shown that the solution may be obtained by solving an uncoupled system of one dimensional, second kind, Volterra integral equations. Some examples of numerical solutions are given using this idea.

v) Sacks and Symes consider the problem of velocity inversion from common offset data for a two dimensional acoustic medium. Find $c(x,z)$ for $z > 0$ given $c(x,0)$ and $u(x+h,h,0,0,t)$ where $u(x,y,z,t)$ satisfies

$$u_{tt} = c^2(x,z)\ \Delta u + \delta(x-h)\ \delta(y)\ \delta(z)\ \delta(t)$$

$$u(x,y,z,x) = 0 \qquad t < 0$$

where $h > 0$ is a fixed half-offset (source-receiver) distance. The assumption of weak inhomegeneity is made (c close to constant) and the corresponding linearized inverse problem is studied with special attention to uniqueness and continuous dependence questions. A constructive solution method is proposed, and the case of multi-offset data is also considered.

D. Evolutionary Equations. K. Ames, Levine and Payne have succeeded in demonstrating that it is possible to obtain Hölder continuous data dependence results for the evolution equation described earlier for small n, ($n \leq 4$). However the general case is still open.

E. Eigenvalue problems. Let $D \subset R^n$ be a bounded domain. Levine and Weinberger have shown that if the domain is convex with a $C^{2+\alpha}$ boundary then $\mu_{k+N} < \lambda_k$, $k = 1,\ldots$ where $\{\lambda_i\}$, $\{\mu_i\}$ are the Dirichlet and Neumann eigenvalues for the Laplacian on D. If the mean curvature is positive then $\mu_{k+1} < \lambda_k$ (Aviles). Also, if, at every point of the boundary, every sum of length $N - R + 1$ of the set of numbers $\{\kappa_1,\ldots, \kappa_{N-1}, (N-1)\ H\}$ is nonnegative, then $\mu_{\kappa+R} < \lambda_\kappa$. (Here

the κ's are the principal curvatures and H is the mean curvature.) Examples and counterexamples are given. Also, a characterization in terms of the equations defining the boundary surface is given.

F. Numerical Solution of Nonlinear Partial Differential Equations. It was shown numerically, that radial, global, positive solutions of the equation

$$\operatorname{div}\left(\frac{\operatorname{grad} u}{(1+|\operatorname{grad} u|^2)^{1/2}}\right)+\lambda u^q - u = 0 \quad (\lambda > 0,\ q > 1).$$

exist for all λ sufficiently large and $1 < q < (n+2/(n-2)$. This was a surprise to Serrin who (with Ni) had shown that no global solutins existed for all $\lambda < \lambda_q = (\,{}^1\!/_2\,(q-1)/q+1))^{\,{}^1\!/_2\,(q-1)}$ and $q < (n+2)/(n-2)$. The computations motivated Serrin and Peletier to prove the existence of such solutions. The computations also show that the nonexistence result is not best possible. It was also shown that global, radial solutions which change sign also exist. (Serrin and Peletier have not shown this theoretically.) The computations raised other interesting questions about the nature of the solutions of this equation.

IV. Publications

I. Papers in refereed journals

[1] Benilan, P., Crandall, M.G., and Sacks, P.E., Some L^1 existence and depandence results for semilinear elliptic equations under nonlinear boundary conditions, to appear in Appl. Math. Opt.

[2] Chen, T.F., On least-squares approximations to compressible flow problems, Num. Meth. P.D.E., 2, 207-228, 1986.

[3] Chen, T.F., Levine, H.A., and Sacks, P.E., Analysis of a convective reaction-diffusion equation, to appear in Nonlinear Anal. Theory, Methods and Applications.

[4] Khalsa, S.N.S., Finite element approximation of a reaction-diffusion equation. Part I: Application of topological techniques to the analysis of asymptotic behavior of the semidiscrete approximations, Quart. Appl. Math., 44, 2, 375-386, 1986.

[5] Levine, H.A., Minimal periods for solutions of semilinear wave equations in exterior domains and the equations of nonlinear elasticity, to appear in J. Math. Anal. Appl.

[6] Levine, H.A., Stability, instability and nonexistence for solutions of an initial-boundary value problem for Burger's equation with a semilinear boundary condition, to appear in SIAM J. Math. Anal.

[7] Levine, H.A. and Protter, M.H., The breakdown of solutions of quasilinear first order systems of partial differential equations, Arch. Rat. Mech. Anal., 95, 3, 253-267, 1986.

[8] Levine, H.A. and Smith, R.A., A potential well theory for the wave equation with a nonlinear boundary condition, J. Reine Angew. Math., 374, 1-23. 1987.

[9] Levine, H.A. and Smith, R.A., A potential well theory for the heat equation with a nonlinear boundary condition, Math. Meth. Appl. Sci., 9,127-136, 1987.

[10] Levine, H.A., and Weinberger, H.F., Inequalities between Dirichlet and Neumann eigenvalues, Arch. Rat. Mech. Anal., 94, 3, 193-208, 1986.

[11] Sacks, P.E., The inverse problem for a weakly inhomogeneous two dimensional acoustic medium, to appear in SIAM J. Appl. Math.

[12] Sacks, P.E. and Santosa, F., A simple computational scheme for determining the sound speed of an acoustic medium from its surface impulse response, SIAM J. Sci. Stat. Comp., 8,4, 501-520, 1987.

[13] Sacks, P.E. and Symes, W.W., Recovery of the elastic parameters of a layered half-space, Geophys. J. Roy. Astr. Soc., 88, 593-620, 1987.

[14] Smith, R.A., On a hyperbolic quenching problem in several dimensions, to appear in SIAM J. Math. Anal.

II. Preprints

[1] Axtell, J.D., Numerical study of the derivatives of solutions of the wave equation with a singular forcing term at quenching, preprint.

[2] Bandle, C. and Levine, H.A., On the existence and nonexistence of global solutions of reaction-diffusion equations in sectorial domains, preprint.

[3] Bube, K., Lailly, P., Sacks, P.E., Santosa, F. and Symes, W.W., Simultaneous determination of a source wavelet and velocity profile using impulsive point-source reflections from a layered fluid, submitted to Geophys. J. Roy. Astr. Soc.

[4] Levine, H.A., Quenching and nonquenching for some nonlinear parabolic equations, preprint.

[5] Levine, H.A., and Quintanilla, R., Some remarks on Saint-Venant's principle. preprint.

[6] Levine, H.A., Payne, L.E., Sacks, P.E., and Straughan, B., Analysis of a convective reaction-diffusion equation (II), submitted to SIAM J. Math. Anal.

[7] Sacks, P.E., Universal decay in nonlinear parabolic equations, submitted to Nonlinear Anal. T.M.A.

[8] Sacks, P.E. and Symes, W.W., Velocity inversion using common offset data, preprint.

III. Conference proceedings and technical reports

[1] Ames, K.A., Levine, H.A., and Payne, L.E., Improved continuous dependence results for a class of evolutionary equations, Proc. Int. Symp. Inverse and Ill-Posed Problems, Academic Press, 1986.

[2] Khalsa, S.N.S., Finite element approximation of a reaction-diffusion equation. Part II: Approximation of the spontaneous bifurcation and error estimates uniform in time, Trans. 4th Army Conf. Appl. Math. Comp., 1157-1172.

[3] Levine, H.A., Some boundary value problems for the wave equation, Proc. Int. Symp. Inverse and Ill-Posed Problems, Academic Press, 1986.

[4] Levine, H.A., A numerical search for ground state solutions of a modified capillary equation and for solutions of the charge balance equation, to appear in proceedings of M.S.R.I. microprogram on nonlinear diffusion.

[5] Levine, H.A., Some remarks on inequalities between Dirichlet and Neumann eigenvalues, Proceedings of Conference on Maximum Principles and Eigenvalue Problems, Univ. of Tenn., Knoxville, Tenn., June 15-19, 1987 Pitman-Longman (in press).

[6] Sacks, P.E., Behavior near t=0 for some nonlinear diffusion equations, Semigroups, Theory and Applications, Brezis et al ed., 205-213, Pitman, 1986.

[7] Sacks, P.E., Qualitative behavior for a class of reaction-diffusion-convection equations, to appear in proceedings of M.S.R.I. microprogram on nonlinear diffusion.

[8] Sacks, P.E., Some linearized inverse problems for acoustic media, to appear in proceedings of IEEE meeting on Decision and Control, 1987.

[9] Sacks, P.E., Computation of the principal part of the linearized map from coefficients to surface value in the P-SV problem, Cornell University Technical Report, 1986.

V. Personnel

A. Senior Staff

	Support (Oct. 1, 1984-Sept. 30, 1987)
1. Professor Howard A. Levine (P.I.)	9 months
2. Associate Professor Paul E. Sacks	9 months
3. Associate Professor Roger K. Alexander	2 months
4. Associate Professor Michael E. Smiley	2 months
5. Assistant Professor Tsu Fen Chen	4 months
6. Assistant Professor Sat Nam Khalsa	2 months

B. Graduate Research Assistants

1. Richard A. Smith — 15 months
   (Smith received his Ph.D. in December 1985. He is now employed by Exxon Corp. in Houston, TX)
   Thesis Title: Theoretical and Numerical Studies of Some Ill Posed Problems in Partial Differential Equations
   (Smith's thesis was considered good enough to earn him a travel grant to the International Congress of Mathematicians in Berkeley. This was awarded by the U.S. Organizing Committee)

2. Thomas K. Evers — 0 months
   (Evers received his M.S. in December 1985. He is now employed by Texas Instruments, Dallas, TX)
   Thesis Title: Numerical Search for Ground State Solutions of a Modified Capillary Equation.
   The numerical results in Ever's M.S. paper formed the basis of two conjectures which Serrin and Peletier have recently established concerning the existence of ground states for

   $$\vec{\nabla}\cdot[(1 + |\vec{\nabla}u|^2)^{-1/2}\,\vec{\nabla}u] + \lambda u^q - u = 0.$$

   (Peletier, L. A. and Serrin, J., Ground states for the prescribed mean curvature equation, Proc. AMS (in press).
   Serrin, J., Positive solutions of a prescribed mean curvature equation (in press).)

3. Jeffrey Anderson (Ph.D. Candidate) — 10.5 months
   Thesis Area: Reaction-diffusion equations with convection. He is currently developing a local L^∞ existence theory.

4. Sang Ro Park (Ph.D. Candidate) — 4.5 months
   Thesis Area: Nonlinear parabolic equations. He has been trying to prove blow up results for the time derivative at quenching in some sngular parabolic problems.

5. Deng Keng (Ph.D. Candidate) — 4.5 months
   Thesis Area: Hyperbolic equations. He is attempting to establish theoretically the numerical results obtained by Axtell.

6. John Axtell (M.S. Candidate) — 3 months
   (Axtell will receive his M.S. in December, 1987. His thesis will be published in an appropriate journal.)
   Thesis Title (tentative): The Blow Up of Derivatives of Solutions of Hyperbolic Problems which Quench in Finite Time.

VI. Interactions

A. Seminar, Colloquium Speakers

1.	Ralph Showalter (University of Texas, Austin)	10/ 2/84
2.	Brian Straughan (University of Wyoming)	10/23/84
3.	Murray H. Protter (University of California, Berkeley)	4/ 8/85
4.	Philip S. Crooke (Vanderbilt University)	4/23/85
5.	Michael Crandall (University of Wisconsin, Madison)	11/19/85
6.	William Symes (Rice University)	3/18/86
7.	Robert Finn (Stanford University)	3/28/86
8.	Stephen Hook (University of California, Berkeley)	4/ 4/86
9.	Jerry Bona (University of Chicago)	4/ 8/86
10.	Pierre Vuillermot (University of Texas, Arlington)	2/15/87 - 2/23/87
11.	Jost Holshof (University of Minnesota (IMA))	3/16/87 - 3/19/87
12.	L.E. Payne (Cornell University)	3/23/87 - 3/25/87
13.	Joyce McLaughlin (Rennsaler Polytechnic University) 7/26/87 - 7/29/87	4/27/87 - 4/29/87
14.	Mohammed Rammaha (University of Nebraska, Lincoln) 7/26/87 - 7/29/87	4/27/87 - 4/29/87
15.	Brian Straughan (University of Glasgow)	9/17/87 - 9/23/87

B. Other Interactions (Supported in part by AFOSR 84-0252).

1. Howard A. Levine

   a) Consulted with L.E. Payne at Cornell University June 22-29, 1985 and held informal discussions at the AFOSR Mathematics and Information Sciences, July, 1985.

   b) Visited L. E. Payne, Cornell University (2/10/86-2/14-/86) and AFOSR, Bolling (2/18/86)

   c) Spoke at U.S.-Alpine Conference on Ill Posed Problems in St. Wolfgang, Austria on periodic solutions of nonlinear wave equations (6/8/86-6/13/86)

   d) Spoke at Microprogram on Nonlinear Diffusion Equations (N.S.F. supported) at M.S.R.I., Berkeley, California on numerical search for ground state solutions of a modified capillary equation (9/2/86-9/6/86).

e) Talked on minimal periods for semilinear wave equations at joint University of Iowa - Iowa State Seminar in Partial Differential Equations at Grinnell College. (11/87).

f) Consulted with L.E. Payne on nonlocal reaction diffusion equations and gave invited talk at J.H. Barrett Memorial Lectures, University of Tennessee. (4/20/87-4/25/87)

g) Consulted with C. Bandle on blowup problems for reaction diffusion equations in unbounded domains and lectured at the University of Basel and the University of Zurich. (5/16/87-5/30/87).

l. Paul E. Sacks

a) Consulted at Cornell University with F. Santosa, W.Symes and others on inverse problems June 24-July 5, 1985.

b) Consulted at Rice University with W.Symes on inverse problems, July 30-August 16, 1985.

c) Attended two M.R.C. conferences on partial differential equations, Madison Wisconsin. (10/85,4/86).

d) Visited I.M.A., University of Minnesota. (12/85).

e) Consulted with M.Crandall at the University of Wisconsin and spoke on recovery of elastic parameters in a layered half space. (2/86).

f) Consulted with F.Santosa at the University of Delaware and talked on nonlinear elliptic problems with a noninear flux condition. (3/86).

g) Consulted with W.Symes at Rice University and spoke on recovery of elastic paramenters in a layered half space. (4/86).

h) Spoke at Fourth Army Conference on Applied Mathematics and Computing, Cornell University on the recovery of elastic parameters in a layered half space. (5/17/86-5/30/86).

i) Spoke at Bay Area Seminar on Inverse Problems, Stanford University, on the recovery of elastic parameters of a layered half space. (8/1/86).

j) Visited Mathematics Department, University of Iowa, gave colloquium on hyperbolic inverse problems. (2/20/87).

k) Attended workshop on nonlinear partial differential equations, Provo, Utah, gave talk on reaction diffusion convection equations. (3/4/87-3/7/87).

l) Consulted with W.Symes at Rice University on hyperbolic inverse problems. (5/31/87-6/3/87).

m) Consulted with W.Symes and F.Santosa at Rice University on hyperbolic inverse problems. (8/9/87-8/11/87).

3. Sat Nam Khalsa

a) Talked on "Application of topological techniques to the analysis of asymptotic behavior of the finite element solutions of a reaction-diffusion equation," at the SIAM Fall meeting, Arizona State University, Tempe, Arizona. (10/28/85-19/30/85).

b) Talked on "L^{∞} error estimates for finite element with "product integration for semilinear elliptic problems" at the Finite Element Circus, Brookhaven National Laboratory, Long Island, New York. (11/14/85-11/15/85).

c) Talked on "Finite element approximation of a reaction-diffusion equation," at the Fourth Army Conference on Applied Mathematics and Computing, Cornell University, Ithaca, New York. (5/27/86-5/30/86).

d) Attended "Microprogram": Nonlinear Diffusion Equations and their Equilibrium States", Math. Science Research Inst., Berkeley. (8/29/86-9/6/86).

4. Tsu Fen Chen

a) Spoke at the Fourth Army Conference on Applied Mathematics and Computing on numerical experiments for a convective reaction diffusion equations, Cornell University. (5/27/86-5/20/86).

b) Attended IMA workshop on Basic Methods of Numerical Analysis and Introduction to State-of-the-Art Research, Minneapolis, MN. (8/18/86-8/29/86.

c) Attended IMA workshop on Computation Fluid Dynamics and Reacting Gas Flows, Minneapolis, MN. (9/15/86-9/26/86).

5. Roger K. Alexander

   a) Attended SIAM Conference on Numerical Combustion, San Francisco. (3/9/87-3/11/87)

   b) Attended AFOSR Contractors' Meeting on Combustion, University Park, Pennsylvania. (6/22/87-6/24/87).

6. Michael E. Smiley

   a) Attended computer experimentation in nonlinear analysis, a conference held at the University of Missouri-Columbia from 6/4/87-6/6/87.

   b) The Second Howard University Symposium on Nonlinear semigroups, Partial Differential Equations, and Attractors, held at Howard University in Washington, D.C. A talk on breathers in several dimensions was given. (8/3/87-8/7/87).

7. Richard A. Smith

   a) Presented a paper on his dissertation at the International Conference on the Theory and Applications at Pan American University, Edinburg, Texas, May 20-23, 1983.