Use of the maximum likelihood method in the analysis of chamber air dives

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Tikuisis P, Nishi RY, Weathersby PK. Use of the maximum likelihood method in the analysis of chamber air dives. Undersea Biomed Res 1988; 15(4):301–313.—The method of maximum likelihood was used to evaluate the risk of decompression sickness (DCS) for selected chamber air dives. The parameters of two mathematical models for predicting DCS were optimized until the best agreement (as measured by maximum likelihood) corresponding to the observed DCS incidents from a series of dives was attained. The decompression data used consisted of 800 man-dives with 21 incidents of DCS and 6 occurrences of marginal symptoms. The first model investigated was based on a nonlinear gas exchange in a series arrangement of four compartments. The second model was based on a monoexponential gas exchange in a parallel arrangement of two compartments. The overall statistical success in describing the 800 man-dives was quite similar for the two models. Predictions of safety for dives not part of the original data differed for the models due to differences in gas kinetics. For short, no-decompression dives, the series arrangement of compartments predicted a lower incidence of DCS. These predictions were more consistent with the outcome of subsequent testing than were predictions of the parallel compartment model. Predictions of the series arrangement model were also similar to those of a single-compartment, two-exponential model that was evaluated with over 1700 man-dives by the U.S. Navy.

The variability in the occurrence of decompression sickness (DCS) makes the prediction of DCS difficult for any given dive profile. Conventional methods for establishing the risk of DCS for a particular dive require a great number of man-dives. For example, if in 100 man-dives for a particular trial, 5 incidents of DCS occurred (5% occurrence), then the 95% confidence interval limits for this occurrence indicate that the probability of DCS can be between 1.64 and 11.28% (1). To "tighten" the prediction of the incidence of DCS for this particular dive trial, many more man-dives would be required. For example, 65 incidents of DCS out of 1300 man-dives (5% occurrence) would narrow the 95% confidence interval to between 4 and 6% for
a particular dive trial. Since most dive trials involve considerably fewer than 100 individuals, a method for combining dive trials in a single analysis is required.

Weathersby et al. (2) suggested using the method of maximum likelihood (3) to evaluate the risk of DCS. This method applies a predictive model for DCS to actual outcomes from a series of dive trials. Inputs to the predictive model are a decompression dose response function and any decompression model that can be stated mathematically. The decompression dose response function is a measure of the cumulative risk of DCS for a dive. The parameters of the predictive model are adjusted until the best agreement or maximum likelihood corresponding to the DCS events of past trials is obtained. Once the parameters of the predictive model have been optimized using any number of dives from different sources, the model can be applied to predict the probability of DCS for any new (tested and untested) dive profile.

In addition, different decompression models can be tested to determine which model best fits the data. In this study, the method of maximum likelihood is used to compare the prediction of DCS between a nonlinear series compartments (NLSC) model (4, 5), featuring four compartments in series, and a monoexponential parallel compartments (MEPC) model, used by Weathersby et al. (6), featuring two compartments in parallel. Although many other models involving more than two compartments in parallel exist (4), they offer little or no statistical advantage over the two-compartment MEPC model in the present analysis (6), but may be justified by larger data sets (7). The method of maximum likelihood has not been previously applied to compartments in series such as the NLSC model. The decompression model that gives the largest maximum likelihood value is defined as the one that best fits the data. For specific classes of models, formal statistical tests allow rigorous comparison of model predictions (2, 3). A desirable feature of the method of maximum likelihood is that any set of dive data can be used for the comparison among models. For instance, the MEPC model can be tested against dive trials that followed the decompression procedures governed by the gas kinetics of the NLSC model, and vice versa.

In summary, a probabilistic model optimized by maximum likelihood can be used to test decompression procedures and to predict the probability of DCS. It can be used as an optimizing tool for determining the parameters of decompression models and as an evaluative tool, but the prediction must be tested and the actual incidence must be established by experiment. This study examines the use of the method of maximum likelihood as applied to the NLSC and MEPC models in optimizing model parameters, evaluating dive profiles, and comparing the prediction of DCS for particular dive tables.

**LIKELIHOOD CRITERION**

The variability in DCS can be described in terms of probabilities. The probability of DCS is a function of the dive profile and the decompression model tested, i.e.,

\[ P_{\text{DCS}} = f(\text{time}, \text{depth}, \text{model}) \]

where \( f \) is the decompression dose response function. It follows that the probability of no DCS is

If \( Y \) is the actual outcome or event, then

\[ Y = \begin{cases} 1 & \text{if DCS} \\ 0 & \text{otherwise} \end{cases} \]

where \( Y = 0 \) if DCS did not occur. Most often, the assignment of \( Y = 0 \) is shown to affect the likelihood criterion.

Assuming that the probability of all the known outcomes is less than 1, the method of maximum likelihood can be used to determine the parameters of the predictive model. The decompression model that gives the largest maximum likelihood value is defined as the one that best fits the data. For specific classes of models, formal statistical tests allow rigorous comparison of model predictions (2, 3). A desirable feature of the method of maximum likelihood is that any set of dive data can be used for the comparison among models.
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\begin{equation}
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\end{equation}

If \( Y \) is the actual outcome of DCS for a dive, then the predicted probability of the
actual outcome or observation is

\begin{equation}
P_{\text{outcome}} = (P_{DCS})^y (1 - P_{DCS})^{1-y},
\end{equation}

where \( Y = 0 \) if DCS does not occur, and \( Y = 1 \) if DCS does occur. For those dives
that defy a definitive diagnosis (sometimes called marginal symptoms or niggles) an
assignment of \( Y = 0.5 \) can be made. In other analyses such a choice has not been
shown to affect the ultimate conclusions of the analysis (2, 8).

Assuming that the outcome of each dive is independent, the total predicted prob-
ability of all the known outcomes is given by the likelihood function,

\begin{equation}
L = \prod P_{\text{outcome}},
\end{equation}

where \( n \) is the total number of man-dives. \( L \) is usually a very small number, but
greater than zero, because it is the product of many numbers less than 1. It is
customary to use the natural logarithm of \( L \), or \( LL \) (log likelihood),

\begin{equation}
LL = \sum \ln P_{\text{outcome}}.
\end{equation}

Since \( L \) is less than 1, \( LL \) is negative. The maximum likelihood value, which defines
the best fit between the model and data, is given by the smallest absolute value of \( LL \)
(or smallest negative value) as shown in Fig. 1. The modified Marquardt algorithm
(9) was used to obtain this value by varying the parameters of the predictive model
until the best agreement with the data was found.

![Graph](image-url)

**Fig. 1.** Log maximum likelihood, \( LL \), plotted against the risk coefficient, \( C \), for the NLSC model. *Solid circles* indicate the 95% confidence limits based on the Likelihood Ratio Test (3).
To use the method of maximum likelihood, the decompression dose response function or $P_{DCS}$ (see Eq. 1) must be defined. The function that was adopted for this study is known as the "risk" function (2, 6, 10),

$$P_{DCS} = 1 - \exp (-\int rdt),$$  \hspace{1cm} (6)

where $r$ is the measure of risk and $t$ is time. If $r$ is large, then over a period of time the probability of DCS approaches 1. For each decompression model examined, $r$ is defined empirically and its value is dependent on the actual decompression profile (see Decompression Models). Generally, $r$ is non-zero when a state of inert gas supersaturation occurs. Depending on the degree and length of this state, $P_{DCS}$ will increase accordingly.

For models that can be expressed as simplified or more general versions of other models, a formal Likelihood Ratio Test can establish whether a model is a significant improvement over another in describing a data set (2, 3). For example, to justify increasing the generality of a model by estimating one, two, three, etc., additional parameters, the $LL$ must increase by at least 1.92, 3.00, 3.91 units, etc., respectively.

DECOMPRESSION MODELS

1. Nonlinear series compartments model

The NLSC model for air diving (4, 5) consists of four compartments in series representing tissues having different rates of gas exchange. The rate of change of gas pressure in the four compartments is described by the following set of 4 nonlinear differential equations:

\[
\frac{dP_i}{dt} = A[(B + P_i + P_{i+1})(P_i - P_{i+1}) - (B + P_i + P_{i+2})(P_i - P_{i+2})], \tag{7}
\]

\[
\frac{dP_2}{dt} = A[(B + P_2 + P_3)(P_2 - P_3) - (B + P_2 + P_4)(P_2 - P_4)], \tag{8}
\]

\[
\frac{dP_3}{dt} = A[(B + P_3 + P_4)(P_3 - P_4) - (B + P_3 + P_4)(P_3 - P_4)], \tag{9}
\]

\[
\frac{dP_4}{dt} = A[(B + P_4 + P_4)(P_4 - P_4)]. \tag{10}
\]

where $P_i$ is the total gas pressure in compartment $i$, $P_0$ is the current ambient pressure, and $A$ and $B$ are constants (0.0026 min$^{-1}$ · ATA$^{-1}$ and 8.31 ATA, respectively). Equations 7–10 are those that control the uptake and elimination of gas for the original Kidd-Stubbs pneumatic analogue decompression computer model (4), the Kidd-Stubbs 1971 decompression tables (11), and the current Defence and Civil Institute of Environmental Medicine (DCIEM) 1983 decompression tables (12). The only differences among these 3 cases are in the safe ascent criterion for each compartment.

The measure of risk for the present likelihood analysis is defined as

$$r = r_1 + r_2 + r_3 + r_4,$$  \hspace{1cm} (11)

where, following the method of Weathersby et al. (6), the risk for compartment $i$ is assumed to be

and where $C$ is proportional to $r^0.79$. If $r < 0$, the actual time is obtained. The method is schematically portrayed in Figure 2 for a single dive. The divers and no in predicted incidence staggering that (12, becomes zero for confidence limit)

2. Exponential

The MEPC model was divided into two compartments, the present data exponential model and

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Fig. 2. Measure of
NLSC model for a dive and the average g
USE OF THE MAXIMUM LIKELIHOOD METHOD

\[ r_i = C(0.79P_i - P_o)/P_o, \]  

(12)

and where \( C \) is the risk coefficient to be estimated. The risk defined by Eq. 12 is proportional to the relative supersaturation of the \( N_2 \) component, hence the factor 0.79. If \( r < 0 \), then \( r \) is set to 0. The value of \( C \) is estimated by fitting the model to the actual time-depth profile data for each dive until the maximum value of \( LL \) is obtained. The model constants, \( A \) and \( B \), can also be optimized using the maximum likelihood method; however, time constraints did not allow this.

Figure 2 graphically demonstrates the NLSC model with \( C = 0.000154 \cdot \text{min}^{-1} \) to a single dive. This dive, to 185 fsw (5.60 ATA) with a bottom time of 125 min, had 4 divers and no incidence of DCS. At the end of decompression \( (t = 827 \text{ min}) \), the predicted incidence of DCS is 5.97%, while \( r \) is still greater than zero owing to the supersaturation of \( N_2 \) in the model compartments. After 1100 min, \( r \) according to Eq. 12, becomes zero and the final predicted incidence of DCS is 7.04 ± 1.55% (95% confidence limit).

2. Mono-exponential, parallel compartments model

The MEPC model is based on an arrangement of single exponential compartments in parallel. Weathersby et al. (6) found that two compartments were adequate to fit the present data in their study. Hence, in this study a two-compartment mono-exponential model [model 3 in (6)] is used. The exchange of inert gas in each of the two compartments is described by the following linear differential equation:

\[ \text{Compartment } i \text{ is} \]

\[ \text{Initial pressure, } P_{i0} \text{, respectively). For the original} \]

\[ \text{Kidd-}

\[ \text{L Civil Institute} \]

\[ \text{1980. The only compartment.} \]

\[ \text{Fig. 2. Measure of risk of DCS, } \text{r,} \text{and the probability of incidence of DCS, } P_{\text{DCS}}, \text{predicted by the NLSC model for a dive to 185 fsw for 125 min plotted as a function of time. The time-depth profile of the dive and the average gas pressure of the four compartments of the NLSC model are also shown.} \]
where $P_i$ is the pressure of the $N_2$ component of gas in compartment $i$ and $k_i$ is the time constant of the $N_2$ uptake and elimination of compartment $i$.

The measure of risk is defined as

$$ r = r_1 + r_2 $$

where

$$ r_i = C_i (P_i - P_0) / P_0, $$

and $C_i$ is the risk coefficient of compartment $i$. The values of $C_i$ and $k_i$ were estimated by fitting the model to the data until the maximum value of $LL$ was obtained.

RESULTS

The dive data were obtained from the CANDID (Canadian Dive Data) data bank (13) and are the same data used in data set "D" in (6). The data used consist of 163 air dive profiles in a hyperbaric chamber totaling 800 man-dives that were conducted during 1967–1968 with the Kidd-Stubbs pneumatic analogue decompression computer and other decompression criteria (4). The dive depths for this data set ranged from 99 to 300 fsw (3.00 to 9.08 ATA) and the bottom times ranged from 6 to 360 min. There were 21 incidents of DCS and 6 occurrences of marginal symptoms of DCS. Incidents of DCS were assigned an outcome of $Y = 1.0$ and marginal cases were assigned an outcome of $Y = 0.5$.

Using these data, the values of $LL$ are plotted against the risk coefficient $C_i$ for the NLSC model in Fig. 1. The maximum value of $LL$ ($= -102.57$) was obtained with $C = 0.000154 \cdot \text{min}^{-1}$. This value was used to generate Fig. 2. For the MEPC model, the maximum value of $LL$ ($= -100.63$) was obtained with $k_1 = 3.91 \text{ min}$, $k_2 = 382 \text{ min}$, $C_1 = 0.00615 \cdot \text{min}^{-1}$, and $C_2 = 0.00126 \cdot \text{min}^{-1}$ (6). The values of likelihood for the 2 models are close enough that the additional three estimated parameters of the MEPC model would not be justified by the Likelihood Ratio Test if it could be applied. Since the models are not suitable for this test, then within current limits of statistical theory, the results can only be declared as similar.

It is interesting to compare the predictions of these 2 models under different categories of risk of DCS. Figure 3 shows the results of the 2 models after separating the predictions of DCS (using Eq. 6) into four categories: <2%, 2–5%, 5–10%, and >10%. For example, the NLSC model predicted a DCS incidence of less than 2% for 312 out of the 800 man-dives in the data set. Of these 312 dives, the average predicted incidence was 1.47% and the average observed incidence was 1.44%. In comparison, the MEPC model predicted a DCS incidence of less than 2% for 238 out of the 800 man-dives, and of these the average predicted incidence was 1.36% while the average observed incidence was 0.84%. No dives were predicted to have a DCS incidence of >10% using the NLSC model in contrast to the 15 man-dives predicted to exceed 10% DCS incidence by the MEPC model. Despite these differences, the chi-square goodness-of-fit test for agreement between model prediction and actual outcome for all 4 categories is 0.60 for the NLSC model and 1.29 for the MEPC model; these low values indicate that both models predict outcome of DCS equally well.

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Application of Theory

Once the parameters of the predictive model have been optimized, the model can be used to predict the probability of DCS for a series of different but related dives. Figure 4 shows the probability of DCS predicted by both the NLSC and MEPC models for the DCIEM no-decompression air diving limits (12) as shown in Table 1. These no-decompression limits are more conservative than those from the USN Standard Air Decompression Tables (14, 15) (also shown in Table 1). For depths of 30 to 40 fsw (0.91 and 1.21 ATA), which have not been tested, the predicted probability of DCS is noticeably higher than that for deeper depths; hence, according to this analysis the no-decompression limits (12) from these shallow depths entail a greater risk of DCS.

Dives deeper than 50 fsw (1.51 ATA) have been tested according to the DCIEM no-decompression air diving limits. No instances of DCS occurred in the 131 man-dives considered here (16), an outcome that has 95% confidence limits of 0 to 2.8% DCS (1). In this depth range the NLSC model predicts a uniformly low probability of DCS averaging 0.2%, which is quite consistent with the outcome. This behavior is not too surprising since the dive limits were obtained by a nonprobabilistic model with the same kinetic constants as the present NLSC model (5, 12). On the other hand, the MEPC model predicts a logarithmically increasing probability of DCS with depth that averages 3.1%, or slightly outside the observed outcome. The reason for the contrasting trend between the 2 models lies with the difference in the kinetics of N₂ gas uptake and elimination in the compartments of the 2 models. The series
TABLE 1
COMPARISON OF NO-DECOMPRESSION AIR DIVING LIMITS

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<th>Depth, fsw</th>
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*These dive limits have been tested from 50 to 180 fsw at DCIEM.

The compartmental structure of the NLSC model leads to a slower rate of gas uptake in the compartments, whereas the "fast" compartment \((k_1 = 3.91\text{ min})\) of the MEPC model saturates quickly. It is this fast compartment that increases the calculated measure of risk substantially more than do the compartments of the NLSC model.

Figure 4 also shows the predictions of model 5 in (6), a single-compartment, two-exponential model evaluated with over 1700 man-dives used to estimate the risk of bends for different air decompression tables (15). In the depth range of greater than 50 fsw (1.51 ATA), the average prediction is 1.2% DCS which, along with the NLSC model, is consistent with the outcome for the present test dives.

Differences in model predictions are not as pronounced for dives with decompression stops because the slower compartments predominate. Dives with decompression stops are usually long enough for the gas pressure in the compartments of the NLSC model to increase the measure of risk to a degree similar to that predicted by the MEPC model. For example, of the 800 man-dives examined earlier, the average predicted incidence of DCS for the NLSC model was 2.92% and that for the MEPC model was 2.97%. However, there are still some important differences between the predictions of the 2 models for dives with decompression stops.

This is demonstrated in Fig. 5, where the probability of DCS predicted by both the NLSC and MEPC models for standard air dives to 150 fsw (4.54 ATA) from the DCIEM 1983 decompression tables (12) is shown as a function of bottom time at 150 fsw. The NLSC model predicts a monotonically increasing probability of DCS with increasing bottom time. The MEPC model, on the other hand, predicts a high probability of DCS for a short bottom time of 7 min, then decreasing probabilities until a minimum is reached thereafter. In a recent study, close agreement was found between the probabilities of different gas kits (17) produced at the 95% confidence level.
gas uptake in the MEPC model predictions differ from those calculated with the NLSC model. The figure shows the risk of DCS greater than 0.1% for the NLSC model predictions, with the MEPC model risk predictions also shown. The model 5 predictions (dotted line) are consistent with the outcome cited above (17). These values follow a similar trend to the MEPC model prediction but with much lower values around 20 min bottom time and progressively much higher values for bottom times beyond 35 min. These differences are attributed to the different configuration (1-exponential 2-compartment vs. 1-compartment 2-exponential) and data set (800 vs. >1700 man-dives) used to establish the MEPC model and model 5 in (6), respectively.

Finally, the method of maximum likelihood is used to compare the predicted risk of DCS among different decompression tables. Figure 6 shows a comparison of the probabilities of DCS predicted by the NLSC model for standard air diving to 150 fsw (4.54 ATA) based on the DCIEM 1983 air decompression tables (12), the Kidd-Stubbs 1971 air decompression tables (11), and the USN standard air tables (14) as a function of the bottom time. (Table 2 lists the decompression times as a function of bottom time.)
time from all 3 tables.) All decompression procedures produce similar predictions for bottom times up to 30 min. The Kidd-Stubbs 1971 decompression times are more conservative than the DCIEM 1983 tables for bottom times exceeding 45 min, but less conservative in the moderate bottom time range. Therefore, one would expect a smaller $P_{DCS}$ for the longer bottom times and a larger $P_{DCS}$ for the moderate bottom times. This predicted behavior is shown in Fig. 6; however, the differences are not very large. Similarly, the predicted $P_{DCS}$ using the DCIEM 1983 tables are lower than that using the USN tables, both by this analysis and that of (15). This is consistent with the longer, and more conservative, decompression times of the DCIEM 1983 tables.

A powerful application of the method of maximum likelihood is the calculation of decompression tables of equal $P_{DCS}$. Weathersby et al. (6, 18) have used this method to establish a set of air decompression tables based on model 5 in (6). A similar set could be developed for any model, including the NLSC model, although this is beyond the scope of the present study.

**SUMMARY AND CONCLUSION**

The method of maximum likelihood was applied to compare two different gas exchange models used to establish safe decompression procedures. These models were the NLSC and MEPC based on compartments in series and in parallel, respectively. The decompression data that were used consisted of 800 man-dives with 21 incidents of DCS and 6 occurrences of marginal symptoms of DCS. A comparison of

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**Fig. 5.** Probability of incidence of DCS as a function of bottom time for standard air dives to 150 fsw from the DCIEM 1983 tables showing the difference between the NLSC model (solid line), the MEPC model (broken line), and model 5 in (6) (dotted line) predictions.

**Fig. 6.** Probability of standard air dives to 15 line, and the USN Sta

<table>
<thead>
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<th>Bottom Time, min</th>
<th>7</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
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</tbody>
</table>
USE OF THE MAXIMUM LIKELIHOOD METHOD

![Graph showing probability of DCS incidence as a function of bottom time for standard air dives to 150 fsw from different tables.](image)

**Fig. 6.** Probability of incidence of DCS predicted by the NLSC model as a function of bottom time for standard air dives to 150 fsw from the DCIEM 1983 tables (solid line), the Kidd-Stubbs 1971 tables (broken line), and the USN Standard Air Tables (dotted line).

<table>
<thead>
<tr>
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<th>KS-1971</th>
<th>USN</th>
</tr>
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<td>7</td>
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<td>146.5</td>
</tr>
<tr>
<td>80</td>
<td>271</td>
<td>355</td>
<td>173.5</td>
</tr>
</tbody>
</table>

**TABLE 2**

**COMPARISON OF TOTAL DECOMPRESSION TIMES FOR STANDARD AIR DIVES AT 150 FSW**

the NLSC and MEPC model predictions for DCS showed no significant difference between the two and so either model appears equally suitable for predicting DCS with this limited data set. Use of a larger data set would improve the predictions.
allow for more optimized parameters, and perhaps statistically distinguish the predictive capability of different models.

A striking result with the use of the maximum likelihood method is the difference in the predicted probabilities of DCS between the 2 models for no-decompression air dives in the range of 50–180 fsw (1.51–5.45 ATA) and short-bottom-time air dives to 150 fsw (4.54 ATA). Because of the nonlinear gas kinetics of the NLSC model, a lower probability of DCS is predicted for these dives with this model than with the MEPC model. Subsequent testing indicated consistency with actual outcome for the NLSC model applied to the no-decompression air dives and for both models applied to the short-bottom-time air dives. Further studies must be conducted to test differences in prediction between the 2 models and to validate the extrapolation of model parameter values to dives that fall outside the range of the data set used to establish their values, as done in this case.

The authors gratefully acknowledge the advice of Dr. B.C. Eatock in the preparation of this paper. DCIEM Publication 88-P-20. —Manuscript received February 1988; accepted May 1988.

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10. Kuhlbein JD, Pr..
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15. Weathersby PK, tables. H. Comp.
16. Lauckner GR. N compressed air.
17. Lauckner GR. N compressed air.
18. Weathersby PK Equal risk air di.

Tikuisis P, Nishi RY, Weathersby PK. Emplo de la méthode de probabilité maximale dans l'étude de plongées à l'air en caisson. Undersea Biomed Res 1988; 15(4):301–313.—La méthode de probabilité maximale fut utilisée pour évaluer le risque de maladie de décompression (MDC) pour un choix de plongées à l'air en caisson. Les paramètres de deux modèles mathématiques pour prédire la MDC furent optimisés jusqu'à l'obtention du meilleur assentiment (tel que mesuré par la probabilité maximale) correspondant aux incidents de MDC observés dans une série de plongées. Les données de décompression employées provenaient de 800 plongées humaines contenant 21 incidents de MDC et 6 cas de symptômes marginaux. Le premier modèle étudié était basé sur le principe des échanges gazeux mono-exponentiels dans 2 compartiments montés en parallèle. La réussite statistique globale à décrire les 800 plongées humaines fut très similaire pour les deux modèles. Les prédicteurs de sécurité pour les plongées ne faisant pas partie des données originales étaient différentes de celles des modèles à cause des variations dans la cinétique des gaz. Pour les plongées courtes, sans décompression, les compartiments montés en série présentaient une incidence de MDC. Ces prédicteurs étaient plus consistants (s'accordaient mieux) avec les essais subéquents que les prédicteurs avec le modèle des compartiments en parallèle. Les prédicteurs du modèle avec compartiment en série étaient aussi similaires à celles obtenues avec le modèle à compartiment simple qui fut évalué par la Marine américaine avec plus de 1700 plongées humaines.

Tikuisis P, Nishi RY, Weathersby PK. Empl el método de probabilidad maxima en el análisis de inmersiones de cámara con aire. Undersea Biomed Res 1988; 15(4):301–313.—Se empleó el método de probabilidad máxima para evaluar el riesgo de enfermedad por descon+ pressión (EPD) en inmersiones de cámara con aire electivas. Se optimizaron los parámetros de dos modelos matemáticos para predecir la EPD, hasta que se alcanzo el mejor criterio (medio por la probabilidad máxima) correspondiente a los incidentes por EPD observados en una serie de inmersiones. Los datos de descon+ pressión empleados consistían de 800 inmersiones de humanos, con 21 casos de EPD y la incidencia de 6 con síntomas marginales. El primer modelo estudiado se basaba en un intercambio gaseoso no lineal en un arreglo en serie de cuatro compartimentos. El segundo modelo se basaba en un intercambio gaseoso monoxponential, en un arreglo en paralelo de dos compartimentos. El éxito estadístico global para describir las inmersiones de 800 humanos fue similar para ambos modelos. Las predicciones de seguridad para inmersiones que no formaban parte de los datos originales, fue distinta para los dos modelos debido a la diferencia de la cinética de los gases. La serie de arreglo en compartimentos predijo una incidencia menor de EPD, para inmersiones de no descon+ pressión cortas. Estas predicciones fueron más consistentes con el resultado de pruebas
USE OF THE MAXIMUM LIKELIHOOD METHOD

subsecuentes, que las del modelo de compartimentos en paralelo. Las predicciones del modelo de arreglo en serie, también resultaron similares a las del modelo de un compartimento sencillo, dos exponenciales, que se evaluó con más de 1700 inmersiones de humanos por la U.S. Navy.

REFERENCES