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BAYESIAN ABILITY ESTIMATION VIA 3PL WITH PARTIALLY KNOWN ITEM PARAMETERS

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Bayesian Ability Estimation via 3PL with Partially Known Item Parameters

ROBERT K. TSUTAKAWA AND JANE JOHNSON

Abstract

The conventional method of measuring ability, which is based on items with true parameters assumed to have values estimated by a pretest, is compared to a Bayesian method which deals with the uncertainties of such items. Computational expressions are presented for approximating the posterior mean and variance of ability under the three–parameter logistic (3PL) model. A 1987 ACT math test is used to demonstrate that the standard practice of using maximum likelihood or empirical Bayes techniques may seriously underestimate the uncertainty in the estimated ability when the pretest sample is only moderately large.

Key Words: ability estimation; Bayesian IRT; calibration; pretest; three–parameter logistic.

INTRODUCTION

A standard practice in mental testing is to score individuals using the responses to a set of test items which have previously been calibrated. When latent trait models are employed, the calibration involves estimating parameters of the model using a moderately large sample. The estimated parameters are then assumed to be the true values when the scoring is performed.

Even when the assumed model is correct, there are two sources of errors in this process. One is due to the responses of the individuals being scored and the other is due to the error in the calibration. Ignoring the second source could lead to inferential errors, particularly when the calibrating sample is not large. In many areas of testing large samples may not be readily available for calibration. Moreover, disclosure laws commonly require public dissemination of tests, making it necessary to have more items while the pool from which to draw the calibrating sample is limited.

This paper deals with the problem of estimating ability when there is uncertainty concerning the item parameters due to the limited size of the calibrating sample. Because of the sequential nature of first calibrating the test and then using it on the target population, the Bayesian paradigm for statistical inference is particularly attractive. This paper discusses how the uncertainty in the item parameters may be incorporated into the estimation and uncertainty of the abilities being measured. (KP)

The main idea will be demonstrated in terms of the three-parameter logistic model (3PL) which was introduced by Birnbaum (1968). The model specified that the probability of a correct response by an individual with real valued ability θ to a given item has the form

$$p_{\xi}(\theta) = c + \frac{1-c}{1+\exp\{-a(b-\theta)\}}, \tag{1}$$

where $\xi = (a,b,c)$ is an unknown item parameter, subject to $a>0$, $-\infty<b<\infty$, and $0<c<1$. It

will be assumed that a test consisting of K items has already been given to a calibrating sample of n individuals and that the posterior mean or mode of the item parameters is already available. (See Mislevy & Bock, 1984, or Tsutakawa, 1988, for algorithms to compute the posterior mode.)

The uncertainty in the calibrated item parameters will be summarized in terms of the posterior covariance matrix, which is approximated by the inverse Hessian of the negative log posterior evaluated at the mode. The mode and covariance matrix will then be used in the approximation of the posterior mean and variance of ability presented by Tsutakawa & Soltys (1988) for the case of the two–parameter logistic model (2PL), a limiting case of 3PL when c=0 in (1).

The method will be illustrated on data from a 1987 American College Testing Program (ACT) math test. The results will be compared with the more conventional approaches using maximum likelihood as implemented by LOGIST (Wingersky, Barton, & Lord, 1982) and empirical Bayes based on item parameters estimated by marginal maximum likelihood (Bock & Aitken, 1981, and Tsutakawa, 1988). The main conclusion of the paper is that when there is uncertainty in the item parameter, both maximum likelihood and empirical Bayes underestimate the variance of ability and therefore produce interval estimates which are too narrow and misleading.

General Setup and Alternative Solutions

Consider a K item test where the items are scored x_j=0 or 1 according as the answer to the jth item is incorrect or correct, j=1,...,K. Assume local independence so that the probability of the response vector $\mathbf{x} = (x_1,\ldots,x_K)$ for an individual with ability θ is

$$p_{\xi}(\mathbf{x}|\theta) = \prod_{j=1}^{K} p_{\xi_j}^{x_j}(\theta)\{1-p_{\xi_j}(\theta)\}^{1-x_j}, \tag{2}$$

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where $\xi = (\xi_1,\ldots,\xi_K)$ is the set of item parameters for the K items.

For calibration, assume there are n individuals with abilities $\boldsymbol{\theta} = (\theta_1,\ldots,\theta_n)$ sampled from a N(0,1) distribution. If $\mathbf{y}_i = (y_{i1},\ldots,y_{iK})$ is the response vector of the ith individual and $\mathbf{y} = (\mathbf{y}_1,\ldots,\mathbf{y}_n)$ the response matrix of the n individuals, the joint distribution of $(\mathbf{y},\boldsymbol{\theta})$ is given by

$$p(\mathbf{y},\boldsymbol{\theta}|\xi) = \prod_{i=1}^{n} p_{\xi}(\mathbf{y}_i|\theta_i)\phi(\theta_i), \tag{3}$$

where $p_{\xi}(\mathbf{y}_i|\theta_i)$ is defined by (2) and ϕ is the N(0,1) pdf. The marginal distribution of $\mathbf{y}$ then has probability function

$$p(\mathbf{y}|\xi) = \prod_{i=1}^{n} \int \phi(\theta_i)p_{\xi}(\mathbf{y}_i|\theta_i)d\theta_i. \tag{4}$$

If $p(\xi)$ is a prior pdf of ξ, the posterior of ξ is simply

$$p(\xi|\mathbf{y}) \propto p(\xi)p(\mathbf{y}|\xi). \tag{5}$$

The main problem to be addressed in this paper is the estimation of the ability θ of a new individual with response vector $\mathbf{x}$, when we are given $\mathbf{y}$, the data from the calibration.

When ξ is known, a standard method of estimating θ is by maximum likelihood (ML), i.e., finding the value of θ which maximizes the likelihood function $\mathcal{L}(\theta|\xi) = p_{\xi}(\mathbf{x}|\theta)$. In this case the variance of the ML estimator $\hat{\theta}$ may be approximated by the inverse of the test information function, which is given for 3PL (Lord, 1980, p. 73) by

$$I(\theta) = \sum_{j=1}^{K} \frac{a_j^2(1-c_j)}{(c_j+e^{z_j})(1+e^{-z_j})^2}, \tag{6}$$

where $z_j = a_j(\theta - b_j)$ and $(a_j, b_j, c_j) = \xi_j$. In the absence of known ξ, a common practice is to replace ξ by $\hat{\xi}_J$, a component of the joint ML estimate $(\hat{\xi}_J, \hat{\theta}_J)$ based on the likelihood function

$$\mathcal{L}(\xi,\theta) = \prod_{i=1}^{n} p_{\xi}(y_i|\theta_i) \tag{7}$$

and to estimate the unknown new θ by the value of θ which maximizes the conditional likelihood function $\mathcal{L}(\theta|\hat{\xi}_J) = p_{\hat{\xi}_J}(x|\theta)$.

When ξ is known, the standard Bayesian method (Birnbaum, 1969) of estimating θ is in terms of posterior mean

$$\tilde{\theta} = E(\theta|x,\xi) = \int \theta p(\theta|x,\xi) d\theta, \tag{8}$$

where, by Bayes theorem,

$$p(\theta|x,\xi) = \frac{p_{\xi}(x|\theta)\phi(\theta)}{\int p_{\xi}(x|\theta)\phi(\theta)d\theta}. \tag{9}$$

In this case the measure of uncertainty is the posterior variance,

$$\tilde{\sigma}^2 = V(\theta|x,\xi) = \int (\theta-\tilde{\theta})^2 p(\tilde{\theta}|x,\xi) d\theta. \tag{10}$$

(See Lord (1986) for an interesting comparison of the posterior mean and ML estimate of θ when ξ is known.)

In the absence of a known ξ, an empirical Bayes (EB) solution would be to replace ξ in (8) and (10) by the marginal maximum likelihood estimate, $\hat{\xi}_M$, based on (4). This type of approach has been criticized by Deeley & Lindley (1981) for its failure to account for the error in $\hat{\xi}_M$.

When ξ is unknown, the Bayesian solution is through the posterior distribution of θ given the data $\mathbf{z} = (\mathbf{x},\mathbf{y})$. The pdf of this distribution is given by

$$p(\theta|\mathbf{z}) = \int p(\theta|\mathbf{z},\xi)p(\xi|\mathbf{z})d\xi, \tag{11}$$

where, from the conditional independence if $\mathbf{x}$ and $\mathbf{y}$ given ξ,

$$p(\xi|\mathbf{z}) = p(\mathbf{x}|\xi)p(\xi|\mathbf{y})/p(\mathbf{x}|\mathbf{y}) \tag{12}$$

and

$$p(\theta|\mathbf{z},\xi) = p_\xi(\mathbf{x}|\theta)\phi(\theta)/p(\mathbf{x}|\xi). \tag{13}$$

Substituting (12) and (13) into (11) we have

$$p(\theta|\mathbf{z}) = \frac{\phi(\theta)}{p(\mathbf{x}|\mathbf{y})} \int p_\xi(\mathbf{x}|\theta)p(\xi|\mathbf{y})d\xi, \tag{14}$$

where we can now see how $p(\xi|\mathbf{y})$ serves as the prior for ξ subsequent to the calibration.

The posterior mean and variance of θ can be similarly expressed as

$$\mu = \int E(\theta|\mathbf{x},\xi)p(\xi|\mathbf{z})d\xi \tag{15}$$

and

$$\sigma^2 = \int E\{(\theta-\mu)^2|\mathbf{x},\xi)p(\xi|\mathbf{z})d\xi. \tag{16}$$

These expressions are difficult to work with numerically. Practical approximations are presented in the next section.

It is instructive to consider a decomposition of the posterior variance (16) in order to identify the sources of variability. Using a well known identity (e.g. DeGroot, 1986, p. 225) we have

$$\begin{aligned}\sigma^2 &= \int V(\theta|\xi,z)p(\xi|z)d\xi + \int\{E(\theta|\xi,z) - E(\theta|z)\}^2 p(\xi|z)d\xi \\ &= \int V(\theta|\xi,x)p(\xi|z)d\xi + \int\{E(\theta|\xi,x) - E(\theta|z)\}^2 p(\xi|z)d\xi, \quad (17)\end{aligned}$$

where we have used the fact that $(\theta,\mathbf{x})$ and $\mathbf{y}$ are conditionally independent given ξ. Thus the posterior variance of θ may be interpreted as the average conditional posterior variance of θ given ξ plus the variance of the conditional posterior expectation of θ given ξ with respect to the posterior uncertainty in ξ. The empirical Bayes variance approximates the first integral in (17) with (10) by replacing ξ with $\hat{\xi}_M$, but ignores the second. The second term is important when ξ is ill–determined after observing $\mathbf{z}$.

Bayesian Approximation

The approximation used to compute the posterior mean and variance of θ under the 2PL model by Tsutakawa & Soltys can be modified for 3PL. The approximation is a special case of Lindley's (1980) approximation to the posterior mean of a function of hyperparameters.

Suppose $w(\xi)$ is a function of the item parameter ξ whose expectation we wish to evaluate. Let w and w_{rs} denote the value of $w(\xi)$ and its second partial derivatives evaluated at the posterior mode $\hat{\xi}$. Let τ_{rs} denote the elements of the approximate posterior covariance matrix of ξ, where the approximation used is the inverse Hessian of

the negative log posterior evaluated at $\hat{\xi}$. The approximation is then given by

$$\overline{w} = w + \frac{1}{2} \Sigma\, w_{rs} \tau_{rs}. \tag{18}$$

When the posterior of ξ is normal, this is Lindley's approximation. For a heuristic justification of $\overline{w}$ and discussion on other Bayesian approximations see Tsutakawa & Soltys.

To approximate the posterior mean of θ, use $w(\xi) = E(\theta|\mathbf{x},\xi)$ in (18). If the approximate mean has value m, the posterior variance of θ is similarly computed by using (18) again with $w(\xi) = E\{(\theta-m)^2|\mathbf{x},\xi\}$.

To apply this approximation in practice, we recommend replacing $p(\xi|\mathbf{z})$ by $p(\xi|\mathbf{y})$ in (15) and (16). This substitution will have a negligible effect since $\mathbf{z}$ contains data on only one additional individual. Moreover updating $p(\xi|\mathbf{z})$ for each new individual is not only costly but would change the scoring criterion from one person to the next so that two individuals with the same response $\mathbf{x}$ could have different estimates of θ. This substitution is obviously not necessary when the θ being estimated is for an individual in the calibration sample. Computational expression for w, w_{rs} and τ_{rs} needed for 3PL are summarized in the Appendices.

Numerical Results

The ability estimation will now be demonstrated using a sample of n=400 for calibration and an additional sample of 100 to estimate 100 θ. Both samples were drawn from a larger sample of 1987 ACT math test results where K=40. Due to the fair number of omitted responses, the samples were selected after deleting examinees who omitted the last item or more than 10% of the items.

The computation of the posterior mode is based on the EM algorithm (Dempster, Laird & Rubin, 1976) as implemented in Tsutakawa (1988) for the case in which the prior

of ξ is derived from a Dirichlet distribution based on data from a similar test given in 1981. This prior assigns a joint distribution to the probability of correct responses at three values of θ. When properly constrained, it induces a distribution on ξ. Although we use the same prior as Tsutakawa (1988), the data used here has been resampled from the same 1987 test after making the deletions mentioned above. We also use the parameterization $\xi_j = (b_j, c_j, d_j)$, where $d_j = \log a_j$, $j=1,\ldots,K$, in order to enhance the asymptotic normality of the posterior distribution of ξ, a condition which would make our approximation closer to Lindley's.

Table 1 lists the posterior modes of the item parameters, together with the approximate standard deviations and within item correlations. The correlations between items are used in the computation but not listed since they are quite numerous. These results were used to compute the means and standard deviations of θ, for the 100 individuals, which are plotted in Figure 1. The standard deviations are lowest when the estimated θ are close to 0.25 and increase quite rapidly as the estimated θ departs from this central location. This general pattern is to be expected since tests of this type are generally designed to assess individuals whose abilities are close to the average.

In order to compare these results to those under ML and EB, the corresponding estimates of θ and their standard deviations were computed using the methods outlined above. Figure 1, which gives a plot of the values computed, shows that the standard deviations under EB tend to be considerably smaller than those under Bayes. For $\theta > .5$, the standard deviations under ML also tend to be considerably smaller than those under Bayes, but for $\theta < .5$, the two procedures have comparable standard deviations on the average.

In Figures 2 and 3 the ML and EB estimates of θ are plotted against the posterior means. There is a general agreement over the interval from about -1.2 to 0. However the posterior means in the interval from 0 to 2 tend to be larger than the other two estimates. For the more extreme values (which are listed but not plotted in Figure 2), there is a

tendency for the Bayes estimate to be pulled more towards the origin relative to ML. In fact there was one individual having a perfect score whose θ cannot be computed under ML.

The inferential effect that the different procedures have on the estimates and their standard deviations may be illustrated in terms of interval estimates, defined here as the estimate of $\theta \pm 2$ standard deviations. The end points of the interval estimates obtained by ML and EB are plotted against the end points of the Bayesian or posterior intervals in Figures 4 and 5 for the first 50 of the 100 examinees. It is quite apparent from these plots that both ML and EB produce substantially shorter intervals than the Bayes intervals in most cases. Since the intervals tend to become quite wide when the estimated θ departs from the origin, the intervals were converted to percentile intervals, with a percentile defined by $100\ \phi(\theta)$ where ϕ is the N(0,1) cdf. The corresponding plots, shown in Figures 6 and 7, further accent the narrower width of the ML and EB intervals.

In order to explain the differences observed in these graphs it is important to consider the differences in the assumptions and information used. The Bayes and EB methods both assume a N(0,1) prior on θ. Thus the observed difference is not due to the prior on θ but the use or nonuse of certain information from the calibration phase. Under EB, the unknown ξ is replaced by its estimate, without accounting for the error in the estimate, resulting in a deflated standard deviation. On the other hand ML makes no distributional assumption about θ, suggesting that its intervals should be wider than Bayes (Lord, 1986). However the fact that the estimated ξ is treated as the true value again deflates the standard deviation, but not to the extent of EB.

Discussion

The main conclusion of this paper is that tests based on calibrations which produce imprecise estimates of item parameters and ignore this imprecision can lead to serious inferential errors. The discrepancy found here between the Bayes and conventional

methods for 3PL is more striking than that reported earlier by Tsutakawa & Soltys (1988) for 2PL.

In large scale testing the sample size for calibration is typically substantially larger than the 400 used here. However, increasing the size of the calibrating sample alone will not increase the precision of the ability estimates. The major component of the uncertainty, whether the inference is Bayesian or frequentist, is the randomness of the individual response pattern **x** for a given θ and ξ. This uncertainty cannot be reduced without increasing the number of items in the test.

There is a need for better approximations which are not only more accurate but simple enough for routine use. Bayesian approximations, which are adaptable to modern computer technology are only beginning to appear and give promise for widespread Bayesian applications in testing.

For inferential purposes it is important to distinguish the sampling variance of the ability estimator for individuals with ability θ and the posterior variance of θ for an individual with response **x**. The former may be interpreted as the variance before observing **x** among these with ability θ and the latter as the (subjective) posterior variance after observing **x**. Since θ is unknown, the former is unknown, but it is common practice to estimate it by replacing θ with its maximum likelihood estimate based on **x**. Since this estimate is unreliable so is this variance estimate. On the other hand the posterior variance is a measure of uncertainty we have about a particular individual's θ after observing **x**. The subjectivity of this measure enters only through the choice of the prior distribution for the item parameters. This subjectivity should not be a controversial issue when the choice of the prior is based on past tests, as in the illustration used here. If one is interested in the probable values of θ, after **x** has been realized, the Bayesian approach is the logical choice.

Appendix A: Expressions for Computing the Empirical Information Matrix

To simplify the notation let $\xi_j = (\xi_{j1}, \xi_{j2}, \xi_{j3}) = (b_j, c_j, d_j)$ and $P_{ij} = p_{\xi_j}^{y_{ij}}(\theta)\{1-p_{\xi_j}(\theta)\}^{1-y_{ij}}$.

Now define

$$g_s(i,j,\theta) = \partial \log P_{ij}/\partial \xi_{js},$$

$$g_{st}(i,j,\theta) = \partial^2 \log P_{ij}/\partial \xi_{js}\partial \xi_{jt},$$

and

$$h_{st}(i,j,\theta) = \{\partial^2 \log P_{ij}/\partial \xi_{js}\partial \xi_{jt}\}/P_{ij},$$

for $i = 1,\ldots,n$; $j = 1,\ldots,K$; $s, t = 1,2,3$. We then have

$$h_{st}(i,j,\theta) = g_{st}(i,j,\theta) + g_s(i,j,\theta)g_t(i,j,\theta). \tag{19}$$

Define, for notational convenience only,

$$z_{\theta j} = \exp(d_j)(\theta - b_j),$$

$$\phi_{\theta j} = \{1 + \exp(-z_{\theta j})\}^{-1},$$

$$\psi_{\theta j} = 1-\phi_{\theta j},$$

$$\lambda_{\theta j} = \{1 + c_j\exp(-z_{\theta j})\}^{-1},$$

$$\eta_{\theta j} = \{c_j + \exp(z_{\theta j})\}^{-1}.$$

Then the first two derivatives of $\log P_{ij}$ may be expressed by

$$g_1(i,j,\theta) = -\exp(d_j)(y_{ij}\lambda_{\theta j} - \phi_{\theta j}),$$

$$g_2(i,j,\theta) = y_{ij}[\eta_{\theta j} - 1/(c_j-1)] + 1/(c_j-1),$$

$$g_3(i,j,\theta) = z_{\theta j}(y_{ij}\lambda_{\theta j} - \phi_{\theta j}),$$

$$g_{11}(i,j,\theta) = \exp(2d_j)(c_j y_{ij}\lambda_{\theta j}\eta_{\theta j} - \phi_{\theta j}\psi_{\theta j}),$$

$$g_{12}(i,j,\theta) = y_{ij}\exp(d_j)\eta_{\theta j}\lambda_{\theta j},$$

$$g_{13}(i,j,\theta) = -\exp(d_j)(y_j\lambda_{\theta j}-\phi_{\theta j}) + z_{\theta j}\exp(d_j)[\phi_{\theta j}\psi_{\theta j}-c_j y_{ij}\lambda_{\theta j}\eta_{\theta j}],$$

$$g_{22}(i,j,\theta) = -[y_{ij}\eta^2_{\theta j} + (1-y_{ij})/(c_j-1)^2],$$

$$g_{23}(i,j,\theta) = -z_{\theta j}y_{ij}\lambda_{\theta j}\eta_{\theta j},$$

$$g_{33}(i,j,\theta) = z^2_{\theta j}(c_j y_{ij}\lambda_{\theta j}\eta_{\theta j}-\phi_{\theta j}\psi_{\theta j}) + z_{\theta j}(y_{ij}\lambda_{\theta j}-\phi_{\theta j}).$$

Then $h_{st}(i,j,\theta)$ may be expressed in terms of the expressions for g_s and g_{st} through equation (19).

Now define the conditional posterior expectations (given ξ) of these functions by

$$\bar{g}_s(i,j) = \int_{-\infty}^{\infty} g_s(i,j,\theta)p(\theta|\mathbf{y}_i,\xi)d\theta,$$

$$\bar{h}_{st}(i,j) = \int_{-\infty}^{\infty} h_{st}(i,j,\theta)p(\theta|\mathbf{y}_i\xi)d\theta,$$

$$\bar{d}_{st}(i,j,j') = \int_{-\infty}^{\infty} g_s(i,j,\theta)g_t(i,j',\theta)p(\theta|\mathbf{y}_i,\xi)d\theta,$$

for $i = 1,\ldots,n$; $j,j' = 1,\ldots,K$; and $s,t = 1,2,3$, where $p(\theta|\mathbf{y}_i,\xi) = p_\xi(\mathbf{y}_i)\phi(\theta)/p(\mathbf{y}_i|\xi)$, the posterior pdf of θ_i given $\mathbf{y}_i = (y_{i1},\ldots,y_{iK})$

Then, finally, the first and second partial derivatives of the loglikelihood function $L(\xi) = \log p(\mathbf{y}|\xi)$ are given by

$$\partial L(\xi)/\partial\xi_{js} = \sum_{i=1}^{n} \bar{g}_s(i,j),$$

and

$$\partial^2 L(\xi)/\partial\xi_{js}\partial\xi_{j't} = \begin{cases} \sum_{i=1}^{n} \{\bar{h}_{st}(i,j)-\bar{g}_s(i,j)\bar{g}_t(i,j)\} \text{ if } j=j' \\ \sum_{i=1}^{n} \{\bar{d}_{st}(i,j,j')-\bar{g}_s(i,j)\bar{g}_t(i,j')\} \text{ if } j\neq j', \end{cases}$$

for $i=1,\ldots,n$; $j,j'=1,\ldots,K$; $s,t = 1,2,3$. The empirical information matrix is the negative of

the 3Kx3K 2nd derivative matrix, i.e.,

$$I(\xi) = [-\partial^2 L(\xi)/\partial\xi_{js}\partial\xi_{j't}].$$

To complete the expressions for the Hessian of the negative log posterior, we must add to $I(\xi)$ the second partials of the negative log prior, expressions for which are summarized in Tsutakawa (1988). A quadrature method for the required numerical integration is given in Tsutakawa (1984).

Appendix B: Expressions for Computing the Second Derivatives of $w(\xi)$

The function $w(\xi)$ used in the approximations have the form,

$$w(\xi) = \int f(\theta_i) p(\theta_i | y_i, \xi) d\theta_i,$$

where $f(\theta_i) = \theta_i$ for the posterior mean and $f(\theta_i) = (\theta_i - m)^2$ for the posterior variance. By interchanging the order of differentiation and integration, the required second derivatives of $w(\xi)$ evaluated at $\xi = \hat{\xi}$ have the form

$$w_{st}(j,j') = \int f(\theta_i) \{ \partial^2 p(\theta_i | y_i, \hat{\xi}) / \partial \xi_{js} \partial \xi_{j't} \} d\theta_i,$$

for j, j' = 1,...,K and s, t = 1,2,3. (Additional subscripts have been introduced here in order to identify the nesting of the item parameters within items.)

Upon evaluating the second derivatives of $p(\theta_i | y_i, \xi)$, the computational expressions reduce to

$$\begin{aligned} w_{st}(j,j) = {} & E\{f(\theta_i) h_{st}(i,j,\theta_i)\} - \bar{g}_t(i,j) E\{f(\theta_i) g_s(i,j,\theta_i)\} - \bar{g}_s(i,j) E\{f(\theta_i) g_t(i,j,\theta_i)\} \\ & + E\{f(\theta_i)\} \{2\bar{g}_s(i,j)\bar{g}_t(i,j) - \bar{h}_{st}(i,j)\} \end{aligned}$$

and, for $j \neq j'$,

$$\begin{aligned} w_{st}(j,j') = {} & E\{f(\theta_i) g_s(i,j,\theta_i) g_t(i,j',\theta_i)\} - \bar{g}_t(i,j') E\{f(\theta_i) g_s(i,j,\theta_i)\} - \bar{g}_s(i,j) E\{f(\theta_i) g_t(i,j',\theta_i)\} \\ & + E\{f(\theta_i)\} \{2\bar{g}_s(i,j)\bar{g}_t(i,j') - \bar{d}_{st}(i,j,j')\} \end{aligned}$$

for j, j' = 1,...,K and s, t = 1,2,3, where E{} denotes expectation with respect to the posterior pdf $p(\theta_i | y_i, \hat{\xi})$.

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TABLE 1

Summary of Posterior Distribution of Item Parameters

Item	Item Score	Posterior Mean b	Posterior Mean c	Posterior Mean d	Posterior SD b	Posterior SD c	Posterior SD d	Posterior Correlation bc	Posterior Correlation bd	Posterior Correlation cd
1	327	-1.43	0.01	0.10	0.87	0.53	0.21	0.97	0.81	0.69
2	325	-0.02	0.56	0.69	0.31	0.09	0.28	0.89	0.77	0.69
3	304	-0.77	0.12	0.32	0.53	0.28	0.22	0.96	0.86	0.78
4	304	-0.05	0.41	1.42	0.11	0.06	0.23	0.67	0.62	0.47
5	298	-0.17	0.36	0.62	0.31	0.13	0.25	0.92	0.82	0.76
6	294	-0.61	0.05	0.63	0.30	0.19	0.20	0.94	0.85	0.78
7	279	0.07	0.35	0.96	0.14	0.07	0.20	0.77	0.61	0.53
8	276	-0.34	0.13	0.57	0.27	0.14	0.21	0.93	0.82	0.76
9	274	0.22	0.39	1.16	0.12	0.05	0.22	0.69	0.55	0.49
10	274	-0.31	0.15	0.48	0.31	0.16	0.21	0.94	0.82	0.77
11	265	-0.49	0.00	0.39	0.34	0.19	0.20	0.95	0.84	0.79
12	254	0.74	0.47	0.74	0.17	0.05	0.27	0.71	0.38	0.58
13	253	0.13	0.26	0.57	0.19	0.08	0.20	0.85	0.64	0.63
14	245	0.18	0.24	0.52	0.23	0.10	0.24	0.90	0.74	0.75
15	240	0.27	0.25	0.29	0.33	0.12	0.27	0.93	0.76	0.78
16	234	0.45	0.31	0.98	0.12	0.05	0.22	0.70	0.50	0.56
17	225	0.13	0.11	0.91	0.11	0.05	0.16	0.72	0.55	0.58
18	224	0.12	0.10	1.01	0.11	0.06	0.18	0.73	0.61	0.65
19	216	0.55	0.28	0.87	0.13	0.06	0.23	0.74	0.51	0.63
20	215	0.42	0.22	0.77	0.14	0.06	0.21	0.77	0.56	0.65
21	213	0.42	0.22	0.98	0.11	0.05	0.21	0.70	0.51	0.61
22	211	0.44	0.21	0.65	0.16	0.07	0.22	0.82	0.58	0.70
23	205	0.37	0.16	1.07	0.09	0.04	0.18	0.64	0.42	0.54
24	195	0.70	0.24	0.61	0.15	0.06	0.25	0.75	0.44	0.69
25	195	0.33	0.09	0.60	0.14	0.06	0.18	0.79	0.55	0.68
26	189	1.03	0.28	0.39	0.20	0.06	0.28	0.68	0.25	0.69
27	187	0.46	0.11	0.49	0.15	0.07	0.20	0.81	0.53	0.70
28	184	0.69	0.21	0.85	0.11	0.05	0.22	0.63	0.32	0.59
29	180	0.59	0.16	0.76	0.12	0.05	0.21	0.71	0.42	0.66
30	178	0.83	0.23	0.73	0.13	0.05	0.23	0.62	0.23	0.61
31	174	0.43	0.00	-0.12	0.56	0.21	0.31	0.97	0.83	0.88
32	172	0.52	0.10	0.80	0.10	0.05	0.18	0.66	0.39	0.64
33	168	1.05	0.29	1.24	0.09	0.03	0.24	0.31	-0.11	0.33
34	166	0.60	0.12	1.05	0.08	0.03	0.16	0.44	0.08	0.36
35	160	0.99	0.21	0.67	0.14	0.06	0.30	0.63	0.32	0.77
36	151	1.87	0.30	0.41	0.30	0.04	0.35	0.01	-0.65	0.52
37	149	0.98	0.20	0.99	0.10	0.03	0.23	0.41	-0.04	0.50
38	147	0.68	0.10	1.18	0.07	0.03	0.16	0.32	-0.03	0.29
39	144	0.79	0.12	0.93	0.09	0.03	0.18	0.44	-0.01	0.47
40	133	0.88	0.09	0.72	0.10	0.04	0.20	0.46	0.00	0.61

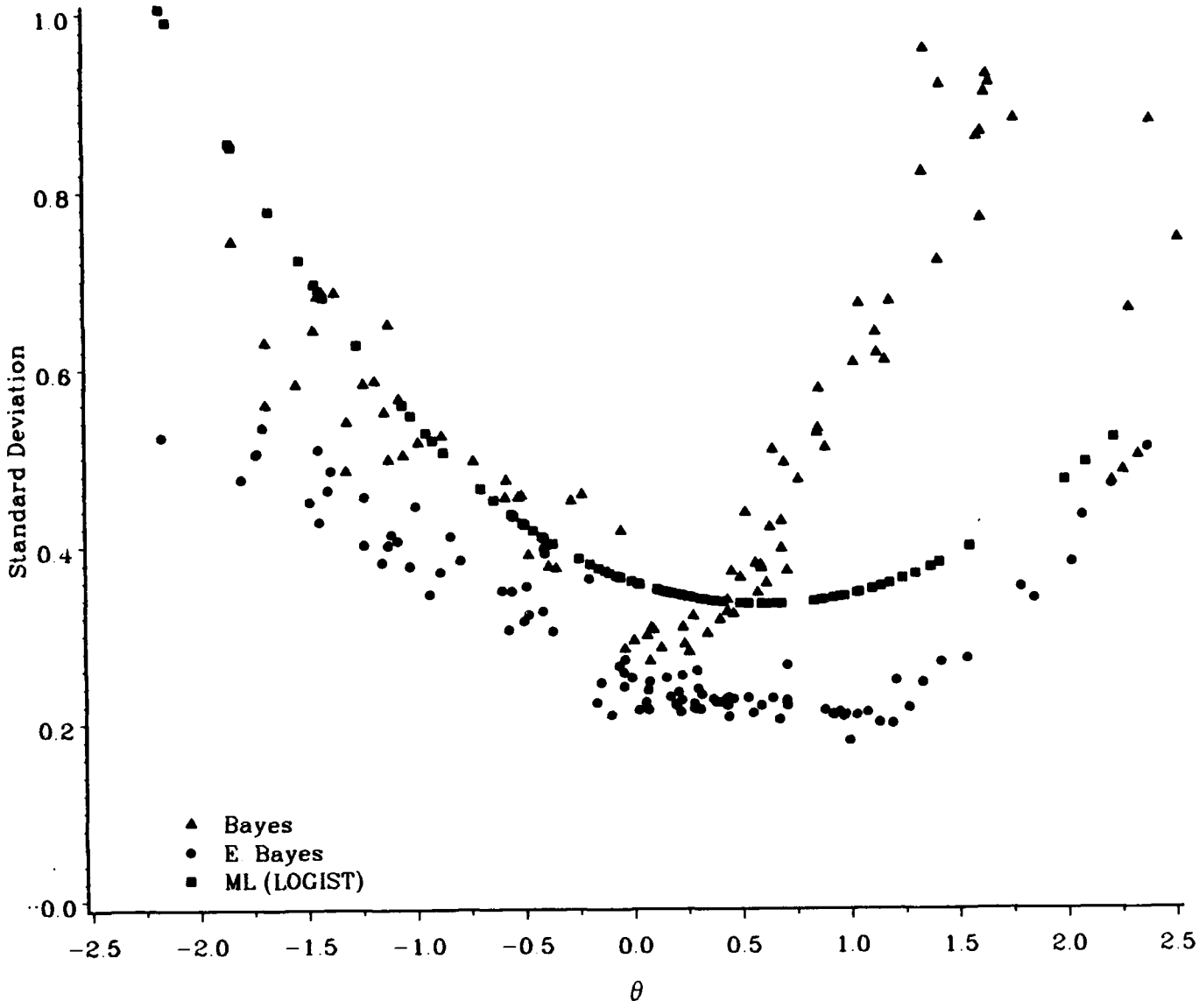

FIGURE 1
Estimated $\hat{\theta}$ and Standard Deviations Under Three Procedures.

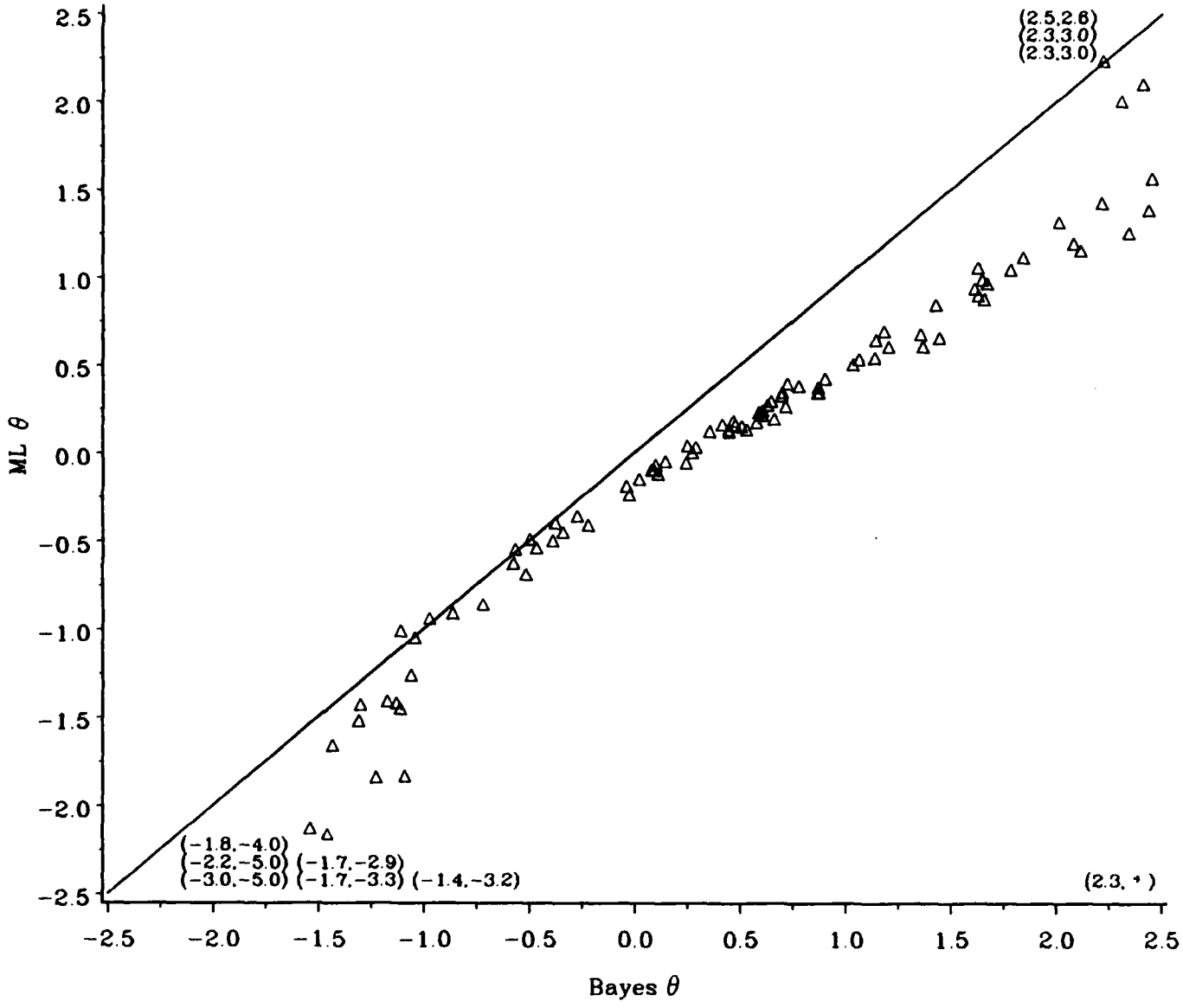

FIGURE 2
Maximum Likelihood vs. Bayes Estimates of θ.

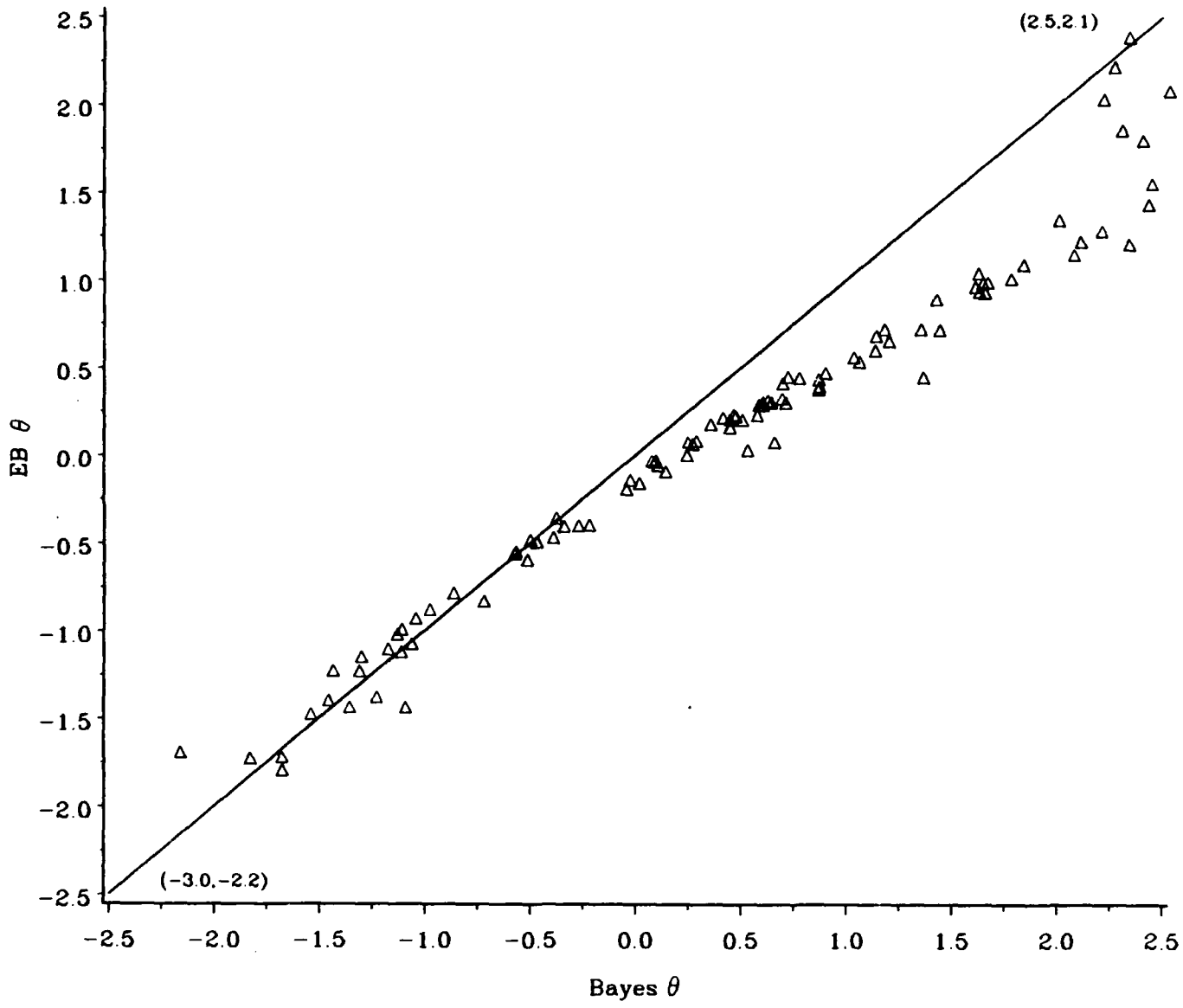

FIGURE 3
Empirical Bayes vs. Bayes Estimates of θ.

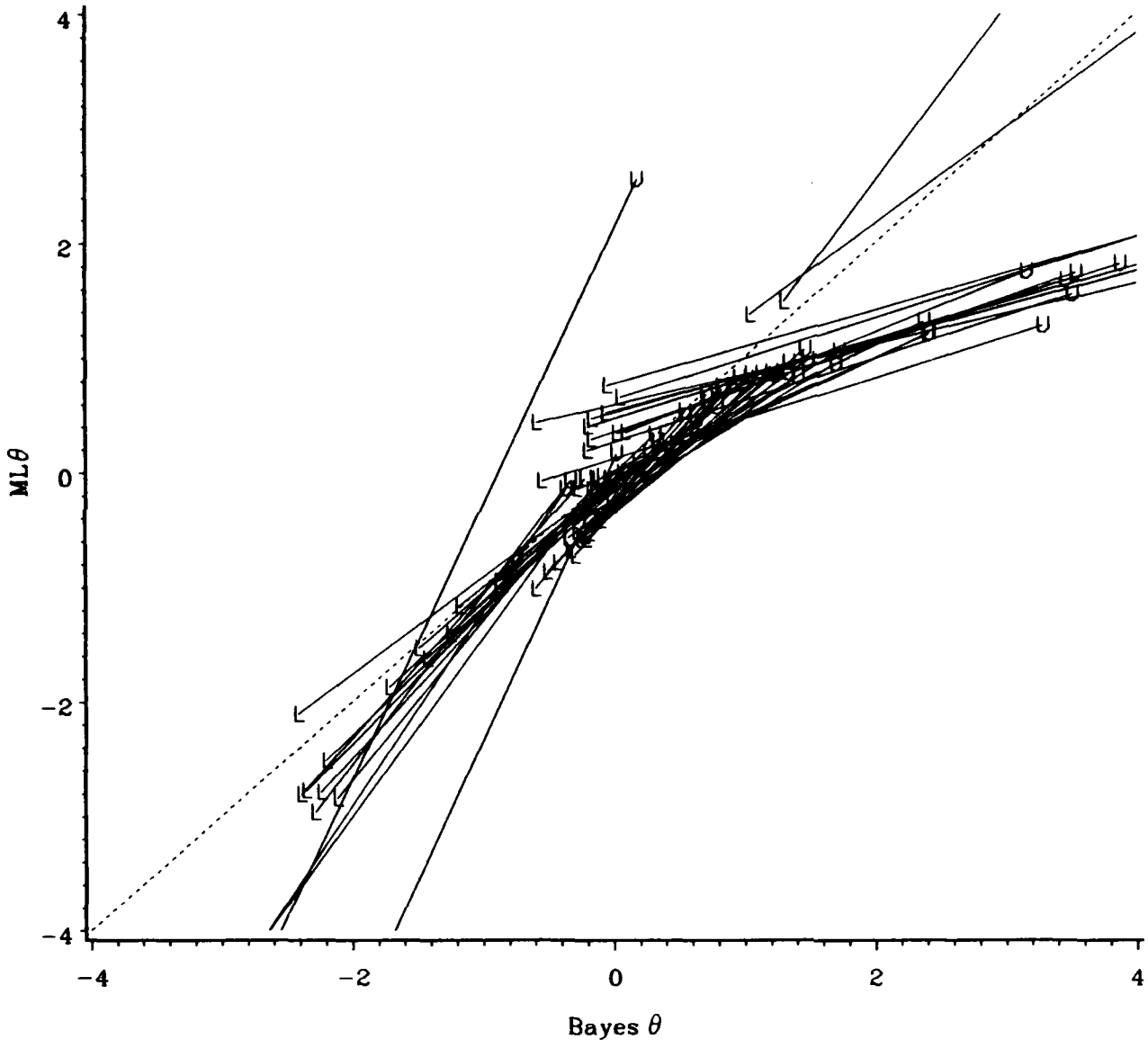

FIGURE 4
Interval Estimates of θ under Maximum Likelihood vs. Bayes for 50 Examinees.

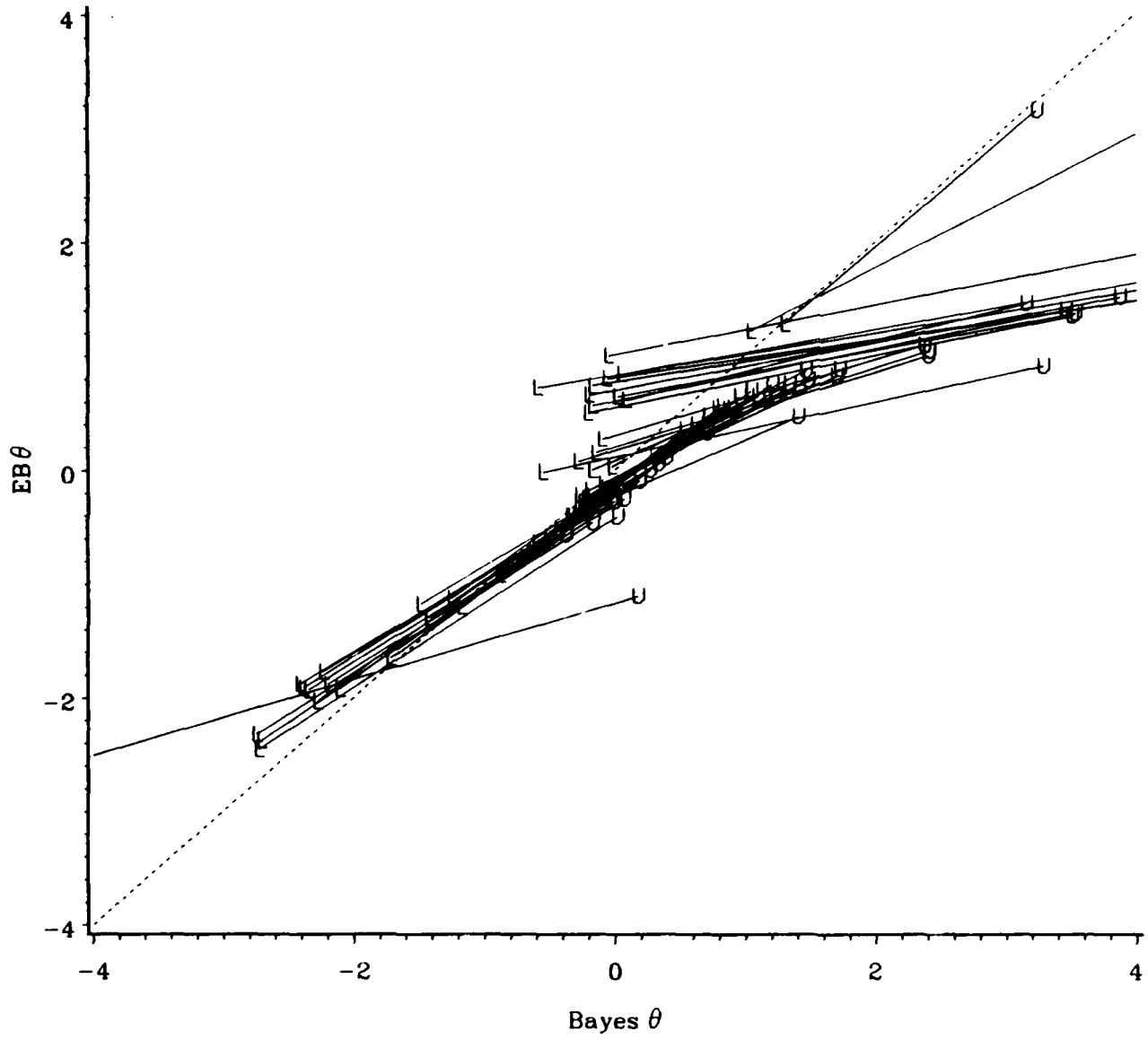

FIGURE 5
Interval Estimates of θ under Empirical Bayes vs. Bayes for 50 Examinees.

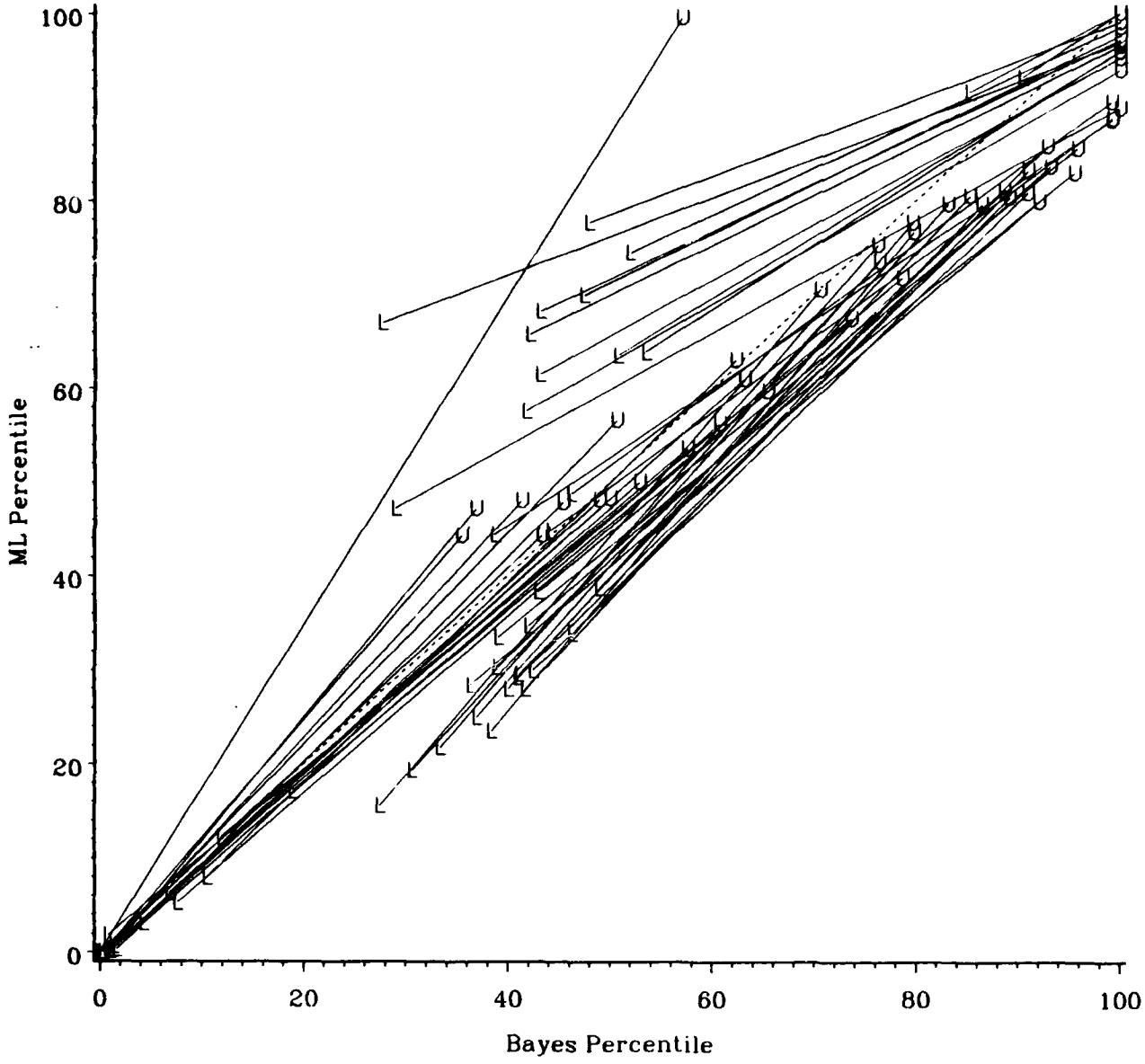

FIGURE 6
Interval Estimates of Percentile under ML vs. Bayes for 50 Examinees.

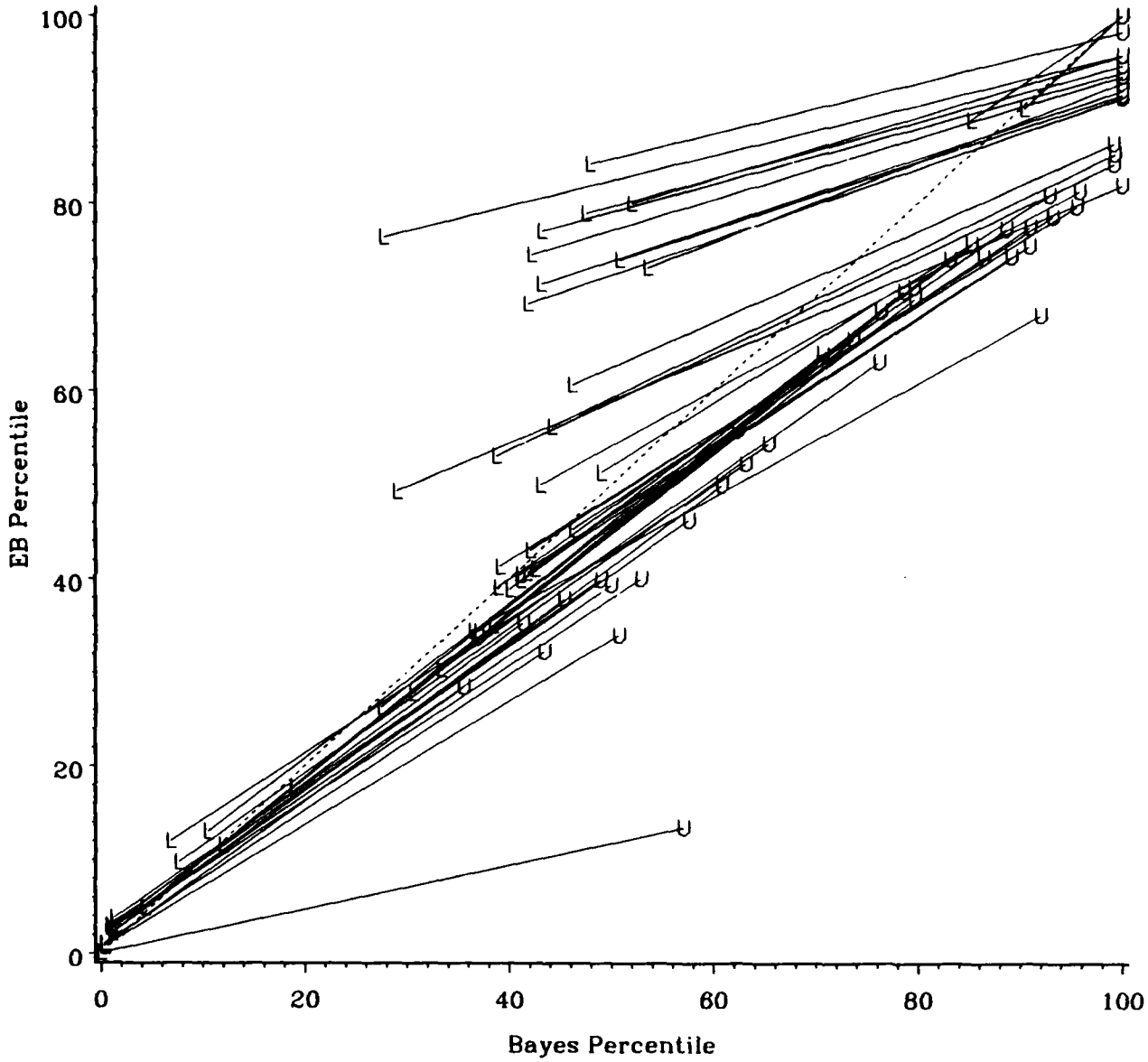

FIGURE 7
Interval Estimates of Percentile under Empirical Bayes vs. Bayes for 50 Examinees.

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