The Propagation of Electromagnetic Waves in Thin Dielectric Slabs

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This report presents the solutions of Maxwell's equations for the TE and TM modes of propagation along a thin dielectric slab. From these, the propagation constants are determined and the electric and magnetic field patterns are plotted. These calculations are done with the use of the dielectric coefficients of LiNO₃ and GaAs. The wavelengths used are 0.82 \( \mu \text{m} \) (diode laser), 1.064 \( \mu \text{m} \) (Nd:YAG laser), and 1.55 \( \mu \text{m} \) (diode laser). The thicknesses of the slabs range from 0.1 to 1.0 \( \mu \text{m} \).
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1. Introduction

Because dielectric waveguides are frequently used as part of the experimental setups in the laboratory, it is of interest to have general programs that calculate their characteristics. These waveguides generally consist of thin sheets (a few wavelengths or less) of a low-loss nonmagnetic dielectric material. The electromagnetic wave is launched at one end of the slab and guided to the sample [1].

In this report, we idealize the situation to a single slab of infinite length in two dimensions, and a thickness, 2d, which will be arbitrary. First, we introduce the appropriate Maxwell's equations for the problem and derive the wave equation for the transverse electric (TE) and transverse magnetic (TM) modes. We obtain the general solutions for both modes inside and outside the dielectric and, by satisfying the boundary conditions at the surfaces of the slab, obtain four transcendental equations which provide the solutions to the problem. There is one equation for each mode, the TE even, TE odd, TM even, and TM odd. Next, we assign values to our parameters and numerically solve the transcendental equations. This procedure is repeated for several thicknesses and indices of refraction. The results are presented in graphical and tabular form. The appendix contains the listings of the programs used.
2. Maxwell’s Equations

Our waveguide extends infinitely in the y and z directions, and from \(-d\) to \(d\) in the x direction (see fig. 1a). The wave is launched in the y direction. The relevant Maxwell equations for the problem (in centimeter-gram-seconds) are

\[ \nabla \times \vec{E} = ik \vec{H} \]  
(1)

and

\[ -\nabla \times \vec{H} = ik \varepsilon \vec{E} , \]  
(2)

where \(\varepsilon\) is the dielectric constant. The time dependence is assumed to be \(\exp(-i\omega t)\), with \(k = \omega/c\). Throughout the problem, \(\mu\) is considered constant and equal to one. In all cases considered, we assume that none of the fields vary in the z direction \((\partial/\partial z = 0)\).

In our first case, the TE mode, \(E_x = 0, E_y = 0,\) and \(E_z \neq 0\). So, Maxwell’s equations yield

\[ ik H_x = \frac{\partial}{\partial y} E_z , \]  
(3)

\[ ik H_y = -\frac{\partial}{\partial x} E_z , \]  
(4)

and

\[ \frac{\partial}{\partial y} H_x - \frac{\partial}{\partial x} H_y = ik \varepsilon E_z . \]  
(5)

Combining these equations gives us the wave equation

\[ \nabla_i^2 E_z = -k^2 \varepsilon E_z \]  
(6)

\[ \varepsilon = \varepsilon \quad |x| < d \]

\[ \varepsilon = 1 \quad |x| > d \]

where \(\nabla_i^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \).
We will assume a dependence in the y direction of the form $\exp(i\beta y)$. We will also assume odd and even solutions of the form $\sin k_1 x$ and $\cos k_1 x$ inside the dielectric, and a decaying exponential, $\exp(-\Gamma x)$, outside the dielectric. For guided waves in a lossless material, we must have $\Gamma$ real and positive.

For even solutions, using equation (6), we have

$$E_z = E_1 \cos k_1 x e^{i\beta y}$$

for $|x| < d$, (7)

$$E_z = E_2 e^{-\Gamma x} e^{i\beta y}$$

for $|x| > d$, (8)

where

$$k_1^2 + \beta^2 = k^2 \varepsilon$$

(9)

$$-\Gamma^2 + \beta^2 = k^2$$

(10)

therefore,

$$\Gamma = \sqrt{k^2 (\varepsilon - 1) - k_1^2}.$$  (11)

We know that $E_y$ and $H_y$ must be continuous at the boundaries $x = d$ and $x = -d$. We need only solve at one of these boundaries, though, because
of the symmetry of the problem. We can find $H_y$ by using equation (4) with equations (7) and (8):

$$H_y = \frac{1}{ik} E_1 k_1 \sin k_1 x e^{i \beta y} \quad |x| < d ,$$  \hspace{1cm} (12)$$

$$H_y = \frac{1}{ik} \Gamma E_2 e^{-\Gamma x} e^{i \beta y} \quad |x| > d .$$  \hspace{1cm} (13)$$

Setting equation (7) equal to (8), and (12) equal to (13) gives us the system of equations

$$E_1 \cos k_1 d = E_2 e^{-\Gamma d} ,$$  \hspace{1cm} (14)$$

$$E_1 k_1 \sin k_1 d = E_2 \Gamma e^{-\Gamma d} ,$$  \hspace{1cm} (15)$$

which is reduced to

$$k_1 \tan k_1 d = \Gamma .$$  \hspace{1cm} (16)$$

Throughout the remainder of our discussion, we will be using the following substitutions:

$$u = k_1 d$$  \hspace{1cm} (17)$$

and

$$A^2 = k^2 (\varepsilon - 1) d^2 .$$  \hspace{1cm} (18)$$

Using equations (11), (17), and (18), equation (16) becomes

$$\tan u = \frac{\sqrt{A^2 - u^2}}{u} .$$  \hspace{1cm} (19)$$

This transcendental equation will be solved in section 3.

For odd solutions, sine replaces cosine in the material, making our wave equation reduce to

$$E_z = E_3 \sin k_1 x e^{i \beta y} \quad |x| < d ,$$  \hspace{1cm} (20)$$

$$E_z = E_4 e^{-\Gamma x} e^{i \beta y} \quad |x| > d .$$  \hspace{1cm} (21)$$
Equations (9), (10), and (11) still hold. Using equation (4) on equations (20) and (21) yields
\[ H_y = -\frac{1}{ik} E_3 k_1 \cos k_1 x e^{i\beta y} \quad |x| < d \]  \hfill (22)
\[ H_y = \frac{1}{ik} E_4 \Gamma e^{-\Gamma x} e^{i\beta y} \quad |x| > d \]  \hfill (23)

At \( x = d \), we have a system of equations which produces
\[-\cot k_1 d = \frac{\Gamma d}{k_1 d} \]  \hfill (24)

Using our substitutions, from equations (11), (17), and (18) we get
\[-\cot \mu = \frac{\sqrt{A^2 - \mu^2}}{\mu} \]  \hfill (25)

This transcendental equation will also be solved in section 3.

The calculations for the TM modes are very similar. We use the same Maxwell’s equations, but now the electromagnetic wave is described by figure 1b:
\[ H_x = 0, H_y = 0, H_z \neq 0 \]

Maxwell’s equations yield
\[ E_x = -\frac{1}{ik\varepsilon} \frac{\partial}{\partial y} H_z \]  \hfill (26)
\[ E_y = \frac{1}{ik\varepsilon} \frac{\partial}{\partial x} H_z \]  \hfill (27)
\[ H_z = \frac{1}{ik} \left( \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \right) \]  \hfill (28)

Combining these equations yields the wave equation
\[ \nabla^2 H_z = -k^2 \varepsilon H_z \]  \hfill (29)

For even solutions, we assume
\[ H_z = H_1 \cos k_1 x e^{i\beta y} \quad |x| < d \]  \hfill (30)
\[ H_z = H_2 e^{-\Gamma x} e^{i\beta y} \quad |x| > d \]  \hfill (31)

\[ 9 \]
where $k_1$ and $\Gamma$ are given in equations (9) and (10). $E_y$ and $H_z$ must be continuous over $x$. Using equation (27) with equations (30) and (31) gives us a system of equations at $x = d$ which yields

$$\tan k_1 d = \frac{\varepsilon \Gamma}{k_1}. \quad (32)$$

Or, after making our usual substitutions, with equations (11), (17), and (18), we get

$$\tan u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u}. \quad (33)$$

This will be numerically solved in section 3 with the other transcendental equations.

For the TM odd mode, a very similar set of calculations arrives at the solution

$$-\cot u = \varepsilon \frac{\sqrt{A^2 - u^2}}{u}. \quad (34)$$

We now have the four transcendental equations providing the even and odd solutions to the TE and TM modes.
3. Numerical Calculation of Solutions

In section 2 we obtained analytical solutions to our problem in the form of these four transcendental equations:

\[
\begin{align*}
\text{TE even} & : \tan u = \frac{\sqrt{A^2 - \mu^2}}{u} \\
\text{TE odd} & : -\cot u = \frac{\sqrt{A^2 - \mu^2}}{u} \\
\text{TM even} & : \tan u = \epsilon \frac{\sqrt{A^2 - \mu^2}}{u} \\
\text{TM odd} & : -\cot u = \epsilon \frac{\sqrt{A^2 - \mu^2}}{u}.
\end{align*}
\]

We wish to numerically calculate all the solutions to these equations using the Newton-Raphson method. We must first assign values to \( A \) and \( \epsilon \). To assign a value to \( A \) we must first break it down to its components. We assume the following:

\[
\begin{align*}
\epsilon &= 12.25 = n^2 \quad \text{(GaAs)} \\
d &= 0.175 \, \mu\text{m} \\
\lambda &= 1.064 \, \mu\text{m} \\
k &= 5.93 \times 10^4 \, \mu\text{m} \\
A &= 3.466 = \sqrt{k^2 (\epsilon - 1) d^2}.
\end{align*}
\]

These values are chosen to illustrate the method of solution to be described, but are not to represent practical values in use in the laboratory [2]. Although there may be some use of waveguides as thin as this, most optical waveguides are in the neighborhood of 1 \( \mu\text{m} \).

Just knowing these parameters is not enough. We must know the number of solutions, and their approximate values, for the Newton-Raphson method to work well. Figure 2a is a plot of \( \tan u, -\cot u, (A^2 - \mu^2)^{1/2}/u \), and \( \epsilon(A^2 - \mu^2)^{1/2}/u \) versus \( u \). The intersections of the trigonometric func-
tions with the latter two functions are the solutions to our problem. Their exact values, so far, are unknown, but they provide a good first guess for the program, as well as a check on the number and range of our solutions. Our initial estimate from figure 2a for the first even solutions was \( \pi/2 - d \). For each successive even solution, our initial guess was the previous final approximation plus \( \pi \). For the first odd solutions, our initial guess was \( \pi - d \). We used the same method for successive solutions (of which there were none) as in the even case. The markers on figure 2a are the values of the solutions generated by the program with the Newton-Raphson algorithm. Their "\( u \)" value is given in table 1.

We now wish to see the result of altering our input parameters--thickness, dielectric coefficient, and wavelength. Table 2 shows values of \( u \) for all possible solutions given these parameters. All combinations were used for thicknesses of 0.1, 0.35, and 1.00 \( \mu \text{m} \), and wavelengths of 0.820, 1.064, and 1.550 \( \mu \text{m} \), with a dielectric coefficient of 12.25. We also did one trial with a dielectric coefficient of 4.80 (\( \text{LiNO}_3 \)), a thickness of 1.00 \( \mu \text{m} \), and a wavelength of 1.064 \( \mu \text{m} \).

We can see from this table that increasing the wavelength, decreasing the thickness, or decreasing the dielectric coefficient will decrease the number of solutions. However, the first even modes (TE and TM) will never disappear. If the value of \( A \) drops below \( \pi/2 \), there will be no odd solutions, and only one of each even solution. If the value of \( A \) is very high, the number of solutions will be approximately \( 4 \times A/\pi \) (\( A/\pi \) of each type). The separation of the \( u \)'s for any two consecutive solutions of the same type approaches \( \pi \) for the lower value modes. These results are illustrated in figures 2a through j. More detailed effects of altering the input parameters are shown in the next section.
Figure 2. Plots of both sides of four transcendental equations (19, 25, 33, 34) presented in text. Circles mark points derived by Newton-Raphson approximation. These points define parameters for viable modes of propagation. Calculations were done for a slab with various thicknesses, dielectric coefficients, and free space wavelengths: (a) 0.35-μm slab thickness, 12.25 dielectric coefficient, and 1.064-μm free space wavelength; (b) 1.00-μm slab thickness, 4.80 dielectric coefficient, and 1.064-μm free space wavelength; (c) 1.00-μm slab thickness, 12.25 dielectric coefficient, and 1.550-μm free space wavelength; (d) 1.0-μm slab thickness, 12.25 dielectric coefficient, and 1.064-μm free space wavelength.
Figure 2. Plots of both sides of four transcendental equations (19, 25, 33, 34) presented in text. Circles mark points derived by Newton-Raphson approximation. These points define parameters for viable modes of propagation. Calculations were done for a slab with various thicknesses, dielectric coefficients, and free space wavelengths (cont'd): (e) 1.00-μm slab thickness, 12.25 dielectric coefficient, and 0.820-μm free space wavelength; (f) 0.10-μm slab thickness, 12.25 dielectric coefficient, and 1.064-μm free space wavelength; (g) 0.10-μm slab thickness, 12.25 dielectric coefficient, and 1.550-μm free space wavelength; (h) 0.10-μm slab thickness, 12.25 dielectric coefficient, and 0.820-μm free space wavelength.
Figure 2. Plots of both sides of four transcendental equations (19, 25, 33, 34) presented in text. Circles mark points derived by Newton-Raphson approximation. These points define parameters for viable modes of propagation. Calculations were done for a slab with various thicknesses, dielectric coefficients, and free space wavelengths (cont'd): (i) 0.35-μm slab thickness, 12.25 dielectric coefficient, and 0.820-μm free space wavelength; and (j) 0.35-μm slab thickness, 12.25 dielectric coefficient, and 1.550-μm free space wavelength.

Table 1. Values of \( u \) for all possible modes with slab thickness of 0.35 μm (\( d = 0.175 \) μm), dielectric constant of 12.25, and free space wavelength of 1.064 μm

<table>
<thead>
<tr>
<th>Mode</th>
<th>TEE</th>
<th>TEO</th>
<th>TME</th>
<th>TMO</th>
</tr>
</thead>
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<td></td>
<td>1.213211536</td>
<td>2.383396864</td>
<td>1.530640006</td>
<td>3.001243353</td>
</tr>
<tr>
<td></td>
<td>3.373444796</td>
<td>—</td>
<td>3.464896202</td>
<td>—</td>
</tr>
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</table>
Table 2. Values of $u$ for all possible modes with physical parameters as specified

<table>
<thead>
<tr>
<th>Parameters</th>
<th>TEE</th>
<th>TEO</th>
<th>TME</th>
<th>TMO</th>
</tr>
</thead>
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<td>2.660983324</td>
<td>1.514054298</td>
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<tr>
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<td>5.168169022</td>
<td>4.462066174</td>
<td>5.585698605</td>
</tr>
<tr>
<td>$\lambda = 1.064 \mu m$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$d = 0.005 \mu m$</td>
<td>0.849031210</td>
<td>—</td>
<td>1.248315096</td>
<td>—</td>
</tr>
<tr>
<td>$\varepsilon = 12.25$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$\lambda = 0.820 \mu m$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
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<td>—</td>
</tr>
<tr>
<td>$\lambda = 1.064 \mu m$</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$d = 0.050 \mu m$</td>
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</tr>
<tr>
<td>$\lambda = 1.550 \mu m$</td>
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</tr>
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</tr>
<tr>
<td>$\lambda = 1.550 \mu m$</td>
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<td>—</td>
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</tr>
<tr>
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<td>9.338629723</td>
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<td>6.552733421</td>
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</tr>
</tbody>
</table>

Note: $d =$ one-half thickness
$\varepsilon =$ dielectric coefficient
$\lambda =$ free space wavelength
4. Calculation of Propagation Parameters

Once we have found values for \( u \), we can calculate \( k_1, \Gamma, \) and \( \beta \). Given \( k \) [3] and \( \Delta k \) (the bandwidth of the source), we can also calculate \( \Delta \beta \) [4] (the bandwidth in the material). We shall first find \( \Delta \beta \) for the TE even mode. Differentiating equations (9), (17), (18), and (19) gives us

\[
k_1 \frac{dk_1}{du} + \beta \frac{d \beta}{du} = \varepsilon k dk
\]

\[
du = dk_1 d
\]

\[
dA = dk d/\sqrt{\varepsilon - 1}
\]

\[
\sec^2 u = \frac{A (dA/du) - u}{u \sqrt{A^2 - u^2}} - \frac{\sqrt{A^2 - u^2}}{u^2}.
\]

Substituting equation (37) for \( dA \) in equation (38) and rearranging yield

\[
du = \frac{Adk}{k_1 \sec^2 u(u \tan u + 1)}. \tag{39}
\]

There are no unknowns on the right side of the equations. We can now introduce equations (35) and (36) and solve for \( d \beta \). We get

\[
d \beta = \frac{kd k}{\beta} \left( \varepsilon - \frac{\varepsilon - 1}{\sec^2 u(u \tan u + 1)} \right). \tag{40a}
\]

For the other solution types, we get

\[
d \beta = \frac{kd k}{\beta} \left( \varepsilon - \frac{\varepsilon - 1}{\csc^2 u(u \cot u + 1)} \right) \quad \text{TE odd} \tag{40b}
\]

\[
d \beta = \frac{kd k}{\beta} \left( \varepsilon - \frac{\varepsilon^2 (\varepsilon - 1)}{\sec^2 u(u \tan u + 1)} \right) \quad \text{TM even} \tag{40c}
\]

\[
d \beta = \frac{kd k}{\beta} \left( \varepsilon - \frac{\varepsilon^2 (\varepsilon - 1)}{\csc^2 u(u \cot u + 1)} \right) \quad \text{TM odd}. \tag{40d}
\]
If we know the propagation parameters $k, k_1, \Gamma$, and $\beta$, we can find expressions for $E_2, E_4, H_2,$ and $H_4$ in terms of $E_1, E_3, H_1,$ and $H_3$. Solving equation (14) for $E_2$ yields

$$E_2 = E_1 \cos k_1 d e^{ird} .$$

(41)

Similar manipulations give the other desired relationships:

$$E_4 = E_3 \sin k_1 e^{ird}$$

(42)

$$H_2 = H_1 \cos k_1 d e^{ird}$$

(43)

$$H_4 = H_3 \sin k_1 d e^{ird} .$$

(44)

These equations are useful for calculating the electric or magnetic field at any point, which will be done later, and for comparing the energy propagated inside the slab to that outside the slab.

The time-averaged energy flux at any point is defined [5] by

$$<S> = \text{Re} \left( \frac{c}{8\pi} \overrightarrow{E_x \ H^*} \right) .$$

(45)

In the TE case, $E_z$ is defined by equation (7) inside the slab, and equation (8) outside the slab. $E_x$ and $E_y$ are both zero. Taking $H^*$ from equations (3) and (4) gives us

$$H_x^* = \frac{\beta}{k} E_1 \cos k_1 x e^{-i\beta y} \ |x| < d$$

(46)

$$H_x^* = \frac{\beta}{k} E_2 e^{-r x} e^{-i\beta y} \ |x| > d$$

(47)

$$H_y^* = \frac{i k_1}{k} E_1 \sin k_1 x e^{-i\beta y} \ |x| < d$$

(48)

$$H_y^* = \frac{i \Gamma}{k} E_2 e^{-r x} e^{-i\beta y} \ |x| > d .$$

(49)

Inside the waveguide, equation (45) becomes

$$<S> = \frac{c}{8\pi} E_1^2 \frac{\beta}{k} \cos^2 k_1 x \ |x| < d ,$$

(50)
and outside we have

\[
<S> = \frac{c}{8\pi} E_2^2 \beta k e^{-2\pi x} \quad |x| > d .
\]  

(51)

We define the efficiency of the waveguide as

\[
eff = \frac{2 \int_{0}^{d} <S> \, dx}{2 \int_{0}^{\infty} <S> \, dx} .
\]  

(52)

The denominator can be expressed as

\[
2 \int_{0}^{\infty} <S> \, dx = 2 \int_{0}^{d} <S> \, dx + 2 \int_{d}^{\infty} <S> \, dx .
\]  

(53)

We first calculate the integral from zero to d (energy inside the slab):

\[
2 \int_{0}^{d} <S> \, dx = \frac{c}{4\pi} \beta \frac{E_1^2}{k} \int_{0}^{\infty} \cos^2 k_1 x \, dx
\]

\[
= \frac{c\beta E_1^2 d}{8\pi k} \left( 1 + \sin \frac{2kd}{2k_1} \right) \quad |x| < d ;
\]  

(54)

and outside the slab we have

\[
2 \int_{d}^{\infty} <S> \, dx = \frac{c}{4\pi} \beta \frac{E_2^2}{k} \int_{d}^{\infty} e^{-2\pi x} \, dx = \frac{c\beta E_2^2}{8\pi k} e^{-2\pi d} \quad |x| > d .
\]  

(55)

Now using equation (41), equation (55) becomes

\[
2 \int_{d}^{\infty} <S> \, dx = \frac{c\beta E_1^2 \cos^2 k_1 d}{8\pi k} ,
\]  

(56)

and the efficiency of the waveguide becomes

\[
eff = \frac{\Gamma d + \sin u}{\Gamma d + 1} 
\]  

(57)
for the TE even modes. Similar calculations yield

$$\eta = \frac{\Gamma d - \sin^2 u}{\Gamma d}$$  \hspace{1cm} (58)$$

for the TE odd modes. These calculations can also be done for the TM modes, giving us

$$\eta = \frac{e\Gamma d + \sin^2 u}{e\Gamma d + 1}$$ \hspace{1cm} (59)$$

for TM even modes, and

$$\eta = \frac{e\Gamma d - \sin^2 u}{e\Gamma d}$$ \hspace{1cm} (60)$$

for the TM odd modes.

We now have a method for determining the propagation parameters of a waveguide, given its physical properties and the wavelength used. We can also determine the frequency bandwidth inside the material for each mode of operation, given the bandwidth of the incident light. The efficiency of the waveguide is also calculable for each mode. These parameters are presented in table 3(a through j) for various thicknesses, wavelengths, and indices of refraction.

Also presented are plots of $E_x$ (or $H_x$ for the TM modes) versus $x$. These plots were calculated using, for the TE even case, equations (7), (8), and (41), and assuming that $E_1 = 1$. For the other cases, we used corresponding equations and assumed either $E_3 = 1$, $H_1 = 1$, or $H_3 = 1$. These plots are figures 3 through 8.

An interesting feature of these plots is the bend in each of the TM curves. This means that there must be a discontinuity in the derivative of $H_x$. This is correct and is caused by the discontinuous polarization current density at the surface of the slab. In calculations for the TM modes, $E_y$ must be continuous. In this case, $E_y$ is defined by

$$E_y = \frac{1}{ik\epsilon} \frac{\partial}{\partial x} H_z.$$  \hspace{1cm} (27)
At $x = d$, $e$ has a jump discontinuity. For $E_y$ to be continuous, $\partial H_z/\partial x$ must also have a jump discontinuity.

Interesting conclusions can also be drawn from Table 3. We see that $\Delta \beta$ is larger than $\Delta k$ and, in many cases, much larger. We see that it is inversely proportional to $\beta$. We also see that $\beta$, as expected, can have a value from $k$ to $(\varepsilon)^{1/2}k$. We can also compare the efficiencies of various modes. Subsequent solutions for one mode type are always less efficient. The first TE even solution is always more efficient than the second TE even solution, and so on. This is true for all four mode types. Also, the TM modes are always more efficient than their corresponding TE modes.

The trends evidenced in the sample data cannot necessarily be generalized for all situations.

Table 3. Propagation Parameters for Slab Waveguides

<table>
<thead>
<tr>
<th>Mode</th>
<th>$k_i$</th>
<th>$\Gamma$</th>
<th>$\beta$</th>
<th>$\Delta \beta$</th>
<th>Efficiency</th>
</tr>
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<tbody>
<tr>
<td>A. Thickness = 1.00000 μm, $\varepsilon = 4.800$, wavelength = 1.06400 μm, bandwidth = 0.00100 μm, $k = 59052$, and $\Delta k = 55.501$</td>
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<td></td>
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<tr>
<td>TEE</td>
<td>26729</td>
<td>111968</td>
<td>126586</td>
<td>123.47</td>
<td>0.99183</td>
</tr>
<tr>
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<td>102074</td>
<td>117925</td>
<td>138.91</td>
<td>0.95812</td>
</tr>
<tr>
<td>TME</td>
<td>30281</td>
<td>111060</td>
<td>125784</td>
<td>125.07</td>
<td>0.99988</td>
</tr>
<tr>
<td>TMO</td>
<td>60284</td>
<td>98067</td>
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<td>0.99931</td>
</tr>
<tr>
<td>TEE</td>
<td>79097</td>
<td>83636</td>
<td>102382</td>
<td>142.57</td>
<td>0.90889</td>
</tr>
<tr>
<td>TEO</td>
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<td>50669</td>
<td>77811</td>
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</tr>
<tr>
<td>TME</td>
<td>89241</td>
<td>72714</td>
<td>93672</td>
<td>167.92</td>
<td>0.99667</td>
</tr>
<tr>
<td>TMO</td>
<td>111714</td>
<td>27773</td>
<td>65257</td>
<td>241.68</td>
<td>0.93811</td>
</tr>
<tr>
<td>B. Thickness = 0.10000 μm, $\varepsilon = 12.250$, wavelength = 0.82000 μm, bandwidth = 0.00001 μm, $k = 76624$, and $\Delta k = 0.934$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>TEE</td>
<td>169806</td>
<td>192919</td>
<td>207579</td>
<td>3.36</td>
<td>0.77780</td>
</tr>
<tr>
<td>TME</td>
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<td>60993</td>
<td>97936</td>
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<td>0.97879</td>
</tr>
<tr>
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<td></td>
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</tr>
<tr>
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<td>132792</td>
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</tr>
<tr>
<td>TME</td>
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<td>24084</td>
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<td>62.91</td>
<td>0.87576</td>
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Table 3. Propagation Parameters for Slab Waveguide (cont’d)

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<tr>
<th>Mode</th>
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<th>$\Gamma$</th>
<th>$\beta$</th>
<th>$\Delta\beta$</th>
<th>efficiency</th>
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<tbody>
<tr>
<td>**D. Thickness = 0.10000 , \mu\text{m}, \varepsilon = 12.250, wavelength = 1.55000 , \mu\text{m},\right.\hspace{1cm}</td>
<td>114343</td>
<td>73567</td>
<td>83996</td>
<td>324.79</td>
<td>0.48295</td>
</tr>
<tr>
<td>\hspace{1cm} \text{bandwidth} = 0.00400 , \mu\text{m}, k = 40537, and $\Delta k = 104.611$</td>
<td>135671</td>
<td>8917</td>
<td>41506</td>
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</tr>
<tr>
<td>**E. Thickness = 0.35000 , \mu\text{m}, \varepsilon = 12.250, wavelength = 0.82000 , \mu\text{m},\right.\hspace{1cm}</td>
<td>73245</td>
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<td>0.98471</td>
</tr>
<tr>
<td>\hspace{1cm} \text{bandwidth} = 0.00001 , \mu\text{m}, k = 76624, and $\Delta k = 0.934$</td>
<td>145209</td>
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<tr>
<td>TEE</td>
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<td>143331</td>
<td>162527</td>
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<tr>
<td>TEE</td>
<td>250183</td>
<td>58822</td>
<td>96599</td>
<td>9.08</td>
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</tr>
<tr>
<td>**F. Thickness = 0.35000 , \mu\text{m}, \varepsilon = 12.250, wavelength = 1.06400 , \mu\text{m},\right.\hspace{1cm}</td>
<td>69326</td>
<td>185539</td>
<td>194710</td>
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<td>0.97115</td>
</tr>
<tr>
<td>\hspace{1cm} \text{bandwidth} = 0.00010 , \mu\text{m}, k = 59052, and $\Delta k = 5.550$</td>
<td>136194</td>
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<tr>
<td>TEE</td>
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<td>99090</td>
<td>115352</td>
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<tr>
<td>TME</td>
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<td>45512</td>
<td>74556</td>
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<tr>
<td>TMO</td>
<td>197994</td>
<td>5415</td>
<td>59300</td>
<td>67.53</td>
<td>0.58392</td>
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<tr>
<td>**G. Thickness = 0.35000 , \mu\text{m}, \varepsilon = 12.250, wavelength = 1.55000 , \mu\text{m},\right.\hspace{1cm}</td>
<td>62477</td>
<td>120759</td>
<td>127382</td>
<td>382.41</td>
<td>0.93218</td>
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<tr>
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<tr>
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Table 3. Propagation Parameters for Slab Waveguide (cont'd)

<table>
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<tr>
<th>Mode</th>
<th>$k_1$</th>
<th>$\Gamma$</th>
<th>$\beta$</th>
<th>$\Delta\beta$</th>
<th>efficiency</th>
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<td>H.</td>
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<td>Thickness = 1.00000 $\mu$m, $\varepsilon = 12.250$, wavelength = 0.82000 $\mu$m, bandwidth = 0.00001 $\mu$m, $k = 76624$, and $\Delta k = 0.934$</td>
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<tr>
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<td>TMO</td>
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<td>6041</td>
<td>76862</td>
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| I.    |        |          |         |               |            |
|-------|        |          |         |               |            |
| Thickness = 1.00000 $\mu$m, $\varepsilon = 12.250$, wavelength = 1.06400 $\mu$m, bandwidth = 0.00010 $\mu$m, $k = 59052$, and $\Delta k = 5.550$ |
| TEE   | 28526  | 196003   | 204706  | 19.58         | 0.99808    |
| TEO   | 56994  | 189691   | 198670  | 20.39         | 0.99127    |
| TME   | 31156  | 195602   | 204322  | 19.65         | 1.00000    |
| TMO   | 62291  | 188018   | 197074  | 20.37         | 0.99999    |
| TEE   | 85339  | 178741   | 188243  | 20.96         | 0.98132    |
| TEO   | 113464 | 162348   | 172754  | 24.22         | 0.95957    |
| TME   | 93376  | 174677   | 184389  | 21.77         | 0.99998    |
| TMO   | 124349 | 154170   | 165093  | 24.32         | 0.99995    |
| TEE   | 141207 | 138894   | 150927  | 25.04         | 0.93603    |
| TEO   | 168205 | 104585   | 120105  | 38.66         | 0.86209    |
| TME   | 155033 | 123270   | 136685  | 29.37         | 0.99986    |
| TMO   | 184392 | 72323    | 93370   | 43.00         | 0.99906    |
| TEE   | 193020 | 44431    | 73900   | 39.62         | 0.70521    |
| TME   | 197898 | 8205     | 59620   | 67.29         | 0.86810    |
Table 3. Propagation Parameters for Slab Waveguide (cont’d)

<table>
<thead>
<tr>
<th>Mode</th>
<th>$k_1$</th>
<th>$\Gamma$</th>
<th>$\beta$</th>
<th>$\Delta\beta$</th>
<th>Efficiency</th>
</tr>
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<tbody>
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</table>

$J$. Thickness = 1.00000 μm, $\varepsilon = 12.250$, wavelength = 1.55000 μm, bandwidth = 0.00400 μm, $k = 40537$, and $\Delta k = 104.611$

Figure 3. Electric field (arbitrary units) versus distance from slab center for two even and two odd TE modes, for a slab of 1 μm thickness, a dielectric coefficient of 4.80, and a free space wavelength of 1.064 μm. Vertical line in this, and all following figures, represents slab edge.
Figure 4. Magnetic field (arbitrary units) versus distance from slab center for two even and two odd TM modes. All parameters are same as for figure 3.

Figure 5. Electric field for TE even mode and magnetic field for TM even mode (only modes possible) for a slab of 0.1 μm thickness and a dielectric coefficient of 12.25. Free space wavelength is (a) 1.550, (b) 1.064, and (c) 0.820 μm.
Figure 6. Electric field versus distance from slab center. Slab thickness of 0.35 \( \mu m \), a dielectric coefficient of 12.25, and free space wavelength of (a) 1.550 \( \mu m \), for one even mode and one odd mode; (b) 1.064 \( \mu m \), for two even modes and one odd mode; and (c) 0.820 \( \mu m \), for two even modes and one odd mode.
Figure 7. Magnetic field versus distance from slab center. Slab has thickness of 0.35 μm, a dielectric coefficient of 12.25, and free space wavelength of (a) 1.550 μm, for one even mode and one odd mode; (b) 1.064 μm, for two even modes and one odd mode; and (c) 0.820 μm, for two even modes and one odd mode.
Figure 8. Highest (fifth) TE even mode for a slab with a thickness of 1 \( \mu \)m and a dielectric coefficient of 12.25, for a free space wavelength of 0.820 \( \mu \)m.

Acknowledgements

Thanks to Clyde Morrison for guidance and direction.

Literature Cited


Appendix A.--FORTRAN Programs Used in Calculating Dielectric Waveguide Characteristics
C$FORT SLABI
C$LINK SLABI
C$PURGE SLABI.EXE
C$DEL SLABI.OBJ;*

THIS PROGRAM FINDS THE SOLUTIONS FOR AN INFINITE PLANAR SLAB WAVEGUIDE WITH DIELECTRIC CONSTANT EPS, THICKNESS THK, AND INCIDENT WAVE LENGTH WL.

PROGRAM NEWRAP
CHARACTER*1 ANS
DIMENSION TEE(500),TEO(500),TWE(500),TMO(500)
TYPE*,* ' THIS PROGRAM SOLVES FOR THE ROOTS TO OUR SLAB WAVEGUIDE PROBLEM.'
TYPE*,* ' ENTER DIELECTRIC CONSTANT,THICKNESS,WAVE LENGTH.'
TYPE*,* ' THIS FILE CONTAINS THE RUN PARAMETERS. IT IS MADE BY THE USER. YOU MUST STORE THEM IN THE FILE LIKE THIS:
EPSILON THICKNESS(cm) WAVELENGTH(cm) BANDWIDTH(cm)
OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
READ(*,*) EPS,THK,WL,BW
THK=THK/2

SET RUN CONSTANTS
EFS=1.0
P1=3.141592654
A=2*PI*THK*(EPS-1)**.5/WL
10=IFIX(A/P1+.5)

IF A IS LESS THEN PI/2 THEN WE HAVE ONLY TWO ROOTS, ONE TE EVEN AND ONE TM EVEN. THERE WILL BE NO ODD ROOTS, SO WE GO TO THE SPECIAL EVEN ONLY ROUTINE.

IF(A.LT.PI/2)GOTO 123

100
FORMAT(18x,4F16.9)

START LOOP, AND SET INITIAL GUESSES FOR THE NEWTON-RAPHSON ROUTINE

l=1
SE=P1/2.0-THK
SEI=SE
SO=P1-THK

IF OUR GUESS IS GREATER THAN A, THE PROGRAM WILL CRASH. SO WE INSURE THAT OUR GUESSES ARE LESS THAN A.
IF (SO .GT. A) SO=A-THK
SO1=SO

USE SUBROUTINES TO FIND ROOTS
EPS IS EPSILON, EFS IS EPSILON OF FREE SPACE

THERE SHOULD BE ONE SOLUTION OF EACH TYPE - TWO ODD, TWO EVEN, OF TM AND TE MODES.
10
IF (SO .GE. A) SO=A-THK
IF (SO1 .GE. A) SO1=SO
CALL ODD(TEO(1),SO,EFS,A)
CALL ODD(TMO(1),SO1,EFS,A)
CALL EVEN(TEE(1),SE,EFS,A)
CALL EVEN(TWE(1),SE1,EFS,A)

SET UPPER AND LOWER LIMITS FOR EACH INDIVIDUAL MODE
A1=P1*(2*l+1)/2.0
A2=P1*(2*l-1)/2.0
A3=P1*(2*l-3)/2.0

31
CHECK THAT ROOT IS WITHIN THE SET LIMITS
THAT IS, CHECK THAT NO ROOTS ARE SKIPPED, OR REPEATED.

1. IF (TEO(I) .GT. AJ .OR. TNO(I) .GT. AJ) IFLG=-IFLG+1
2. IF (TEE(I) .GT. AJ .OR. TME(I) .GT. AJ) JFLG=-JFLG+1
3. IF (TEO(I) .LT. AJ .OR. TNO(I) .LT. AJ) KFLG=-KFLG+1
4. IF (TEE(I) .LT. AL .OR. TME(I) .LT. AL) LFLG=-LFLG+1

TYPE 100, TEE(I), TEO(I), TME(I), TNO(I)

CHECK TO SEE IF WE'RE DONE.

IF (I .LE. 10) GOTO 10

CHECK TO SEE IF WE HAVE ANOTHER PAIR OF EVEN ROOTS.

IF (A-(I-1)*PI .LE. 0) GOTO 987

FIND PAIR OF EVEN ROOTS

SE=A-THK
SE1=SE+THK/2

TYPE*10*PI, SE, A
CALL EVEN(TEE(I), SE, EPS, A)
CALL EVEN(TME(I), SE1, EPS, A)

IXFL=1

IF ANY OF THE FLAGS BELOW ARE NOT ZERO, A MODE WAS SKIPPED OR REPEATED.

TYPE*, IFLG, JFLG, KFLG, LFLG

OPEN FILE FOR EZ-GRAPH PLOTTING, AND FILE FOR PRINT OUT

OPEN (UNIT=2, TYPE='NEW', NAME='ALROOT.EZG')
OPEN (UNIT=1, TYPE='NEW', NAME='ALROOT.DAT')
WRITE(1,*) ' ' ' ' ' ' ' ' ' '
DO 20 I=1,10,1
WRITE(1,100) TEE(I), TEO(I), TME(I), TNO(I)

WRITE TO EZG FILE

WRITE(2,*) TEE(I), TN
WRITE(2,*) TEO(I), CN
WRITE(2,*) TME(I), TNI
WRITE(2,*) TNO(I), CN1

CONTINUE

IF (IXFL .NE. 1) GOTO 789

WRITE(2,*) 0.0,999999
WRITE(1,234) TEE(I), TME(I)

WRITE(2,*) TEE(I), TN
WRITE(2,*) TNO(I), TN1
WRITE(2,*) TME(I), TNI
WRITE(2,*) 0.0,999999
WRITE(2,*) TME(I), TN

STOP
END

SUBROUTINE ODD(F,X,E,A)

THIS SUBROUTINE FINDS THE ROOTS OF ODD SOLUTION MODES
DEPENDING ON E, IT WILL SOLVE FOR TWO, OR THREE

DOUBLE PRECISION Q,F2,F3,D1

Q=1/(A*A-X*X)**.5

TYPE*,X,D1

F2=TAN(X)+X*Q/E

F3=COS(X)**(-2)+Q/E+X*X*Q**3/E

D1=-F2/F3

IF (ABS(D1) .LE. .00001) GOTO 310

X=X+D1

GOTO 320

310 F=X

X=X+3.141592654

RETURN

END

SUBROUTINE EVEN(F,X,E,A)

DOUBLE PRECISION Q,F2,F3,D1

420 Q=(A*A-X*X)**.5

TYPE*,X,D1

F2=TAN(X)-Q*E/X

F3=COS(X)**(-2)+Q*E/(X*X)+E/Q

D1=-F2/F3

IF (ABS(D1) .LE. .00001) GOTO 410

X=D1+X

GOTO 420

410 F=X

X=X+3.141592554

RETURN

END

THIS SUBROUTINE FINDS THE ROOTS FOR EVEN SOLUTION MODES
THIS PROGRAM CALCULATES THE PROPAGATION PARAMETERS FOR THE INDIVIDUAL SOLUTIONS TO THE SLAB WAVEGUIDE PROBLEM.  

PROGRAM BETAWD 
CHARACTER*5 MD(4) 
REAL K,K1,EFF(4) 
DATA PI/3.1415926536/ 

 THESE CHARACTER STRINGS REPRESENT THE FOUR DIFFERENT MODE TYPES.  

 MD(1)= ' TEE ' 
 MD(2)= ' TEO ' 
 MD(3)= ' TME ' 
 MD(4)= ' TMO ' 

 OPEN THE RUN PARAMETER FILE (CREATED BY THE USER).  
 OPEN (UNIT=9, TYPE=' OLD ', NAME=' SLABRUN.DAT ')  

 READ (9,* ) EPS, THK, WL, DW 
 DK=2*PI*DW/(WL**2-DW**2) 
 D=THK/2 
 K=2*PI/WL 

 OPEN SOLUTION DATA FILE, CREATED BY SLAB1 PROGRAM  
 OPEN (UNIT=2, TYPE=' OLD ', NAME=' ALROOT.EZG ')  

 OPEN OUTPUT FILE FOR PROPAGATION PARAMETERS, AND WRITE FILE HEADER  
 OPEN (UNIT=1, TYPE=' NEW ', NAME=' PARAMS.DAT ')  
 WRITE (1,* ) ' Table. Propagation parameters for slab waveguide  
 + with' 
 WRITE (1,10) THK, EPS 
 10 FORMAT(' thickness=',F7.5,' um',' epsilon=',F7.3)  
 WRITE (1,12) WL, DW 
 12 FORMAT(' wavelength=',F7.5,' um',' bandwidth=',  
 +F7.5,' um')  
 WRITE (1,14) K, dk 
 14 FORMAT(' k=',F7.0,' delta k=',F7.3)  

 WRITE (1,*) ' mode k gamma beta  
 + delta beta efficiency ' 
 WRITE (1,*) ' 1'  
 WRITE (1,*) 

 999 FORMAT(A5,3F10.0,F11.2,F13.5)  

 MODE TYPE IS KEPT TRACK OF WITH MTP (0 BEFORE PROGRAM STARTS)  
 MTP=0  

 READ (2,* , END=777) U, DUMMY  
 MTP=MTP+1  
 IF (MTP .GT. 4) MTP=4  
 IF (U.EQ. 0) GOTO 100  

 CALCULATE K1, GAMMA  
 K1=U/D  
 GAM=(K*K*(EPS-1)-K1*K1)**.5  

 CALCULATE EFFICIENCY AND BETA "B"  
 B=(GAM**2+K**2)**.5  
 SU=SIN(U)  
 CU=COS(U)  
 SC=SU*CU  
 GD=GAM*D  
 GDP=GD*EPS  
 EFF(1)=(GD+SU**2)/(GD+1)
EFF(2) = (GD - SU**2) / GD
EFF(3) = (GED + SU**2) / (GED + 1)
EFF(4) = (GED - SU**2) / GED

C CALCULATE THE BANDWIDTH OF BETA
CSU2 = 1 / SU**2
SCU2 = 1 / CU**2
tu = su / cu
c1 = 1 / tu
Q = 1
IF (MTP .GT. 2) Q = 1 / EPS**2
IF (MTP .EQ. 2 * INT(MTP / 2)) GOTO 987
f1 = scu2
f2 = tu
GOTO 986
987
f1 = csu2
f2 = ct
986
db = k * dk *(eps - q * (eps - 1) / (f1 * (u * f2 + 1))) / b
C WRITE TO OUTPUT FILE
WRITE (1, 999) MD(MTP), K1, GAM, B, DB, EFF(MTP)
C GOTO 100
777
CLOSE (UNIT = 1)
STOP
END
THIS PROGRAM CALCULATES THE VALUES OF THE CURVES REPRESENTED
BY THE FOUR TRANSCENDENTAL EQUATIONS FOR OUR SLAB WAVEGUIDE
PROBLEM. THE FOUR CURVES NEEDED ARE:

\[ F_1(U) = \tan(U) \quad F_2(U) = -\cot(U) \quad F_3(U) = (A^2 - U^2)^{0.5} \quad F_4(U) = \varepsilon \phi(U) \]

PROGRAM RTGRA
DIMENSION F(4)
DATA U, WL, PI/0.0, 0.001063.1415926536/

OPEN RUN PARAMETER FILE (CREATED BY THE USER) AND READ PARAMETERS
OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
READ(9,*) EPS, THK, WL, BW

CALCULATE D(HALF THICKNESS) AND A(DESCRIBED IN PAPER)
D = THK/2
A = 2*PI*D*(EPS-1)**.5/WL
N = 0

OPEN FILE FOR OUTPUT OF CURVES. THE OUTPUT IS OF THE FORM:
U F(1) F(2) F(3) F(4)
OPEN(UNIT=1,TYPE='NEW',NAME='SMROOT.DAT')

BEGIN CURVE CALCULATING LOOP
DO 10 1 = 0, 999

WE CALCULATE IN BLOCKS OF PI/2 TO AVOID A VERTICLE LINE WHEN
WE PLOT THE TRIGONOMETRIC CURVES WITH EZGRAPH.
U = U + DU + PI/2.
IF(U .GT. A) GOTO 11
IF (INT(2*U/PI) .EQ. 2*U/PI) GOTO 10
F(1) = TAN(U)
F(2) = -1/F(1)
F(3) = (A^2 - U^2)^{0.5}/U
F(4) = EPS * F(3)
DO 20 J = 1, 4
IF(ABS(F(J)) .LT. 100) GOTO 20
IF(ABS(F(J)) .LT. 100) GOTO 20

FOR PLOTTING PURPOSES, WE DON'T WANT ANY OF THE FUNCTIONS VERY HIGH
IT DOESN'T MATTER MUCH EITHER WAY
F(J) = 99.9999*ABS(F(J))/F(J)
20 CONTINUE
WRITE(1,100) U, F(1), F(2), F(3), F(4)
10 CONTINUE
100 FORMAT(5F15.10)
N = N + 1
CLOSE(UNIT=1)
IF(U .LT. A ) GOTO 9
STOP
END
THIS PROGRAM CALCULATES THE VALUES OF THE ELECTRIC OR MAGNETIC FIELDS FOR ANY X, INSIDE OR OUTSIDE THE SLAB. IT ASSUMES A MAXIMUM POSSIBLE FIELD OF ONE, OR E_1=1, AND Y=2*PI*n.

PROGRAM MODEPL
REAL K,K1
DATA PI/3.1415926536/
COMMON U,D,K1,B,GAM,EGD,EPSCU,SU,EO

OPEN FILE CREATED BY USER WITH RUN PARAMETERS
OPEN(UNIT=9,TYPE='OLD',NAME='SLABRUN.DAT')
READ(9,*) EPS,THK,WL,BW

CALCULATE PARAMETERS USED FOR FINDING FIELDS
D=THK/2
K=2*PI/WL

OPEN FILE WITH SOLUTIONS TO TRANSCENDENTALS, CONTAINING VALUES FOR U
THIS FILE WAS CREATED BY SLAB
OPEN (UNIT=2,TYPE='OLD',NAME='ALROOT.EZG')

KEEP TRACK OF MODE TYPE WITH MTP
MTP=-1
100 READ(2,*,END=777) U,DUMMY
MTP=MTP+1
IF(U.EQ.0) GOTO 100
CU=-COS(U)
SU=SIN(U)

CALCULATE PROPAGATION PARAMETERS
K1=U/D
GAM=(K*K*(EPS-1)-K1*K1)**.5
B=(GAM**2+K*K)**.5
EGD=EXP(GAM*D)

OPEN FILE FOR OUTPUT
OPEN(UNIT=1,TYPE='NEW',NAME='SMODE.DAT')
DX=10.*D/1000.

LOOP TO CALCULATE FIELDS
DO 200 I=0,1000
X=I*DX
EO=MTP-2*IFIX(FLOAT(MTP)/2.0)
CALL MODCLC(X,V)
WRITE(1,*)X*10000.,V
200 CONTINUE
CLOSE(UNIT=1)
GOTO 100
777 STOP
END

SUBROUTINE MODCLC(X,V)

THIS SUBROUTINE CALCULATES THE FIELD DEPENDING ON WHICH TYPE OF IS BEING PLOTTED
REAL K,K1
COMMON U,D,K1,B,GAM,EGD,EPSCU,SU,EO

CHECK FOR INSIDE OR OUTSIDE OF SLAB
IF (X.LT. D) GOTO 9
C CHECK FOR EVEN OR ODD MODE
   IF(EO) 2,2,1
   AM=CU
   GOTO 3
1   AM=SU
3   V=AM*EGD*EXP(-X*GAM)
   GOTO 4
9   IF (EO) 6,6,5
6   V=COS(K1*X)
   GOTO 4
5   V=SIN(K1*X)
4   RETURN
END
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