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Technical Report

AN ALGORITHM FOR SELECTING
TRANSDUCER ELEMENT
ARRAY POSITIONS

by

King W. Wiemann

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AN ALGORITHM FOR SELECTING TRANSDUCER ELEMENT ARRAY POSITIONS

by

King W. Wiemann

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June 1988

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A two-part technique to determine the placement of transducer elements in an array which minimizes the impact of element tolerances on the directional beam pattern has been developed. This technique has been used to place in an array elements with tolerances of ±9% in amplitude and ±11°

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The first half of the technique selects four elements at a time from a larger selection pool, and places them in symmetric positions about the center of the array in a manner that partially cancels their respective amplitude and phase variations. The second half of the technique uses a Permutation Search algorithm which rearranges the initial placement of elements in and out of the array looking for improvements in the array response.

A lumped-parameter equivalent circuit of a tonpilz transducer is used to predict element amplitude and phase tolerances for different radiation loadings, based on in-air measurements of the transducers. Relationships among the measurable parameters of a transducer and its performance characteristics have been explored. Preliminary estimates of acceptable tolerances for each circuit parameter have been established.

ABSTRACT

A two-part technique to determine the placement of transducer elements in an array which minimizes the impact of element tolerances on the directional beam pattern has been developed. This technique has been used to place in an array elements with tolerances of ±9% in amplitude and ±11° in phase. The resulting theoretical array response achieved side lobe levels that were within 1 dB of the -40 dB design.

The first half of the technique selects four elements at a time from a larger selection pool, and places them in symmetric positions about the center of the array in a manner that partially cancels their respective amplitude and phase variations. The second half of the technique uses a Permutation Search algorithm which rearranges the initial placement of elements in and out of the array looking for improvements in the array response.

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LIST OF SYMBOLS

A_m	Amplitude Response of Element m
a_m	Phase Response of Element m
a	Dimension of Square Piston
B	Set of Constrained Angles
c	Speed of Sound
C	Capacitance
C_M	Mechanical Compliance of Tonpilz Transducer
C_o	Clamped Capacitance of Transducer
C_t	Total Measured Capacitance of Transducer
d	Distance Between Sound Sources
E	Voltage
E_{out}	Open Circuit Receive Output Voltage
F_1, F_2	Half-Power Frequencies
F_y	Motional Resonance Frequency
G_y	Conductance of Transducer at $F = F_y$
I	Current
i	Square Root of -1
J_1	First Order Bessel Function of the First Kind
k	Wavenumber
l'	Extra Distance Travelled by Wave to Second Source
L	Inductance
M_e	Open Circuit Output Voltage Receive Sensitivity
M_M	Mechanical Mass
M_1	First Order Struve Function
P_{in}	Incident Sound Pressure
Q_m	Contribution of mth Element to Total Array Response

LIST OF SYMBOLS (Continued)

R	Resistance
R_A	Acoustic Resistance, Real Part of Radiation Loading
R_M	Mechanical Resistance of Transducer
S	Area of Head of Transducer
v_i	Velocity of ith transducer
X_A	Acoustic Reactance, Imaginary Part of Radiation Loading
X_m, Y_m	x and y Coordinate of Array Position m
Z_A	Acoustic Impedance
Z_M	Mechanical Impedance
Z_m^{ij}	Mutual Impedance Coefficient for Transducers i and j
Z_s^i	Self Impedance of ith Transducer
Z_T^i	Total Impedance on i^{th} Transducer
$\Delta\theta$	Separation, in Degrees, of Constrained Angles, in θ Direction
$\Delta\phi$	Separation, in Degrees, of Constrained Angles, in ϕ Direction
θ	Incident Angle, Measured from + Z Axis
θ_o	Real Part of Impedance on Rectangular Piston, Resistance
λ	Wavelength
ρ	Density
σ	Standard Deviation
ϕ	Roll Plane Angle, Measured from the + X Axis
Φ	Electro-Mechanical Transformation Ratio
X_o	Imaginary Part of Impedance on Rectangular Piston, Reactance
ω	Radian Frequency

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CHAPTER ONE

INTRODUCTION

1.1 PROBLEM DEFINITION

An underwater acoustic array is a specific spatial arrangement of a set of transducer elements. It can be used as either a transmitter or a receiver. Each of these modes of operation places certain requirements on the performance of the array. The degree to which these requirements are met determines the effectiveness and usefulness of the array.

The directional characteristics of such an array are controlled by the shading values at each position in the array. These shading values, or weights, are developed under the assumption that all the elements have identical performance characteristics, expressed in terms of amplitude, (or gain), and phase over a broad frequency range.

The variations found in actual elements refute this assumption. The introduction of element errors degrades the performance characteristics of the array. Because of this, the side-lobe levels may deviate significantly from the design specifications.

The amplitude and phase characteristics of the response from each element are also influenced by the element's location within the array. This positional dependence results from the acoustic interaction with all the other elements in the array.

A reasonably effective two part method of reducing the amount of error introduced into the response pattern by elements with different tolerances was developed by Kendig [1]. Some of this method's drawbacks have been solved by Schafer [2]. In addition, Schafer has developed a computer version of this method.

Originally, the first part of the Kendig method used indirect measurements of frequency and capacitance to estimate the amplitude and phase errors of each element. The second part of the Kendig method determined the placement of elements in the array. The Kendig Scatter Diagram Method involves the pairing of elements in a manner that partially cancels out their respective errors. With the development of an in-air current/velocity measurement technique, Schafer [2] was able to use a lumped-parameter equivalent circuit to predict more accurately the amplitude and phase characteristics of a tonpilz transducer that is subjected to an arbitrary radiation loading. Since the mutual interactions will manifest themselves as changes in radiation loading, the positional dependence can be accounted for.

The success of Permutation Search in solving similar problems known as Travelling Salesman Problems indicates it may be an effective tool for determining array positions for transducer elements. It is a straighforward and effective search technique. For the problem of element placement, it rearranges the elements in the array, searching for those permutations which improve upon the response of the current array.

1.2 GOALS OF THIS STUDY

The goal of this thesis is to develop an algorithm which determines the placement of elements in an array so that the impact of the transducer element errors on the array response is minimized. This technique must be applicable to all four-fold symmetric arrays, odd and even. To accomplish this goal, several objectives must be met:

1. Analyze the effects of tolerances in the lumped-parameter circuit values on an element's performance characteristics.

2. Study the effects of mutual interaction between elements and the effect of incidence angle on the mutual interaction.

3. Adapt the Permutation Search algorithm to the problem of element placement, and determine its effectiveness.

4. Enhance the Kendig Scatter Diagram Method by using a three dimensional model of array response.

5. Determine the effectiveness of using the Neo-Kendig selection scheme as a starting point for the Permutation Search algorithm.

The lumped-parameter equivalent circuit and a study of the effects of parameter tolerances is presented in Chapter Two. In Chapter Three is a discussion of the effect of the acoustic coupling between elements on an element's performance. Chapter Four is a presentation of Permutation Search and a discussion of its results. The derivation of

the Neo-Kendig method is in Chapter Five. The results of combining the Neo-Kendig method and Permutation Search are presented in Chapter Six.

CHAPTER TWO

THE TRANSDUCER ELEMENTS

2.1 THE EQUIVALENT CIRCUIT*

In order to study the response characteristics of a tonpilz transducer, (shown in Figure 2.1), an equivalent electrical circuit is often used. The two types of equivalent circuits used are distributed-parameter and lumped-parameter. The distributed-parameter is used primarily during the design of a transducer. This study uses the lumped-parameter model because the circuit parameters can be determined from the measurement of transducer elements. In this manner, the element tolerances can be accounted for as variations in the circuit parameters.

With the lumped-parameter model, the performance of a tonpilz transducer operated in the vicinity of its fundamental resonance can be readily predicted. In this circuit, the mechanical properties of the transducer, mass and compliance, are represented by linear electrical parameters, inductance and capacitance. Mechanical resistance is directly analogous to electrical resistance. This circuit is valid only in the vicinity of the fundamental resonance of the transducer.

The input/output relationships for a transducer element are determined by an analysis of the equivalent circuit. Shown in Figure 2.2 is the lumped-parameter equivalent circuit. For the receive case, the output voltage sensitivity, M_e, for a transducer is given by

* The material in this section is a synopsis of Reference [2].

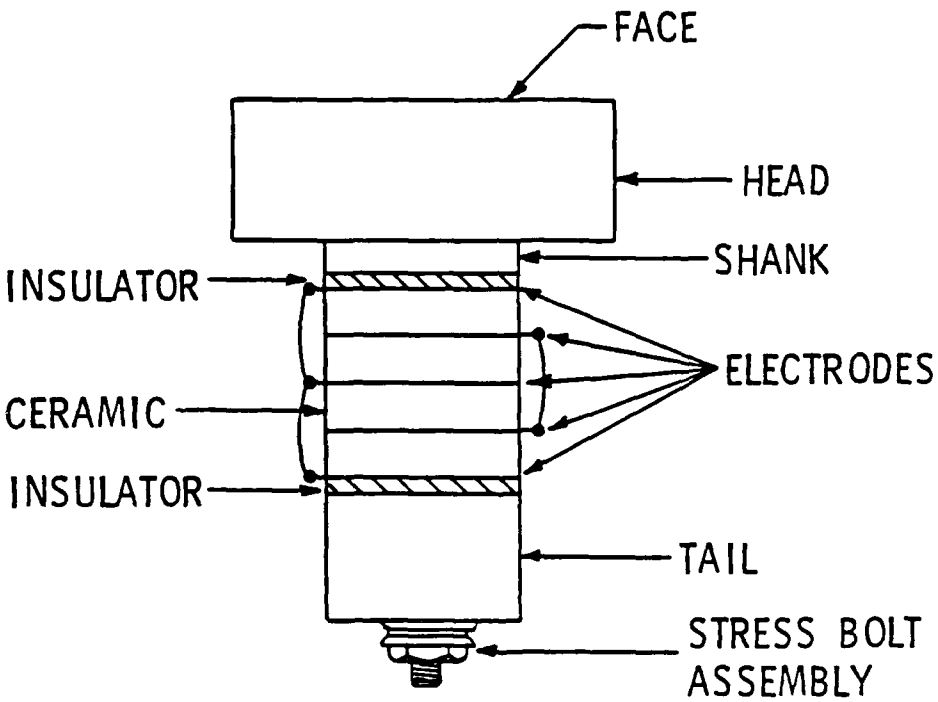
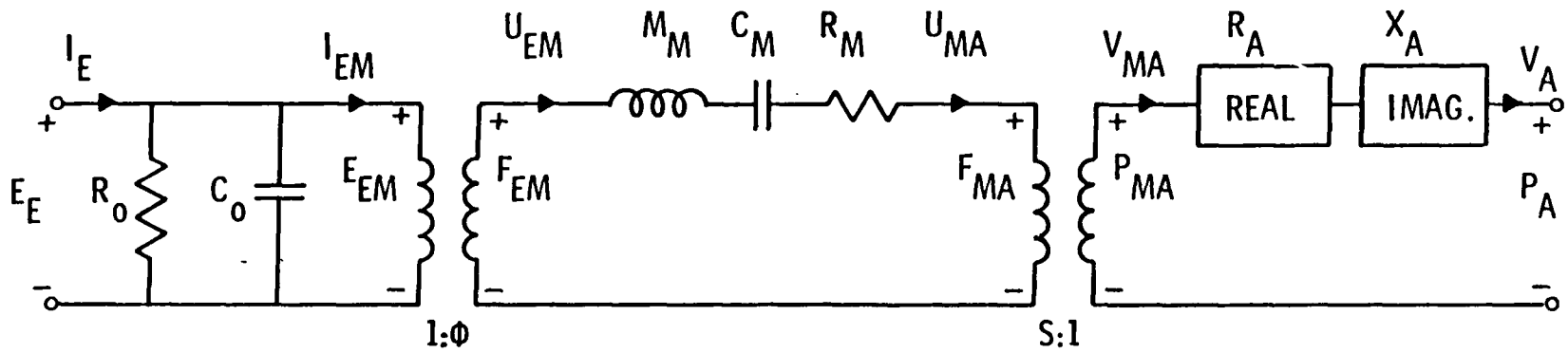


Figure 2.1 A Tonpilz Type Transducer

(a) Tonpilz Transducer Equivalent Circuit



(b) Transformed to Receive Case

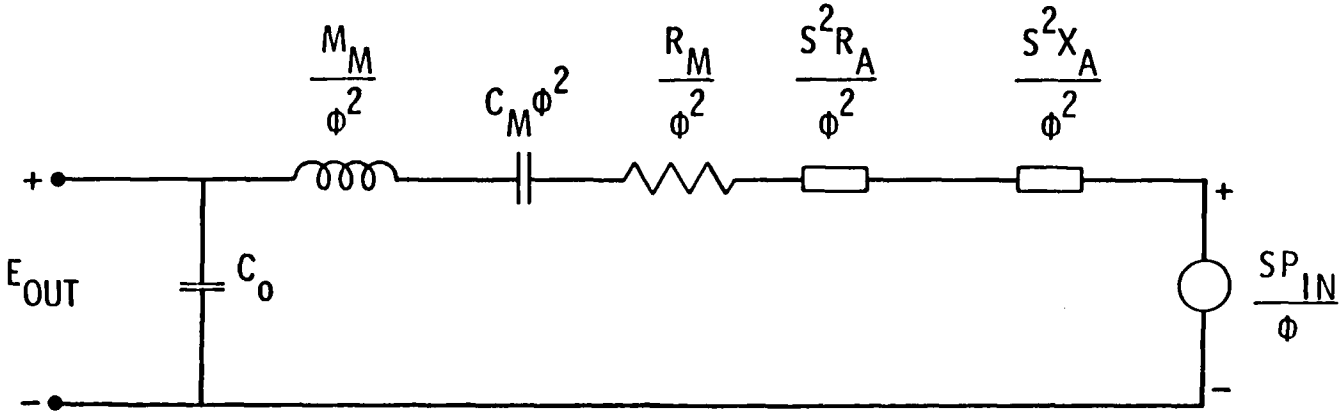


Figure 2.2 The Equivalent Circuit

$$\left|M_e\right|_k = \left|\frac{E_{out}}{P_{in}}\right| \tag{2-1}$$

where E_{out} is the open circuit output voltage for an incident sound pressure P_{in}. From the equivalent circuit, it can be shown that

$$\left| M_e \right|_k = \frac{\Phi S}{[(\Phi^2 - \omega^2 C_o M_M + C_o/C_M - \omega C_o S^2 X_A)^2 + \omega^2 C_o^2 (R_M + S^2 R_A)^2]^{1/2}} \tag{2-2}$$

$$\mathrm{Arg}(M_e)_k = \tan^{-1} \left[\frac{C_o(R_M + S^2 R_A)}{\Phi^2 - \omega^2 C_o M_M + C_o/C_M - \omega C_o S^2 X_A} \right] . \tag{2-3}$$

Equations 2-2 and 2-3, the input/output relationships, are used to determine the amplitude and phase responses of each element. It is seen, in Equation 2-1, that the magnitude of the open circuit voltage is proportional to both the receive voltage sensitivity and the incident sound pressure. In other words, the output voltage is proportional to the displacement of the head of the transducer, i.e., amplitude.

Included in the equivalent circuit is the analytically derived complex acoustic radiation impedance, Z_A, (= $R_A + iX_A$), where the real part, R_A, is the resistance, and the imaginary part, X_A, the reactance. The mechanical impedance experienced by the transducer, Z_M, is related to the acoustic impedance by a factor of $1/S^2$: $Z_A = Z_M/S^2$, where S is the area of the face of the transducer. In the equivalent circuit, the acoustic impedance in the acoustic domain is transformed into mechanical impedance in the mechanical domain by an ideal transformer with a turns ratio of S:1.

The input/output relationships show that a transducer element will have different responses for different loadings. This complicates the array design process because an element placed in one position in the array will not experience the same radiation loading when placed in a different, nonsymmetric position. This is a result of the acoustic interaction among transducer elements in an array. Each element position in the array has a set of spatial relationships with all the other element positions in the array. Those positions which have the same relationships with all the other array positions are symmetric positions.

2.1.1 CIRCUIT PARAMETERS

A reduced form of the lumped-parameter circuit, shown in Figure 2.3 is used to determine the circuit parameters for each element. The relationships between the two circuits are shown below.

$$C_o = C_o \quad C = C_M\Phi^2 \quad L = M_M/\Phi^2 \quad R = R_M/\Phi^2 \ . \tag{2-4}$$

In the reduced form, dielectric losses, R_o, are assumed negligible. Since the measurements are made in air, the radiation impedance is also assumed negligible. By measuring F_y, the motional resonance frequency; F_1 and F_2, the half-power frequencies; C_t, the total capacitance; and G_y, the conductance at $F = F_y$, the circuit parameters of the reduced model are given by

$$\begin{aligned} R &= 1/G_y & C &= G_y\,(\omega_2 - \omega_1)/\omega_y^2 \\ L &= 1/(C\cdot\omega_y^2) & C_o &= C_t - C, \end{aligned} \tag{2-5}$$

where $\omega = 2\pi F$. The transformation ratio, Φ, represents the coupling between the mechanical and electrical domains of the equivalent circuit.

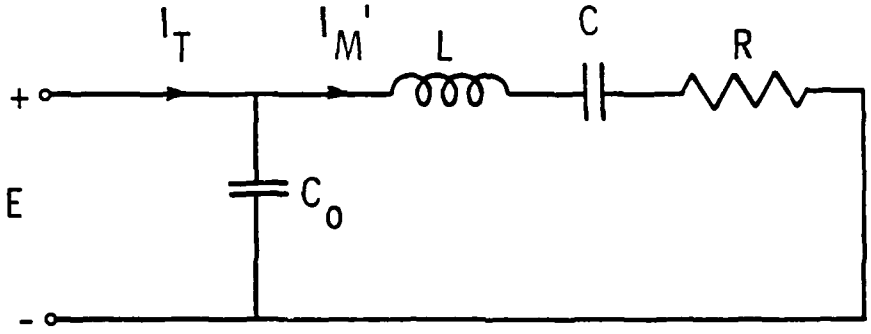


Figure 2.3 The Reduced Form of the Equivalent Circuit

Since it has the units of current/velocity it can be determined by the simultaneous measurement of both these quantities. A more detailed discussion of this procedure and the procedures for determining the other parameters is discussed in Reference [2]. The six measured parameters, F_y, F , F , G_y, C_t, and Φ, are all that are needed to predict a transducer element's performance for an arbitrary radiation loading.

2.1.2 A MEASURED SET

In order to simulate realistic sets of elements to test the selection process, the relationships, if any, among the measured circuit parameters had to be determined. A set of 66 tonpilz transducers were constructed and measured. Correlations and distributions in measured data were tested by plotting each parameter versus every other parameter. In all the following plots, the parameter values have been normalized to the mean values of the measured set.

There is a linear correlation between F_y and both F_1 and F_2, shown in Figures 2.4 and 2.5, respectively. There is also a linear correlation between F_1 and F_2, as seen in Figure 2.6. However, there is no correlation between F_y, and $F_2 - F_1$, as seen in Figure 2.7. Neither is there a correlation between F_1 or F_2 and $F_2 - F_1$. It was determined that

$$F_2 = F_y + (F_2 - F_1)/2$$

$$F_1 = F_y - (F_2 - F_1)/2 \qquad (2\text{-}6)$$

is a good representation of the relationships between F_y, F_1 and F_2, (within .01%).

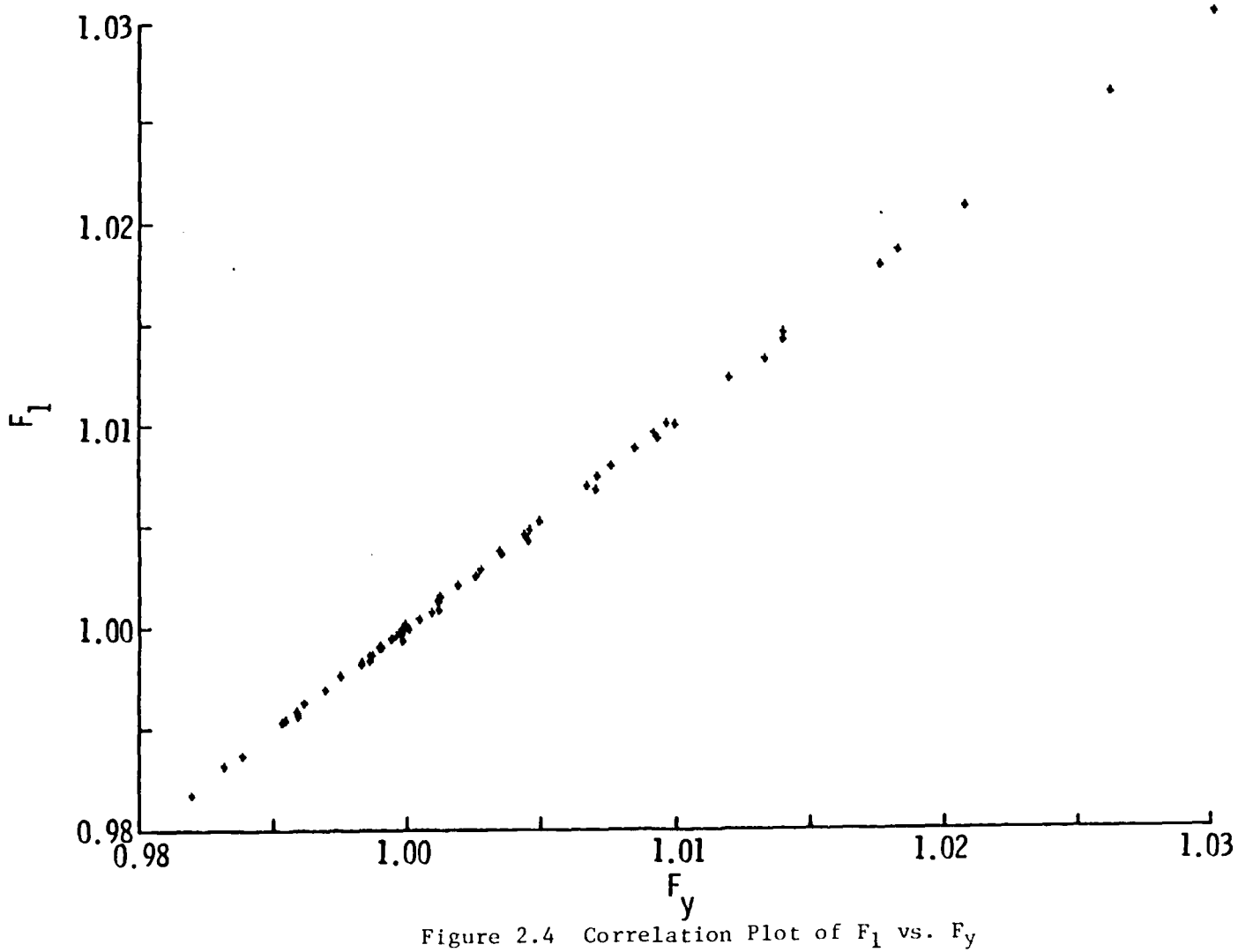


Figure 2.4 Correlation Plot of F_1 vs. F_y

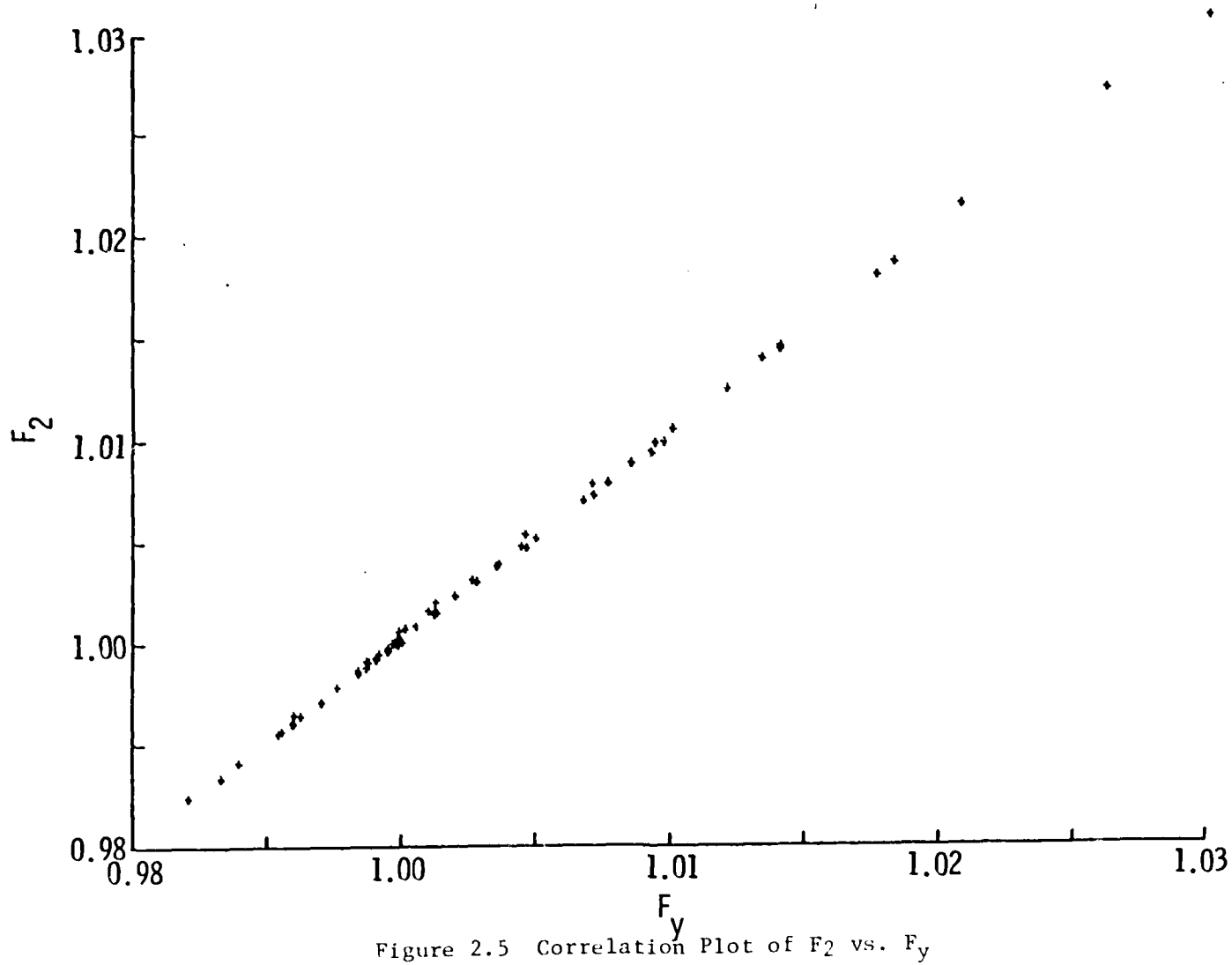


Figure 2.5 Correlation Plot of F_2 vs. F_y

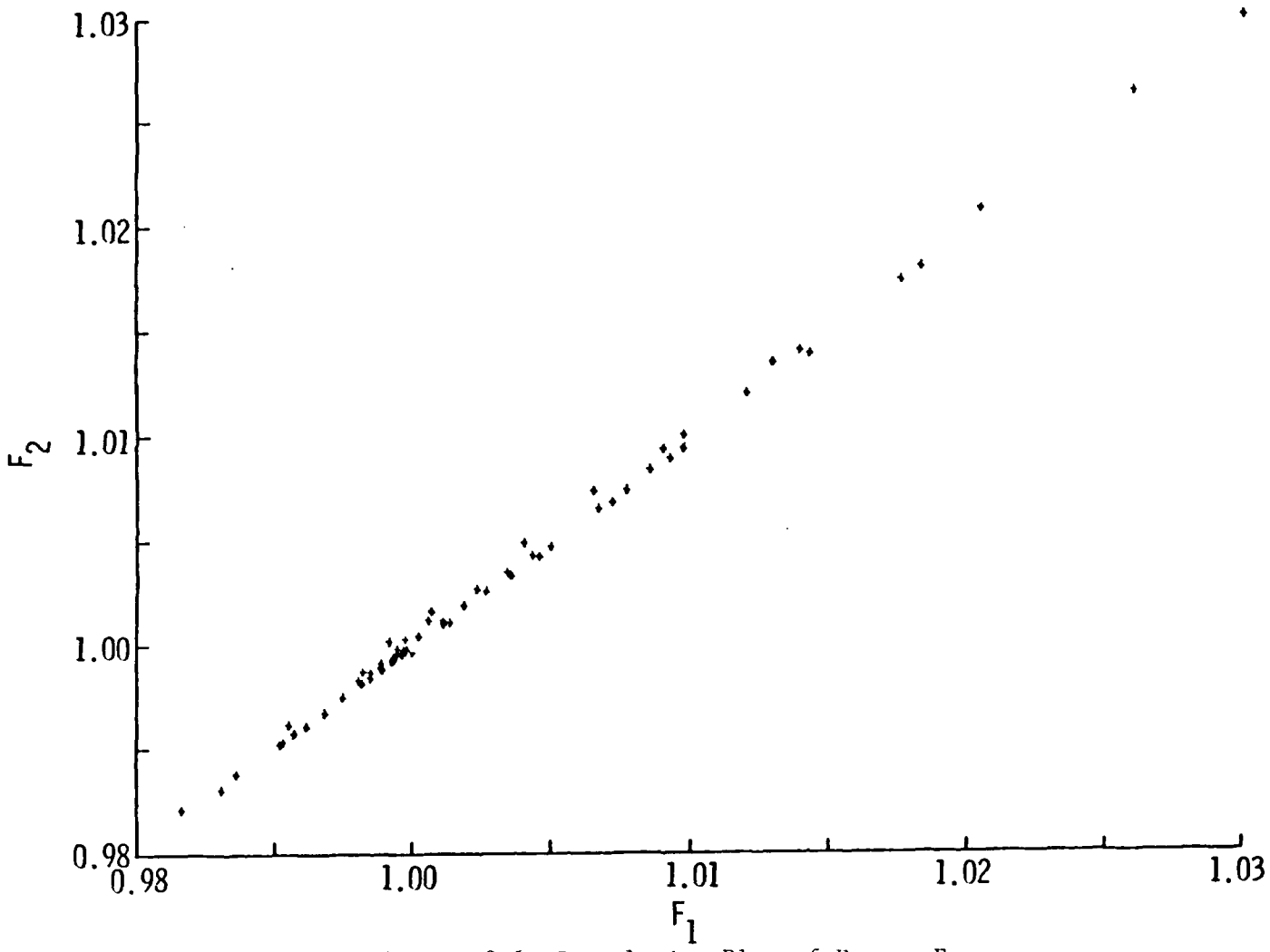


Figure 2.6 Correlation Plot of F_2 vs. F_1

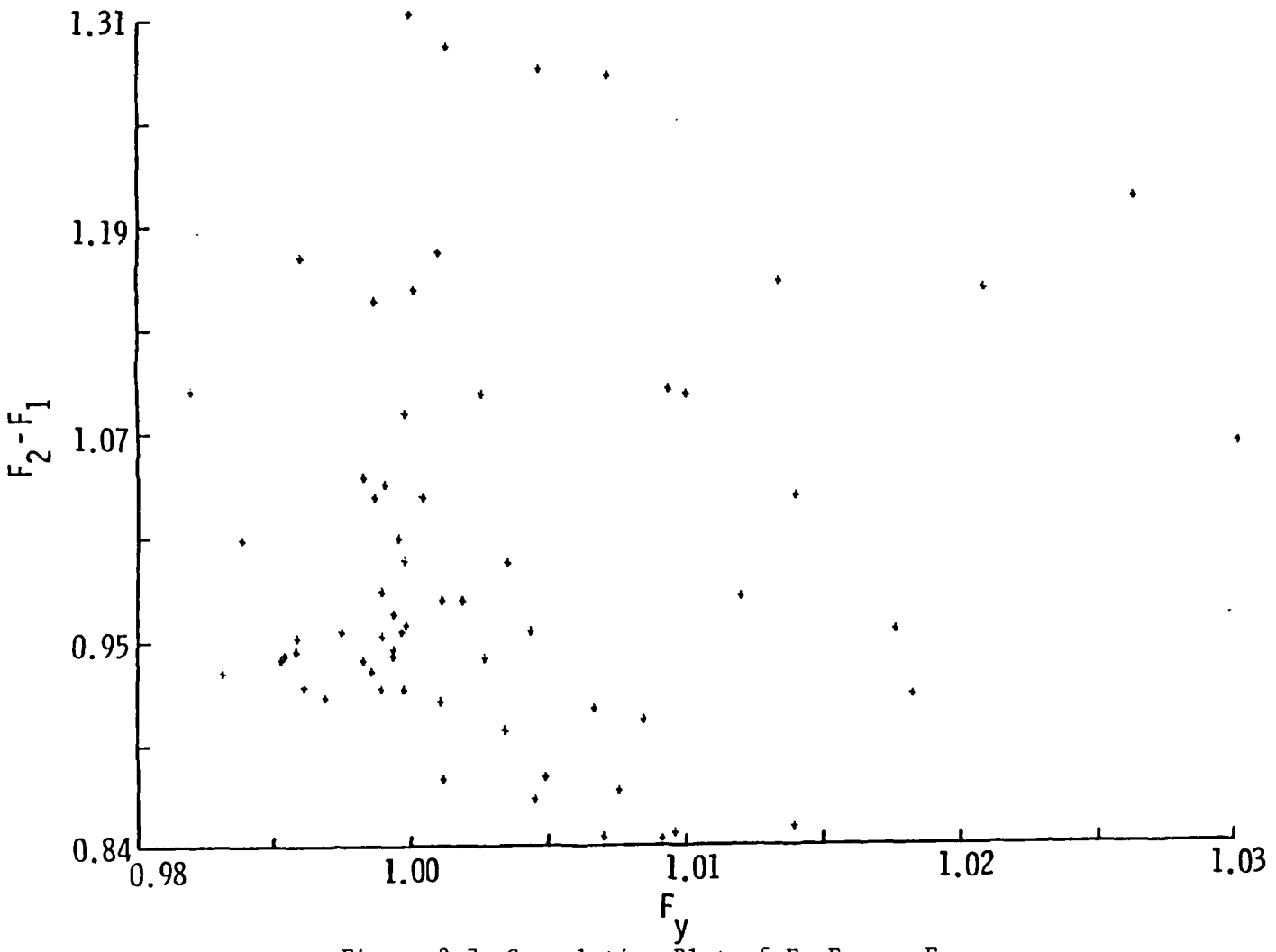


Figure 2.7 Correlation Plot of F_2-F_1 vs. F_y

A correlation was found between G_y and $F_2 - F_1$, (Figure 2.8). Equation 2-5, rewritten below, (Equation 2-7), shows that $F_2 - F_1$ is inversely proportional to G_y,

$$\omega_2 - \omega_1 = C \cdot \omega_y^2/G_y. \tag{2-7}$$

The two curves shown in Figure 2.8 represent the least squares fit to the data and a linear approximation to this curve. The equation for the least sqaures fit is,

$$F_2 - F_1 = .5415/G_y + .4402 \tag{2-8}$$

and the linear approximation is,

$$F_2 - F_1 = -.627 \cdot G_y + 1.627. \tag{2-9}$$

This relationship is shown in Figure 2.9, a plot of conductance, G, versus frequency. As the value of G at resonance, G_y, increases, the resonance peak becomes narrower. Therefore, an increase in G_y produces a decrease in $F_2 - F_1$.

These plots, along with Figures 2.10 and 2.11, have shown that G_y, F_y, and C_t have approximately Gaussian distributions. Additionally, it is seen that Φ has a random distribution. This is not surprising since Φ represents the coupling between the electrical and mechanical domains of the transducer. Random construction errors show up as random fluctuations in Φ.

From the relationships expressed in Equations 2-6 and 2-7, it is evident that instead of the six measured parameters, there are only four independent parameters required to describe the performance of a transducer: G_y, F_y, Φ, and C_t.

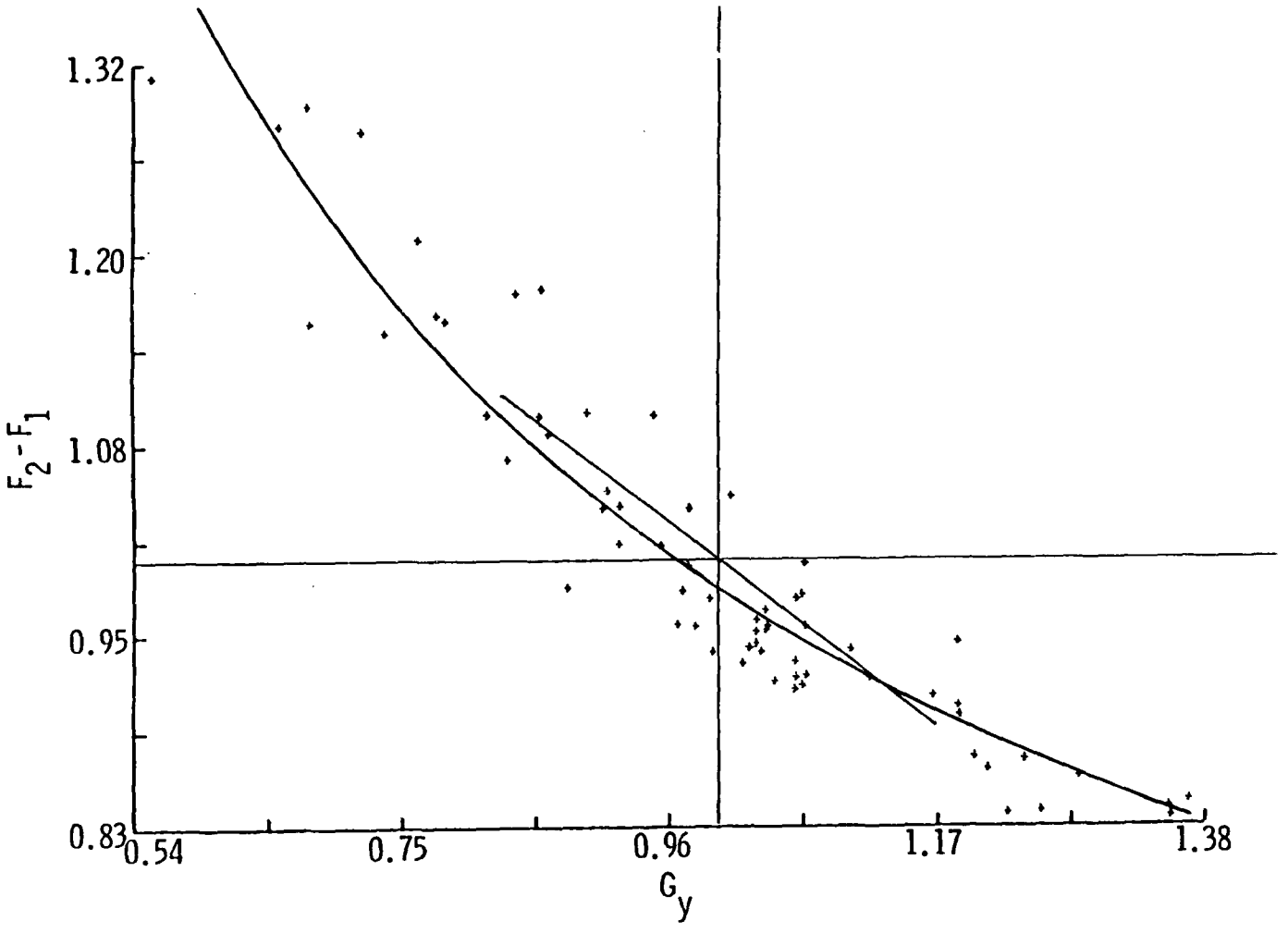


Figure 2.8 Correlation Plot of F_2-F_1 vs. G_y

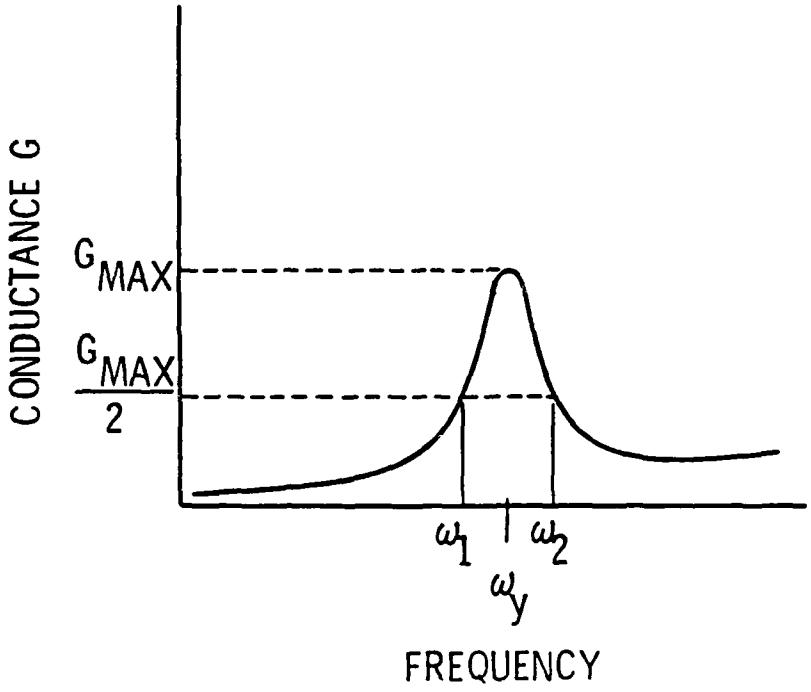


Figure 2.9 Conductance vs. Frequency

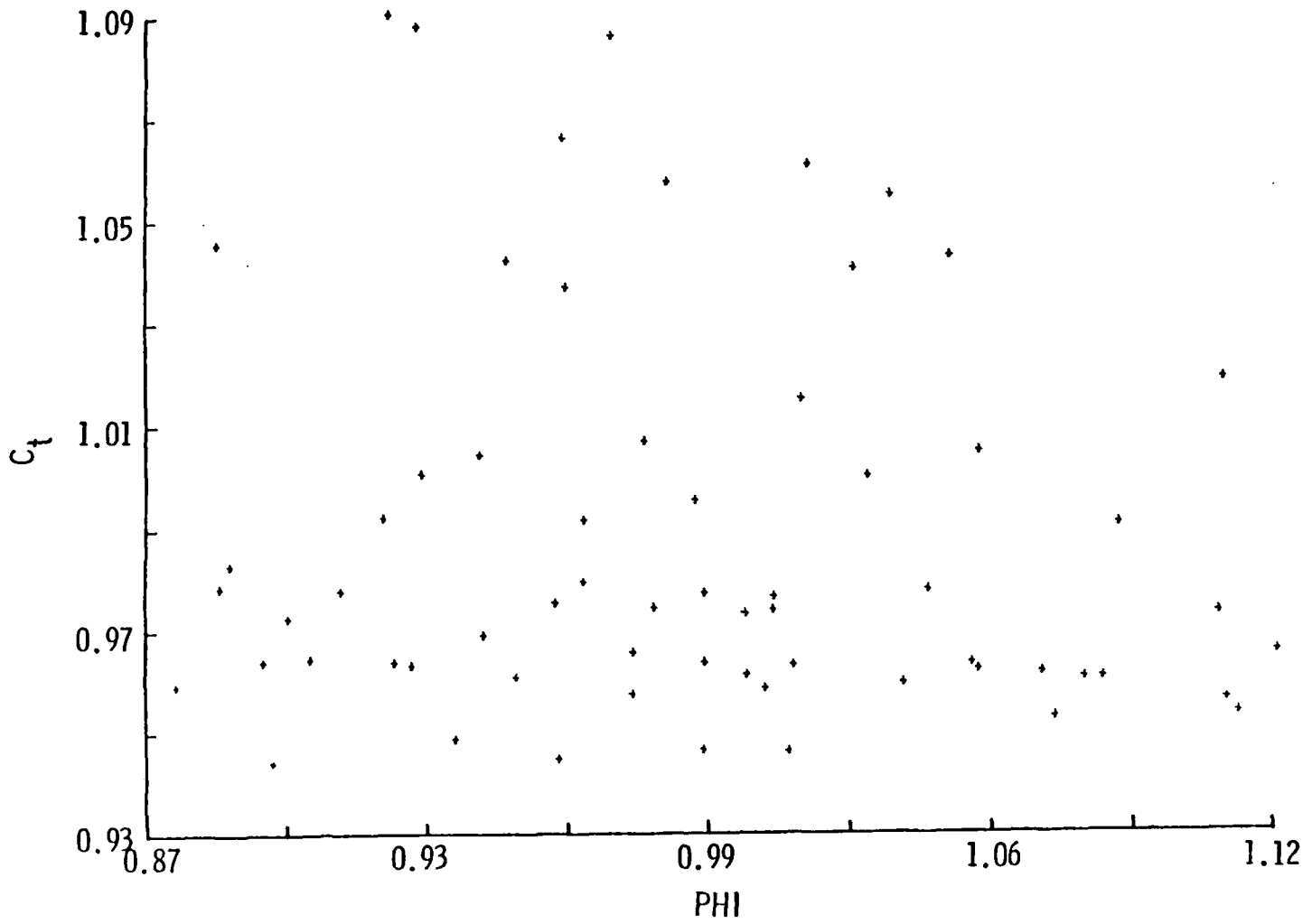


Figure 2.10 Correlation Plot of C_t vs. Phi

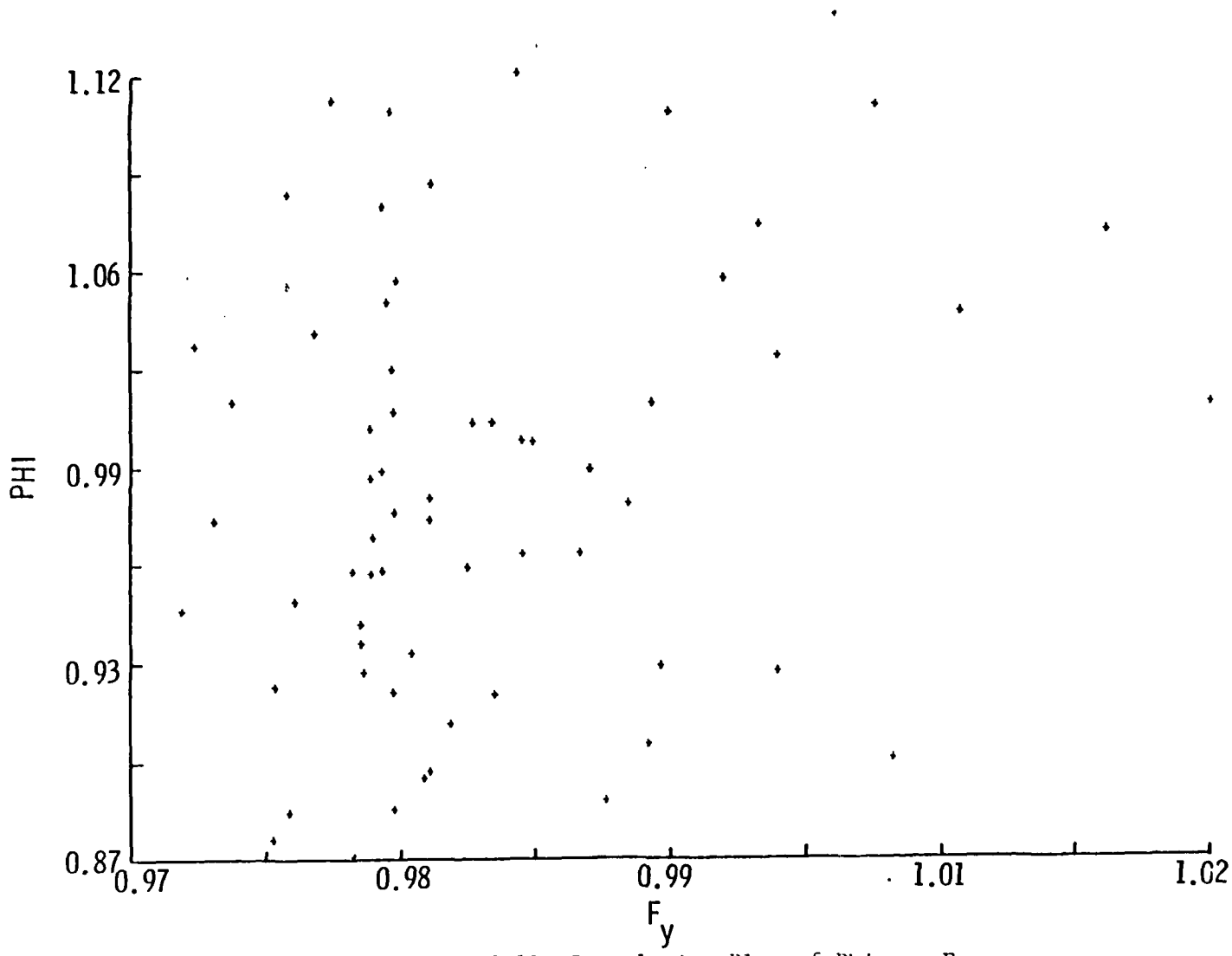


Figure 2.11 Correlation Plot of Phi vs. F_y

A simulated set of elements was generated using the results of this study. F_y, G_y, and C_t were generated by a Gaussian number generator with standard deviations, σ, expressed as percentages of the measured means, of 0.9%, 17.4%, and 3.9%, respectively. A random number generator and a ±12% distribution was used to generate values for Φ. The resulting selection pool, when represented in a scatter plot of amplitude vs. phase, was indistinguishable from the real selection pool. The actual range of values for each parameter is ±4% for F_y, ±45% for G_y, ±11% for C_t, and ±13% for Φ. These ranges, in turn, produce amplitude and phase tolerances of ±9% and ±11°, respectively.

2.2 A TOLERANCE STUDY

One of the obvious results of this thesis is that better response patterns can be obtained if the amplitude and phase responses of the elements do not vary greatly from one another. Thus, it is advantageous to remove from the selection pool those elements whose amplitude and phase responses do not fall within a specified tolerance limit of the mean values.

Calculating the amplitude and phase responses of each element in every array position is a lengthy process. If relationships between circuit parameter values and element performance characteristics can be found, these relationships could be used as a criterion for element suitability, thereby avoiding lengthy calculations.

Using the input/output relationships of the equivalent circuit, it is possible to estimate the effects of extreme values for each parameter, and establish acceptable parameter value limits. The amplitude and phase responses of a nonspecific transducer are calculated as one parameter value is changed a specified percentage above and below a measured mean value and the other parameters are held constant. The mean value is obtained from the measurement of a set of elements. The specified percentage is based on the percent differences between the mean value and the maximum and minimum values of the measured set. To see how the parameter values interact, three lines are plotted on each graph. Each line represents different fixed values of an additional parameter. This discussion is limited to the effects on an element's open circuit voltage amplitude response.

Those parameters for which the amplitude, (and phase), responses of the elements do not change significantly over their entire ranges are Φ and G_y, (Figure 2.12). The amplitude response of an element does not vary significantly for a $\pm 30\%$ variation in Φ, or a $\pm 80\%$ in G_y; large changes in Gy or Φ will not produce large changes in amplitude. In contrast, Figure 2.13 shows the effect of a $\pm 30\%$ variation in C_t, and $\pm 20\%$ variation in F_y. Large changes in either of these parameters will produce a large change in amplitude response.

Although the four circuit parameters are independent, one parameter can influence the effect another will have on an element's response. Figure 2.14 shows how the amplitude response of an element changes as a function of G_y, with F_y as the additional parameter. The relationships mentioned in the previous paragraph are clearly visible.

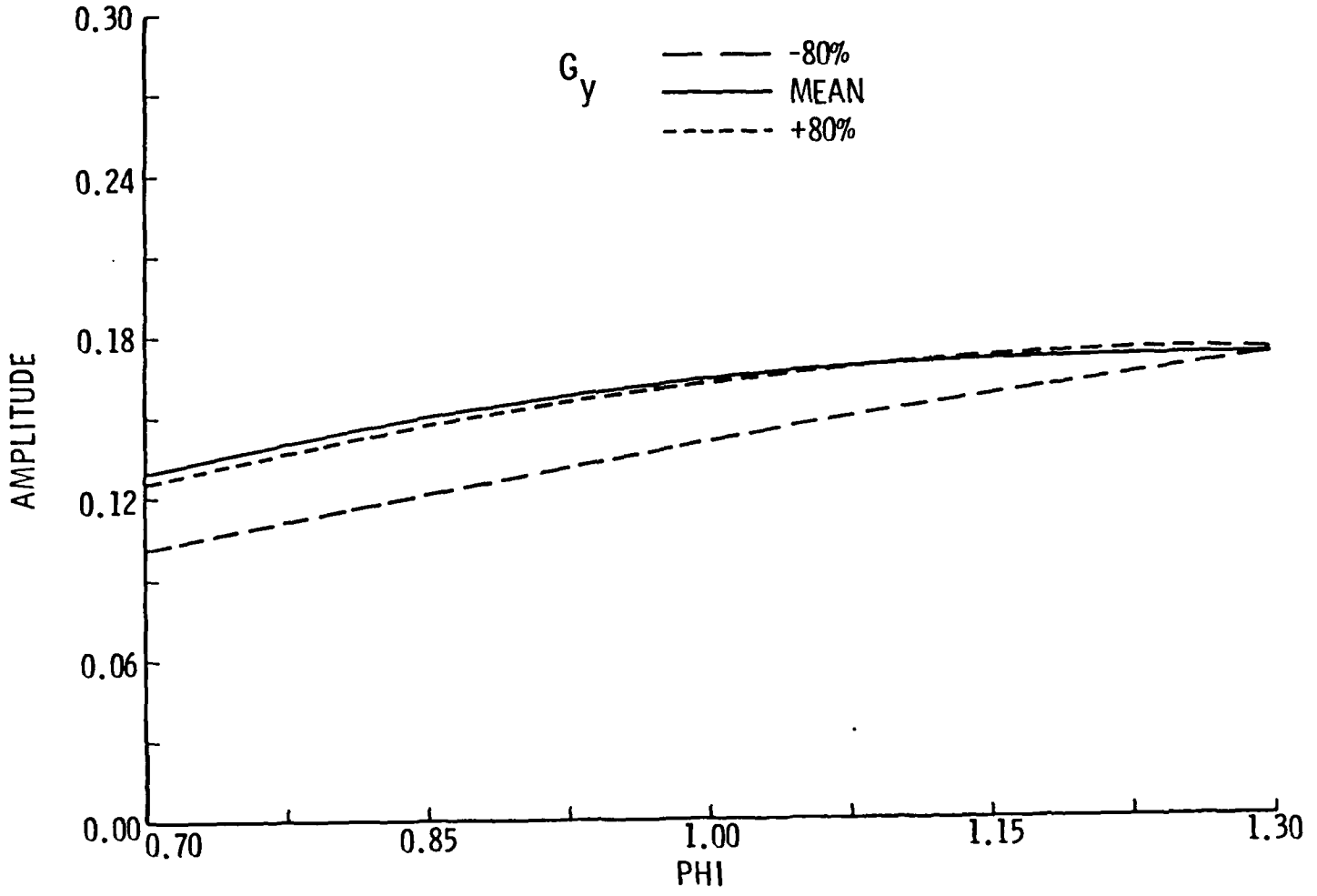


Figure 2.12 Phi and G_y vs. Amplitude

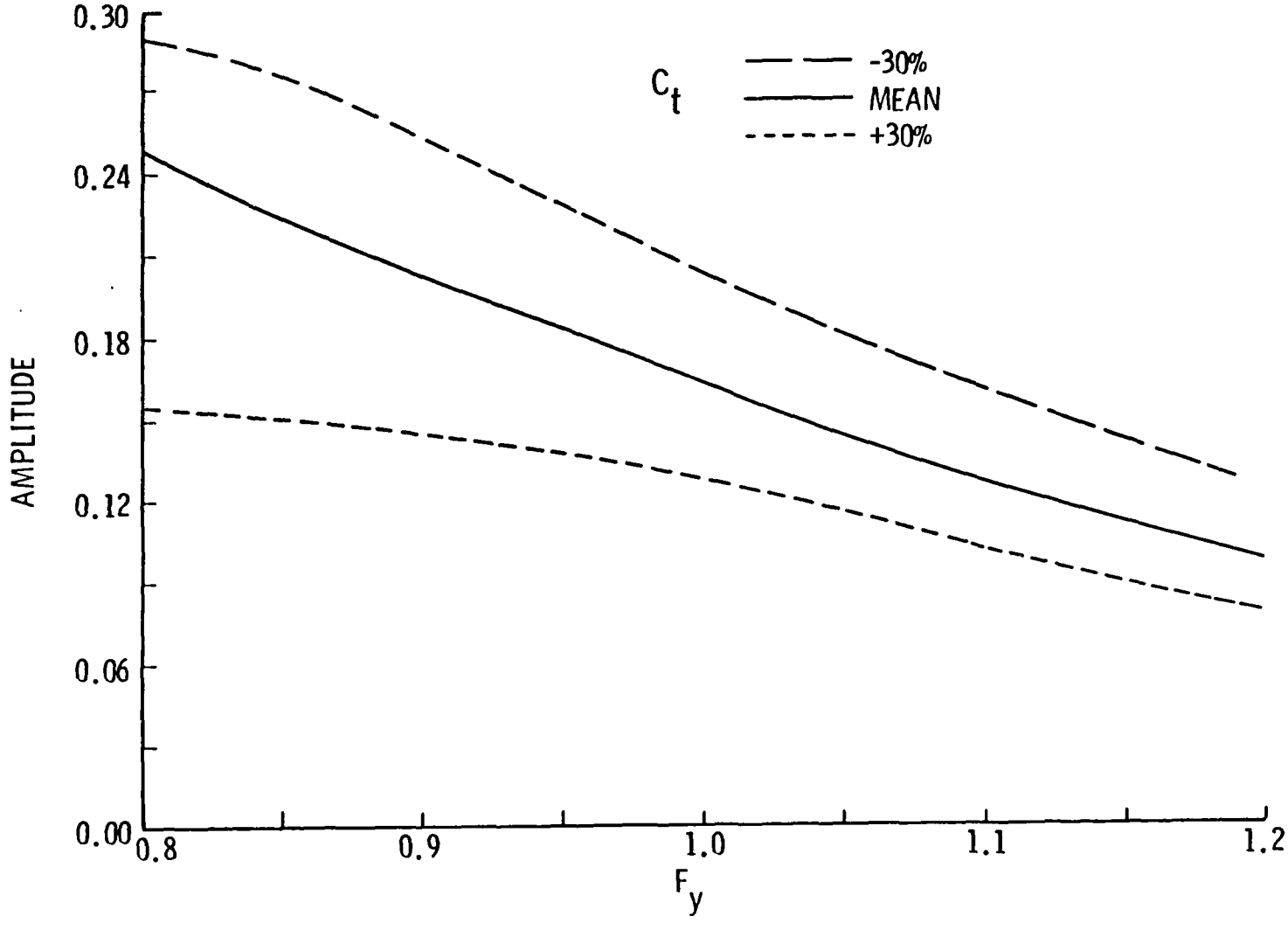


Figure 2.13 F_y and C_t vs. Amplitude

Note that near the lower extreme of G_y there is a sudden change in slope for the curve indicating F_y at five percent below its mean value. This indicates that the effect of G_y on amplitude is somewhat coupled to the value of F_y.

Figure 2.15 is a similar plot, but F_y is allowed to vary over a larger range. As seen, for F_y at twenty percent below its measured mean value, the coupling between G_y and F_y is more pronounced. This results in a reduced range of acceptable values for G_y. Therefore, a greater tolerance in F_y reduces the range of acceptable values of G_y. Transducers with F_y and G_y values in this range may be unsuitable for the array.

Although a ±20% tolerance on F_y violates the lumped-parameter equivalent circuit assumption, this plot is a valid representation of the coupling of F_y and G_y. And, even though the limits on the parameter values in these plots far exceeds those found in the measured set, these plots do show the relative importance of each parameter. The results indicate that the values of Φ and G_y are not as influential on the response characteristics as are F_y and C_t. It follows that greater uniformity of element performance characteristics can be achieved by closely controlling the tolerances on F_y and C_t.

Since only one set of elements was measured, it is not possible to set absolute or relative limits on the parameter values for other sets based on this set. However, this method has shown the effects each of the circuit parameters has on an element's amplitude and phase response, and on each other.

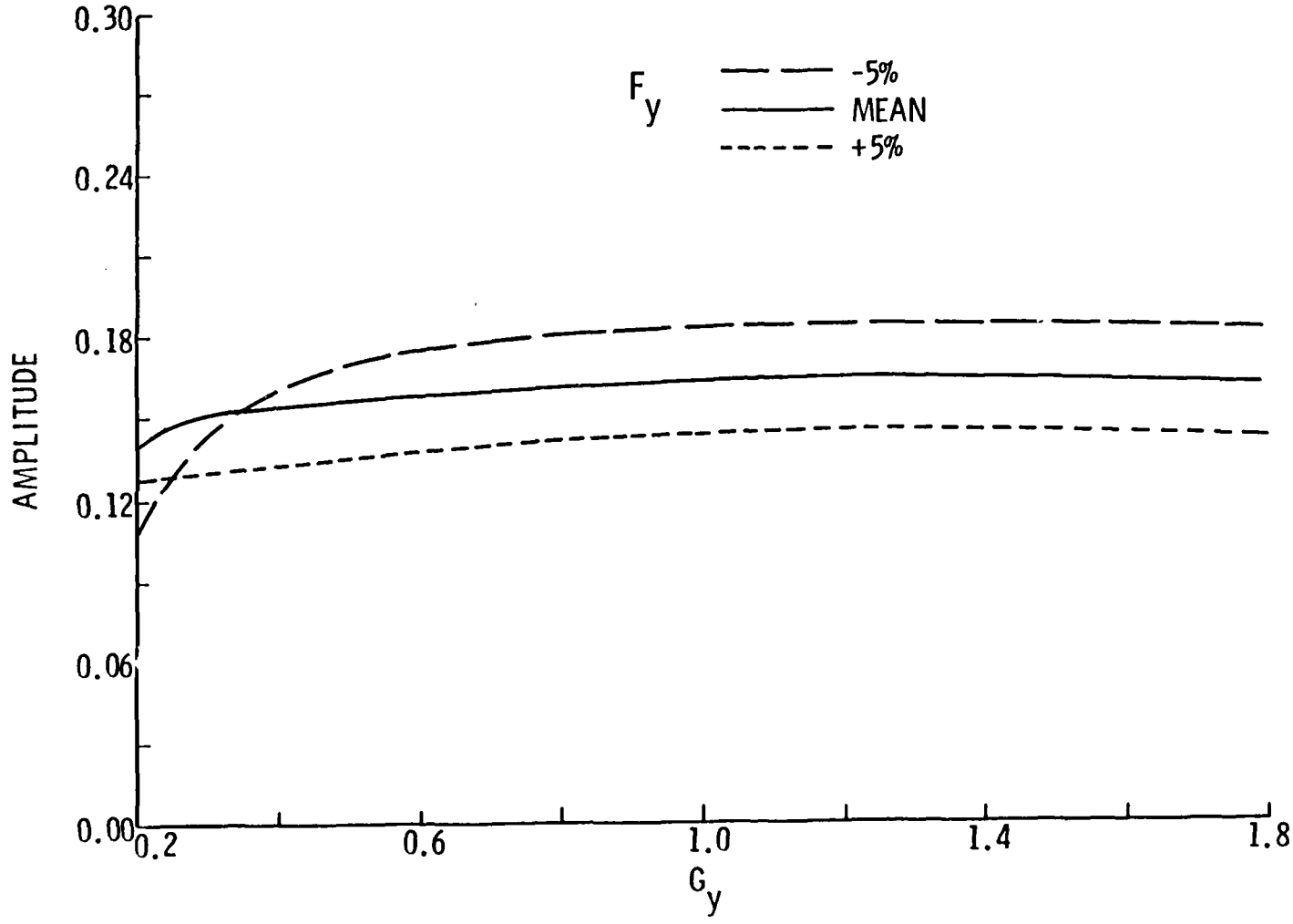


Figure 2.14 F_y (5%) and G_y vs. Amplitude

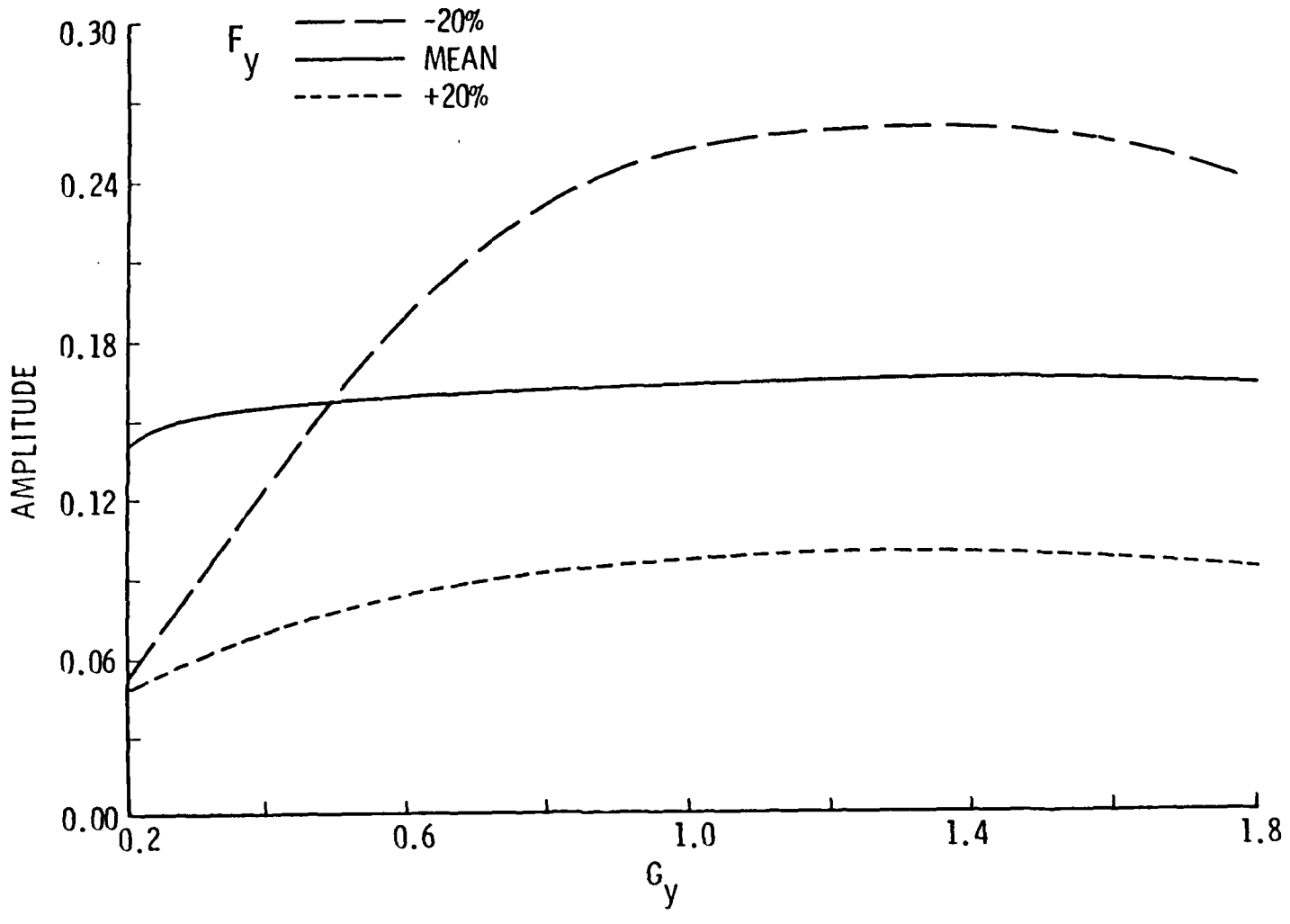


Figure 2.15 F_y (20%) and G_y vs. Amplitude

2.3 ARRAY RESPONSE AND ELEMENT TOLERANCES

Prior to this study, a rule of thumb relationship between array response and constituent element tolerances was to set the tolerances on amplitude and phase equal to the desired side lobe level. For example, to achieve a -40 dB side lobe level, a .01 tolerance in amplitude and phase is required. This translates into a ±1% tolerance in amplitude and a ±0.6° tolerance in phase. These tolerances are very stringent and very difficult to obtain.

The selection process presented in this thesis changes the above mentioned tolerances by an order of magnitude. Elements with ±9% amplitude and ±11° phase tolerances, when randomly placed in an array produce -30 dB side lobe levels, as predicted by the rule of thumb relationship. However, using the same elements, the selection process can produce arrays with side lobe levels that are within 1 dB of the design level of -40 dB side lobes, (see Chapter Six).

CHAPTER THREE

IMPEDANCE AND INTERACTION

3.1 INTRODUCTION

Mechanical impedance, Z_M, is defined as the ratio of the force applied to an object and the resulting velocity at the point of application. In general, impedance is a complex quantity. The real part, referred to as resistance, is related to the power lost in the system. The imaginary part, reactance, represents energy which is stored in the system.

In acoustics, a more useful quantity is the acoustic impedance, Z_A. It is defined as the ratio of the pressure on a surface and the volume velocity of the medium which results from the motion of the surface. Since pressure and force are related by a factor of area, as are volume velocity and particle velocity, mechanical impedance and acoustic impedance are related, $Z_M = S^2 \cdot Z_A$, where S is the area of the face of the transducer.

For a plane wave incident on a piston, the force exerted on the piston is obtained by averaging the pressure over the surface area of the piston. In this way, the acoustic impedance is transformed into mechanical impedance. In the equivalent circuit, the coupling of the mechanical domain with the acoustic domain is represented by an ideal transformer with a turns ratio equal to the area of the transducer face.

3.1.1 SELF IMPEDANCE

The head of a tonpilz transducer can be modelled as a rigid square piston in an infinite baffle. For a piston which is small compared to wavelength, the pressure on the face of the piston will vary with position. This being the case, the acoustic impedance of the transducer is no longer a simple ratio.

Using the Green's function for radiation from a planar surface, Morse and Ingard [3] derived an expression for the acoustic radiation impedance of a square piston of side a. The expression is given below.

$$Z(\omega) \cong \rho c a^2 [\theta_o(ka) + j\chi_o(ka)] \tag{3-1}$$

where

$$\chi_o = M_1(ka) = \frac{4}{\pi} \int_0^{\pi/2} \sin((ka)\cos(q))\sin^2(q)dq$$

$$\theta_o = 1 - 2J_1(ka)/ka$$

3.1.2 MUTUAL IMPEDANCE

For the two square pistons shown in Figure 3.1, the pressure on the face of one piston is a function of its own interaction with the medium and of the pressure field of the other transducer. Simultaneously, the other piston is experiencing the same effects as a result of the pressure from the first piston. The motion of each piston is, therefore, coupled with the other.

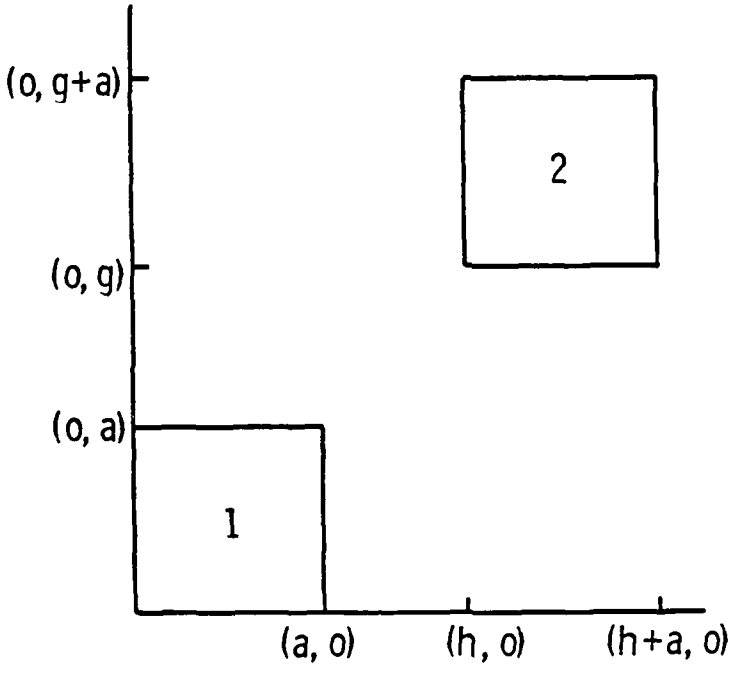


Figure 3.1 Two Square Pistons

As expressed below, the total impedance on one transducer is the combination of its self impedance, Z_s^1, and the mutual impedance with the other transducer.

$$Z_T^1 = Z_s^1 + (v_2/v_1) \cdot Z_m^{1\ 2} \tag{3-2}$$

$Z_m^{1\ 2}$ is called the mutual impedance coefficient. v_2/v_1 represents the of the velocity amplitudes of the two transducers. The superscripts refer to the elements involved.

For an array of M transducers, the total loading on one element is a function of its self impedance and the weighted sum of the mutual impedances with all the other elements.

$$Z_T^j = Z_s^j + \sum_{\substack{i=1 \\ i \neq j}}^{M} (v_i/v_j) \cdot Z_m^{ij} \tag{3-3}$$

If all the $Z_m^{i,j}$ can be determined, as well as Z_s^j, it would be possible to determine the acoustic radiation loading on every element in an array. The ratio, v_i/v_j, representing the ratio of velocity amplitudes, is equivalently a ratio of shading coefficients.

Using the expression for self impedance, Equation 3-1, and an expression for the mutual impedance, Reference [4], Schafer [2] has developed a computer routine, "MUTUAL", to compute the radiation loadings on all the positions of an 8 by 8 array. For this study, this program has been expanded to handle all types of four-fold symmetric arrays up to 12 by 12 in size.

3.2 INTERACTION FOR THE TRANSMIT AND RECEIVE CASES

Since the mutual interaction depends on a ratio of the velocities of two elements vibrating in phase, a change in shading values will affect this interaction. When an array is used in the transmit mode, the elements' amplitudes are shaded in a manner to produce a specified directional response. Each set of shading values corresponding to a different beam pattern will subject the array elements to different mutual interaction effects.

The receive case is different. Although shading values are still used in beamforming, they have no effect on the acoustic interaction between elements. The shading values are applied to the output electrical signals and therefore do not influence the impedance on or the motion of the transducers.

3.2.1 THE INFLUENCE OF THE INCIDENT ANGLE

Unfortunately, the receive case can not be simply modelled by equating it to the transmit case with unity shading. For a plane wave with an incident angle $\theta \neq 0°$, the elements separated by $d = \lambda/2$ will not vibrate in phase. If the elements are modelled as point sources placed at their respective centers, the responses from two elements separated by a distance d will be out of phase by $k \cdot d \cdot \sin\theta$, (see Figure 3.2).

If the phase relationship between two elements changes, so will their interaction. The interaction is reflected in the radiation resistance and the power radiated. To see this, consider two sources separated by a distance d. If these two elements vibrate 180° out of

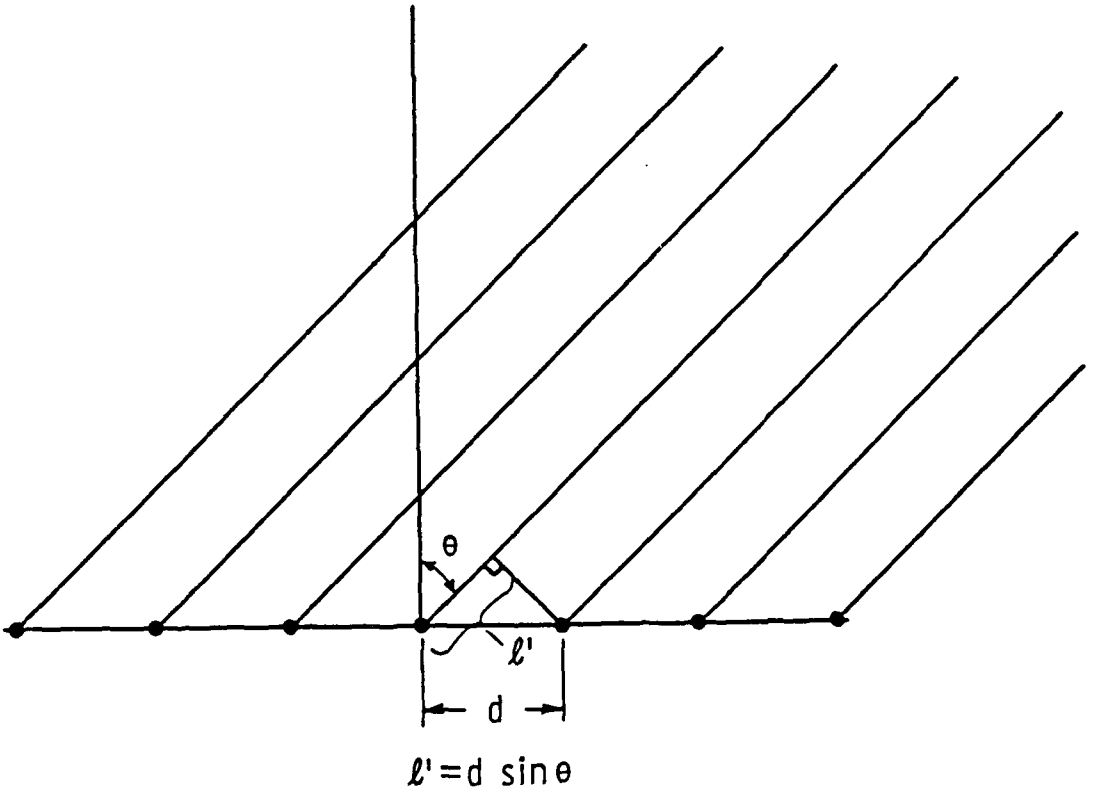


Figure 3.2 Phase Relationship Among Point Sources

phase, as in a dipole, the total power radiated is, for $k \cdot d = 1$, one third of that for a single source. As $k \cdot d$ changes, so does the amount of power radiated. When two elements vibrate 90° out of phase, the total power radiated is twice that of a single source. Similarly, for two sources vibrating in phase, the total power radiated is four times that of a single source [5].

It is easy to extend this example to the receive case. The result is that as the incident angle changes, so do the phase relationships, and the elements' mutual interactions. Consequently, each element's performance characteristics will change with the incident angle.

This is disastrous news for the problem of placing elements in an array. If the phase relationships are determined for a particular incident angle, the radiation loadings and element responses can be determined. An array can then be designed which has minimal error for that incident angle. However, the errors at other incident angles are not accounted for by this element placement.

3.3 TRADITIONAL AND NEW ASSUMPTIONS

Before the inclusion of mutual interaction effects became feasible, it was assumed that the entire array experienced ρc loading. Since the results based on this assumption have been acceptable, the mutual interaction effects were considered second order effects. Through the use of "MUTUAL", it is shown that mutual interaction effects on individual elements are not second order.

In the transmit case, the loading on the centermost elements differs from the loading on a single square element, as computed using Equation 3-1, by 6% in the resistive component and -83%, reactive. This error increases to 18%, resistive, and -97%, reactive for elements farthest from the center. In the receive case, for a 0° incident angle, the errors are 9%, resistive and -85%, reactive for centermost and 4.5%, resistive and -52%, reactive for outermost.

These errors are far from what may reasonably be considered second order. Although the loading on every element is dramatically different from ρc loading, the loading on the entire array, with interaction effects included, can still be reasonably approximated by ρc loading, [6]. The larger the array, the more accurate is the approximation.

For the purpose of this study, it is assumed that for plane waves incident at angles $\neq 0°$, the radiation loadings on the elements of an array do not differ significantly from the loadings for 0° incidence. Thus, the radiation loadings for all the incident angles are assumed to be the same as the loadings for 0° incidence.

The following section illustrates how changes in radiation loading do not significantly affect element performance. Therefore, the small changes in radiation loading which occur over different incident angles will not produce significant changes in element response.

3.4 THE EFFECT OF ELEMENT TOLERANCES ON MUTUAL INTERACTION

In order to determine the radiation loading on every position in an array, it is necessary to know the shading values for each position.

These radiation loadings are then used to determine the elements' performance tolerances. However, the use of ideal shading values assumes that each shaded element is identical. But this is not true! It is important to ask, and answer, what effects do element tolerances have on the radiation loading?

An array was designed using the ideal radiation loadings to calculate element tolerances. The normalized mean amplitude values of the elements in each symmetric array position were then multiplied by the shading values for each position to produce new shading values for each position. It was assumed that the phase errors of each element are small and were not included. These new shading values were then used to re-evaluate the radiation loadings for each array position.

For those elements which comprise this array, new amplitude and phase characteristics were determined. A new response pattern for this array was then determined using the newly computed element characteristics. The resulting response pattern did not exhibit significant changes from the original pattern. The new radiation loading that the elements experienced due to their neighbors' new tolerances was also computed.

The first recalculation of the radiation loadings on each position showed changes ranging from 0% to 3% in resistance and 1.5% to 90% in reactance. Yet these changes resulted in a maximum change of only 3% in amplitude and 0.5° in phase. It should be noted that the greatest changes occur for radiation loadings corresponding to the array positions farthest from the array center. The errors for the closer

positions are an order of magnitude smaller. The second recalculation of radiation loadings produced even smaller changes. This translates into smaller errors in the amplitude and phase.

Additionally, these new performance characteristics were used to redesign the array from the original selection pool. It was assumed that the errors in each symmetric array position would not change significantly from those of the first array. (This was tested and shown to be valid.) A comparison of this new response pattern with the first also showed no significant degradation or improvement. The normalized mean amplitude values for each position were used to recalculate the amplitude and phase characteristics of each element. These new characteristics were not significantly different from the original values.

In conclusion, the errors introduced into the radiation loadings by element tolerances are small enough to be neglected.

CHAPTER FOUR

PERMUTATION SEARCH

4.1 INTRODUCTION

The process of placing in an array elements which have different tolerances involves combining the elements in such a way as to minimize the total impact of their errors on the directional beam pattern of the array. This process resembles a class of problems in industrial engineering known as the Travelling Salesman Problem. Travelling Salesman Problems involve ordering a set of elements in different ways in an attempt to meet a certain objective. The elements could represent the stops a salesman has to make during a trip and the objective could be to minimize the number of miles travelled. Or, in the case of array design, each element would represent a transducer element placed in an array position, and the objective would be to minimize the error impact on the array response.

For small problems, i.e., those involving a small number of elements, there are solution techniques which will give a globally optimum solution. But with large problems, the amount of computer time these methods require becomes unreasonable. For an array of 36 positions and 36 elements, there are 36! possible permutations of the elements. There are other methods of solution which may not produce a globally optimum solution, but do offer a considerable reduction in necessary computer time. One such method is called Permutation Search.

4.1.1 PROBLEM DEFINITION

A permutation, denoted $[P^n]$, is any ordering of a set of elements. For each permutation, there is an associated objective function, $E(P^n)$. Additionally, there may be constraints on how these elements may be arranged in the permutation, denoted as the set of feasible permutations, G. The problem solved by Permutation Search can be formulated as

$$\begin{aligned} &\text{Find } [P^n] \text{ to minimize} \\ &E(P^n), \quad \text{such that} \\ &P^n \in G. \end{aligned} \tag{4-1}$$

4.1.2 THE ELEMENT PLACEMENT PROBLEM

For the process of creating an array with M positions, using a set of N transducer elements, where $N > M$, the problem formulation is as follows.

Let $X(m,n)$ be the decision variable associated with element m and position n, such that

$$\begin{aligned} X(m,n) &= 1 \quad \text{if element m is in position n} \\ &= 0 \quad \text{otherwise.} \end{aligned} \tag{4-2}$$

There are two constraints on the placement of the elements. The first is that every array position must be filled and by one element only. For those humans who know that two elements may not occupy the same position simultaneously, the second part of this constraint may seem unnecessary. However, computers have no way of knowing this unless they are told so.

$$\sum_{m=1}^{N} X(m,n) = 1 \qquad n = 1,2,\ldots,M \tag{4-3}$$

The second constraint is that each element may either be assigned to one array position, or unassigned.

$$\sum_{n=1}^{M} X(m,n) \leq 1 \qquad m = 1,2,\ldots,N \tag{4-4}$$

The array response, $R(\theta,\phi)$, at incident angles θ and ϕ, is

$$R(\theta,\phi) = 20 \log \left| \left[\sum_{m=1}^{M} \sum_{n=1}^{N} X(m,n)\, A(m,n) \cdot \exp\{i[ku_n + a(m,n)]\} \right] \right| , \tag{4-5}$$

where

$X(m,n)$ = Decision variable
$A(m,n)$ = $S(n) \cdot \text{Real}\{\text{Elemnt}(m,n)\}$
$S(n)$ = Shading value at position n
N = Number of array positions
M = Number of elements in the selection pool
$a(m,n)$ = $\text{Imaginary}\{\text{Elemnt}(m,n)\}$
u_n = $k[C(n) \cdot \sin\theta \cos\phi + D(n) \cdot \sin\theta \sin\phi]$
$C(n)$ = x coordinate of position n
$D(n)$ = y coordinate of position n
$\text{Elemnt}(m,n)$ = Amplitude, (real) and Phase, (imaginary) response of element m at position n.

The objective is to minimize the total error, defined as the sum over all constrained angles of the error at each angle:

$$E\{X(m,n)\} = \sum_{B} \{L(\theta,\phi) - R(\theta,\phi)\} \cdot \varepsilon, \tag{4-6}$$

where

$L(\theta,\phi)$ = Desired level at angles θ and ϕ
B = The set of all constrained angles
ε = 1 if $L(\theta,\phi) > R(\theta,\phi)$
= 0 if $L(\theta,\phi) < R(\theta,\phi)$.

4.2 THE SEARCH PROCESS*

Permutation Search is a process which searches for a locally optimum permutation. It looks in the neighborhood of an initial base permutation for permutations which improve the objective function. The neighborhood of a permutation is defined as the set of permutations which can be obtained through a series of element exchanges. A base permutation is any permutation from which a new permutation is formed. The initial base permutation, or the first permutation in a series of permutations, is denoted $[P^o]$. For example, a permutation, $[P^i]$, of 5 elements,

$$[P^i] = 4\ 3\ 2\ 5\ 1$$

is in the neighborhood of the permutation

$$[P^o] = 5\ 4\ 3\ 2\ 1$$

since it can be reached from $[P^o]$ through a sequence of one for one adjacent element exchanges, viz.,

$$[P^o] = 5\ 4\ 3\ 2\ 1$$

$$[P^1] = 4\ 5\ 3\ 2\ 1$$

$$[P^2] = 4\ 3\ 5\ 2\ 1$$

$$[P^3] = 4\ 3\ 2\ 5\ 1.$$

When the final element in the permutation is exchanged, the next exchange involves the first two elements of the existing permutation. In the ample above, the next two permutations would be

$$[P^4] = 4\ 3\ 2\ 1\ 5$$

$$[P^5] = 3\ 4\ 2\ 1\ 5.$$

* The Permutation Search procedure presented in this section is a synopsis of reference [7]

Exchanges can be the interchanging of two elements, or shifts of large blocks of elements. The nature of the problem dictates the appropriate type of exchange. For the transducer array, one for one exchanges of adjacent elements are performed.

4.2.1 EVALUATING THE PERMUTATIONS

Since Permutation Search looks for those permutations which improve the value of the objective function, $E(P^n)$, E must be determined for every permutation. It is obvious that not every exchange will result in a permutation with an improved objective function. If an improvement is found, then the next exchange is performed on the newly achieved permutation, at the position adjacent to the previous exchange. If there is no improvement, the next exchange is performed on the previous permutation, also at the adjacent position. Any permutation obtained as a result of an element exchange performed on a base permutation is called a trial permutation. If this trial permutation improves the value of the objective function it becomes the current base permutation. Therefore, a sequence of bases represents a continuous improvement in the objective function. This sequence will converge to a locally optimum solution in the neighborhood of the initial base permutation.

4.3 ARRAY DESIGN SIMPLIFICATION

The Permutation Search formulation of element placement problems can be set up in such a way as to sidestep the two constraints. Let the permutation array, (note, not permutation), be a vector with as

many components, or slots, as there are transducer elements in the selection pool. Each slot will represent either a position in the transducer array, or a position on a workbench, i.e., an unassigned position. A permutation consists of each transducer element assigned to only one slot in the permutation array. Thus, every slot in the permutation array is filled. This arrangement automatically ensures that any permutation in the neighborhood of the first permutation satisfies the constraints.

Finding $R(\theta,\phi)$ can also be simplified through the implementation of the permutation array. Let there be a function assigned to each slot of the permutation array. This function operates on the amplitude and phase characteristics of the element assigned to that slot in a manner which is unique to the array position that the slot represents. The uniqueness of this function is derived from the x and y coordinates of each position as well as the shading value at the position. R is found by summing the value of this function over all the slots. Thus, the problem reduces to

$$\begin{aligned}&\text{Given } [P^o],\\&\text{Find } [P^n] \text{ to minimize}\\&E(P^n).\end{aligned} \tag{4-7}$$

4.3.1 PERMSER

The application of Permutation Search to the problem of element placement in arrays is best described by an example. This example demonstrates how it is implemented in the FORTRAN routine, "PERMSER", (see

Appendix D for a listing and section 4.4 for a discussion of this program). There are seven elements in the selection pool, numbered 1 through 7. There are, therefore, seven slots in the permutation array,

$$[P\ ] = \underset{1}{\_}\ \underset{2}{\_}\ \underset{3}{\_}\ \underset{4}{\_}\ \underset{5}{\_}\ /\ \underset{6}{\_}\ \underset{7}{\_}\ . \qquad (4\text{-}8)$$

The "transducer array" to be designed has five positions, represented by the first five slots in the permutation array. "Workbench" positions, slots 6 and 7, indicate an element unassigned to an array position. Normally, transducer elements are assigned a number to distinguish them. For this example, the "response" of each "transducer element" will be represented by its assigned number. The unique function at each slot, representing the contribution to the array response of each element, will be the product of the slot index and the "response" of each "transducer element". "Workbench" elements do not contribute to the response of an array.

The response of the array is the sum of the values of the function at each slot over all the "transducer array position" slots. For this example the objective will be to maximize $E(P^n)$.

Starting with the initial base permutation,

$$[P^o] = \underline{1}\ \underline{4}\ \underline{2}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7} \qquad E[P^o] = 64$$

the first exchange gives a trial permuation,

$$[P_t] = \underline{4}\ \underline{1}\ \underline{2}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7}$$

with an objective function $E[P_t] = 61$. This permutation does not result in an improved objective function, so the next exchange is performed at the adjacent permutation array slot in $[P^o]$,

$$[P_t] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7}\ .$$

The objective function is $E[P_t] = 66$. Since this is an improvement over the value of the objective of the current base point, this trial permutation becomes the current base point,

$$[P^1] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7} \qquad E[P^1] = 66.$$

Exchanging cyclically through the permutation array, the process converges to a locally optimum solution. The sequence of successive bases is given below.

$$[P^o] = \underline{1}\ \underline{4}\ \underline{2}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7} \qquad E[P^o] = 64$$

$$[P^1] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{6}\ \underline{5}\ /\ \underline{3}\ \underline{7} \qquad E[P^1] = 66$$

$$[P^2] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{5}\ \underline{6}\ /\ \underline{3}\ \underline{7} \qquad E[P^2] = 67$$

A local optimum has been reached. At this point, "PERMSER", will initiate what is called a new start. A new permutation is generated by randomly rearranging the unassigned elements, and leaving the elements which are assigned to "transducer array position" slots where they are. It is called a new start because the new permutation represents a new, untested neighborhood and retains the same level of achievement of the objective function.

The new start, shown below, has the same objective function value as the current base permutation, but is a different permutation.

$$[P^3] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{5}\ \underline{6}\ /\ \underline{7}\ \underline{3} \qquad E[P^3] = 67$$

From here, there is only one more base permutation that will improve the objective function. The new local optimum is

$$[P^4] = \underline{1}\ \underline{2}\ \underline{4}\ \underline{5}\ \underline{7}\ /\ \underline{6}\ \underline{3} \qquad E[P^4] = 72.$$

The global optimum,

$$[P^n] = \underline{3}\ \underline{4}\ \underline{5}\ \underline{6}\ \underline{7}\ /\ \underline{1}\ \underline{2} \qquad E[P^n] = 85$$

is not attainable in either of the two neighborhoods searched.

In this example, the elements in the permutation array slots representing the workbench were never in a "transducer array position" slot for a base permutation, (except for the final permutation). This particular type of permutation array is very restrictive in the sense that there is only one entry point into the "transducer array position" slots for elements that are in "workbench" slots. A much more effective permutation array, which has many entry points, is illustrated below.

$$[P\ ] = \overline{1}\quad \overset{*}{\overline{2}}\quad \overline{3}\quad \overline{4}\quad \overset{*}{\overline{5}}\quad \overline{6}\quad \overline{7} \tag{4-9}$$

For this permutation array, "transducer array positions" are represented by slots 1, 3, 4, 6, 7, and the "workbench" slots are 2, and 5. This arrangement gives the "transducer elements" in the "workbench" slots a greater chance of being exchanged into a "transducer array position" slot.

The emphasis on the ease of getting elements from the workbench into the transducer array is justified. Due to the way the responses of elements in symmetric array positions interact, it is important that as many new combinations as possible be tried, (see section 5.4.1). This requires that elements initially on the workbench be included as often as elements initially in the transducer array.

4.4 TEST RESULTS

"PERMSER", the FORTRAN implementation of Permutation Search, has been successfully used to place elements in both 3 by 3 and 6 by 6 arrays from scratch, as well as 7 by 7, 8 by 8, and 9 by 9 arrays with non-random starting points. It can handle any four-fold symmetric array as well as most other types of arrays. There are several parameters which are used to control the Permutation Search algorithm, each with different effects on the outcome. They can influence run time, quality of result, both or neither. The following parameters are those which are not directly specified by the problem.

4.4.1 ZLOLIM

The computer variable ZLOLIM represents the desired or acceptable amount of error in the response pattern of the array being designed. A comparison of ZLOLIM with the value of the objective function at each base permutation determines if the desired error has been achieved. ZLOLIM controls both the run time and the quality of results. By setting ZLOLIM very low, the run time increases, and the quality of

results will usually improve. The improvement in results is a reflection of the number and size of the neighborhoods searched. If more time is used, more neighborhoods can be searched.

The value of ZLOLIM can be determined by using, in the YVALUE routine of "PERMSER", the parameters of an array whose response is already known. An evaluation of how well this array meets the design specifications at the constrained angles will give a ballpark figure for ZLOLIM.

An alternative use of ZLOLIM is to set it equal to zero. The run time is then controlled by other search routine control parameters. This method results in a loss of control over the amount of acceptable error, but eliminates the need to determine ZLOLIM in advance. This is the first of the two control parameters which are capable of stopping the execution of the Permutation Search algorithm.

4.4.2 CONSTRAINED ANGLES

Another parameter which controls the Permutation Search algorithm is the number and location of the constrained angles. As the number of constrained angles increases, so does the run time. The only limits on the maximum number of constraint points are those imposed by the computer facilities and the amount of time allowed. However, there are limits to the fewest number of constraints which can be specified. If too few angles are constrained, the results are generally poor since the response is controlled in too few places, (see Appendix C). Similarly, if the constrained angles are concentrated in specific areas, the response at other points will tend to be quite poor.

It has been found that a loose set of constraints, such as increment angles of $\Delta\theta = 5°$ and $\Delta\phi = 20°$ give results nearly as good as those for a tighter set of constraints, such as $\Delta\theta = 2°$ and $\Delta\phi = 5°$, but with a 40% to 75% reduction in run time. The range of reduction in run time is influenced mainly by the number of new starts. These constraints are valid for element spacing, d, equal to $\lambda/2$.

It is obvious that with tighter constraints, a better solution will most likely be found, but the trade off is an unavoidable increase in run time.

4.4.3 MAXITE

MAXITE controls the maximum number of iterations of the Permutation Search algorithm. It influences both the amount of computer time and the quality of the final results. The larger MAXITE, the longer the run time. For smaller arrays, 3 by 3 and 6 by 6, 20 iterations was the maximum used in this study. Ten iterations was the maximum used for larger arrays, 7 by 7 up to 9 by 9. Fewer iterations are used for larger arrays because of the longer run time associated with them. MAXITE is the other parameter which can stop program execution. The quality of result is affected by MAXITE through its association with the effects of NCOUNT.

4.4.4 NCOUNT

NCOUNT controls the maximum number of new starts that "PERMSER" will use during its search. It may seem that setting NCOUNT very high ensures that many neighborhoods will be searched, thereby improving the

results. In practice, this is not the case. For NCOUNT values greater than 5, the amount of improvement for each new start decreases with each successive new start. Thus enters the problem of diminishing returns!

This may seem to imply that the current solution is near the global optimum. Unfortunately, this is not usually so. If two arrays are designed from the same selection pool of simulated elements, the results can vary greatly. The elements selected and their array locations will be entirely different. If both attempts are random starts, and the search covers the same number of neighborhoods, the resulting arrays can have good response patterns, or bad response patterns. Therefore, diminishing returns do not necessarily herald proximity to a global optimum. An accurate interpretation is that the quality of the locally optimum solution depends on the amount of area in the solution space that has been searched.

The wide variation in the quality of responses obtained does not discourage the use of Permutation Search. The ratio of good responses to bad responses for a given selection pool is so great that with only a few runs of "PERMSER", an acceptable result will be found.

4.4.5 OTHER INFLUENCES

As mentioned earlier, there are parameters which result from the problem type which can influence "PERMSER". The number of elements in the selection pool directly influences the run time, and to a lesser extent, the quality of results. An increase in the number of elements will increase the run time and perhaps improve the results due to a

greater number of permutations available. The size of the array influences the run time in a manner similar to the size of the selection pool.

4.4.6 THE STARTING POINT

Several tests were done to determine the effects of different starting points. The different types of starting points used included random arrangements, sequential arrangements, and group arrangements of elements in the permutation array. Sequential arrangements are those in which elements adjacent to each other in the transducer array are adjacent to each other in the permutation array. Group arrangements are those in which elements that occupy symmetric array positions are adjacent in the permutation array. Many variations on these different arrangements were also tried. Results indicate that the best array response patterns are obtained from randomly arranged initial permutations. Those arrays which result from group arrangements are not much poorer than those from the random arrangements. They differ most significantly in run time and efficiency. Efficiency is defined as the average amount of improvement per base permutation. This does not mean that the quality of the initial base permutation has no effect on the outcome. It means the arrangement of elements in the permutation array does not affect the objective function of the initial base permutation.

The most significant and obvious result is that the best results are obtained from those initial permutations with the best initial response patterns. Therefore, if permutations with good response patterns can be easily found, better results can be obtained.

4.4.7 FLOW CHART OF "PERMSER"

The following is a discussion of the operation of "PERMSER". A flow chart for "PERMSER" is shown in Figure 4.1. The element parameters and Permutation Search control parameters are read in subroutine DATAINN. Control passes from DATAINN to NEARIN, where the element parameters are written into an output file. This file will be used by "NEARPRES", (see Appendix F), to compute and print the response pattern for the initial permutation. The value of the objective function is determined in ZVALUE and is used as the initial reference point. The interchanging of elements occurs in PERMUTE. Several evaluations are also made in PERMUTE, but for simplicity, are depicted as occuring outside of PERMUTE. EVAL determines whether the trial permutation improves the objective function. To do this, it calls ZVALUE. ZVALUE calls YVALUE and INITIALIZE to determine the response of each trial permutation at the constrained angles. INITIALIZE determines the response at $\theta = 0°$, and $\phi = 0°$, i.e., the main beam response. YVALUE determines the response at every constrained angle and subracts it from the main beam response to determine how many dB down the response is at each constraint.

At this point, there are two possibilities. Either, the trial permutation becomes a new base permutation, or the trial permutation is unsuccessful. PERMUTE will continue exchanges of elements either on the new base permutation, or on the previous base permutation. When a complete cycle of exchanges does not present an improvement, NEWSTART is called. If the maximum number of new starts has not been exceeded, a new start is initiated. If the maximum number has been exceeded, the

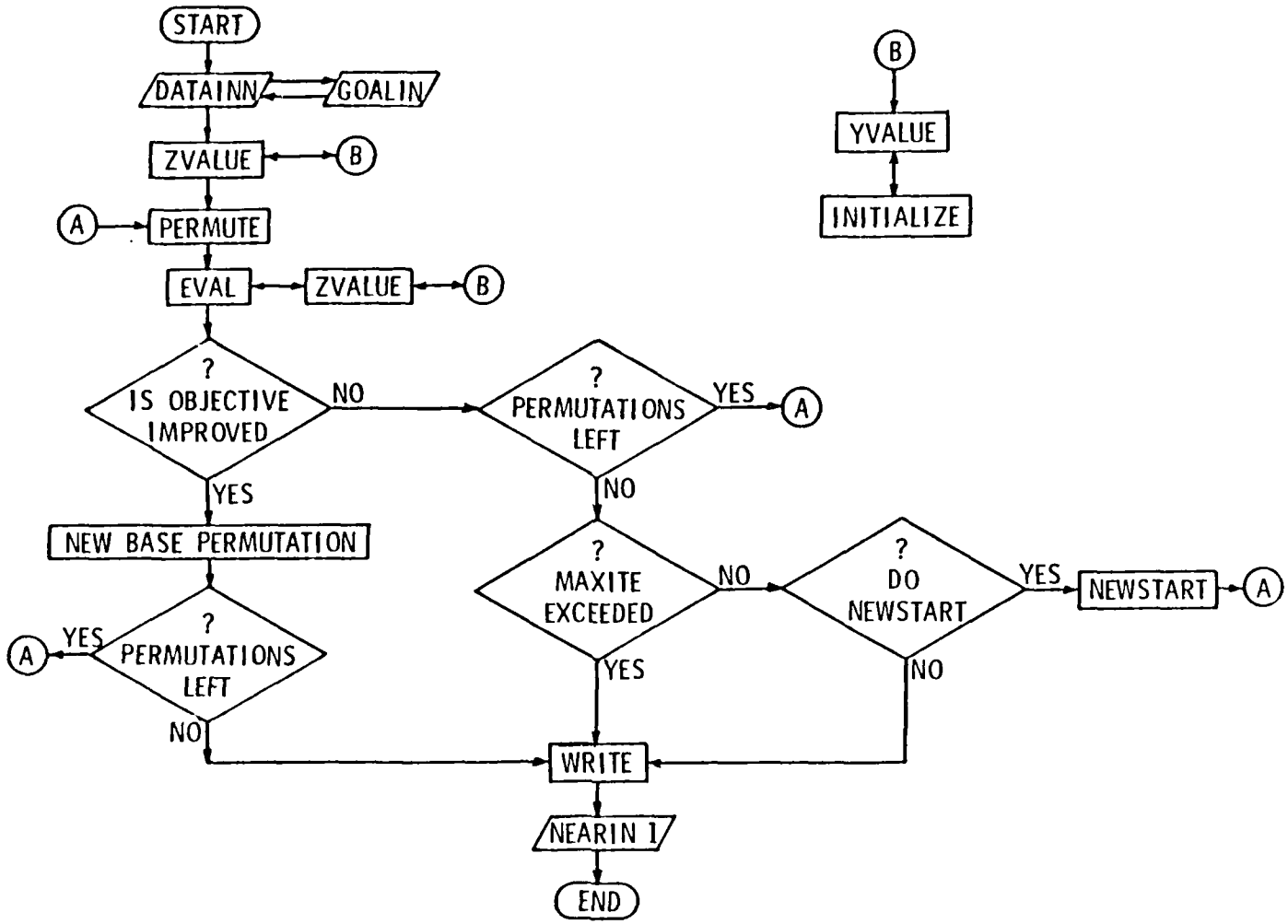


Figure 4.1 Flowchart for "PERMSER"

search process stops. Additionally, after a complete cycle of unsuccessful exchanges has been made, PERMUTE determines if MAXITE has been exceeded. If it has, the search process is terminated.

Once the search process is terminated, the results are printed out. The parameters of the completed array are written into a file to be used by "NEARPRES" to print a response pattern.

CHAPTER FIVE

THE NEO-KENDIG SELECTION SCHEME

5.1 INTRODUCTION

As mentioned in Chapter Four, if an initial base permutation yields a good response pattern, Permutation Search will find a permutation with a better response pattern. The problem, then, lies in finding a good starting point.

Since the Kendig Scatter Diagram Method has been used successfully to design arrays with reduced response pattern errors, it is a logical good starting point. This chapter presents a new selection scheme based on the Kendig method. Since the derivation in this chapter follows Kendig's derivation, and presents the same idea of grouping elements to cancel their errors, this new method is called the Neo-Kendig Selection Scheme. However, this new scheme avoids the errors inherent in the Kendig method by using a three-dimensional model of array performance instead of the two-dimensional model used previously.

In his report [1], Kendig presents a technique to minimize the impact of the individual transducer element errors on the entire transducer array response. In essence, the Kendig Scatter Diagram Method involves the pairing of elements which will occupy symmetric positions along a line through the center of the array in such a way as to partially cancel their respective amplitude and phase errors. This technique was implemented in a BASIC routine by Schafer [2]. It represented an improvement over the more tedious hand selection version previously used.

The Neo-Kendig method represents a substantial improvement over the existing versions of the Kendig Scatter Diagram Method. It involves the simultaneous selection of four elements to occupy symmetric array positions, instead of pairs of elements as previously done.

5.2 DERIVATION

In this derivation, the transducers are treated as point elements located at the center of the transducer faces. The contribution of an element, m, located at a position (X_m, Y_m) in a planar array, see Figure 5.1, to the array response is

$$Q_m = A_m \exp\{i(k \cdot U_m + a_m)\} , \tag{5-1}$$

where A_m is the product of the shading for position m and the amplitude response of the transducer element in that position, and a_m is the sum of the phase shading for position m and the element's phase response. U_m is a function of the x and y coordinates for position m and the angles of incidence θ and ϕ, viz.,

$$U_m = X_m \cdot \cos\phi \sin\theta + Y_m \cdot \sin\phi \sin\theta$$

or

$$U_m = X_m \cdot ucos + Y_m \cdot usin \quad \text{where} \quad \begin{aligned} ucos &= \cos\phi \sin\theta \\ usin &= \sin\phi \sin\theta . \end{aligned} \tag{5-2}$$

Expanding the exponential into sin and cos form yields

$$\begin{aligned} \exp\{i(k \cdot U_m + a_m)\} &= \cos(k \cdot U_m)\cos(a_m) - \sin(k \cdot U_m)\sin(a_m) \\ &\quad + i[\sin(k \cdot U_m)\cos(a_m) + \cos(k \cdot U_m)\sin(a_m)]. \end{aligned} \tag{5-3}$$

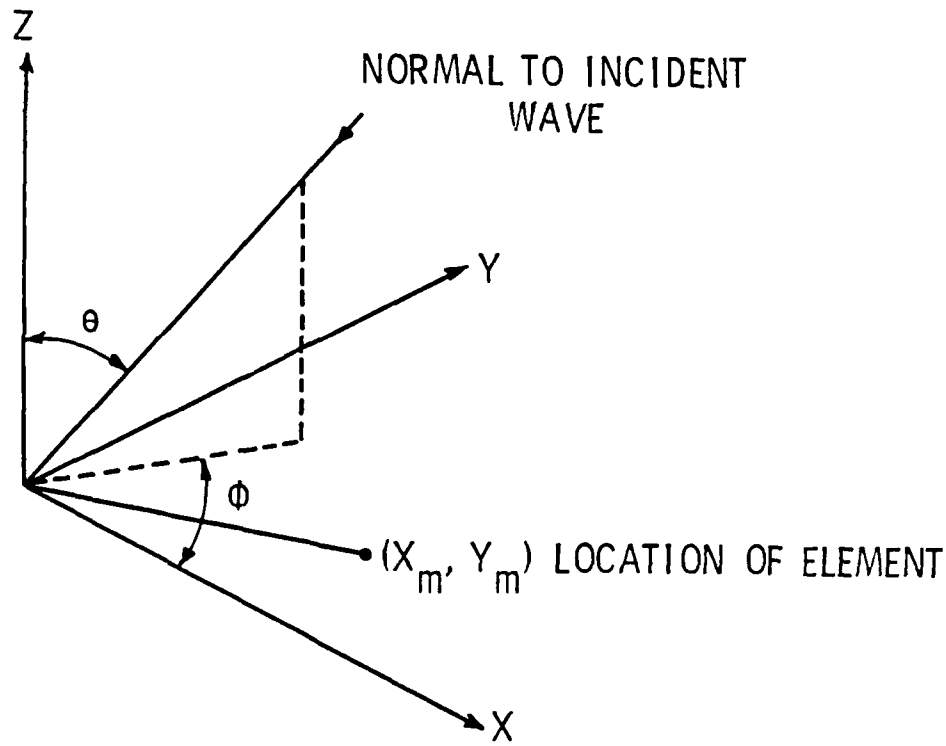


Figure 5.1 Location of a Position in an Array

Using Equation 5-2, the sin and cos terms of Equation 5-3 can be further expanded into

$$\begin{aligned}\cos(k\cdot U_m) &= \cos(k\cdot X_m\cdot u\cos + k\cdot Y_m\cdot u\sin)\\ &= \cos(k\cdot X_m\cdot u\cos)\cos(k\cdot Y_m\cdot u\sin)\\ &\quad - \sin(k\cdot X_m\cdot u\cos)\sin(k\cdot Y_m\cdot u\sin)\end{aligned} \tag{5-4}$$

and

$$\begin{aligned}\sin(k\cdot U_m) &= \sin(k\cdot X_m\cdot u\cos + k\cdot Y_m\cdot u\sin)\\ &= \sin(k\cdot X_m\cdot u\cos)\cos(k\cdot Y_m\cdot u\sin)\\ &\quad + \cos(k\cdot X_m\cdot u\cos)\sin(k\cdot Y_m\cdot u\sin).\end{aligned} \tag{5-5}$$

Consider four positions located symmetrically about the center of an even array, (Figure 5.2). The contribution of these four elements is

$$\begin{aligned}Q_4 = A_1\cdot\exp\{i(k\cdot U_1 + a_1)\} + A_2\cdot\exp\{i(k\cdot U_2 + a_2)\}\\ + A_3\cdot\exp\{i(k\cdot U_3 + a_3)\} + A_4\cdot\exp\{i(k\cdot U_4 + a_4)\},\end{aligned} \tag{5-6}$$

where

$$\begin{aligned}U_1 &= X_1\cdot u\cos + Y_1 u\sin \qquad & U_3 &= -X_1\cdot u\cos - Y_1 u\sin\\ U_2 &= -X_1\cdot u\cos + Y_1 u\sin \qquad & U_4 &= X_1\cdot u\cos - Y_1 u\sin\end{aligned} \tag{5-7}$$

Note that $U_1 = -U_3$ and $U_2 = -U_4$. It is assumed there are no positional errors when the elements are placed in the array.

Breaking the response up into real and imaginary components, the real part can now be expressed as

$$\begin{aligned}\mathrm{Re}\{Q_4\} = A_1[\cos(k\cdot U_1)\cos a_1 - \sin(k\cdot U_1)\sin a_1] +\\ A_2[\cos(k\cdot U_2)\cos a_2 - \sin(k\cdot U_2)\sin a_2] +\\ A_3[\cos(k\cdot U_3)\cos a_3 - \sin(k\cdot U_3)\sin a_3] +\\ A_4[\cos(k\cdot U_4)\cos a_4 - \sin(k\cdot U_4)\sin a_4].\end{aligned} \tag{5-8}$$

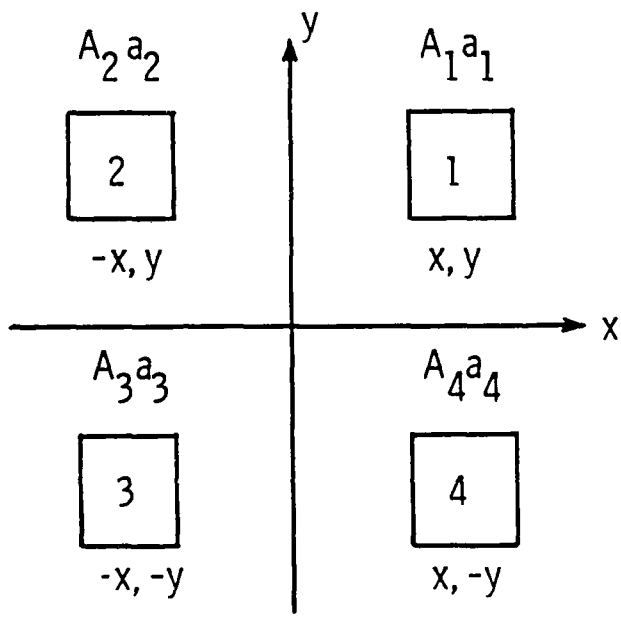


Figure 5.2 Four Symmetric Positions in an Even Array

For small phase, i.e., a $\ll 1$, (in radians), the following approximations can be made,

$$\cos a_i = 1 \text{ and } \sin a_i = a_i \ . \tag{5-9}$$

Substituting Equations 5-7 and 5-9 into Equation 5-8, the real and imaginary parts of the contribution can be written as shown in Equations 5-10 and 5-11, (Figure 5.3). The terms in square brackets will be referred to as the Kendig Terms and distinguished by the number appearing above each term. The sin and cos products will be referred to as the Kendig Coefficients.

This preceding derivation is valid for all four-fold symmetric even arrays. The derivation for odd arrays does not differ substantially and produces an equivalent result. It is presented in Appendix B.

5.3 THE KENDIG TERMS

In Equations 5-10 and 5-11, terms 3, 4, 7, and 8 include both the phase and amplitude responses of the elements, while terms 1, 2, 5, and 6 represent contributions from amplitude only. If all the elements have ideal response characteristics, e.g., $A_i = 1.0$ and $a_i = 0.0$, only one term, term 1, would be non-zero. Thus, terms 2 through 8 represent the error introduced into the array response. Minimizing the total error entails minimizing all seven of these terms.

$$Q_4 = \text{Real} + j \cdot \text{Imag}$$

<u>Real Part</u>

$$\overset{1}{[A_1+A_2+A_3+A_4]}\cos(kxu\cos)\cos(kyu\sin) + \overset{2}{[A_2+A_4-A_1-A_3]}\sin(kxu\cos)\sin(kyu\sin)$$

$$+ \overset{3}{[A_3a_3+A_4a_4-A_1a_1-A_2a_2]}\cos(kxu\cos)\sin(kyu\sin) + \overset{4}{[A_3a_3+A_2a_2-A_1a_1-A_4a_4]}\sin(kxu\cos)\cos(kyu\sin) \tag{5-10}$$

<u>Imaginary Part</u>

$$\overset{5}{[A_1+A_2-A_3-A_4]}\cos(kxu\cos)\sin(kyu\sin) + \overset{6}{[A_1+A_4-A_3-A_2]}\sin(kxu\cos)\cos(kyu\sin)$$

$$+ \overset{7}{[A_1a_1+A_2a_2+A_3a_3+A_4a_4]}\cos(kxu\cos)\cos(kyu\sin) + \overset{8}{[A_1a_1+A_3a_3-A_4a_4-A_2a_2]}\sin(kxu\cos)(kyu\sin) \tag{5-11}$$

Figure 5.3 The Neo-Kendig Equation

A look at terms 3 and 4 reveals a dilemma. Minimizing term 3 means

$$A_3a_3 + A_4a_4 = A_1a_1 + A_4a_4$$

or

$$A_3a_3 - A_1a_1 = A_2a_2 - A_4a_4 \quad , \tag{5-12}$$

while minimizing term 4 means

$$A_3a_3 + A_2a_2 = A_1a_1 + A_4a_4$$

or

$$A_3a_3 - A_1a_1 = A_4a_4 - A_2a_2 \ . \tag{5-13}$$

Excluding the uncommon case of $A_2a_2 = A_4a_4$, minimizing term 3 will necessarily prevent an equal reduction of the value of term 4. This relationship between terms 3 and 4 exists between all combinations of terms 3, 4 and 8, as well as combinations of terms 2, 5, and 6, (see Table 5.1).

Consider the Kendig Coefficients. For any array, x and y are known for every position. Since ucos and usin are functions of the incident angles, the Kendig Coefficients can be determined for specific incident angles. If elements are then chosen in such a way as to minimize the sum at each symmetric position of the product of the terms and coefficients, the error at this angle is reduced. However, at a different incident angle which is not nearby the first, this minimization no longer necessarily holds because the incident angles result in different Kendig Coefficient values. For a more detailed discussion of the implications of this, see Appendix C.

Kendig Terms

Table 5.1

Term 2	$A_1 + A_3 = A_2 + A_4$
Term 5	$A_1 - A_3 = A_4 - A_2$
Term 6	$A_1 - A_3 = A_2 - A_4$
Term 3	$A_3a_3 - A_1a_1 = A_2a_2 - A_4a_4$
Term 4	$A_3a_3 - A_1a_1 = A_4a_4 - A_2a_2$
Term 8	$A_1a_1 + A_3a_3 = A_2a_2 + A_4a_4$

The solution to reducing the error impact lies in reducing each term independently of the others, (in this case, independently means without regard to the interaction of the Kendig Coefficients). If each term is as small as possible, the product with the coefficients will also be small. Thus, the error impact will be reduced over all angles.

5.4 THE SELECTION ROUTINE

As just discussed in the previous section, reducing the total error introduced into the array response involves minimizing the Kendig Terms individually. In order to determine which terms to minimize and in what order, the Kendig terms of Equations 5-10 and 5-11 were programmed into a BASIC routine on an HP9825B calculator. This routine requires, as an input, the radiation loading for each symmetric array position. Using these and the equivalent circuit parameters of each element, it computes the amplitude and phase response for every element in each symmetric position. Using one of the Kendig terms as a criterion, the routine searches through the set of responses for a chosen position, and finds the group of four that gives the lowest value for the specified term. Since terms 3, 4, 7, and 8 represent the error due to both the phase and amplitude responses of an element, combinations of these terms were the Kendig terms used. Although terms 2, 5, and 6 appear to depend on amplitude only, the phase responses are still represented in these terms through the approximation $\cos a_i = 1$. These terms are not used because the phase influence is much less.

5.4.1 TEST RESULTS

Many different tests of the selection process were done. Using the results of section 2.1.2, several selection pools were generated on the computer. Different combinations of terms 3, 4, 7, and 8 were used to test the selection process. These selection pools were used in various ways to design many arrays. Combinations of one, two , three or four term(s) were used for the selection criteria.

It has been found that use of any two terms for the selection criteria gives the best results. It is not surprising that minimizing two terms would give better results than minimizing only one. However, it is surprising that minimizing three or four terms does not present additional improvement. There are no apparent causes for this phenomenon.

For a collection of selection pools, there is no particular combination of terms 3, 4, and 8 that will give the best response for every selection pool. Any combination of terms will choose the same four elements for each symmetric position, but will arrange them differently within the array. These different arrangements, or permutations, can sometimes result in vastly different responses, and other times give comparatively equivalent responses. In addition, if two arrays are created from a large selection pool, the combination of terms which results in a good first array does not necessarily produce a good second array.

Term 7 has been excluded from consideration because it does not have the same form as terms 3, 4, and 8; it offers no information on the arrangement of elements in the groups selected. Additionally, for small amplitude and phase variances, the range of values for term 7 is

not as great as for the other terms. Greater reduction of error impact can be achieved by striving for the minimum value of the other terms than for the minimum value of term 7.

5.5 THE NEO-KENDIG COMPUTER ROUTINE, "HKV"

The following is a brief description of the actual selection process. For an even array, the radiation loading for the symmetric position closest to the array center is used to determine the amplitude and phase responses of all the elements in the selection pool. For these, the means and variances are found. Elements that are farthest from a reference point are temporarily removed and new means and variances are found. Again, the elements farthest from the means are removed. This process is repeated until there are between 10 and 14 elements remaining in the pool.

Several different methods of searching the selection pool for the 10 to 14 initial elements were also tried. These included using the initial means of the entire selection pool as the reference point for the entire array, or using no reference point. It was determined that use of the first four chosen elements as a reference point for all subsequent positions produces the best results. The reference point to choose the first four is the moving mean just described.

The value of the first Kendig term is then determined for every possible permutation of four elements, a total of 24024 trials for 14 elements. The lowest 40 of these are kept. The value of the second term is then ascertained for these 40 best permutations. The permutation that produces the minimum response for the second term is the

permutation for the first position. Next, the chosen four elements are permanently removed from the selection pool and those temporarily removed are restored.

The means and variances of these four elements are determined and used as a reference point for all subsequent selections. The selection process for every successive position follows the same procedure except that the elements removed are those farthest from the means of four elements first chosen. Consequently, it is not necessary to recalculate the means and variances of the elements in the selection pool, once the first symmetric position is filled.

The routine for an odd array is different in one respect. Since the array has one unsymmetric position at the center of the array, the element nearest to means of the group of 10 to 14 elements is chosen and placed in the center position. The amplitude and phase response of this element is used as the reference point for all subsequent selections.

5.5.1 FLOW DIAGRAM FOR "HKV"

The Neo-Kendig method has been implemented into a FORTRAN computer routine called "HKV", (a listing is in Appendix E). This routine allows any combination of Kendig Terms 3, 4, and 8 to be used as the selection criteria. A flow diagram is shown in Figure 5.4.

The element circuit parameters, and selection control parameters are read in by the routine DATAININ. The element responses for the first symmetric array position, (always closest to the center), are computed in RESPONSE. The mean amplitude and phase responses for the

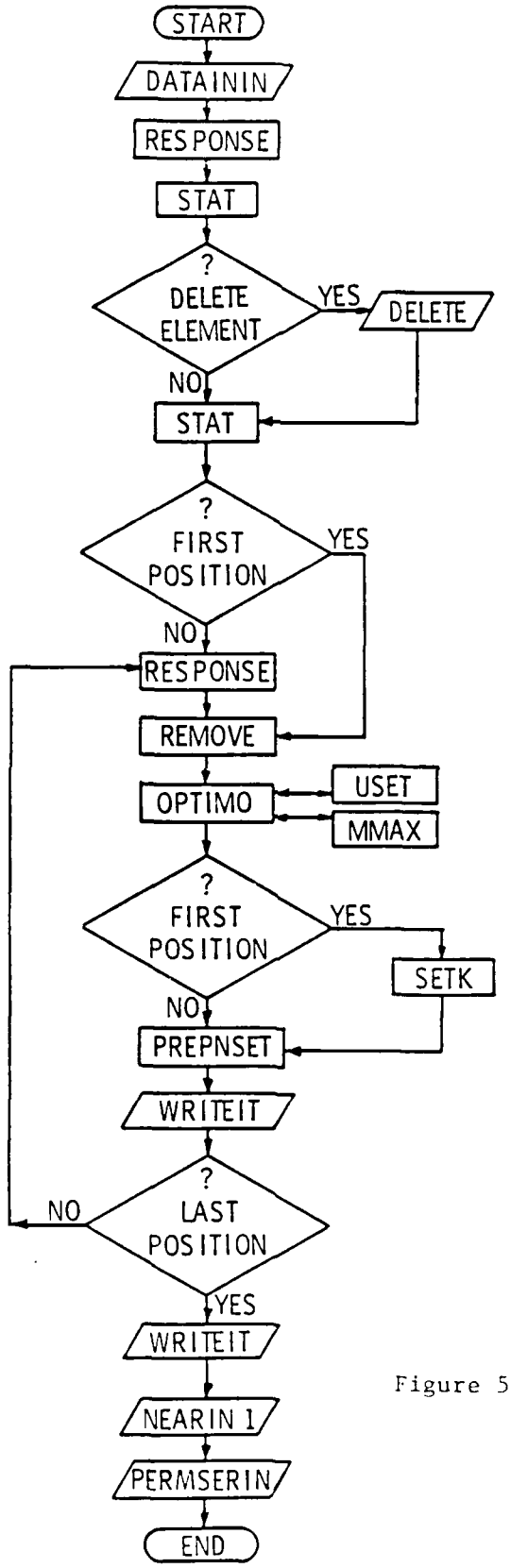


Figure 5.4 Flowchart for "HKV"

selection pool are computed and stored for reference in STAT. Those elements which are farther than a specified tolerance from the selection pool means are permanently removed from the selection pool in DELETE.

Routines REMOVE and OPTIMO temporarily remove unsuitable elements from the selection pool and search for those permutations which give minimal values for the Kendig Terms used as the search criteria. OPTIMO calls USET to determine the values of the Kendig Terms being analyzed. Once such a permutation is found, the elements are assigned to their respective array positions, and permanently removed from the selection pool. If the array position just filled is the centermost position, the chosen elements' means are computed and stored for reference in SETK. In PREPNSET, those elements temporarily removed are now returned to the selection pool to be considered for the next array position. The process loops back to RESPONSE to calculate the element responses for the next array position, and the selection process repeats. Upon completion of the selection process, WRITEIT prints a map of the completed array, showing where each element is situated. Routines NEARIN1 and PERMSERIN are used to write the parameters of the completed array into files for use by "NEARPRES" and "PERMSER".

CHAPTER SIX

THE SELECTION PROCESS

6.1 THE COMBINATION

As mentioned in Chapter Four, Permutation Search will look for better permutations in the neighborhood of an initial permutation. However, without a good starting point, the results are not always acceptable. With the Neo-Kendig method, good results can be found, but not guaranteed. Several different combinations of two terms must be tried in order to find an acceptable result.

Combining the Permutation Search and Neo-Kendig methods results in a cancellation of their respective disadvantages. If the solutions obtained from the Neo-Kendig method are used as initial permutations for "PERMSER", the results are superior to those obtained by either method alone. Any pairing of terms 3, 4, and 8 produces results which are acceptable initial permutations for "PERMSER". As discussed in Chapter Four, a random arrangement of elements in the permutation array gives better results than placing elements occupying symmetric positions adjacent to each other. The results of the methods described in Chapters Four and Five are presented in this chapter to facilitate comparison. Figure 6.1 shows one half of the response pattern for a 6 by 6 array of ideal elements and a desired side lobe level of -40 dB. The shading values are from the Dolph-Chebyshev method for line arrays [12] and the second product theorem [13]. This map represents the response of the

array to a signal at any incident angle, θ, between 0° and -90° and roll plane, ϕ , between 0° and 180°. The reference level is the response of the array to the same signal at θ = 0° and ϕ = 0° incidence. The dB levels represesent negative decibels. A colon, (:), following a number indicates a phase greater than 90° different from the response at the peak of the main beam, referred to as a phase change. Figure 6.2 shows the same portion of the response pattern for a random arrangement of non-ideal elements. The selection pool was the set of 66 transducers mentioned in Chapter Two. The regions of the response pattern that meet the 40 dB level requirement are outlined. Note how the locations of the phase changes have been altered significantly. Presented in Figure 6.3 is the result obtained after 17 iterations of the Permutation Search algorithm for θ constrained every 5° and ϕ constrained every 20° in the side-lobe region. This result was obtained from a random initial permutation. There are only a few locations which do not meet the desired 40 dB level. In addition to the amount of area that meets the 40 dB requirement, a good indication of the quality of the result is that the locations of the phase changes approximate those of the ideal case.

The same selection pool that produced the response just mentioned was also used by the Neo-Kendig Selection Scheme. Terms 3 and 4 were the selection criteria. The results are shown in Figure 6.4. Again, the locations of the phase changes approximate those of the ideal array. In this case, the worst response is 38 dB, only a 2 dB error.

Finally, Figure 6.5 shows the response pattern obtained by using the results of the Neo-Kendig method as the initial starting point for "PERMSER". The only deviations from the desired 40 dB level are 1 dB in magnitude and not concentrated in one area. This result was obtained after 12 iterations, during which two new starts were done with a run time of 31 cpu minutes on a VAX 11/782.

Similar results for an 8 by 8 array with three elements removed from each corner are shown in Figures 6.6 - 6.9. The selection pool for this array contained 140 elements and was generated as described in Chapter Two. The shading values for the desired side lobe levels of -40 dB were obtained by Wilson [10]. Figure 6.6 is the response for ideal elements. Figure 6.7 is for a random arrangement of elements. Figure 6.8 is the response pattern obtained by the Neo-Kendig method. Figure 6.9 is the final result, obtained after 11 iterations of the Permutation Search algorithm, three new starts, and 80 cpu minutes.

The maximum tolerance on the elements was 9% in amplitude, and 11° in phase. However, the tolerances on elements in the selection pool do not indicate which elements will be selected. The Neo-Kendig method will use only those elements which are close to the reference means. Therefore, those elements farthest from the means will not be selected. But, "PERMSER" disregards an element's distance from a reference mean. Any element may enter the array if its presence produces better results. It is possible, and has happened, that elements with large deviations will be placed in an array alongside elements with small deviations.

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	99:67	56 52	52 55	63 82	99 83	99 82	63 55	52 52	56 67	99:67	56 52	52 55	63 82	99 83	99 82	63 55	52 52	56 67	99:
-88.00	82 66	56 52	52 55	63 82	99 83	99 82	63 55	52 52	56 66	82 66	56 52	52 55	63 82	99 83	99 82	63 55	52 52	56 66	82
-86.00	70 63	55 52	52 55	63 82	99 83	99 82	63 55	52 52	55 63	70 63	55 52	52 55	63 82	99 83	99 82	63 55	52 52	55 63	70
-84.00	63 59	54 51	51 55	63 82	99 82	99 82	63 55	51 51	54 59	63 59	54 51	51 55	63 82	99 82	99 82	63 55	51 51	54 59	63
-82.00	58 56	52 50	51 55	63 81	97 82	97 81	63 55	51 50	52 56	58 56	52 50	51 55	63 81	97 82	97 81	63 55	51 50	52 56	58
-80.00	54 53	51 49	50 54	63 81	95 81	95 81	63 54	50 49	51 53	54 53	51 49	50 54	63 81	95 81	95 81	63 54	50 49	51 53	54
-78.00	51 50	49 48	50 54	62 81	94 81	93 81	62 54	50 48	49 50	51 50	49 48	50 54	62 81	93 81	93 81	62 54	50 48	49 50	51
-76.00	49 46	47 47	49 54	62 81	92 80	92 81	62 54	49 47	47 48	49 48	47 47	49 54	62 81	92 80	92 81	62 54	49 47	47 48	49
-74.00	47 46	46 47	49 53	62 82	92 80	92 82	62 53	49 47	46 46	47 46	46 47	49 53	62 82	92 80	92 82	62 53	49 47	46 46	47
-72.00	45 45	45 46	48 53	63 85	94 80	94 85	63 53	48 46	45 45	45 45	45 46	48 53	63 85	94 80	94 85	63 53	48 46	45 45	45
-70.00	43 43	44 45	48 53	63 92	99 80	99 92	63 53	48 45	44 43	43 43	44 45	48 53	63 92	99 80	99 92	63 53	48 45	44 43	43
-68.00	42 42	43 44	48 53	64 91	:99:80	99:91	:64 53	48 44	43 42	42 42	43 44	48 53	64 91	:99:80	99:91	:64 53	48 44	43 42	42
-66.00	41 41	42 44	48 54	66 79	:88:81	88:79	:66 54	48 44	42 41	41 41	42 44	48 54	66 79	:88:81	88:79	:66 54	48 44	42 41	41
-64.00	40 41	42 44	48 55	71 73	:82:84	82:73	:71 55	48 44	42 41	40 41	42 44	48 55	71 73	:82:84	82:73	:71 55	48 44	42 41	40
-62.00	40 40	42 44	49 57	95 69	:77:88	77:69	:95 57	49 44	42 40	40 40	42 44	49 57	95 69	:77:88	77:69	:95 57	49 44	42 40	40
-60.00	40 41	42 45	50 61	69	:65:73:99	73:65	:69:61	50 45	42 41	40 41	42 45	50 61	69	:65:73:99	73:65	:69:61	50 45	42 41	40
-58.00	41 41	43 46	53 72	62	:62:71:99	:71:62	:62:72	53 46	43 41	41 41	43 46	53 72	62	:62:71:99	:71:62	:62:72	53 46	43 41	41
-56.00	42 43	45 49	59 64	:58:60	:70:95	70:60	:58:64	:59 49	45 43	42 43	45 49	59 64	:58:60	:70:95	70:60	:58:64	:59 49	45 43	42
-54.00	44 45	48 55	73	:56:55	:59:71	:78 71	:59:55	:56:73	:55 48	45 44	45 48	55 73	:56:55	:59:71	:78 71	:59:55	:56:73	:55 48	45 44
-52.00	49 50	56 74	:55:52	:53:58	:80:68	80:58	:53:52	:55:74	:56 50	49 50	56 74	:55:52	:53:58	:80:68	80:58	:53:52	:55:74	:56 50	49
-50.00	67 96	59:52	:49:49	:51:59	:69 60	69 59	:51:49	:49:52	:59:96	67 96	59:52	:49:49	:51:59	:69 60	69 59	:51:49	:49:52	:59:96	67
-48.00	51:50	:48:47	:46:47	:52:66	:58 54	58 66	:52:47	:46:47	:48:50	:51:50	:48:47	:46:47	:52:66	:58 54	58 66	:52:47	:46:47	:48:50	:51:
-46.00	41:44	:44:44	:44:47	:54:62	51 49	51 62	54:47	:44:44	:44:44	:44:44	:44:44	:44:47	:54:62	51 49	51 62	54:47	:44:44	:44:44	:44:
-44.00	41:41	:41:42	:44:49	:72:50	45 44	45 50	72:49	:44:42	:41:41	:41:41	:41:41	:42:44	:49:72	:50 45	44 45	50 72	:49:44	:42:41	:41:41:
-42.00	40:40	:41:43	:47:59	:50 43	40 39	40 43	50 59	:47:43	:41:40	:40:40	:41:43	:47:59	:50 43	40 39	40 43	50 59	:47:43	:41:40	:40:
-40.00	41:41	:43:47	:60:49	41 38	36 35	36 38	41 49	60:47	:43:41	:41:41	:43:47	:60:49	41 38	36 35	36 38	41 49	60:47	:43:41	:41:
-38.00	47:46	:54:57	45 39	35 33	32 32	33 35	39 45	57 54	:48:47	:46:54	:57 45	39 35	33 32	32 32	33 35	39 45	57 54	:48:47	:
-36.00	47 46	42 39	35 33	31 29	28 28	28 29	31 33	35 39	42 46	47 46	42 39	35 33	31 29	28 28	28 29	31 33	35 39	42 46	47
-34.00	33 34	33 31	30 28	27 26	25 25	25 26	27 28	30 31	33 34	34 34	33 31	30 28	27 26	25 25	25 26	27 28	30 31	33 34	34
-32.00	28 28	27 26	25 24	23 22	22 22	22 23	24 25	26 27	28 28	28 28	27 26	25 24	23 22	22 22	22 22	23 24	25 26	27 28	28
-30.00	23 23	23 22	21 21	20 20	19 19	19 20	20 21	21 22	23 23	23 23	23 22	21 21	20 20	19 19	19 20	20 21	21 22	23 23	23
-28.00	19 19	19 19	18 18	17 17	17 17	17 17	18 18	19 19	19 19	19 19	19 18	18 17	17 17	17 17	17 17	18 18	19 19	19 19	19
-26.00	16 16	16 16	15 15	15 14	14 14	14 15	15 15	16 16	16 16	16 16	16 15	15 15	14 14	14 14	14 15	15 15	16 16	16 16	16
-24.00	13 13	13 13	13 13	12 12	12 12	12 12	13 13	13 13	13 13	13 13	13 13	13 13	12 12	12 12	12 12	12 12	13 13	13 13	13 13
-22.00	11 11	11 11	11 10	10 10	10 10	10 10	11 11	11 11	11 11	11 11	11 11	10 10	10 10	10 10	10 10	10 10	11 11	11 11	11
-20.00	9 9	9 9	9 9	9 8	8 8	8 8	9 9	9 9	9 9	9 9	9 9	9 9	8 8	8 8	8 9	9 9	9 9	9 9	9
-18.00	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7
-16.00	6 6	6 5	5 5	5 5	5 5	5 5	5 5	5 5	6 6	6 6	6 5	5 5	5 5	5 5	5 5	5 5	5 5	6 6	6
-14.00	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4
-12.00	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3
-10.00	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2
-8.00	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1
-6.00	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1
-4.00	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0
-2.00	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0
0.00	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0

Figure 6.1 Response Pattern, 6 by 6 Array, Ideal Elements

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	44:42:42:42	40	38 37	36 35	36 37	38 40	39 36	34 32	30 30	31 34	36	40	38 39:43:50:50:44:40:	38:37:39:	40 41	41 44			
-88.00	44:42:42:42	40	38 37	36 35	36 37	38 40	39 36	34 32	30 30	31 34	38	40	38 39:43:50:50:44:40:	38:37:39:	40 41	41 44			
-86.00	45:42:42:42	40	38 37	36 35	36 37	38 39	39 36	34 32	30 30	31 34	38	40	39 39:43:50:50:44:40:	38:37:39:	40 40	41 44			
-84.00	45:43:43:42	40	38 37	36 35	36 37	38 39	38 36	33 31	30 30	31 34	36	40	39 39:43:50:50:45:40:	38:37:39:	40 40	41 43			
-82.00	46:43:43:42	40	39 37	36 35	36 37	38 39	38 36	33 31	30 30	31 34	38	40	39 40:43:50:50:45:40:	38:37:39:	40 40	41 43			
-80.00	47:43:43:42	41	39 37	36 36	36 37	38 39	38 36	33 31	30 30	31 34	38	40	39 40:43:50:51:45:40:	38:37:38	40 40	40 42			
-78.00	48:44:43:43	41	39 38	36 36	36 37	38 39	38 35	33 31	30 30	31 34	36	40	39 40 44:50:51:45:41:	38:38:38	39 39	40 41			
-76.00	49:45:44:43	41	39 38	37 36	36 37	38 36	37 35	33 31	30 30	31 33	37	40	40 40 44:50:52:46:41:	38:38:38	39 39	39 40			
-74.00	50:45:44:43	42	40 38	37 36	36 37	38 38	37 35	32 30	29 29	31 33	37	40	40 41 44:50:52:46:41:	38:38:38	39 38	38 39			
-72.00	50:46:44:43	42	41 39	37 36	36 37	38 38	36 34	32 30	29 29	30 33	37	40	41 41 44:50:52:46:41:	39:38:38	38 38	38 39			
-70.00	49 46:44:44	43	41 40	38 37	36 37	37 37	36 34	32 30	29 29	30 33	36	40	41 42 45:50:52:47:42:	39:38:38	38 38	37 38			
-68.00	48 46:45:44	44	42 40	39 37	37 37	37 37	36 34	31 30	29 29	30 32	36	41	42 42 45 50 52 47:42:	39:38 38	38 37	37 37			
-66.00	47 46:45:44	45	44 42	39 38	37 37	37 37	35 33	31 30	29 29	30 32	36	41	43 43 46 50 52 47 42:	39:38 38	38 37	36 36			
-64.00	46 46:45:45	45	45 43	40 38	37 37	37 36	35 33	31 30	29 29	30 32	36	41	43 44 47 50 51 47 42	40 38	38 38	36 36			
-62.00	45 45:44:44	46	47 45	42 39	36 37	37 36	35 33	31 29	29 29	30 32	36	41	44 45 48 50 49 47 43	40 39	38 38	36 35			
-60.00	45 45:44:44	45	46 47	43 40	38 37	37 36	35 33	31 30	29 29	30 32	36	42	46 47 48 49 48 46 43	40 39	38 38	36 35			
-58.00	45 44:43:42	43	46 49	45 41	39 38	37 36	35 33	31 30	29 29	30 33	37	42	47 48 49 48 46 45 43	41 39	39 38	36 35			
-56.00	45:43:41:40	41	44 48	47 42	39 38	37 36	35 33	31 30	29 30	31 33	37	43	49 50 49 46 45 44 43	41 40	39 38	37 36			
-54.00	44:41:39:38	39	41 45	47 43	40 38	37 36	35 33	32 30	30 30	32 34	38	44	50 51 48 45 43 43 42	41 40	40 39	38 36			
-52.00	41:39:37:37	37	39 43	46 43	40 38	37 36	35 33	32 31	31 31	32 35	39	45	51 51 47 43 42 41 42	41 41	41 40	39 37			
-50.00	39:37:35:35	36	37 41	44 43	40 38	37 36	35 34	33 32	32 32	33 36	40	46	51 49 45 41 40 40 40	41 42	42 41	40 38			
-48.00	37:35:34:34	34	36 39	43 42	39 38	36 36	35 34	33 33	33 33	35 37	41	45	49 46 42 39 38 38 39	40 42	43 43	41 39			
-46.00	35:33:33:33	33	35 38	42 41	39 37	36 35	35 34	34 33	34 34	36 38	41	45	46 44 40 37 36 36 37	39 41	43 43	42 40			
-44.00	33:32:32:32	33	35 38	41 40	38 36	35 35	35 34	34 34	34 35	37 39	41	44	44 40 37 35 34 34 35	37 39	41 42	41 39			
-42.00	33:32:32:32	33	35 39	41 38	36 35	34 34	34 34	34 35	35 36	37 39	41	42	40 37 35 33 32 32 33	34 36	39 40	40 38			
-40.00	33:32:32:33	34	37 40	39 36	34 33	33 33	33 34	34 35	35 36	37 39	40	39	37 34 32 31 30 30 31	32 34	35 37	37 37			
-38.00	35:34:34:35	37	39 39	35 33	31 30	30 31	32 32	33 34	35 36	37 37	37	35	33 31 29 28 28 28 28	29 31	32 34	34 34			
-36.00	38:37:37:38	39	37 34	31 29	28 28	28 28	29 30	31 32	33 33	34 33	32	31	29 28 26 26 25 25 26	26 28	29 30	31 31			
-34.00	37 37 37 35	33	31 29	27 26	25 25	25 25	26 27	28 29	29 30	30 29	28	27	26 25 24 23 23 23 23	24 25	26 26	27 28			
-32.00	30 30 29 29	27	26 25	24 23	22 22	22 23	23 24	24 25	25 26	26 25	24	24	23 22 21 21 20 20 21	21 22	22 23	24 24			
-30.00	24 24 24 23	23	22 21	20 20	20 19	20 20	20 21	21 22	22 22	22 21	21	20	20 19 19 18 18 18 18	19 19	20 20	20 21			
-28.00	20 20 20 19	19	18 18	17 17	17 17	17 17	17 18	18 18	19 19	19 18	16	17	17 17 16 16 16 16 16	16 17	17 17	18 18			
-26.00	16 16 16 16	16	16 15	15 15	14 15	15 15	15 15	16 16	16 16	15 15	15	15	15 14 14 14 14 14 14	14 14	15 15	15 15			
-24.00	14 14 14 13	13	13 13	13 12	12 12	12 13	13 13	13 13	13 13	13 13	13	12	12 12 12 12 12 12 12	12 12	12 13	13 13			
-22.00	11 11 11 11	11	11 11	10 10	10 10	10 10	11 11	11 11	11 11	11 11	10	10	10 10 10 10 10 10 10	10 10	10 10	10 10			
-20.00	9 9 9 9	9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9	9	6 8 8 8 8 8 8	8 8	8 8	9 9			
-18.00	7 7 7 7	7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7 7	7	7	7 7 7 7 7 7 7	7 7	7 7	7 7			
-16.00	6 6 6 6	6	6 6	5 5	5 5	5 5	5 6	6 6	6 6	6 6	5	5	5 5 5 5 5 5 5	5 5	5 5	5 5			
-14.00	4 4 4 4	4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4 4	4	4	4 4 4 4 4 4 4	4 4	4 4	4 4			
-12.00	3 3 3 3	3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3	3	3 3 3 3 3 3 3	3 3	3 3	3 3			
-10.00	2 2 2 2	2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2	2	2 2 2 2 2 2 2	2 2	2 2	2 2			
-8.00	1 1 1 1	1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1	1	1 1 1 1 1 1 1	1 1	1 1	1 1			
-6.00	1 1 1 1	1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1	1	1 1 1 1 1 1 1	1 1	1 1	1 1			
-4.00	0 0 0 0	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0	0 0 0 0 0 0 0	0 0	0 0	0 0			
-2.00	0 0 0 0	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0	0 0 0 0 0 0 0	0 0	0 0	0 0			
0.00	0 0 0 0	0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0	0 0 0 0 0 0 0	0 0	0 0	0 0			

Figure 6.2 Response Pattern, 6 by 6 Array, Random Arrangement of Non-Ideal Elements

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	45:47:47:44	41	40:42:45:43	39:38:38	41:44:47	47	47	46	47	50	61	51:43	39:38:39	41:40	38	38	38	40	42	45	47	47	45
-88.00	45:47:47:44	41	40:42:45:43	39:38:38	41:44:47	47	47	46	47	50	60	51:43	39:38:39	41:40	38	38	38	40	42	45	47	47	45
-86.00	45:[illegible]46	44	41	40:42:45:43	39:38:38	41:44:47	47	46	46	47	49	59	51:43	39:38:39	41:40	38	38	38	40	42	45	47	47	45
-84.00	45:47:46	44	41	40:42:46:43	39:38:39	41:44:47	47	46	45	46	49	57	52:43	39:38:39	41:40	38	38	38	40	42	45	47	47	45
-82.00	45:47:46	43	[illegible]	40:42:46:44	39:38:39	41:44:47	47	45	45	45	46	55	52:43	39:38:39	40:40	38	38	38	40	42	45	46	46	45
-80.00	45 46	46	43	41	41:42:46:43	40:38:39	41:44:46	46	45	44	44	46	53	53:43	39:38:39	40:40	39:38	39	40	42	44	46	46	45
-78.00	45 46	45	43	41	[illegible]:47:44	40:38:39	41:44:46	46	44	43	43	45	51	53:43	39:38:38	40:40	39:38	39	40	42	44	45	45	44
-76.00	44 45	44	43	41	41:44:47:45	40:39:39	41:44:46	45	43	42	42	44	49	53:44	39:38:38	40:40	39:38	39	41	43	44	45	45	44
-74.00	43 44	44	42	41	41:[illegible]:47:45	41:39:39	41:44:46	45	43	42	42	43	48	52:44	39:38:38	40:40	39:38	39	41	43	44	44	44	43
-72.00	43 43	43	42	41	41:43:46:46	41:39:40	41:44:46	44	42	41	41	42	46	51 44	39:38:38	39:40	39:39	39	41	43	44	43	43	42
-70.00	42 42	42	41	41	41:43:46:47	42:40:40	42:44:46	44	41	40	40	41	45	50 44	39:38:38	39:40	39:39	40	42	43	43	43	42	42
-68.00	41 42	42	41	41	41:43:49:48	42:40:40	42:44:46	44	41	40	39	41	44	49 43	39:37:38	39:40	39:39	40	42	43	43	42	41	41
-66.00	40 41	41	41	41	41:43:49:48	43:41:41	42:44:46	43	41	39	39	40	43	48 43	39:37:38	39:40	39:39	41	43	44	43	42	41	40
-64.00	40 41	41	41	41	41:43:49:49	43:41:41	42:44:46	44	41	39	39	40	43	47 43	39:37:38	39:40	39:40	41	43	44	43	42	40	40
-62.00	40 40	41	41	41	41:44:49:50	44:42:41	42:44:46	44	41	39	38	40	43	47 42	39:37:37	39:40	40:40	42	44	45	44	42	40	40
-60.00	40 40	41	41	41	42:44:49:51	45:42:42	42:44:46	44	41	39	39	40	43	46 42	38:37:37	39:40	40:40	42	45	47	44	42	41	40
-58.00	40 41	41	42	41	42:44:49:52	46:43:42	42:44:46	45	42	40	39	41	44	46 42	38:37:38	39:40	40:40	42	46	48	46	43	41	41
-56.00	41 42	42	42	42	42:44:49:53	47:44:42	42:43:46	46	44	41	41	42	45	46 41	38:37:38	39:40	40:40	42	46	50	48	45	43	42
-54.00	43 43	43	43	42	42:44:49:53	48:44:43	42:43:45	47	45	43	43	44	48	46 41	38:37:38	39:40	40:40	42	45	50	51	48	45	44
-52.00	45 45	44	43	42	42:44:49:53	49:45:43	42:42:43	45	47	46	46	48	51	45 40	38:38:38	40:41	40:40	42	44	48	53	52	49	48
-50.00	46 45	44	42	42	42:44:49:50	50:46:43	42:41:42	43	46	48	51	50	52	44 40	38:38:39	41:41	41:40	41	43	46	49	52	52	51
-48.00	45 44	42	41	41	42:45:51	61:52:47	44:42:41	40	42	44	47	51	50	49 43	40:39:39	41:43	42:41	40	41	42	44	45	47	47	47:
-46.00	42:42:41	41	41	43	47:50:59	51 49	46:43:41	40	40	42	44	46	48	46 43	41:40:42	44:46	43:41	40	40	41	42	43	43	43	43:
-44.00	41:40:40	41	42	45	52:55 49	47 47	47:45:42	40	40	41	42	44	45	45:44:43	43:47:53	48 43	40 40	40	40	41	41	41	41	40:	
-42.00	40:40:41	42	45	52	52 45 42	41 42	45 47	44:42	41	41	42	43	45	47:47:48	52:59 47	42 39	38 38	38	39	40	40	40	39	39:	
-40.00	41:42:44	47	60	49 42	38 37	36 37	39 42	46	45	44	43	44	46	50:58	63 51	45 41	38 36	35	34	35	35	37	38	39	39:39:39:
-38.00	46:48:53	51	43	39 35	33 32	32 32	34 36	39	42	44	45	46	47	47 44	42 39	36 34	32 31	31	31	31	32	33	35	37	38 39 39:
-36.00	41 41	39	37	35	32 30	29 28	28 28	29	31	32	34	36	37	37 37	36 35	34 32	31 29	28	27	27	27	28	29	30	31 33 34 35 36
-34.00	33 32	32	30	29	27 26	25 25	25 25	26	26	27	29	30	30	31 31	30 29	28 27	26 25	25	24	24	24	25	26	27	28 29 30 31
-32.00	27 27	26	25	24	24 23	22 22	22 22	23	23	24	25	25	26	25 25	25 24	23 23	22 21	21	21	21	21	22	23	24	24 25 26 26
-30.00	23 22	22	21	21	20 20	19 19	19 19	19	20	20	21	21	21	21 21	21 21	20 20	19 19	19	18	18	18	19	19	20	20 21 21 22 22
-28.00	19 19	18	18	18	17 17	17 16	16 16	17	17	17	17	18	18	18 18	18 18	17 17	17 16	16	16	16	16	16	17	17	17 18 18 18 19
-26.00	16 16	15	15	15	15 14	14 14	14 14	14	14	15	15	15	15	15 15	15 15	14 14	14 14	14	14	14	14	14	14	14	15 15 15 15 16
-24.00	13 13	13	13	13	12 12	12 12	12 12	12	12	13	13	13	13	13 12	12 12	12 12	12 12	12	12	12	12	12	12	13	13 13 13 13
-22.00	11 11	11	11	10 10	10 10	10 10	10 10	10	10	10	10	10	10	10 10	10 10	10 10	10 10	10	10	10	10	10	10	10	11 11 11
-20.00	9 9	9	9	8 8	8 8	8 8	8 8	8	8	8	8	8	8	8 8	8 8	8 8	8 8	8	8	8	8	8	8	8	9 9 9
-18.00	7 7	7	7	7 7	7 7	7 7	7 7	7	7	7	7	7	7	7 7	7 7	7 7	7 6	6	6	7	7	7	7	7	7 7 7 7
-16.00	6 5	5	5	5 5	5 5	5 5	5 5	5	5	5	5	5	5	5 5	5 5	5 5	5 5	5	5	5	5	5	5	5	5 5 5 5
-14.00	4 4	4	4	4 4	4 4	4 4	4 4	4	4	4	4	4	4	4 4	4 4	4 4	4 4	4	4	4	4	4	4	4	4 4 4 4
-12.00	3 3	3	3	3 3	3 3	3 3	3 3	3	3	3	3	3	3	3 3	3 3	3 3	3 3	3	3	3	3	3	3	3	3 3 3 3
-10.00	2 2	2	2	2 2	2 2	2 2	2 2	2	2	2	2	2	2	2 2	2 2	2 2	2 2	2	2	2	2	2	2	2	2 2 2 2
-8.00	1 1	1	1	1 1	1 1	1 1	1 1	1	1	1	1	1	1	1 1	1 1	1 1	1 1	1	1	1	1	1	1	1	1 1 1 1
-6.00	1 1	1	1	1 1	1 1	1 1	1 1	1	1	1	1	1	1	1 1	1 1	1 1	1 1	1	1	1	1	1	1	1	1 1 1 1
-4.00	0 0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0	0 0 0 0
-2.00	0 0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0	0 0 0 0
0.00	0 0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0 0	0 0	0 0	0 0	0	0	0	0	0	0	0	0 0 0 0

Figure 6.3 Response Pattern, 6 by 6 Array, Non-Ideal Elements Placed by "PERMSER"

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	44 45 46 48:50:52:56:64:52	48 47 49 49 46	43 42	47 44 48	53 51	46 49	52 53:52:51:49:48:48:50:49	46	45	45	45:44:								
-88.00	44 45 46 48:50:52:56:64:52	48 47 49 49 46	43 42	42 44 48	53 50	48 49	52 53:52:51:49:48:48:50:49	46	44	45	45:44:								
-86.00	44 45 46 48 50:52:56:64:52	48 47 49 49 46	43 42	42 44 48	52 50	48 49	52 53:52:51:49:48:48:50:49	46	44	44	45:44:								
-84.00	44 45 46 48 50:52:56:64:52	48 47 49 49 46	43 42	42 44 47	51 49	48 49	52 54:52:51:50:48:49:50:49	46	44	44	44:44:								
-82.00	44 44 46 48 50:52:56:64:52	48 47 49 49 46	43 42	42 43 46	49 48	47 49	52 54:53:51:50:49:49:50:48	46	44	44	44:44:								
-80.00	43 44 46 48 50:52:55:64:52	48 48 49 49 46	43 42	42 43 46	48 47	47 49	52 55:53:52:50:49:49:50:48	45	44	44	44 44:								
-78.00	43 44 46 48 50:52:55:63:53	48 48 49 49 46	43 42	41 42 44	46 46	46 48	53 56:54:52:50:49:49:50:48	45	44	43	44 44:								
-76.00	42 44 46 48 51:52:54:63:53	48 48 49 49 46	43 41	41 42 43	45 45	46 48	53 58:55:53:51:49:49:50:48	45	43	43	44 44:								
-74.00	42 43 46 48 51:52:54:62:53	49 48 49 49 46	43 41	41 41 42	43 44	45 48	53 60:56:54:52:50:49:50:46	45	43	43	43 44								
-72.00	41 43 45 49 51:51:53:61:54	49 48 49 49 46	43 41	40 40 41	42 43	44 47	54 63:57:54:52:50:50:50:48	45	43	42	43 44								
-70.00	41 43 45 49 51:51:52:61:55	49 48 49 49 46	43 41	40 40 40	41 42	43 47	54 69:58:55:53:51:50:50:48	45	42	42	42 43								
-68.00	40 42 45 49 52:51:52:60:55	49 48 49 49 46	43 41	39 39 39	40 41	43 46	54 83:59:56:54:52:51:50:48	45	42	41	41 42								
-66.00	40 42 45 50 52:50:51:59:56	50 48 49 49 47	43 41	39 39 39	39 40	42 46	54 68 58:56:55:52:51:51:48	45	42	41	41 42								
-64.00	40 42 45 51 52:49:50:59:57	50 48 49 49 47	43 41	39 38 38	39 40	42 46	54 62 57:56:56:53:52:51:49	45	42	41	41 41								
-62.00	40 42 46 52 51:48:49:57:58	50 48 49 49 47	44 41	39 38 38	38 40	42 47	54 58 55:55:56:54:52:51:49	45	42	41	40 41								
-60.00	41 43 48 54 50:47:49:56:59	50 48 49 50 48	45 41	39 38 38	38 40	43 47	55 56:53:54:56:55:53:51:49	45	42	41	40 41								
-58.00	42 45 50 55:48:46:48:55:58	50 48 49 50 49	46 42	40 39 38	39 40	43 48	55 53:52:53:56:56:54:51:49	46	43	41	41 41								
-56.00	44 47 55 52:47:45:47:54:56	49 47 46 50 50	47 44	41 40 39	40 41	45 51	55 52:51:52:54.57:54:51:49	46	43	42	41 42								
-54.00	47 53 61:49:45: 4:47:54:53	48 46 48 50 51	49 45	43 41 40	41 43	47 54	54 50:50:51:53:57:56:52:49	46	44	42	42 42								
-52.00	55 56:51:46:43: 4:46:53:50	46 45 47 50 52	51 48	45 43 43	43 46	51 59	52:49:49:50 52 57:56:52:48	46	44	43	42:42:								
-50.00	54:50:46:43:42: 3:47:52 47	44 44 45 49 52	53 51	48 46 46	47 51	62 57	51:49:48 49 50 55 63:53:48	46	44	43	42:42:								
-48.00	46:44:43:41:41: 3:47:49 44	42 41 43 47 51	53 52	51 50 50	52 57	59:53:50:49:48 47 48 51 62	56:48:45:43:42:41:41:												
-46.00	42:41:41:40:41: 4:48:45 41	39 39 41 44 49	53 51	50:49:50:51:52:51:51:50:48	46 45 44 46 52 63:50:45:42:41:40:39:														
-44.00	40:40:40:40:42: 6:46 41	38 36 36 38 41	45 51	50:48:47:47:47:48:50:51:51:47	44 41 41 42 45 52 53:46:43:41:39:39:														
-42.00	40:38:40:42:46: 7 41	37 35 34 34 35 37	41 46	50 48:47:46:47:48:52:57:49	43 40 38 37 38 40 44 50 49:44:41:40:39:														
-40.00	42:42:44:46:48 41	36 33 32 31 31 32 34	37 40	45 48 48:48:49:54:63 49	42 38 36 34 34 34 36 38 42 47 47:44:42:41:														
-38.00	51:54:58 46 39	35 32 30 29 28 28 29	30 32	35 38 41 44 46 47 46	43 39 36 34 32 31 30 31 32 33 36 39 43 46 46:46:														
-36.00	42 41 39 36 33	30 28 27 26 25 26 26	27 29	30 33 35 36 37 38 37	35 33 31 30 28 28 27 27 26 29 31 33 35 37 39 41														
-34.00	32 32 31 29 28	26 25 24 23 23 23 23	24 25	27 28 29 30 31 31 30	29 28 27 26 25 24 24 24 25 26 27 28 29 31 32 32														
-32.00	27 26 26 25 24	23 22 21 20 20 20 21	21 22	23 24 25 25 26 26 26	25 24 23 23 22 22 21 22 22 22 23 24 25 26 26 27														
-30.00	22 22 22 21 20	20 19 18 18 18 18 18	19 19	20 21 21 22 22 22 22	21 21 20 20 19 19 19 19 19 20 20 21 21 22 22 22														
-28.00	19 18 18 18 17	17 16 16 16 16 16 16	16 17	17 18 18 18 18 18 18	18 17 17 17 16 16 16 17 17 17 18 18 18 19 19														
-26.00	15 15 15 15 15	14 14 14 13 13 13 14	14 14	15 15 15 15 15 15 15	15 15 15 14 14 14 14 14 14 15 15 15 15 16 16														
-24.00	13 13 13 12 12	12 12 12 12 12 12 12	12 12	12 13 13 13 13 13 13	13 12 12 12 12 12 12 12 12 12 13 13 13 13 13														
-22.00	11 11 10 10 10	10 10 10 10 10 10 10	10 10	10 10 11 11 11 11 11	10 10 10 10 10 10 10 10 10 10 11 11 11 11														
-20.00	9 9 9 8 8	8 8 8 8 8 8 8	8 8	8 9 9 9 9 9 9	9 8 8 8 8 8 8 8 8 8 9 9 9 9 9														
-18.00	7 7 7 7 7	7 7 7 6 6 7 7	7 7	7 7 7 7 7 7 7	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7														
-16.00	5 5 5 5 5	5 5 5 5 5 5 5	5 5	5 5 5 5 5 5 5	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5														
-14.00	4 4 4 4 4	4 4 4 4 4 4 4	4 4	4 4 4 4 4 4 4	4 4 4 4 4 4 4 4 4 4 4 4 4 4 4														
-12.00	3 3 3 3 3	3 3 3 3 3 3 3	3 3	3 3 3 3 3 3 3	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3														
-10.00	2 2 2 2 2	2 2 2 2 2 2 2	2 2	2 2 2 2 2 2 2	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2														
-8.00	1 1 1 1 1	1 1 1 1 1 1 1	1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1														
-6.00	1 1 1 1 1	1 1 1 1 1 1 1	1 1	1 1 1 1 1 1 1	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1														
-4.00	0 0 0 0 0	0 0 0 0 0 0 0	0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0														
-2.00	0 0 0 0 0	0 0 0 0 0 0 0	0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0														
0.00	0 0 0 0 0	0 0 0 0 0 0 0	0 0	0 0 0 0 0 0 0	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0														

Figure 6.4 Response Pattern, 6 by 6 Array, Non-Ideal Elements Placed by Neo-Kendig Selection Scheme

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	42 43 45 50 65	48:43:41:42:45	53:52:44:41:40	41 42 44 47	51 52 50 49	50 52 54 56	57:57:56:58:58:52	47 45:43:42:	
-88.00	42 43 45 50 65	48:43:41:42:45	53:52:44:41:40	41 42 44 47	50 51 50 49	50 52 54 56	57:57:56:58:58:52	47 45:43:42:	
-86.00	42 43 45 50 64	48:43:41:42:45	53:53:44:42:41	41 42 44 47	50 51 49 49	50 52 54 56	57:57:56:57:58:52	47 45:43:42:	
-84.00	42 43 45 50 64	48:43:41:42:44	53:53:45:42:41	41 42 44 47	50 50 49 49	50 52 54 56	57:57:56:57:57:51	47 44:43:42:	
-82.00	42 42 45 50 63	48:43:41:42:44	52:53:45:42:41	41 42 44 46	49 49 48 48	50 52 54 55	57:57:56:57:57:51	47 44:43:42:	
-80.00	42 42 44 50 62	47:43:41:42:44	52:54:45:42:41	41 42 43 46	48 48 47 47	49 51 53 55	56:56:55:56:56:51	47 44:43:43:	
-78.00	41 42 44 50 61	47:43:41:42:44	52:54:45:42:41	41 42 43 45	47 46 46 47	49 51 53 54	56:56:55:56:55:50	46 44 43:43:	
-76.00	41 42 44 50 60	47:42:41:42:44	51:55:46:42:41	41 42 43 44	45 45 45 46	48 51 52 54	55:55:55:55:55:50	46 44 43 43:	
-74.00	41 42 44 50 59	46:42:41:41:44	51:56:46:43:42	41 42 43 43	44 44 44 46	48 50 52 53	54:55:54:54:54:49	46 43 43 43	
-72.00	40 41 44 50 58	46:42:41:41:44	51:57:47:43:42	42 42 42 43	43 43 44 45	47 50 51 52	54:54:54:54:53:49	45 43 42 42	
-70.00	40 41 44 50 57	46:42:41:42:44	50:58:47:44:42	42 42 42 42	42 43 44 47	49 51 52	53:54:53:53:52:48	45 43 42 42	
-68.00	40 41 44 51 55	45:42:41:42:44	50:59:48:44:43	42 42 41 41	41 42 44 47	49 50 51	52:53:53:52:52:48	45 43 42 42	
-66.00	39 41 44 52 54	45:42:41:42:44	50:60:49:45:44	43 42 41 40	40 41 42 44	46 49 49 50	52:53:52:52:51:48	44 42 41 42	
-64.00	39 41 44 53 52	44:42:41:42:44	49:60:50:46:44	43 42 41 40	40 40 41 44	46 48 49 49	51:52:52:51:50:47	44 42 41 41	
-62.00	40 41 45 57 51	44:42:41:42:44	49:60:51:47:45	44 42 41 40	39 40 41 44	46 48 48 49	50:51:51:50:49:47	44 42 41 41	
-60.00	40 42 47 66 49	43:41:41:42:45	49:60:51:48:47	45 43 41 40	40 40 42 44	47 48 48 48	50:51:51:50:49:47	44 42 41 42	
-58.00	41 43 49 62	47:43:41:42:43	45:50:59:52:48	48:47 45 42	41 40 41 42	45 47 47 47	48:49:51:51:49:48	46 44 42 42	
-56.00	43 46 53 53	45:42:41:42:43	46:50 60:52:49	49:49 47 44	42 41 42 44	46 47 47 47	47 48:51:51:49:47	46 44 43 42	
-54.00	46 50 55	48:43:42:47:43	44:46 50 62:53	49:49:52:51	46 44 43 44	46 48 46 47	46 47 48:50:51:49	47 45 44 43	43 44
-52.00	49 52	49:44:42:41:42	44:46:47 50 63	54:48:48:51	57 52 48 46	47 49 50 48	47 46 46 48	50:52:49:46	44:44:43:43:45:
-50.00	48	47:43:42:41:41	43:46:47 47 49	58 56:48:47	48:53:71:57	53 54 54 51	48:47 46 46 47	50 54:49:46	44:43:42:43:44:
-48.00	44	43:41:40:40	41:45:46:47 45	46 51 60:49	46:48:48:52	57:63:63:56	51:48:46 45 45	46 49 56:52	46:43:42:41:42:42:
-46.00	41	40:40:39:40	43:48:50 44 42	43 46 54 52	46:45:45:46	48:50:51:51	50:48:46 44 43	44 46 52 59	48:43:41:41:40:41:
-44.00	39	39:39:39:41	46:56 45 40	39 39 41 46	56 49:45:44	44:45:46:48	50:51:49 45 42	41 41 42 46	56 52:45:42:40:40:40:
-42.00	39	39:39:41:45	62:46 39 36	35 35 37 40	47 56:48:45	44:44:45:48	52:53 47 42	39 38 37 38	40 45 54 49:44:41:40:40:
-40.00	41	41:42:46:61	45 38 35	33 32 32 33	36 39 46 59	52:48:48:49	55:58 47 41	38 35 34 34	34 36 38 43 49 49:45:42:42:
-38.00	44	50:55:49 41	36 33 31	29 29 29 30	31 34 37 42	47 54 58 54	48 43 39 36	33 32 31 30	31 32 33 36 40 44 49 51:50:
-36.00	41	42 39 36	33 31 29	27 26 26 26	27 28 29 32	34 36 38 39	39 37 35 33	31 29 28 27	27 27 28 29 31 33 35 38 40 42
-34.00	33	32 31 30	28 26 25	24 23 23 23	24 24 26 27	29 30 31 31	31 31 29 28	27 26 25 24	24 24 25 26 27 28 30 31 32 33
-32.00	27	26 26 25	24 23 22	21 21 20 20	21 21 22 23	24 25 26 26	26 26 25 24	23 23 22 22	21 22 22 22 23 24 25 26 26 27
-30.00	22	22 22 21	20 20 19	18 18 18 18	18 19 19 20	21 21 22 22	22 22 21 21	20 20 19 19	19 19 19 19 20 21 21 22 22 22
-28.00	19	18 18 18	17 17 16	16 16 16 16	16 16 17 17	18 18 18 18	18 18 18 18	17 17 17 16	16 16 17 17 17 18 18 18 19 19
-26.00	16	15 15 15	15 14 14	14 14 13 13	14 14 14 15	15 15 15 16	16 16 15 15	15 15 15 14	14 14 14 14 14 15 15 15 15 16 16
-24.00	13	13 13 12	12 12 12	12 12 11 12	12 12 12 12	12 13 13 13	13 13 13 13	12 12 12 12	12 12 12 12 12 13 13 13 13 13
-22.00	11	11 10 10	10 10 10	10 10 10 10	10 10 10 10	10 11 11 11	11 11 10 10	10 10 10 10	10 10 10 10 10 11 11 11 11
-20.00	9	9 9 8	8 8 8	8 8 8 8	8 8 8 8	8 9 9 9	9 9 9 9	9 8 8 8	8 8 8 8 8 9 9 9 9 9
-18.00	7	7 7 7	7 7 7	6 6 6 6	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7	7 7 7 7 7 7 7 7 7 7
-16.00	5	5 5 5	5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5	5 5 5 5 5 5 5 5 5 5
-14.00	4	4 4 4	4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4	4 4 4 4 4 4 4 4 4 4
-12.00	3	3 3 3	3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3	3 3 3 3 3 3 3 3 3 3
-10.00	2	2 2 2	2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2	2 2 2 2 2 2 2 2 2 2
-8.00	1	1 1 1	1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1 1 1 1 1 1 1
-6.00	1	1 1 1	1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1	1 1 1 1 1 1 1 1 1 1
-4.00	0	0 0 0	0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0 0 0 0 0 0 0
-2.00	0	0 0 0	0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0 0 0 0 0 0 0
0.00	0	0 0 0	0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0	0 0 0 0 0 0 0 0 0 0

Figure 6.5 Response Pattern, 6 by 6 Array, Neo-Kendig Placement used as Starting Point for "PERMSER"

MAP OF ARRAY OUTPUT

THETA	PHI (DEGREES)																		
	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	99:84	99:59:48:42:40:42:51:76:51:42:40:42:48:59:99:84									99:84	99:59:48:42:40:42:51:76:51:42:40:42:48:59:99:84							99:
-88.00	99 83	99:59:48:42:40:42:51:76:51:42:40:42:48:59:99:83									99 83	99:59:48:42:40:42:51:76:51:42:40:42:48:59:99:83							98
-86.00	86 80	99:59:48:42:40:42:51:72:51:42:40:42:48:59:99:80									86 80	99:59:48:42:40:42:51:72:51:42:40:42:48:59:99:80							86
-84.00	79 76	99 59:48:42:40:42:51:68:51:42:40:42:48:59:99									76 79	76 99 59:48:42:40:42:51:68:51:42:40:42:48:59:99							76 79
-82.00	73 73	95 58:47:42:40:42:50:65:50:42:40:42:47:58:95									73 74	73 95 58:47:42:40:42:50:65:50:42:40:42:47:58:95							73 74
-80.00	70 70	87 58:47:42:40:42:50:62:50:42:40:42:47:58:87									70 70	70 87 58:47:42:40:42:50:62:50:42:40:42:47:58:87							70 70
-78.00	66 67	81 57:47:42:40:42:50:60:50:42:40:42:47:57:81									67 66	67 81 57:47:42:40:42:50:60:50:42:40:42:47:57:81							67 66
-76.00	63 64	75 57:47:42:40:42:49:58:49:42:40:42:47:57:75									64 63	64 75 57:47:42:40:42:49:58:49:42:40:42:47:57:75							64 63
-74.00	61 62	71 57:47:42:40:42:49:57:49:42:40:42:47:57:71									62 61	62 71 57:47:42:40:42:49:57:49:42:40:42:47:57:71							62 61
-72.00	59 59	66 58:47:42:40:43:49:57:49:43:40:42:47:58:66									59 59	59 66 58:47:42:40:43:49:57:49:43:40:42:47:58:66							59 59
-70.00	56 57	62 60:48:42:41:43:50:57:50:43:41:42:48:60:62									57 56	57 62 60:48:42:41:43:50:57:50:43:41:42:48:60:62							57 56
-68.00	51 55	59 64:49:43:42:44:51:59:51:44:42:43:49:64:59									55 54	55 59 64:49:43:42:44:51:59:51:44:42:43:49:64:59							55 54
-66.00	52 52	55 99:51:45:43:46:53:64:53:46:43:45:51:99:55									52 52	52 55 99:51:45:43:46:53:64:53:46:43:45:51:99:55							52 52
-64.00	50 50	57 62 54:47:45:48:58:83 58:48:45:47:54:62									52 50	50 50 52 62 54:47:45:48:58:83 58:48:45:47:54:62							52 50 50
-62.00	48 48	50 55 65:50:48:52:78:59 78:52:48:50:65:55									50 48	48 48 50 55 65:50:48:52:78:59 78:52:48:50:65:55							50 48 48
-60.00	46 46	47 50 60 59:54:63:57 53 57 63:54:59:60									50 47	46 46 46 47 50 60 59:54:63:57 53 57 63:54:59:60							50 47 46 46
-58.00	45 45	45 47 51 60 69 58 50 48 50 58 69 60 51									47 45	45 45 45 47 51 60 69 58 50 48 50 58 69 60 51							47 45 45 45
-56.00	43 43	43 44 46 50 51 49 46 45 46 49 51 50 46									44 43	43 43 43 43 43 44 46 50 51 49 46 45 46 49 51 50 46							44 43 43 43
-54.00	42 42	42 42 43 45 46 45 43 43 43 45 46 45 43									42 42	42 42 42 42 42 43 45 46 45 43 43 43 45 46 45 43							42 42 42 42
-52.00	42 41	41 41 41 42 43 42 42 41 42 42 43 42 41									41 41	41 42 41 41 41 41 42 43 42 42 41 42 42 43 42 41							41 41 41 42
-50.00	41 41	40 40 40 41 41 41 40 40 40 41 41 41 40									40 40	41 41 41 40 40 40 41 41 41 40 40 40 41 41 41 40							40 40 41 41
-48.00	41 41	40 40 40 40 40 40 40 40 40 40 40 40 40									40 40	41 41 41 40 40 40 40 40 40 40 40 40 40 40 40 40							40 40 41 41
-46.00	41 42	41 41 40 40 40 40 40 40 40 40 40 40 40									41 41	42 43 42 41 41 40 40 40 40 40 40 40 40 40 40 40							41 41 42 43
-44.00	45 45	43 42 42 41 42 42 42 42 42 42 42 41 42									42 43	45 45 45 43 42 42 41 42 42 42 42 42 42 42 41 42							42 42 43 45 45
-42.00	52 51	48 46 45 45 45 45 46 46 46 45 45 45 45									46 48	51 52 51 48 46 45 45 45 45 46 46 46 45 45 45 45							46 48 51 52
-40.00	59:63:74	58 54 53 54 56 58 59 58 56 54 53 54 58									74 63:59:63:74	58 54 53 54 56 58 59 58 56 54 53 54 58							74 63:59:
-38.00	46:47:48:51:53:54:53:51:50:50:50:51:53:54:53:51:48:47:46:47:48:51:53:54:53:51:50:50:50:51:53:54:53:51:48:47:46:																		
-36.00	41:42:43:43:44:44:44:43:43:43:43:44:44:44:43:43:42:41:42:43:43:44:44:44:43:43:43:43:43:44:44:44:43:43:42:41:																		
-34.00	40:40:41:41:41:41:41:41:40:40:40:41:41:41:41:41:41:40:40:40:41:41:41:41:41:41:40:40:40:41:41:41:41:41:41:40:40:																		
-32.00	41:42:42:42:43:42:42:41:41:41:41:41:42:42:43:42:42:42:41:42:42:42:43:42:42:41:41:41:41:41:42:42:43:42:42:42:41:																		
-30.00	57:57:58:60:60:58:55:53:52:51:52:53:55:58:60:60:58:57:57:57:58:60:60:58:55:53:52:51:52:53:55:58:60:60:58:57:57:																		
-28.00	38 38 37 37 38 38 38 39 39 39 39 39 38 38 38 37 37 38 38 38 37 37 38 38 38 39 39 39 39 39 38 38 38 37 37 38 38																		
-26.00	28 28 28 28 28 29 29 29 29 29 29 29 29 29 28 28 28 28 28 28 28 28 28 29 29 29 29 29 29 29 29 29 28 28 28 28 28																		
-24.00	22 22 22 23 23 23 23 23 23 23 23 23 23 23 23 23 22 22 22 22 22 23 23 23 23 23 23 23 23 23 23 23 23 23 22 22 22																		
-22.00	18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18 18																		
-20.00	14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14																		
-18.00	11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11																		
-16.00	9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9																		
-14.00	7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7 7																		
-12.00	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5																		
-10.00	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3																		
-8.00	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2																		
-6.00	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1																		
-4.00	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1																		
-2.00	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0																		
0.00	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0																		

Figure 6.6 Response Pattern, 8 by 8 Array, Ideal Elements

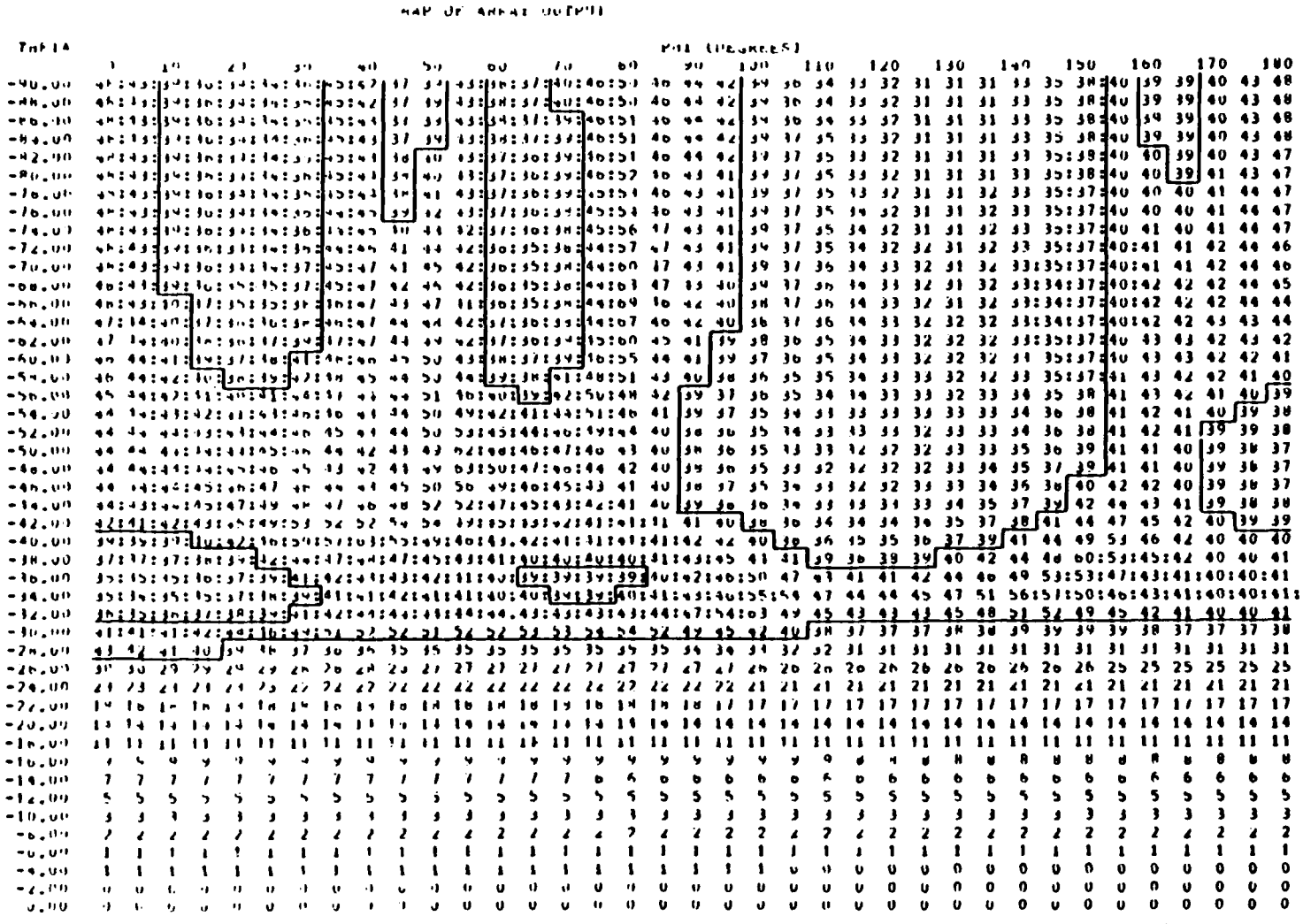


Figure 6.7 Response Pattern, 8 by 8 Array, Random Arrangement of Non-Ideal Elements

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	57 56 52 47 43 41 41 46 61 52 57 43 39 39 41 44 46 49 56 56 51 52 71 49 45 47 52 50 43 39 38 42 47 49 52 56 57																		
-88.00	57 56 52 47 43 41 41 46 61 52 57 43 39 39 41 44 46 49 56 56 51 53 71 49 45 47 52 50 43 39 38 42 47 49 52 56 57																		
-86.00	57 56 52 47 43 41 41 46 62 53 57 43 39 39 41 43 46 49 56 56 52 53 71 49 45 47 52 50 43 39 38 42 47 49 52 56 57																		
-84.00	57 56 52 47 43 41 41 46 62 53 57 43 39 39 41 43 46 49 56 56 52 53 70 49 45 47 51 49 42 38 38 42 47 49 52 56 58																		
-82.00	56 56 52 47 43 40 41 46 62 53 57 43 39 39 41 43 45 49 55 56 52 53 69 49 45 47 51 49 42 38 38 42 47 49 52 56 58																		
-80.00	56 56 53 47 43 41 41 46 61 53 57 43 39 39 41 43 45 49 55 56 52 54 67 49 45 47 51 48 42 38 38 42 47 49 52 56 59																		
-78.00	55 57 53 47 43 41 41 46 61 53 58 44 40 39 41 43 45 48 54 56 53 55 65 49 45 47 50 48 42 38 38 41 47 49 52 57 59																		
-76.00	54 57 54 46 43 41 41 46 60 53 59 44 40 40 41 43 45 48 54 56 53 55 64 49 45 47 50 48 42 39 38 41 47 49 52 57 60																		
-74.00	53 56 55 46 43 41 42 47 59 53 61 45 40 40 41 43 45 46 53 56 53 56 64 49 46 47 50 47 42 39 38 41 47 50 52 56 60																		
-72.00	52 55 56 49 44 42 42 48 58 52 64 46 41 40 42 43 45 46 53 56 54 56 65 50 46 47 50 47 42 39 39 41 47 50 52 56 60																		
-70.00	51 53 56 51 45 42 43 49 56 51 64 47 42 41 42 44 45 48 52 56 54 56 70 51 47 48 50 48 43 39 39 42 47 50 51 55 58																		
-68.00	50 52 55 53 47 44 45 51 53 49 58 49 43 42 43 44 46 48 52 55 53 55 61 53 49 50 51 46 43 40 40 42 47 50 51 54 56																		
-66.00	49 50 53 56 49 46 47 54 50 47 53 53 45 43 44 45 47 49 52 55 53 53 63 57 52 53 53 49 44 41 40 43 48 50 51 52 54																		
-64.00	47 46 50 56 53 49 51 55 47 45 49 60 46 46 46 47 48 49 53 55 52 51 56 69 59 58 54 51 46 42 42 44 49 50 50 51 52																		
-62.00	46 46 48 52 58 55 56 51 45 43 46 54 54 50 49 49 50 51 53 54 50 49 51 57 66 59 53 52 48 44 43 46 50 50 49 49 50																		
-60.00	45 44 45 47 51 54 52 46 42 41 43 46 55 55 55 54 53 53 54 53 49 47 48 50 52 51 50 51 51 47 46 46 51 49 48 46 48																		
-58.00	44 43 43 44 46 47 46 43 40 40 41 44 48 52 57 69 59 55 54 51 47 45 45 46 47 47 47 49 53 51 50 52 51 48 47 47 46																		
-56.00	42 42 41 42 42 43 42 40 39 38 39 41 43 46 50 55 61 56 53 50 46 44 43 43 43 43 44 46 51 59 59 55 49 46 45 45 45																		
-54.00	42 41 40 40 40 40 39 38 37 37 37 39 40 42 45 49 54 54 52 48 45 42 41 41 41 41 42 43 47 54 59 52 47 45 45 44 44																		
-52.00	41 40 39 38 38 38 38 37 36 36 36 37 38 40 42 45 49 51 50 47 44 41 40 39 39 40 40 42 44 48 51 48 46 44 44 44 43																		
-50.00	41 39 38 37 37 37 36 36 35 35 35 36 37 38 40 43 47 50 49 46 43 41 39 38 38 38 39 41 43 46 47 46 45 44 44 44 43																		
-48.00	41 39 38 37 36 36 35 35 35 35 35 35 36 37 39 42 46 49 49 46 43 40 39 38 38 38 39 40 42 44 46 46 45 45 45 45 44																		
-46.00	42 40 38 37 36 36 35 35 35 35 35 35 36 37 39 42 46 49 50 46 43 41 39 38 38 38 39 41 43 45 46 47 47 47 47 47 45																		
-44.00	45 42 39 38 37 36 36 36 35 35 35 36 36 37 39 43 47 52 53 51 47 43 41 39 39 39 40 42 44 47 49 50 51 52 53 52 49																		
-42.00	51 46 42 40 38 37 37 37 37 37 37 37 38 39 41 45 51 57 55 55 52 47 43 42 41 42 43 46 49 53 59 66 68 66 63 63 62																		
-40.00	57 55 47 43 41 40 39 39 39 39 39 39 40 41 44 50 61 52 49 50 55 56 50 47 46 47 50 53 55 55 53 51 49 48 47 48 50																		
-38.00	47 54 60 49 46 44 43 43 43 43 43 43 44 46 50 66 51 46 44 45 47 52 60 63 60 59 54 50 47 45 44 44 43 42 42 42 43																		
-36.00	42 46 51 56 53 50 50 50 50 50 50 49 50 54 72 53 46 43 42 42 43 45 48 49 49 48 46 44 42 41 41 40 40 39 39 39 40																		
-34.00	41 43 47 51 53 54 54 56 59 60 59 58 59 69 59 50 45 43 42 41 42 43 45 46 45 44 43 42 41 40 39 39 39 38 38 38 39																		
-32.00	41 47 51 54 54 53 53 54 55 55 54 53 52 54 61 60 51 47 46 45 46 47 48 46 48 46 45 43 42 42 41 41 40 40 40 40 41																		
-30.00	57 51 47 44 43 42 42 42 42 42 41 41 41 41 42 43 45 47 50 51 51 50 50 50 51 53 58 65 63 58 55 54 53 53 53 55 59																		
-28.00	36 35 34 34 34 33 33 33 33 33 33 33 33 33 33 33 34 34 34 35 35 35 35 35 35 35 36 36 37 37 37 37 37 37 37 37 37																		
-26.00	29 27 27 27 27 27 27 27 27 27 27 27 26 26 26 27 27 27 27 27 27 27 27 27 27 26 28 28 28 28 28 28 28 28 28 28 28																		
-24.00	22 22 22 22 22 22 22 22 22 22 22 22 21 21 21 21 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22 22																		
-22.00	18 18 18 18 18 18 18 17 17 17 17 17 17 17 17 17 17 17 17 17 17 16 18 16 18 18 18 18 18 18 18 18 18 18 18 18 18																		
-20.00	14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14 14																		
-18.00	11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11 11																		
-16.00	9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9 9																		
-14.00	6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6 6																		
-12.00	5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5																		
-10.00	3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3																		
-8.00	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2																		
-6.00	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1																		
-4.00	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1																		
-2.00	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0																		
0.00	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0																		

Figure 6.8 Response Pattern, 8 by 8 Array, Non-Ideal Elements Placed by Neo-Kendig Selection Scheme

MAP OF ARRAY OUTPUT

THETA \ PHI (DEGREES)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180
-90.00	49:56	51 46	44:42	40:42	49:59	50:42	39:39	40:41	41:40	40:42	42 42	43 43	40:39	41:41	43 47	43:40	40:42	48:57	49
-88.00	49:56	51 46	44:42	40:42	49:59	50:42	39:39	40:41	41:40	40:42	42 42	43 43	40:39	41:41	43 47	43:40	40:42	48:57	49
-86.00	48:56	52 46	44:42	40:42	49:56	49:42	39:38	40:41	41:40	40:42	42 42	43 43	40:39	41:41	43 47	43:40	40:42	48:57	49
-84.00	48:55	52 46	44:42	40:42	49:57	49:41	39:38	40:41	41:40	40:42	42 42	43 42	40:39	41:41	43 47	42:40	39:42	48:57	49
-82.00	48:55	53 47	45:42	40:42	48:56	49:41	38:38	39:41	41:40	40:41	42 42	43 42	40:39	41:41	43 47	42:40	39:42	48:57	49
-80.00	47:54	54 47	45:42	40:42	48:55	48:41	38:38	39:41	41:40	40:41	42 42	43 42	39:39	41:41	43 46	42:39	39:42	48:58	50
-78.00	47:53	55 46	45:42	40:42	48:54	48:41	38:38	39:41	41:40	40:41	42 42	43 42	39:39	41:42	43 46	42:39	39:42	47:58	51
-76.00	47:52	56 49	46:42	40:42	47:53	47:41	38:38	39:41	41:40	40:41	42 42	43 42	40:39	41:42	44:46	42:39	39:42	47:58	51
-74.00	46:51	58 50	46:42	41:42	47:52	47:41	38:38	39:41	41:40	40:41	42 42	43 42	40:39	41:42	44:46	42:39	39:42	47:56	52
-72.00	46:50	59 51	47:43	41:42	47:52	47:41	38:38	39:41	41:40	40:41	42 42	43 42	40:40	41:42	44:46	42:40	40:42	47:57	53
-70.00	45:49	58 52	48:44	42:43	47:51	47:41	39:38	40:41	41:40	40:41	41 42	43 42	40:40	41:42	44:46	42:40	40:42	47:57	55
-68.00	45:48	56 54	49:45	42:44	48:51	47:42	39:39	40:41	41:40	40:41	41 41	42 42	41:40	42:43	44:46	43:41	40:42	47:57	57
-66.00	44:47	54 56	51:46	44:45	49:52	47:42	40:39	41:42	41:40	40:40	41 41	42 43	41:41	42:43	44:46	44:43	41:43	48:58	59
-64.00	44:47	52 55	54 49	46:47	51:53	48:43	41:40	42:43	42:40	40:40	41 41	42 43	42:42	43:43	44 47	45:43	42:44	49:59	61
-62.00	44:46	50 53	56 54	49:50	54:56	50:45	42:42	43:44	42:41	40:40	41 41	42 43	43:43	43:43	44 47	47:44	44:46	50:62	61
-60.00	44:45	48 56	53 63	56:55	58:56	53:47	44:44	45:45	43:41	40:40	40 40	41 43	44:44	44 43	44 47	49:47	47:48	53:70	59
-58.00	43 44	46 47	49 54	65 60	56 56	56:51	47:47	49:47	44:41	40:40	40 40	41 43	44 44	44 43	43 45	49:50	50:52	59:70	57
-56.00	43 44	44 45	46 48	52 52	51 51	54 54	52:53	55:49	44:41	40:40	40 39	40 42	43 44	43 42	42 44	47 51	54:58	65:60	54
-54.00	43 43	43 43	43 45	46 47	47 48	50 53	56 61	60 49	44:41	40:40	39 39	39 41	42 43	42 41	41 42	44 48	51 54	56 55	52
-52.00	43 42	42 42	41 42	44 44	45 46	47 49	51 52	51 47	44 41	40 40	39 39	39 40	41 42	42 41	40 41	42 44	47 49	51 52	51
-50.00	42 42	41 41	40 41	42 43	43 44	45 46	47 47	47 45	43 41	40 40	39 39	39 40	41 41	41 40	40 40	41 42	44 46	48 50	50
-48.00	43 42	41 40	40 40	41 42	43 44	44 45	45 45	45 44	43 41	40 40	40 39	39 40	41 41	41 40	40 40	41 42	44 47	50	51
-46.00	43 42	41 40	40 40	41 42	43 44	45 45	45 45	45 44	43 41	41 41	41 40	40 41	42 42	41 40	40 40	41 42	44 47	50	53
-44.00	44 43	42 41	41 41	42 43	44 46	47 47	46 46	45 44	43 42	41:42	42 42	42 42	43 44	44 43	42 41	41 41	42 44	47 51	55
-42.00	44:43	43 42	42 43	44 45	47 49	51 52	50 49	47 45	43:42	41:42	43 44	45 46	48 49	48 47	45 43	42 42	43 45	47:49	51:
-40.00	43:42	42:43	43 44	45 47	48:50	54:63	61 52	48:45	43:41	41:41	43:45	48:51	53:52	51:50	46:46	44 44	44:44	45:45	46:
-38.00	41:41	41:42	43:44	44:45	45:45	47:49	50:48	46:43	41:40	39:40	41:43	45:46	45:45	45:45	45:45	44:44	43:43	42:42	42:
-36.00	39:39	40:40	41:42	42:42	42:42	43:44	44:44	43:41	40:39	38:38	39:40	41:41	41:41	41:41	42:42	42:42	42:41	41:40	40:
-34.00	39:39	39:40	41:41	41:41	41:41	42:42	43:43	42:40	39:38	38:38	38:39	39:39	39:39	39:39	40:41	41:42	41:41	40:40	40:
-32.00	41:41	41:42	43:44	44:44	44:44	45:45	45:44	43:42	41:40	40:40	40:40	41:40	40:40	40:40	40:41	43:44	44:44	44:43	43:42:
-30.00	45 44	44 45	45 47	49 52	55 54	50 47	45 44	43 43	43 44	45:46	48:51	52:51	49:48	48:50	54:65	60 53	50 49	49 48	48
-28.00	35 35	35 34	34 35	35 35	35 35	35 34	34 34	34 34	34 35	35 36	37 37	38 36	39 39	39 38	38 37	36 36	35 35	35 35	35
-26.00	27 27	27 27	27 27	27 27	27 27	27 27	27 27	27 27	27 28	28 28	28 29	29 29	29 29	28 28	28 28	27 27	27 27	27	27
-24.00	22 22	22 22	22 22	22 22	22 22	22 22	22 22	22 22	22 22	22 22	22 23	23 23	23 23	22 22	22 22	22 22	22 22	22	22
-22.00	18 17	17 17	17 17	17 17	17 17	17 17	17 17	17 17	18 18	18 18	18 18	18 18	18 18	18 18	18 18	18 18	18 18	18	18
-20.00	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14 14	14	14
-18.00	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11 11	11	11
-16.00	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9 9	9	9
-14.00	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 6	6 7	7 7	7 7	7 7	7 7	7 6	6 6	6	6
-12.00	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5 5	5	5
-10.00	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3 3	3	3
-8.00	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2 2	2	2
-6.00	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1	1
-4.00	0 0	0 0	0 0	0 0	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1 1	1	1
-2.00	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0
0.00	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0 0	0	0

Figure 6.9 Response Pattern, 8 by 8 Array, Neo-Kendig Placement used as Starting Point for "PERMSER"

CHAPTER SEVEN

SUMMARY AND CONCLUSIONS

7.1 SUMMARY

The goal of this thesis was to develop an element selection technique which would place elements in an array in a manner that minimizes the impact of individual transducer element errors on the array's directional beam pattern. The combination of this technique and the accurate measurement system developed by Schafer [2] is a powerful tool which can reduce costs and production time during the development of a transducer array.

Chapter Two discusses various aspects of the lumped-parameter equivalent circuit. The correlations between the measured parameters have been established. A preliminary set of tolerances on the measured circuit parameters has been established, based on a measured set of transducers. These tolerances indicate which circuit parameters have the most influence on an element's response.

Chapter Three discusses the mutual interaction of transducer elements in an array. The effect of incidence angle is discussed, as well as the effects of element tolerances on the mutual interaction.

The application of Permutation Search is described in Chapter Four. The control parameters and the type of results that can be expected are discussed. The Neo-Kendig selection technique is derived in Chapter Five. It is an effective element placement technique by

itself. When it is used as a starting point for the Permutation Search algorithm, the results are spectacular. This combination has designed arrays, 6 by 6 and 8 by 8, using elements with maximum tolerances of ±9% in amplitude and ±11° in phase, that meet a required 40 dB side lobe level in all but a very few locations, and fail at those points by less than 2 dB.

7.2 FUTURE CONSIDERATIONS

Permutation Search is the most time consuming part of the entire placement process. A reduction of the run time, without a concurrent loss of effectiveness or a reduction of result quality, would represent an additional improvement of the process.

The derivation of the Neo-Kendig technique has shed light on a mechanism that plagues the design of shading values. The use of the Kendig Coefficients for a given array may reduce the time necessary for developing shading values by eliminating some computations. Additionally, if the shading values are considered variables, and the Kendig Coefficients constants, the problem of achieving a desired beam pattern becomes a set of linear equations which may be solvable by linear programming.

The selective use of weighting may improve the effectiveness of Permutation Search. By assigning greater weighting values at regions in the response plane where it is more crucial that the specified side lobe level be attained, greater control over these areas may be achieved.

A similar result might be achieved through the use of Fuzzy Math, [9]. Assigning membership functions to different regions of the response pattern would provide more information on the acceptabilty of the current base permutation than is provided by the value of the objective function. For example, Fuzzy Math would allow the case where achieving a response better than 40 dB in a certain region would outweigh the negative effects of not achieving 40 dB in another region.

Different methods of error analysis may improve both the quality of results and run time. One example would be to minimize the maximum error rather than the total error, as is the case in this work.

The effects of incident angle on the interactions between elements was mentioned in Chapter Two. If the assumption of nominal phase effects could be validated, this selection process would stand on firmer ground. For every incident angle, the mutual interactions present N equations and N unknowns, where N is the array size. Through matrix manipulation, it is possible to solve for all the interaction coefficients, and confirm or refute that assumption.

The testing of more sets of transducer elements would enable the establishment of more reliable equivalent circuit parameter tolerances. This, in turn, would allow more uniformity in element performance characteristics to be achieved. With increased uniformity comes increased array performance.

APPENDIX A

ARRAY PARAMETERS AND TERMINOLOGY

The arrays considered in this study are four-fold symmetric, (or quadrature symmetry), even and odd arrays. An odd array is one which has an element at the center of the array, at $X_1 = 0.0$, $Y_1 = 0.0$. An even array has no center element. There are several parameters used to describe these types of arrays. Accompanying this description are examples of an even and an odd array. In order to implement easily the Neo-Kendig Selection Scheme and the Permutation Search algorithm, it was necessary to create an array numbering system which reflects the unique symmetry of these arrays.

The parameter M describes the number of positions in a full array, i.e., with no corner elements removed. The number of positions in one quadrant is described by IM. For even arrays, IM = M/4. For odd arrays, IM = (M - 1)/4. IM1 is the number of positions in one quadrant that will be occupied by a transducer element. If three elements are removed from each corner, then IM1 = IM - 3.

Each position in the array has a unique number assigned to it. The array is numbered sequentially, as shown in Figures A.1 and A.2. For the purposes of assigning shading values, each position in the first quadrant, (and the center element in an odd array), is assigned a letter representing the appropriate shading value. Examples of shading values are given for desired side lobe levels of -30 dB and -40 dB. These are obtained from the Dolph-Chebyshev [12] shading values for a line array and the second product theorem [13].

32	31	30	29	13	14	15	16
28	27	26	25	9	10	11	12
24	23	22	21	5	6	7	8
20	19	18	17	1	2	3	4
36	35	34	33	49	50	51	52
40	39	38	37	53	54	55	56
44	43	42	41	57	58	59	60
48	47	46	45	61	62	63	64

M = 64
IM = 16

IM1 = 13, if 3 corner elements are removed

Examples of Symmetric Array Positions

3, 19, 35, 51
7, 23, 39, 55

Shading
(Dolph-Chebyshev)

D	G	J	K
C	F	H	J
B	E	F	G
A	B	C	D

Side lobe level

Position	-30dB	-40dB
A	1.0000	1.0000
B	.8120	.7594
C	.5187	.4178
D	.2622	.1460
E	.6593	.5767
F	.4212	.3173
G	.2129	.1109
H	.2690	.1746
J	.1360	.0610
K	.0687	.0213

Figure A.1

Parameters of an Even Array, 8 by 8

25	22	19	16	11	12	13
24	21	18	15	8	9	10
23	20	17	14	5	6	7
28	27	26	1	2	3	4
31	30	29	38	41	44	47
34	33	32	39	42	45	48
37	36	35	40	43	46	49

M = 49
IM = 12

IM1 = 9, if 3 corner elements are removed

Examples of Symmetric Array Positions

2, 17, 26, 38
7, 19, 31, 43

Shading
(Dolph-Chebyshev)

D	G	J	K
C	F	H	J
B	E	F	G
A	B	C	D

Side lobe level

Position	-30dB	-40dB
A	1.0000	1.0000
B	.8738	.8397
C	.5683	.4793
D	.2642	.1594
E	.7635	.7051
F	.4966	.4025
G	.2309	.1338
H	.3230	.2297
J	.1501	.0764
K	.0698	.0254

Figure A.2

Parameters of an Odd Array, 7 by 7

The term "symmetric array position" refers to those elements, one in each quadrant which have the relationship described in equation 5-7, (even), or equation B-1, (odd). In the even array, (this holds true for the odd array as well), the symmetric array position numbers are arithmetically separated by the value of IM. For example, for the 8 by 8 array in Figure A.1, positions 2, 18, 34, and 50 are symmetric. Their arithmetic differences are 16, which is IM for an 8 by 8 array. All symmetric array positions have the same shading values.

APPENDIX B

THE ADJUSTMENT TO NEO-KENDIG FOR ODD ARRAYS

There are two types of symmetry present in an odd array. Consider the first two quadrants of positions in an odd array.

25	22	19	16	11	12	13
24	21	18	15	8	9	10
23	20	17	14	5	6	7
			1	2	3	4

Figure B.1

First Two Quadrants of an Odd Array

The first type is reflective in nature. Position 6 is symmetric about the y-axis with position 20.

$$(X_{20},Y_{20}) = (-X_6,Y_6) \qquad \text{(B-1)}$$

Those positions which do not lie along the axes of the array have three other symmetric positions, for a total of four. However, difficulties arise with this type of symmetry when considering position 16. Since it lies along one of the axes, (the y-axis in this case), there is only one other position symmetric with it.

The other type of symmetry present is rotational. Position 6 is symmetric with position 18 through a 90° rotation about the origin. The symmetry is such that the x and y coordinates are interchanged, viz.,

$$(X_{18},Y_{18}) = (-Y_6,X_6). \tag{B-2}$$

Note that the arithmetic relationship between these two positions is the same as between two symmetric positions in an even array, i.e., 18 - 6 = 12, which is IM for a 7 by 7 array. Figure B.2 is the odd array equivalent of Figure 5.2.

Since the Neo-Kendig Selection Scheme takes advantage of the unique symmetry of an even array, it would be advantageous to extend the selection scheme to odd arrays with as little change as possible. A selection scheme for odd arrays based on the first type of symmetry would not resemble the scheme for even arrays since there are positions which are not four-fold symmetric. The second type of odd symmetry more closely resembles the even symmetry of Chapter Five. It is this type of symmetry which is used for odd arrays.

Paralleling the derivation of Chapter Five, equation B-3 is the odd array form of equation 5-7.

$$\begin{aligned} U_1 &= X_1 \cdot u\cos + Y_1 \cdot u\sin & U_3 &= -X_1 \cdot u\cos - Y_1 \cdot u\sin \\ U_2 &= -Y_1 \cdot u\cos + X_1 \cdot u\sin & U4 &= +Y_1 \cdot u\cos - X_1 \cdot u\sin \end{aligned} \tag{B-3}$$

Therefore,

$$U_1 = -U_3 \qquad U_2 = -U_4 \tag{B-4}$$

is the same as for an even array.

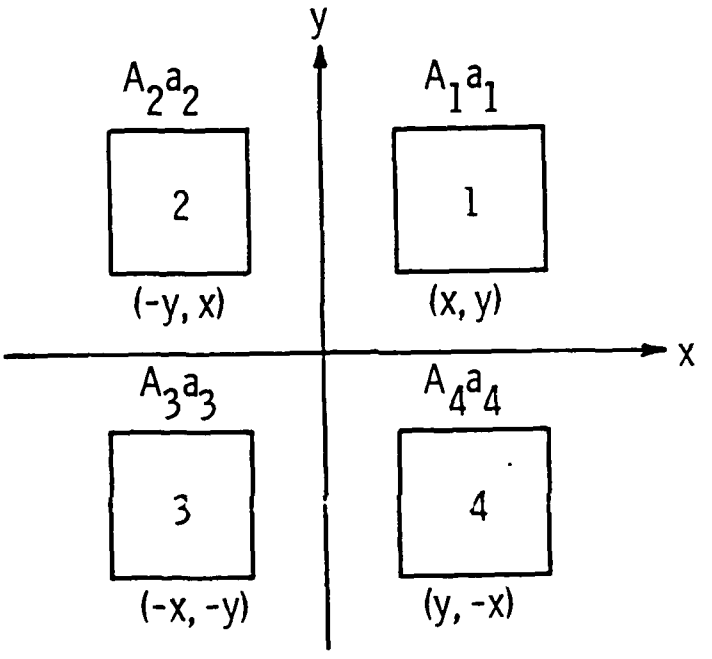


Figure B.2

Four Symmetric Positions of an Odd Array

The difference between an even and an odd array shows up in the odd array equivalent of equations 5-4 and 5-5, which represent the Kendig Coefficients. Since the coefficients do not affect the selection process, the rest of the derivation is not presented.

The Neo-Kendig Selection Scheme for even arrays, outlined in Chapter Five, can be applied, with a small modification to deal with the center element, directly to odd arrays.

The Neo-Kendig technique is based on the inherent symmetry of the arrays considered. If an array with a different type of symmetry is considered, the results of Chapter Five are not applicable. However, a similar derivation, reflecting the different symmetry, would yield a Neo-Kendig technique for that symmetry. The derivation is straightforward and not lengthy.

APPENDIX C

KENDIG COEFFICIENTS AND ARRAY RESPONSE

The Dolph-Chebyshev [12] method is a well known technique used to determine the shading values to give an optimally narrow beam width for a specified uniform side-lobe level for line arrays. The second product theorem [13] extends this result to planar arrays. However, the shading values for other types of response patterns are not generated as easily. Computer routines which use linear and goal programming techniques can be used to generate these values [10], [11].

The use of these programs has revealed a curious phenomenon. Constraining a certain area of the response pattern to a very low side-lobe level will result in an increase in side-lobe level at another area in the response pattern, resembling that which happens to a water balloon when pushed in at one point; it bulges out at another point. The following is a description of the mechanism for this phenomenon, as enlightened by the Neo-Kendig coefficients.

Without loss of generality, the following assumptions can be made.

1) Only ideal elements are considered.
2) There is no phase shading.
3) $k = 1.0$.

Term 1 is the only non-zero Kendig Term. For a 6 by 6 array and an incident angle of $\phi = 75°$ and $\theta = 45°$, the Kendig Coefficients for each

symmetric position are shown in Figure C.1. Each of the coefficients for the symmetric positions is multiplied by four times the shading value for that position, (once for each quadrant). The response of the array is twenty times the logarithm, base 10, of the sum of these products over the symmetric positions with the main beam response as the reference level. If it is assumed, for example, that the response at this incident angle is very low, i.e., a null, the sum of these products over all the symmetric positions is small. The logarithm of a small number, < 1, is a negative number, and in this case, represents the dB down from the main beam response.

At a different incident angle, $\phi = 5°$ and $\theta = 60°$, which is not in the neighborhood of the first, the Kendig Coefficients at each symmetric position will have different values, (Figure C.2). Therefore, the sum of the products at the second angle will not be the same as the sum for the first angle. If the first sum was very small, i.e., near zero, this second sum will be, in most cases including this example, greater than the first. The logarithm of this larger sum will result in a response level which is not as low as for the smaller sum.

This explains why driving the response to a very low level in one region of the response plane may degrade the response in another region.

PHI = 75^0
ucos = .1830

THETA = 45^0
usin = .6830

k = 1

2.5	-.1357	-.1312	-.1223
1.5	.5232	.5001	.4661
y=.5	.9383	.9070	.8454
	x=.5	1.5	2.5

Figure C.1

Kendig Coefficients for $\theta = 45°$, $\phi = 75°$

PHI = 5^0 THETA = 60^0 k = 1

ucos = .8627 usin = .0755

2.5	.8923	.2684	-.5432
1.5	.9026	.2715	-.5495
y=.5	.9078	.2730	-.5526
	x=.5	1.5	2.5

Figure C.2

Kendig Coefficients for $\theta = 60°$, $\phi = 5°$

APPENDIX D

LISTING OF PERMUTATION SEARCH COMPUTER PROGRAM

```
C*********************************************************************
C                          PERMSER                                  *
C        PERMUTATION SEARCH COMPUTER ALGORITHM                      *
C               WRITTEN BY KING W. WIEMANN                          *
C*********************************************************************
      COMMON/COMM02/ Y
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM05/ IEV
      COMMON/COMM11/ NACH,NPRIOR,NOBJ
      COMMON/COMM12/ ZBEST
      COMMON/COMM17/ MM,NPOOL,IJ
      DIMENSION ZBEST(10),Y(3000),IMR(200),Z(10)
      INTEGER RER(200),RERR(200)
      CALL DATAINN
C  DETERMINE THE INITIAL ACHIEVEMENT LEVEL.
      CALL ZVALUE(Z)
      DO 10 KK=1,NPRIOR
      ZBEST(KK)=Z(KK)
 10   CONTINUE
C  WRITE THE INITIAL VALUES OF THE SEARCH.
      WRITE(4,500)
 500  FORMAT(1X,'THE INITIAL PERMUTATION IS'//5X,'ELEMENT NUMBER',5X,
     1'POSITION')
      DO 600 II=1,IJ
      WRITE(4,501) RERR(RER(II)),IMR(II)
 501  FORMAT(T11,I3,T26,I3)
 600  CONTINUE
      WRITE(4,507) NACH
 507  FORMAT(1X,'THE NUMBER OF OBJECTIVES IS',2X,I6)
      WRITE(4,502)
 502  FORMAT(1X,'THE INITIAL ACHIEVEMENT VECTOR IS'//5X,'PRIORITY LEV
     1EL',3X,'ACHIEVEMENT')
      DO 601 II=1,NPRIOR
      WRITE(4,503) II,ZBEST(II)
 503  FORMAT(9X,I2,13X,F10.4)
 601  CONTINUE
C  BEGIN THE SEARCH BY CALLING SUBROUTINE PERMUTE.
      CALL PERMUTE(&100,&200,&300)
C  WHEN THE SEARCH ROUTINE IS TERMINATED, CONTROL RETURNS TO THE MAIN
C  ROUTINE WITH A VALUE OF KK1, INDICATING THE DEGREE OF SUCCESS.
C  KK1=1 IS WHEN A COMPLETE CYCLE OF PERMUTATIONS HAS BEEN COMPLETED
C  WITHOUT AN IMPROVEMENT.
 100  KK1=1
```

```
      GOTO 110
C  KK1=2 IS WHEN THE NUMBER OF CYCLES EQUALS THE MAXIMUM NUMBER OF
C  ITERATIONS, MAXITE.
 200  KK1=2
      GOTO 110
C  KK1=3 IS WHEN THE ACHIEVEMENT VECTOR EVALUATES TO ZERO, e.g. ALL
C  THE OBJECTIVES HAVE BEEN COMPLETELY SATISFIED.
 300  KK1=3
 110  CALL WRITE(KK1)
      STOP
      END

      SUBROUTINE PERMUTE(*,*,*)
C  THIS ROUTINE CONTROLS THE EXCHANGE OF ADJACENT ELEMENTS IN THE
C  PERMUTATION ARRAY.
C  K IS THE PERMUTATION INDEX DURING EACH PASS THROUGH THE PERMUTATION.
C  ARRAY. IEV=1 IS THE INDICATOR OF SUCCESS OF EACH TRIAL POINT.
C  L INDICATES THE NUMBER OF PASSES THROUGH THE PERMUTATION ARRAY.
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM05/ IEV
      COMMON/COMM06/ MAXITE
      COMMON/COMM12/ ZBEST
      COMMON/COMM15/ L,KK
      COMMON/COMM16/ NPRINT,NCOUNT
      COMMON/COMM17/ MM,NPOOL,IJ
      COMMON/COMM18/ LSEED,ZLOLIM
      DIMENSION S(200),IMR(200),ZBEST(10)
      INTEGER RER(200),S,P,RERR(200),P1,RERRT(200)
      KK=1
      IEV=1
      L=1
      S(L)=0
C  THE FIRST EXCHANGE
 1    P=RER(KK)
      RER(KK)=RER(KK+1)
      RER(KK+1)=P
      CALL EVAL(&10,&200)
 10   KK=KK+1
      IF(IEV.EQ.1) S(L)=S(L)+1
      IF(KK.EQ.IJ) GOTO 100
      IF(IEV.EQ.1) GOTO 50
C  IF THE PREVIOUS EXCHANGE RESULTED IN NO IMPROVEMENT, THE PREVIOUS BASE
C  POINT IS RETURNED TO BEFORE THE NEXT EXCHANGE IS EFFECTED.
      P=RER(KK-1)
      RER(KK-1)=RER(KK)
      RER(KK)=RER(KK+1)
      RER(KK+1)=P
      CALL EVAL(&10,&200)
C  IF THE PREVIOUS EXCHANGE RESULTED IN A NEW BASE POINT, THE NEXT
C  EXCHANGE IS FROM THE NEW BASE POINT.
```

```
 50   P=RER(KK)
      RER(KK)=RER(KK+1)
      RER(KK+1)=P
      CALL EVAL(&10,&200)
C  IF THE EXCHANGE BETWEEN THE LAST TWO ELEMENTS RESULTS IN NO NEW
C  BASE POINT, THE PREVIOUS BASE POINT IS RETURNED TO.
 100  IF(IEV.EQ.1) GOTO 120
      P=RER(KK-1)
      RER(KK-1)=RER(KK)
      RER(KK)=P
 120  L=L+1
      IF(L.GT.MAXITE) GOTO 175
      IF(S(L).EQ.S(L-1)) GOTO 150
      KK=1
      GOTO 1
C  IF A COMPLETE CYCLE OF PERMUTATIONS HAS NOT BEEN SUCCESSFUL
 150  IF(L.NE.2) GOTO 152
C  IF THE PERMUTATIONS FROM THE INITIAL BASE POINT DO NOT RESULT IN
C  ANY IMPROVEMENTS;
      WRITE(4,501)
 501  FORMAT(1X,'THERE IS NO IMPROVEMENT ON THE INITIAL PERMUTATION')
C  IF THE BEST SOLUTION IS NOT LESS THAN ZLOLIM, A NEW START IS
C  INTIATED. THIS ROUTINE IS CALLED ONLY 6 TIMES. THE SIXTH
C  CALL CAUSES THE PROGRAM TO TERMINATE.
 152  IF(ZBEST(1).GT.ZLOLIM) CALL NEWSTART(&160)
      RETURN 1
 160  IF(NPRINT.NE.1) GOTO 503
      WRITE(4,502)
 502  FORMAT('0A NEW SEARCH PATTERN IS NOW BEING USED.'/' THE ELEMENTS A
     1RE IN THE FOLLOWING POSITIONS;'/5X,'ELEMENT NUMBER',5X,'POSITION N
     2UMBER')
      DO 503 II=1,IJ
      WRITE(4,504) RERR(RER(II)),IMR(II)
 504  FORMAT(T11,I3,T26,I3)
 503  CONTINUE
      KK=1
      GOTO 1
C  IF THE MAXIMUM NUMBER OF PERMUTATIONS HAVE BEEN PERFORMED.
 175  RETURN 2
C  IF THE ACHIEVEMENT VECTOR HAS ZERO VALUE FOR ALL LEVELS.
 200  RETURN 3
      END

      SUBROUTINE NEWSTART(*)
C  THIS SUBROUTINE EFFECTS A NEW START, KEEPING THE ELEMENTS
C  IN THE SAME TRANSCUCER ARRAY POSITIONS, BUT REARRANGING THE
C  PERMUTATION ARRAY.
      DIMENSION IMR(200),NN(200),IMRR(200)
      INTEGER RER(200),RERT(200)
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM16/ NPRINT,NCOUNT
```

```
      COMMON/COMM17/ MM,NPOOL,IJ
      COMMON/COMM18/ LSEED,ZLOLIM
      DATA NCOUNT/0/
      IF(NCOUNT.GT.4) RETURN
      NCOUNT=NCOUNT+1
      DO 10 II=1,IJ
      NN(II)=0
10    CONTINUE
      LL=1
      ITER=0
1     ITER=ITER+1
      Y=FLOAT(IJ)*RAN(LSEED)+1.0
      NN1=INT(Y)
      IF(LL.NE.1) GOTO 21
      NN(LL)=NN1
      LL=LL+1
      GOTO 1
21    IF(ITER.GT.10*IJ) GOTO 92
      DO 22 II=1,LL
      IF(NN1.EQ.NN(II)) GOTO 1
22    CONTINUE
      NN(LL)=NN1
      LL=LL+1
      IF(LL.GT.IJ) GOTO 94
      GOTO 1
92    ITER=5*IJ
      GOTO 1
94    DO 30 II=1,IJ
      IMRR(II)=IMR(NN(II))
      RERT(II)=RER(NN(II))
30    CONTINUE
      DO 40 II=1,IJ
      RER(II)=RERT(II)
      IMR(II)=IMRR(II)
40    CONTINUE
      RETURN 1
      END

      SUBROUTINE EVAL(*,*)
C  THIS ROUTINE DETERMINES IF THE CURRENT TRIAL POINT GIVES A BETTER
C  RESPONSE THAN THE CURRENT BASE POINT. IF IT DOES, THE TRIAL POINT
C  BECOMES THE NEW BASE POINT.
C  THIS ROUTINE, WITH ZVALUE, AND SOME OF THE VARIABLE NAMES
C  ARE FROM CHARLES ALVORD'S DOCTORAL THESIS.
      COMMON/COMM04/ K,M,AK
      COMMON/COMM05/ IEV
      COMMON/COMM11/ NACH,NPRIOR,NOBJ
      COMMON/COMM12/ ZBEST
      COMMON/COMM15/ L,KK
      COMMON/COMM16/ NPRINT,NCOUNT
      DIMENSION ZBEST(10),Z(10)
      CALL ZVALUE(Z)
```

```
 58   CONTINUE
C  EVALUATION BY COMPARISON TO THE BEST BASE POINT THUS FAR.
 21   DO 20 JJ=1,NPRIOR
      IF(Z(JJ).GT.ZBEST(JJ)) GOTO 50
      IF(Z(JJ)+.0001.LT.ZBEST(JJ)) GOTO 30
 20   CONTINUE
C  THE TRIAL POINT RESULTS IN AN EQUAL REPONSE TO THE CURRENT BASE POINT.
      IEV=0
      GOTO 50
C  THE TRIAL POINT RESULTS IN A BETTER RESPONSE.
 30   DO 35 JJ=1,NPRIOR
      IF(Z(JJ).NE.0.0) GOTO 40
 35   CONTINUE
C  IF ALL THE OBJECTIVES HAVE BEEN SATISFIED.
      GOTO 60
C  INTITIALIZE THE NEW BASE POINT.
 40   DO 45 JJ=1,NPRIOR
      ZBEST(JJ)=Z(JJ)
 45   CONTINUE
      IEV=1
      IF(NPRINT.NE.1) GOTO 600
      WRITE(4,500) L,KK
 500  FORMAT('0A NEW BASE POINT HAS BEEN FOUND.'/' THE SUCCESSFUL PERMUT
     1ATION OCCURRED DURING CYCLE #',I2,' AND AT PERMUTATION POSITION',I
     23/1X,'THE ACHIEVEMENT VECTOR IS'//5X,'PRIORITY LEVEL',3X,'ACHIEVEM
     3ENT')
      DO 600 II=1,NPRIOR
      WRITE(4,501) II,ZBEST(II)
 501  FORMAT(8X,I4,8X,F10.4)
 600  CONTINUE
      RETURN 1
 50   IEV=0
      RETURN 1
C  CONTROL RETURNS TO PERMUTE INDICATING THAT ALL THE OBJECTIVES HAVE
C  BEEN SATISFIED.
 60   RETURN 2
      END

      SUBROUTINE ZVALUE(Z)
C   THIS ROUTINE DETERMINES THE LEVEL OF ACHIEVEMENT OF THE OBJECTIVES.
      COMMON/COMM02/ Y
      COMMON/COMM11/ NACH,NPRIOR,NOBJ
      COMMON/COMM12/ ZBEST
      COMMON/COMM13/ SIGN,IROW,WEIGHT,RHS,MPRI
      DIMENSION Y(3000),ZBEST(10),Z(10),SIGN(2000),IROW(2000)
      DIMENSION WEIGHT(2000),RHS(2000),MPRI(2000)
      CALL YVALUE
      DO 10 KK=1,NPRIOR
      Z(KK)=0.0
 10   CONTINUE
      DO 20 KK=1,NACH
```

```
      IF(SIGN(KK).LT.0.0) GOTO 40
      DEV=Y(IROW(KK))-RHS(IROW(KK))
      GOTO 50
 40   DEV=RHS(IROW(KK))-Y(IROW(KK))
 50   IF(DEV.LT.0.0) DEV=0.0
      Z(MPRI(KK))=Z(MPRI(KK))+WEIGHT(KK)*DEV
 20   CONTINUE
      RETURN
      END

      SUBROUTINE DATAINN
C  THIS ROUTINE READS IN THE TRANSDUCER ARRAY PARAMETERS.
C  THERE ARE 2 MAIN LOOPS TO READ IN THE DATA, ONE FOR SYMMETRIC
C  AND ONE FOR NON-SYMMETRIC
C  IF THE ARRAY IS FOUR-FOLD SYMMETRIC, THERE
C  ARE 2 ALTERNATE WAYS TO INPUT THE DATA, DEPENDING ON THE EXISTENCE
C  OF AN ARRAY POSITION AT X=0,Y=0.
      COMMON/COMM01/ IM,NEVEN,NROUTE
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM06/ MAXITE
      COMMON/COMM08/ C,D
      COMMON/COMM10/ A,AMP,ALPHA
      COMMON/COMM14/ DOLAM,WAVE
      COMMON/COMM16/ NPRINT,NCOUNT
      COMMON/COMM17/ MM,NPOOL,IJ
      COMMON/COMM18/ LSEED,ZLOLIM
      DIMENSION A(200),AMP(200,200),ALPHA(200,200),C(200),D(200),IMR(200)
      INTEGER RER(200),RERR(200)
      DATA D2RAD/1.74532925 E-02/
      RAD2D=1.0000/D2RAD
      READ(3,499) DOLAM,WAVE
      READ(3,500) M,MAXITE,NROUTE,NPRINT,NPOOL
      IF(NPOOL.EQ.1) READ(3,506) MM
      PI=4.0*DATAN(1.0D00)
      AK=2*PI*DOLAM
C  IF NROUTE.NE.1, THE ARRAY IS ASSYMMETRIC; IF NROUTE.EQ.1, THE ARRAY
C  IS SYMMETRIC.
      IJ=M
      IF(NPOOL.EQ.1) IJ=MM
      IK=IJ
      IF(NROUTE.EQ.1) THEN
        READ(3,504) NEVEN,IM
C  IF THE ARRAY IS 'ODD', (i.e. THERE IS AN ELEMENT AT THE CENTER OF THE
C  ARRAY X=0,Y=0), THE PARAMETERS ARE READ IN DIFFERENTLY FOR A EVEN
C  ARRAY. NEVEN.EQ.1 IS AN ODD ARRAY.
        IF(NEVEN.EQ.1) THEN
          IK=IM+1
C  COORDINATES OF THE CENTER ELEMENT AND THE SHADING VALUE ARE READ IN
C  FIRST.
          READ(3,502) C(1),D(1),A(1)
C  THE COORDINATES AND SHADING VALUES FOR THE SYMMETRIC POSITIONS ARE
```

```
C  READ IN.
          DO 13 II=2,IK
          READ(3,502) C(II),D(II),A(II)
C  THE ENTIRE ARRAY VALUES ARE INITIALIZED USING THE EVEN SYMMETRY.
          C(II+IM)=-D(II)
          D(II+IM)=C(II)
          C(II+2*IM)=-C(II)
          D(II+2*IM)=-D(II)
          C(II+3*IM)=D(II)
          D(II+3*IM)=-C(II)
          A(II+IM)=A(II)
          A(II+2*IM)=A(II)
          A(II+3*IM)=A(II)
 13       CONTINUE
C  IF THE ARRAY IS EVEN.
C  THE COORDINATES AND SHADING VALUES OF THE SYMMETRIC POSITIONS ARE
C  READ IN. THEN THE ENTIRE ARRAY VALUES ARE INITIALIZED ACCORDING
C  TO THE EVEN SYMMETRY.
        ELSE
 12       IK=IM
          DO 17 II=1,IK
          READ(3,502) C(II),D(II),A(II)
          C(II+IM)=-C(II)
          C(II+2*IM)=-C(II)
          C(II+3*IM)=C(II)
          D(II+IM)=D(II)
          D(II+2*IM)=-D(II)
          D(II+3*IM)=-D(II)
          A(II+IM)=A(II)
          A(II+2*IM)=A(II)
          A(II+3*IM)=A(II)
 17       CONTINUE
        ENDIF
C  IF THE ARRAY IS NOT 4-FOLD SYMMETRIC THE COORDINATES, AND SHADING
C  VALUES OF THE ENTIRE ARRAY ARE READ IN.
      ELSE
 11     DO 10 II=1,IJ
        READ(3,502) C(II),D(II),A(II)
 10     CONTINUE
      ENDIF
C  IF THERE ARE ELEMENTS IN THE 'POOL', THEIR EQUIVALENT ARRAY POSITION
C  AND SHADING VALUES MUST ALSO BE SPECIFIED, (i.e. SET EQUAL TO ZERO.)
 14   IF(NPOOL.NE.1) GOTO 22
      DO 21 II=M+1,MM
      C(II)=0.0
      D(II)=0.0
      A(II)=0.0
 21   CONTINUE
C  ELEMENT TOLERANCES ARE READ IN IDENTICALLY FOR ALL LOOPS.
C  THE OUTER LOOP CONTROLS INPUT ACCORDING TO ELEMENT, THE INNER
```

```
C  ACCORDING TO POSITION.
 22   IF(NROUTE.NE.1) THEN
        DO 20 II=1,IJ
        READ(3,506) RERR(II)
        DO 20 JJ=1,IK
        READ(3,503) AMP(II,JJ),ALPHA1
C  CONVERT TOLERANCES FROM DEGREES TO RADIANS.
        ALPHA(II,JJ)=ALPHA1*D2RAD
 20     CONTINUE
      ELSE
        DO 27 II=1,IJ
        READ(3,506) RERR(II)
        DO 28 JJ=1,IK
        READ(3,503) AMP(II,JJ),ALPHA1
        ALPHA(II,JJ)=ALPHA1*D2RAD
C  SINCE THE RESPONSE OF AN ELEMENT WILL BE THE SAME FOR
C  ANY OF THE FOUR SYMMETRIC POSITIONS, ARRAY POSITION MUST BE
C  INITIALIZED.
        IF(NEVEN.EQ.1.AND.JJ.EQ.1) GOTO 28
        AMP(II,JJ+IM)=AMP(II,JJ)
        AMP(II,JJ+2*IM)=AMP(II,JJ)
        AMP(II,JJ+3*IM)=AMP(II,JJ)
        ALPHA(II,JJ+IM)=ALPHA(II,JJ)
        ALPHA(II,JJ+2*IM)=ALPHA(II,JJ)
        ALPHA(II,JJ+3*IM)=ALPHA(II,JJ)
 28     CONTINUE
        DO 27 KK=4*IM+1,MM
        AMP(II,KK)=0.0
        ALPHA(II,KK)=0.0
 27     CONTINUE
      ENDIF
C  READ IN THE STARTING POSITIONS FOR EACH ELEMENT.
      DO 40 JJ=1,IJ
      READ(3,504) IMR(JJ),RER(JJ)
 40   CONTINUE
 42   CALL GOALIN
      READ(3,505) LSEED,ZLOLIM
      CALL NEARIN
 499  FORMAT(2F9.5)
 500  FORMAT(5I5)
 502  FORMAT(3F9.5)
 503  FORMAT(2F9.5)
 504  FORMAT(2I5)
 505  FORMAT(I10,F9.4)
 506  FORMAT(I5)
      RETURN
      END

      SUBROUTINE GOALIN
C  THIS ROUTINE READS IN MATH PROGRAMMING PARAMETERS, SUCH AS THE
C  THETA AND PHI ANGLES THAT ARE CONSTRAINED.
```

```
C  THE GOAL PROGRAMMING PARAMETERS, SUCH AS THE RHS VALUES, THE
C  WEIGHTS, AND SIGN OF THE DEVIATION VARIABLES ARE ALSO INPUT
C  HERE.
      COMMON/COMM07/ THETA,DELTHE
      COMMON/COMM09/ PHI,NPHI
      COMMON/COMM11/ NACH,NPRIOR,NOBJ
      COMMON/COMM13/ SIGN,IROW,WEIGHT,RHS,MPRI
      DIMENSION RHS(2000),SIGN(2000),IROW(2000),WEIGHT(2000)
      DIMENSION MPRI(2000),THETA(100),PHI(40)
      DATA RAD2D/57.2958/,D2RAD/1.74532925 E-02/
C  READ IN THE PHI ANGLES CONSTRAINED.
      READ(3,99) NREAD1
 99   FORMAT(I3)
      IF(NREAD1.EQ.1) THEN
        READ(3,99) NPHI
        DO 103 II=1,NPHI
        READ(3,101) PHI2
        PHI(II)=PHI2*D2RAD
 103    CONTINUE
      ELSE
        READ(3,100) PHI1,DELPHI
        TEM=(360.0-PHI1)/DELPHI
        NPHI=TEM
        TEM1=INT(TEM)/2
        IF(TEM/2.0.NE.TEM1) NPHI=TEM+1.0
        PHI(1)=PHI1*D2RAD
        DELPHI=DELPHI*D2RAD
        DO 11 II=2,NPHI
        PHI(II)=PHI(1)+(II-1)*DELPHI
 11     CONTINUE
      ENDIF
      READ(3,101) DELTHE
      NACH=0
C  READ IN THE THETA VALUES CONSTRAINED FOR EACH PHI.
      DO 10 II=1,NPHI
      READ(3,101) THETA(II)
C  SUM OVER ALL ANGLES CONSTRAINED TO DETERMINE THE NUMBER
C  OF OBJECTIVES.
      NACH=NACH+(90.0-THETA(II))/DELTHE+1
      THETA(II)=THETA(II)*D2RAD
 10   CONTINUE
C  CONVERT DEGREES TO RADIANS.
      DELTHE=DELTHE*D2RAD
      NOBJ=NACH
      READ(3,102) NSTED
C  (NSTED.EQ.1) MEANS THAT SIGN,WEIGHT,MPRI,RHS ARE ALL THE SAME FOR EACH
C  OBJECTIVE.
      IF(NSTED.NE.1) THEN
C  READ IN THE VALUES FOR EACH OBJECTIVE.
        READ(3,102) NPRIOR
        DO 30 II=1,NOBJ
        READ(3,104) MPRI(II),WEIGHT(II),IROW(II),SIGN(II),RHS(II)
```

```
 30     CONTINUE
      ELSE
C  READ IN THE SIDE LOBE LEVEL AND SPECIFY ALL THE OTHER VALUES
        NPRIOR=1
        READ(3,101) RHS1
        DO 60 JJ=1,NOBJ
        MPRI(JJ)=1
        WEIGHT(JJ)=1
        SIGN(JJ)=-1.0
        RHS(JJ)=RHS1
        IROW(JJ)=JJ
 60     CONTINUE
      ENDIF
 100  FORMAT(2F8.4)
 101  FORMAT(F8.4)
 102  FORMAT(I2)
 104  FORMAT(I2,F8.4,I4,2F8.4)
      RETURN
      END

      SUBROUTINE WRITE(KK1)
C  THIS ROUTINE IS CALLED ONLY AFTER THE SEARCH HAS BEEN TERMINATED. IT
C  WRITES OUT ALL PERTINENT DATA BASED ON THE RESULTS OF THE SEARCH.
      COMMON/COMM01/ IM,NEVEN,NROUTE
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM05/ IEV
      COMMON/COMM10/ A,AMP,ALPHA
      COMMON/COMM11/ NACH,NPRIOR,NOBJ
      COMMON/COMM12/ ZBEST
      COMMON/COMM15/ L,KK
      COMMON/COMM16/ NPRINT,NCOUNT
      COMMON/COMM17/ MM,NPOOL,IJ
      DIMENSION IMR(200),AMP(200,200),ALPHA(200,200),ZBEST(10),A(200)
      INTEGER RER(200),RERR(200)
      DATA D2RAD/1.74532925 E-02/
      RAD2D=1.0000/D2RAD
C  KK1 INDICATES THE SUCCESS OF THE SEARCH.
C  THE RESULTS ARE PRINTED ACCORDING TO THE SUCCESS OF THE SEARCH.
      GOTO(10,20,30),KK1
C  IF A COMPLETE CYCLE OF PERMUTATIONS HAS BEEN COMPLETED AND
C  NO IMPROVEMENTS WERE MADE.
 10   WRITE(4,100)
 100  FORMAT(1X,'A COMPLETE CYCLE OF PERMUTATIONS HAS BEEN MADE WITHOUT
     1 ANY'/1X,'IMPROVEMENTS ON THE PREVIOUS BASE POINT.'/)
      GOTO 50
 20   WRITE(4,200)
 200  FORMAT(1X,'THE MAXIMUM NUMBER OF CYCLES OF THE PERMUTATION ALGORITH
     1M HAVE BEEN COMPLETED.')
      GOTO 50
 30   WRITE(4,300)
 300  FORMAT(' THE ACHIEVEMENT FUNCTION IS 0.0.  ALL THE OBJECTIVES HAVE
```

```
     1BEEN SATISFIED'/)
      GOTO 60
50    IF(NPRINT.EQ.1) GOTO 51
      WRITE(4,111) L,NCOUNT
111   FORMAT(' THE NUMBER OF CYCLES COMPLETED IS ',I2,'.  THE NUMBER OF
     1NEW SEARCH PATTERNS USED IS ',I2,'.')
51    WRITE(4,101)
101   FORMAT(1X,'THE ACHIEVEMENT FUNCTION IS'/T15,'PRIORITY LEVEL',5X,'A
     1CHIEVEMENT')
      DO 1 II=1,NPRIOR
      WRITE(4,102) II,ZBEST(II)
102   FORMAT(T17,I2,T33,F12.2)
1     CONTINUE
60    WRITE(4,103)
103   FORMAT(1X,'THE ELEMENTS ARE IN THE FOLLOWING POSITIONS'/5X,'ELEMEN
     1T NUMBER',5X,'POSITION NUMBER')
      DO 2 II=1,IJ
      WRITE(4,104) RERR(RER(II)),IMR(II)
104   FORMAT(T11,I3,T30,I3)
2     CONTINUE
C
C  THIS WRITES THE ELEMENT TOLERANCES INTO THE FILE CREATED BY THE
C  SUBROUTINE NEARIN, SO THAT NEARPRES CAN READ THAT FILE DURING
C  EXECUTION. THE PORTION OF THE FILE WRITTEN HERE IS THE BEST SOLUTION
C  FOUND BY THE PERMUTATION SEARCH ALGORITHM.
C
500   DO 601 II=1,IJ
      WRITE(2,602) IMR(II)
601   CONTINUE
602   FORMAT(I4)
      DO 600 II=1,IJ
      WRITE(2,700) AMP(RER(II),IMR(II)),ALPHA(RER(II),IMR(II))*RAD2D
700   FORMAT(F9.5,',',F9.5)
600   CONTINUE
900   WRITE(2,403)
403   FORMAT('0')
      RETURN
      END

      SUBROUTINE YVALUE
C  THIS ROUTINE DETERMINES THE RESPONSE OF THE ARRAY FOR EACH THETA AND
C  PHI THAT IS CONSTRAINED, AS MEASURED AGAINST THE MAIN LOBE RESONSE.
      COMMON/COMM02/ Y
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM07/ THETA,DELTHE
      COMMON/COMM08/ C,D
      COMMON/COMM09/ PHI,NPHI
      COMMON/COMM10/ A,AMP,ALPHA
      COMMON/COMM17/ MM,NPOOL,IJ
      COMMON/COMM19/ DNORM
```

```
      DIMENSION IMR(200),Y(3000),C(200),D(200),A(200),AMP(200,200)
      DIMENSION ALPHA(200,200),PHI(40),THETA(100)
      INTEGER RER(200)
      DATA RAD2D/57.2958/,D2RAD/1.74532925 E-02/
      REAL*8 SUMR,SUMI,DCOS,DSIN,U
C  ZZ1 REPRESENTS THE RESPONSE OF THE MAIN BEAM. ZZ IS THE RESPONSE AT A
C  SPECIFIC THETA AND PHI. ZZ1-ZZ REPRESENTS THE DIFFERENCE (IN dB )
C  BETWEEN THEM.
      CALL INITIALIZE(ZZ1)
      KK=0
      I=1
      DO 50 JJ=1,NPHI
C  EACH THETA(I) REPRESENTS THE THETA ANGLE WHERE THE SIDE LOBE STARTS.
C  FOR EACH1 PHI CONSTRAINED.
      THETA1=THETA(I)
 11   SUMR=0.0
      SUMI=0.0
      USIN=SIN(THETA1)*SIN(PHI(JJ))
      UCOS=SIN(THETA1)*COS(PHI(JJ))
C  SUM OVER ALL THE ELEMENTS TO DETERMINE THE RESPONSE OF THE ARRAY.
      DO 20 II=1,IJ
      IF(A(IMR(II)).EQ.0.0) GOTO 20
      U1=C(IMR(II))*UCOS
      U2=D(IMR(II))*USIN
      U=DBLE(AK*(U1+U2)+ALPHA(RER(II),IMR(II)))
      H=A(IMR(II))*AMP(RER(II),IMR(II))
      SUMR=SUMR+H*DCOS(U)
      SUMI=SUMI+H*DSIN(U)
 20   CONTINUE
      KK=KK+1
      AMPL=SNGL(SUMR*SUMR+SUMI*SUMI)
      AMPL=SQRT(AMPL)
      SAMP=AMPL/DNORM
      IF(ABS(SAMP).GT.1.0592E-5) THEN
        ZZ=20*ALOG10(SAMP)
      ELSE
 13     ZZ=-99.0
      ENDIF
C  SUBTRACT THE SIDE LOBE RESPONSE FROM THE MAIN BEAM RESPONSE.
C  THIS GIVES THE VALUE OF THE LEFT HAND SIDE OF THE OBJECTIVE.
 14   Y(KK)=ZZ1-ZZ
C  INCREMENT THE THETA ANGLE.
      THETA1=THETA1+DELTHE
C  IF THETA EXCEEDS 90.0 DEGREES, INCREMENT THE PHI ANGLE.
      IF(THETA1*RAD2D.GT.90.0) GOTO 45
      GOTO 11
C  INCREMENT THE PHI ANGLE AND THE THETA ANGLE INDEX, I.
 45   I=I+1
 50   CONTINUE
C  IF THE PHI ANGLE EXCEEDS 360.0 DEGREES, THE RESPONSE AT ALL
```

```
C  CONSTRAINED ANGLES HAS BEEN DETERMINED.
      RETURN
      END

      SUBROUTINE INITIALIZE(ZZ1)
C  THIS SUBROUTINE DETERMINES THE RESPONSE OF THE ARRAY ON THE MAINBEAM
C  i.e. THETA = 0.0, AND PHI = 0.0
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM08/ C,D
      COMMON/COMM10/ A,AMP,ALPHA
      COMMON/COMM17/ MM,NPOOL,IJ
      COMMON/COMM19/ DNORM
      DIMENSION IMR(200),C(200),D(200),A(200),AMP(200,200)
      DIMENSION ALPHA(200,200)
      INTEGER RER(200)
      DATA RAD2D/57.2958/,D2RAD/1.74532925 E-02/
      DATA INDEX/0/
      REAL*8 SUMR,SUMI,DCOS,DSIN,U
      KK=1
      IF(INDEX.GT.0) GOTO 23
      DNORM=0.0
      DO 23 II=1,M
      DNORM=DNORM+A(II)
      INDEX=INDEX+1
 23   CONTINUE
      THETA2=0.0
      PHI2=0.0
      SUMR=0.0
      SUMI=0.0
      USIN=SIN(THETA2)*SIN(PHI2)
      UCOS=SIN(THETA2)*COS(PHI2)
C  SUM OVER ALL THE ELEMENTS TO DETERMINE THE RESPONSE OF THE ARRAY.
      DO 20 II=1,IJ
      IF(A(IMR(II)).EQ.0.0) GOTO 20
      U1=C(IMR(II))*UCOS
      U2=D(IMR(II))*USIN
      U=DBLE(AK*(U1+U2)+ALPHA(RER(II),IMR(II)))
      H=A(IMR(II))*AMP(RER(II),IMR(II))
      SUMR=SUMR+H*DCOS(U)
      SUMI=SUMI+H*DSIN(U)
 20   CONTINUE
      AMPL=SNGL(SUMR*SUMR+SUMI*SUMI)
      AMPL=SQRT(AMPL)
      SAMP=AMPL/DNORM
      IF(ABS(SAMP).GT.1.0592E-5) THEN
        ZZ1=20*ALOG10(SAMP)
      ELSE
 13     ZZ1=-99.0
      ENDIF
      RETURN
      END
```

```
      SUBROUTINE NEARIN
C  NEARIN WRITES THE TRANSCUCER ARRAY PARAMETERS IN AN OUTPUT FILE IN A
C  FORMAT WHICH IS COMPATIBLE WITH THE INPUT FORMAT FOR THE NEARPRES
C  PROGRAM. THIS ROUTINE CONTROLS ONLY THE INITIAL PARAMETERS.
C  THE PARAMETERS OF THE FINAL SOLUTION ARE PRINTED BY THE WRITE
C  SUBROUTINE THIS ROUTINE INCORPORATES THE SYMMETRY OF THE ARRAY INTO
C  THE OUTPUT FILE.
      COMMON/COMM01/ IM,NEVEN,NROUTE
      COMMON/COMM03/ RER,IMR,RERR
      COMMON/COMM04/ K,M,AK
      COMMON/COMM08/ C,D
      COMMON/COMM10/ A,AMP,ALPHA
      COMMON/COMM14/ DOLAM,WAVE
      COMMON/COMM17/ MM,NPOOL,IJ
      DIMENSION IMR(200),A(200),AMP(200,200),ALPHA(200,200),C(200),D(200)
      INTEGER RER(200)
      DATA D2RAD/1.74532925 E-02/
      RAD2D=1.0000/D2RAD
      IF(NPOOL.EQ.1) THEN
        WRITE(2,400) NPOOL,MM
 400    FORMAT('1,1,1'/'-90.0,90.0,2.0'/'0.0,180.0,5.0'/I2/I3)
      ELSE
        WRITE(2,401) NPOOL
 401    FORMAT('1,1,1'/'-90.0,90.0,2.0'/'0.0,180.0,5.0'/I2)
      ENDIF
 500  WRITE(2,711) NROUTE,M
 711  FORMAT(' 2, 2,',I2,',',I2,', 1')
C  WRITE THE ARRAY PARAMETERS USING FOUR-FOLD SYMMETRY.
      IF(NROUTE.EQ.1) THEN
        N1L=INT(SQRT(FLOAT(IM)))
        IF(NEVEN.EQ.1) THEN
          N2L=N1L+1
        ELSE
 714      N2L=N1L
        ENDIF
 719    WRITE(2,712) IM,NEVEN,N1L,N2L
 712    FORMAT(I2,',',I2,',',I2,',',I2)
        IF(NEVEN.NE.1) THEN
C  WRITE THE ARRAY PARAMETERS USING EVEN SYMMETRY.
C  WRITE THE SHADING
          DO 601 II=1,IM
          WRITE(2,702) A(II)
 702      FORMAT(F8.4)
 601      CONTINUE
C  WRITE THE ELEMENT TOLERANCES.
 901      DO 902 II=1,IJ
          WRITE(2,302) IMR(II)
 902      CONTINUE
          DO 603 II=1,IJ
          WRITE(2,700) AMP(RER(II),IMR(II)),ALPHA(RER(II),IMR(II))*RAD
```

```
     12D
 603      CONTINUE
 700      FORMAT(F9.5,',',F9.5)
C  WRITE THE COORDINATES.
          DO 599 II=1,IM
          WRITE(2,701) C(II),D(II)
 701      FORMAT(F6.3,',',F6.3)
 599      CONTINUE
          WRITE(2,725) WAVE,DOLAM
 725      FORMAT(F8.4/F8.4)
C  WRITE THE ARRAY PARAMETERS USING ODD SYMMETRY.
        ELSE
 710      DO 721 II=1,IM+1
          WRITE(2,702) A(II)
 721      CONTINUE
          DO 903 II=1,IJ
          WRITE(2,302) IMR(II)
 903      CONTINUE
 302      FORMAT(I4)
          DO 724 II=1,IJ
          WRITE(2,700) AMP(RER(II),IMR(II)),ALPHA(RER(II),IMR(II))*RAD
     12D
 724      CONTINUE
C  WRITE THE COORDINATES.
          DO 723 II=1,IM+1
          WRITE(2,701) C(II),D(II)
 723      CONTINUE
          WRITE(2,725) WAVE,DOLAM
        ENDIF
      ELSE
C  WRITE THE PARAMETERS FOR A NON-SYMMETRIC ARRAY.
        DO 801 II=1,IJ
        WRITE(2,702) A(II)
 801    CONTINUE
        DO 909 II=1,IJ
        WRITE(2,302) IMR(II)
 909    CONTINUE
        DO 904 II=1,IJ
        WRITE(2,700)  AMP(RER(II),IMR(II)),ALPHA(RER(II),IMR(II))*RAD
     12D
 904    CONTINUE
        DO 803 II=1,IJ
        WRITE(2,701)  C(II),D(II)
 803    CONTINUE
        WRITE(2,804) WAVE
 804    FORMAT(F8.4)
      ENDIF
 900  RETURN
      END
```

APPENDIX E

LISTING OF NEO-KENDIG COMPUTER SELECTION PROGRAM

```
C*********************************************************************
C                              H K V                                  *
C  COMPUTER IMPLEMENTATION OF THE NEO-KENDIG SELECTION TECHNIQUE      *
C                  WRITTEN BY KING W. WIEMANN                         *
C*********************************************************************
      INTEGER U(200,2)
      REAL A(2),V(2)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM07/ A,V
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      CALL DATAININ
      CALL RESPONSE(1)
      CALL STAT(1)
      CALL DELETE
      CALL STAT(1)
      WRITE(4,500) 100.*A(1),A(2)
 500  FORMAT(5X,'AMPLITUDE   MEAN   = ',F12.8,' PHASE   MEAN   = ',F12.8
     1)
      WRITE(4,501) 100.*V(1),V(2)
 501  FORMAT(5X,'AMPLITUDE VARIANCE = ',F12.8,' PHASE VARIANCE = ',F12.8
     1/)
      WRITE(4,502)
 502  FORMAT(90X,'GROUP',5X,'POSITION :',3X,'1',5X,'2',5X,'3',5X,'4')
      DO 10 II=1,IM1
      IF (II.GT.1) CALL RESPONSE(II)
      CALL REMOVE(II)
      IF(II.EQ.1.AND.NEVEN.EQ.1) GOTO 10
      CALL OPTIMO(II)
      CALL WRITEIT(II)
 10   CONTINUE
      CALL WRITEIT(0)
      CALL NEARIN1
      CALL PERMSERIN
      STOP
      END

      SUBROUTINE CMAX(D1,S)
C  ROUTINE TO DETERMINE THE MAXIMUM VALUE OF C. USED FOR SORTING THE
C  REMAINING ELEMENTS.
      INTEGER S
      DIMENSION C(20)
      COMMON/COMM17/ C
      D1=1.0E-12
      DO 10 LL=1,S
      IF (C(LL).GT.D1) D1=C(LL)
 10   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE DATAININ
C   ROUTINE TO READ IN THE PARAMETERS NECESSARY TO IMPLEMENT THE
C   NEO-KENDIG TECHNIQUE.
      REAL*16 S(200,9)
      INTEGER U(200,2),OSS(40),TERM(3),RER(200),RERR(200),IMR(200)
      REAL RA(40),XA(40),X(70),Y(70),SHADE(70)
      CHARACTER*1 NAME(40)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM06/ OSS,TERM
      COMMON/COMM6A/ NAME
      COMMON/COMM08/ RA,XA,F,SIZE
      COMMON/COMM8A/ S
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      COMMON/COMM14/ WAVE,DOLAM
      COMMON/COMM15/ X,Y,SHADE
      READ(3,500) MM,M,IM,IM1,NEVEN
 500  FORMAT(5I5)
C  OSS IS THE ORDER OF SEARCH.  IT SPECIFIES IN WHICH ORDER THE ARRAY
C  IS TO BE FILLED.
      READ(3,503) (OSS(JJ),JJ=1,IM1)
 503  FORMAT(20I3)
C  TERM CONTROLS WHICH KENDIG TERMS ARE TO BE THE CRITERIA.
C  TERMS 3, 4 AND 8 ARE ALLOWED IN ANY GROUPING OF 1, 2, OR 3 TERMS.
      READ(3,505) (TERM(JJ),JJ=1,3)
 505  FORMAT(3I2)
      READ(3,501) F,SIZE
 501  FORMAT(2F10.4)
      READ(3,508) LSEED
 508  FORMAT(I9)
C  U IS THE NUMBER ASSIGNED TO EACH ELEMENT; S CONTAINS THE
C  EQUIVALENT CIRCUIT VALUES.
      DO 10 JJ=1,MM
      U(JJ,2)=0
      READ(3,502) U(JJ,1),(S(JJ,LL),LL=2,7)
 502  FORMAT(I5,6D15.9)
 10   CONTINUE
C  RA, AND XA ARE THE REAL AND IMAGINARY RADIATION LOADINGS FOR
C  EACH POSITION. NAME IS THE LETTER ASSIGNED TO EACH SYMMETRIC
C  POSITION.
      DO 30 JJ=1,IM1
      READ(3,504) RA(OSS(JJ)),XA(OSS(JJ)),NAME(OSS(JJ))
 504  FORMAT(2E12.4,A1)
 30   CONTINUE
C  N1L AND N2L SPECIFY THE LENGTH AND WIDTH OF THE FIRST QUADRANT,
C  (IN NUMBER OF ELEMENTS).
      N1L=INT(SQRT(FLOAT(IM)))
      IF(NEVEN.EQ.1) THEN
        N2L=N1L+1
      ELSE
        N2L=N1L
      ENDIF
```

```
C  RER IS THE SEQUENTIAL NUMBER OF EACH ELEMENT OF THE ARRAY IN ARRAY
C  ORDER.
C  RERR IS THE ASSIGNED NUMBER OF EACH ELEMENT OF THE ARRAY IN ARRAY
C  ORDER.
C  IMR IS THE ORDER OF TRANSDUCER ARRAY POSITIONS IN THE PERMUTATION
C  ARRAY.
      DO 40 KK=1,MM
      RER(KK)=0
      RERR(KK)=0
      IMR(KK)=KK
 40   CONTINUE
      N=0
      IF(NEVEN.EQ.1) N=1
C  X AND Y ARE THE X AND Y COORDINATES OF EACH SYMMETRIC POSITION,
C  AND SHADE IS THE SHADING VALUE FOR EACH POSITION.
      DO 50 JJ=1,IM+N
 50   READ(3,507) X(JJ),Y(JJ),SHADE(JJ)
 507  FORMAT(3F8.4)
      READ(3,508) WAVE,DOLAM
 508  FORMAT(2F9.4)
      RETURN
      END

      SUBROUTINE DELETE
C  ROUTINE TO PERMANENTLY DELETE ELEMENTS FROM THE SELECTION POOL BEFORE
C  THE SELECTION PROCESS BEGINS.
      INTEGER U(200,2)
      COMPLEX*16 ELEMNT(200)
      REAL A(2),V(2)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM07/ A,V
      COMMON/COMM16/ ELEMNT
      DO 10 JJ=1,MM
      T1=DREAL(ELEMNT(JJ))-A(1)
      T2=DIMAG(ELEMNT(JJ))-A(2)
      IF (T1.LT.4.0*V(1)) GOTO 10
      IF (T2.LT.4.0*V(2)) GOTO 10
      WRITE(4,100) U(JJ,1)
 100  FORMAT(' ELEMENT NUMBER ',I3,' WAS DELETED FROM CONSIDERATION')
      U(JJ,1)=0
 10   CONTINUE
      RETURN
      END

      SUBROUTINE DMAX(D1,S)
C  ROUTINE TO FIND THE MAXIMUM VALUE OF THE D ARRAY.
C  USED TO FIND THE BEST PERMUTATIONS THAT MEET THE KENDIG TERM
C  UNDER CONSIDERATION.
      INTEGER S
      DIMENSION D(40)
      COMMON/COMM10/ D
```

```
      D1=1.0E-12
      DO 10 KK=1,S
      IF (D(KK).GT.D1) D1=D(KK)
 10   CONTINUE
      RETURN
      END

      SUBROUTINE NEARIN1
C  NEARIN1 WRITES THE TRANSCUCER ARRAY PARAMETERS IN AN OUTPUT FILE IN A
C  FORMAT WHICH IS COMPATIBLE WITH THE INPUT FORMAT FOR THE NEARPRES
C  PROGRAM.
      REAL X(70),Y(70),SHADE(70)
      INTEGER U(200,2),RER(200)
      COMPLEX*16 ELEMNT(200)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      COMMON/COMM14/ WAVE,DOLAM
      COMMON/COMM15/ X,Y,SHADE
      COMMON/COMM16/ ELEMNT
      WRITE(2,500) MM
 500  FORMAT('0,1,1'/'-90.0,90.0,2.0'/'0.0,180.0,5.0'/'1'/I3)
      WRITE(2,501) M
 501  FORMAT('1,0,1,',I2,',1')
C  WRITE THE ARRAY PARAMETERS USING FOUR-FOLD SYMMETRY.
      WRITE(2,502) IM,NEVEN,N1L,N2L
 502  FORMAT(I2,',',I2,',',I2,',',I2)
      IF(NEVEN.NE.1) THEN
C  WRITE THE ARRAY PARAMETERS USING NON-EVEN SYMMETRY.
C  WRITE THE SHADING
        DO 100 JJ=1,IM
        WRITE(2,503) SHADE(JJ)
 503    FORMAT(F7.5)
 100    CONTINUE
C  WRITE THE ELEMENT TOLERANCES.
        DO 110 LL=1,M
        IF(RER(LL).EQ.0) THEN
          WRITE(2,507)
 507      FORMAT(' 0.00000,  0.00000')
          GOTO 110
        ENDIF
        DO 110 KK=1,MM
        IF(U(KK,1).NE.RER(LL)) GOTO 110
        WRITE(2,504) 100.*DREAL(ELEMNT(KK)),DIMAG(ELEMNT(KK))
 504    FORMAT(F8.5,',',F9.5)
 110    CONTINUE
C  WRITE THE COORDINATES.
        DO 130 JJ=1,IM
        WRITE(2,505) X(JJ),Y(JJ)
 505    FORMAT(F5.3,',',F5.3)
 130    CONTINUE
```

```
      WRITE(2,506) WAVE,DOLAM
506   FORMAT(F8.4/F8.4/'0')
C  WRITE THE ARRAY PARAMETERS USING EVEN SYMMETRY.
     ELSE
      DO 140 JJ=1,IM+1
140   WRITE(2,503) SHADE(JJ)
      DO 150 LL=1,M
      IF(RER(LL).EQ.0) THEN
        WRITE(2,507)
        GOTO 150
      ENDIF
      DO 150 KK=1,MM
      IF(U(KK,1).NE.RER(LL)) GOTO 150
      WRITE(2,504) 100.*DREAL(ELEMNT(KK)),DIMAG(ELEMNT(KK))
150   CONTINUE
      DO 170 JJ=1,IM+1
      WRITE(2,505) X(JJ),Y(JJ)
170   CONTINUE
      WRITE(2,506) WAVE,DOLAM
     ENDIF
     RETURN
     END

     SUBROUTINE OPTIMO(II)
C  SUBROUTINE TO SELECT FOUR ELEMENTS AT A TIME FOR AN ARRAY POSITION
C  BASED ON THE VALUES OF THE KENDIG TERMS SPECIFIED.
     INTEGER Q,P,O(8,40),TERM(3),U(200,2),RER(200),RERR(200),OSS(20)
     INTEGER T(4),BR(200),S
     DIMENSION IMR(200)
     REAL K(8),A(2),V(2),D(40),AMP(200),PHASE(200)
     COMMON/COMM01/ M,MM,IM,IM1
     COMMON/COMM02/ U
     COMMON/COMM03/ AMP,PHASE
     COMMON/COMM3A/ BR
     COMMON/COMM04/ Q
     COMMON/COMM06/ OSS,TERM
     COMMON/COMM09/ T
     COMMON/COMM10/ D
     COMMON/COMM12/ RER,RERR,IMR
     P=0
     S=1
     D1=0.0
C  GENERATE ALL POSSIBLE PERMUTATIONS OF 4 ELEMENTS FROM THE
C  ELEMENTS REMAINING IN THE POOL AND FIND THE 40 BEST PERMUTATIONS.
     DO 10 K1=1,Q
     DO 20 L1=1,Q
     IF (L1.EQ.K1) GOTO 20
     DO 30 M1=1,Q
     IF (M1.EQ.L1.OR.M1.EQ.K1) GOTO 30
     DO 40 N1=1,Q
     IF (N1.EQ.M1.OR.N1.EQ.L1.OR.N1.EQ.K1) GOTO 40
     P=1+P
```

```
      CALL SETU(S,U1,K1,L1,M1,N1)
      IF (P.LE.40) THEN
        D(P)=U1
        O(1,P)=K1
        O(2,P)=M1
        O(3,P)=L1
        O(4,P)=N1
        GOTO 40
      ENDIF
 50   CALL DMAX(D1,40)
      IF (U1.LE.D1) THEN
        DO 60 JJ=1,40
        IF (D(JJ).EQ.D1) THEN
          D(JJ)=U1
          O(1,JJ)=K1
          O(2,JJ)=M1
          O(3,JJ)=L1
          O(4,JJ)=N1
          GOTO 40
        ENDIF
 60     CONTINUE
      ENDIF
 40   CONTINUE
 30   CONTINUE
 20   CONTINUE
 10   CONTINUE
C  THIS STEP IS PERFORMED ONLY IF TWO TERMS ARE THE SELECTION CRITERIA
      IF (TERM(3).EQ.0) THEN
        IN=40
        DO 239 JJ=1,40
        O(5,JJ)=O(1,JJ)
        O(6,JJ)=O(2,JJ)
        O(7,JJ)=O(3,JJ)
        O(8,JJ)=O(4,JJ)
 239    CONTINUE
        GOTO 200
      ENDIF
      S=1+S
      P=0
C  GENERATE AND TEST THE VALUES OF THE SECOND TERM FOR ALL THE
C  PERMUTATIONS OF THE FIRST TERM.
      D1=0.0
      DO 80 JJ=1,40
      CALL SETU(S,U1,O(I,JJ),O(3,JJ),O(2,JJ),O(4,JJ))
      P=P+1
      IF (P.LE.8) THEN
        D(P)=U1
        O(5,P)=O(1,JJ)
        O(6,P)=O(2,JJ)
        O(7,P)=O(3,JJ)
        O(8,P)=O(4,JJ)
```

```
        GOTO 80
      ENDIF
      CALL DMAX(D1,8)
      IF (U1.LE.D1) THEN
        DO 100 KK=1,8
        IF (D(KK).EQ.D1) THEN
          D(KK)=U1
          O(5,KK)=O(1,JJ)
          O(6,KK)=O(2,JJ)
          O(7,KK)=O(3,JJ)
          O(8,KK)=O(4,JJ)
          GOTO 80
        ENDIF
 100    CONTINUE
      ENDIF
 80   CONTINUE
      IN=8
C  BASED ON THE VALUES OF THE FIRST TERM, (OR FIRST TWO TERMS),
C  SELECT FOUR ELEMENTS ON THE BASIS OF THE LAST TERM.
 200  W=1.0E12
      S=1+S
      DO 110 JJ=1,IN
      CALL SETU(S,U1,O(5,JJ),O(7,JJ),O(6,JJ),O(8,JJ))
      IF (U1.LT.W) THEN
        W=U1
        T(1)=O(5,JJ)
        T(2)=O(6,JJ)
        T(3)=O(7,JJ)
        T(4)=O(8,JJ)
         ENDIF
 110  CONTINUE
C  FOR THE FIRST SELECTION, SPECIFY THE REFERENCE POINT.
      IF (II.EQ.1) CALL SETK
C  RESTORE ALL THOSE ELEMENTS WHICH WERE TEMPORARILY REMOVED.
      CALL PREPNSET
C  STORE BOTH THE SELECTED ELEMENTS' SEQUENTIAL AND ASSIGNED NUMBERS
C  IN ARRAY ORDER.
      RER(OSS(II))=BR(T(1))
      RER(OSS(II)+IM)=BR(T(2))
      RER(OSS(II)+2*IM)=BR(T(3))
      RER(OSS(II)+3*IM)=BR(T(4))
      NN=0
      DO 120 LL=1,4
      DO 130 KK=1,MM
      IF(U(KK,1).EQ.BR(T(LL))) THEN
        RERR(OSS(II)+(LL-1)*IM)=KK
        IF(NN.EQ.3) GOTO 140
        NN=NN+1
        GOTO 120
      ENDIF
 130  CONTINUE
 120  CONTINUE
```

```
140   RETURN
      END

      SUBROUTINE PERMSERIN
C  ROUTINE TO WRITE THE ELEMENT AND ARRAY PARAMETERS TO A FILE
C  TO BE READ BY PERMSER.
      INTEGER RER(200),RERR(200),IMR(200),U(200,2)
      REAL AMP1(200,40),PHASE1(200,40),X(70),Y(70),SHADE(70)
      REAL THETA(40)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM11/ AMP1,PHASE1
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      COMMON/COMM14/ WAVE,DOLAM
      COMMON/COMM15/ X,Y,SHADE
      WRITE(7,500) DOLAM,WAVE,M,MM,NEVEN,IM
500   FORMAT(F8.4,F8.4/I3,',10,1,0,1'/I3/I2,',',I3)
      N=0
      IF(NEVEN.EQ.1) N=1
      DO 5 JJ=1,IM+N
  5   WRITE(7,501) X(JJ),Y(JJ),SHADE(JJ)
501   FORMAT(F5.3,',',F5.3,',',F7.5)
      DO 20 JJ=1,MM
      WRITE(7,509) U(JJ,1)
509   FORMAT(I3)
      DO 20 LL=1,IM+N
      WRITE(7,504) 100.*AMP1(JJ,LL),PHASE1(JJ,LL)
504   FORMAT(F8.5,',',F9.5)
20    CONTINUE
      CALL SETIMR
      DO 30 KK=1,MM
      WRITE(7,508) IMR(KK),RERR(KK)
508   FORMAT(I3,',',I3)
30    CONTINUE
      READ(3,600) PHI1,DELPHI,DELTHE
600   FORMAT(3F10.5)
      WRITE(7,505) PHI1,DELPHI,DELTHE
505   FORMAT('0'/F6.2,',',F6.2/F6.2)
      TEM=(360.0-PHI1)/DELPHI
      NPHI=TEM
      TEM1=INT(TEM)/2
      IF(TEM/2.0.NE.TEM1) NPHI=TEM+1.0
C  READ IN THE THETA VALUES CONSTRAINED FOR EACH PHI.
      DO 10 II=1,NPHI
10    READ(3,601) THETA(II)
601   FORMAT(F10.5)
      DO 15 II=1,NPHI
15    WRITE(7,506) THETA(II)
506   FORMAT(F6.2)
      READ(3,602) ZLOLIM
602   FORMAT(F12.5)
```

```
      WRITE(7,507) ABS(LSEED/10),ZLOLIM
 507  FORMAT(I9,',',F8.2)
      RETURN
      END

      SUBROUTINE PREPNSET
C  ROUTINE TO RESTORE THOSE ELEMENTS THAT WERE TEMPORARILY
C  DELETED FROM THE SELECTION POOL FOR USE IN SELECTION FOR THE
C  NEXT POSITION.
      INTEGER U(200,2),BR(200),T(4)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM3A/ BR
      COMMON/COMM09/ T
C  PERMANENTLY DELETE THOSE ELEMENTS JUST SELECTED.
      DO 10 JJ=1,MM
      IF (U(JJ,1).EQ.BR(T(1)).OR.U(JJ,1).EQ.BR(T(2)).OR.U(JJ,1).EQ.BR(T(
     13)).OR.U(JJ,1).EQ.BR(T(4))) THEN
      U(JJ,2)=-1
      ENDIF
 10   CONTINUE
      DO 20 JJ=1,MM
      IF (U(JJ,2).EQ.-2) U(JJ,2)=0
 20   CONTINUE
      RETURN
      END

      SUBROUTINE REMOVE(II)
C   ROUTINE TO REMOVE ELEMENTS THAT ARE AWAY FROM THE CURRENT MEAN
C   VALUES FOR ALL BUT THE FIRST POSITION, THESE MEANS ARE THOSE OF THE
C   ELEMENTS OCCUPYING THE FIRST POSITION.
      INTEGER U(200,2),Q,P
      COMPLEX*16 ELEMNT(200)
      REAL K(8)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM04/ Q
      COMMON/COMM05/ K,E
      COMMON/COMM16/ ELEMNT
      DATA K(8)/0.0/
C  K(5-8) ARE THE PARAMETERS THAT CONTROL THE REMOVAL OF ELEMENTS
C  FROM THE SELECTION POOL BASED ON THEIR DISTANCE FROM THE REFERENCE
C  MEAN, WITH A SCALE OF THE REFERENCE VARIANCE.
      K(5)=.25
      K(6)=.10
      K(7)=.05
 1    F=25.0+K(8)
C  P INDICATES WHETHER ELEMENTS HAVE BEEN REMOVED DURING AN ITERATION
C  OF THE REMOVAL CYCLE.
 5    P=0
      DO 10 JJ=1,MM
```

```
      IF (U(JJ,2).NE.0.OR.U(JJ,1).EQ.0) GOTO 10
      T1=(DREAL(ELEMNT(JJ))-K(3))**2
      T2=(DIMAG(ELEMNT(JJ))-K(4))**2
      IF (SQRT(T1+T2).LE.F*E) GOTO 10
      U(JJ,2)=-2
      P=1+P
 10   CONTINUE
      CALL SETQ
C  REPLACE IS CALLED IF THEIR ARE TOO FEW ELEMENTS REMAINING IN THE
C  SELECTION POOL.
      IF (Q.LT.10) CALL REPLACE(&1)
      IF (Q.LE.14) GOTO 50
C  IF THIS IS THE SELECTION FOR THE FIRST POSITION, THE MEAN AND
C  VARIANCE OF THE GROUP MUST BE DETERMINED AFTER ELEMENTS HAVE
C  BEEN REMOVED.
      IF (II.NE.1) GOTO 40
      IF (P.EQ.0) GOTO 40
      CALL STAT(II)
      GOTO 5
 40   IF (F.GT.1.75) F=F-K(5)
      IF (F.LE.1.75.AND.F.GT.1.0) F=F-K(6)
      IF (F.LE.1.0) F=F-K(7)
      GOTO 5
 50   WRITE(4,500) F
 500  FORMAT(5X,'FINAL F = ',F6.2)
      WRITE(4,501) Q
 501  FORMAT(5X,'NUMBER OF ELEMENTS REMAINING = ',I3)
      CALL SORT
      RETURN
      END

      SUBROUTINE REPLACE(*)
C  ROUTINE TO RESTORE ALL ELEMENTS THAT WERE TEMPORARILY DELETED BECAUSE
C  10 OR FEWER ELEMENTS WERE LEFT. ADDITIONALLY, THE INCREMENTS ARE
C  DECREASED.
      REAL K(8)
      INTEGER U(200,2)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM05/ K,E
      K(8)=K(8)+5.0
      K(5)=.1
      K(6)=.075
      K(7)=.025
      DO 10 KK=1,MM
      IF (U(KK,2).EQ.-2) U(KK,2)=0
 10   CONTINUE
      RETURN 1
      END

      SUBROUTINE RESPONSE(L)
C  ROUTINE TO DETERMINE EACH ELEMENTS AMPLITUDE AND PHASE RESPONSE BASED
```

```
C  ON ITS EQUIVALENT CIRCUIT VALUES AND THE RADIATION LOADING FOR
C  EACH ARRAY POSITION.
      INTEGER U(200,2),OSS(40)
      REAL RA(40),XA(40)
      REAL*16 R1,R2,R3,R4,R5,R6,R7,R8,R9,R10,R11,R12,R13
      REAL*16 S(200,9),PI
      COMPLEX*16 ELEMNT(200)
      REAL AMP1(200,40),PHASE1(200,40)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM06/ OSS,TERM
      COMMON/COMM08/ RA,XA,F,SIZE
      COMMON/COMM8A/ S
      COMMON/COMM11/ AMP1,PHASE1
      COMMON/COMM16/ ELEMNT
      PI=4.0Q0*QATAN(1.0Q0)
      DO 20 JJ=1,MM
      R1=S(JJ,2)/(S(JJ,5)-S(JJ,4))
      R2=S(JJ,3)/(2.0Q3*PI*S(JJ,2)*R1)
      R3=1.0Q0/(R2*(2.0Q0*PI*S(JJ,2))**2.0Q0)
      R4=(S(JJ,6)/1.0Q12)-R2
      R5=1.0Q03/S(JJ,3)
      R6=S(JJ,7)
      R7=2.0Q0*PI*QEXT(F)
      R8=(QEXT(SIZE))**2.0Q0
      R9=R8**2.0Q0
      R10=1.0Q0-(R7**2.0Q0)*R3*R4+(R4/R2)-R7*R4*QEXT(XA(OSS(L)))*R9
      R11=R7*R4*(R5+QEXT(RA(OSS(L)))*R9/(R6**2.0Q0))
      R12=R8/(R6*QSQRT(R10**2.0Q0+R11**2.0Q0))
      R13=QATAN2D(R11,R10)
C  AMP1 AND PHASE1 ARE USED AS INPUT TO THE PERMSER FILE.
C  ELEMNT IS USED DURING THE SELECTION PROCESS.
      AMP1(JJ,OSS(L))=SNGL(R12)
      PHASE1(JJ,OSS(L))=SNGL(R13)
      IF (U(JJ,2).EQ.-1) GOTO 20
      ELEMNT(JJ)=DCMPLX(R12,R13)
 20   CONTINUE
      RETURN
      END

      SUBROUTINE SETIMR
C   ROUTINE TO RANDOMIZE THE PERMUTATION ARRAY FOR USE IN PERMSER
C   PROGRAM.
C   THIS ROUTINE APPEARS IN THE NEWSTART SUBROUTINE OF PERMSER.
      INTEGER RER(200),RERR(200),IMR(200),RERT(200),IMRT(200),RERRT(200)
      INTEGER U(200,2),T(200)
      DIMENSION NN(200)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      DO 10 KK=1,MM
```

```
      NN(KK)=0
      T(KK)=U(KK,2)
 10   CONTINUE
      LL=1
      ITER=0
 1    ITER=ITER+1
      Y=FLOAT(MM)*RAN(LSEED)+1.0
      NN1=INT(Y)
      IF(LL.NE.1) GOTO 21
      NN(LL)=NN1
      LL=LL+1
      GOTO 1
 21   IF(ITER.GT.10*MM) GOTO 92
      DO 22 KK=1,LL
      IF(NN1.EQ.NN(KK)) GOTO 1
 22   CONTINUE
      NN(LL)=NN1
      LL=LL+1
      IF(LL.GT.MM) GOTO 94
      GOTO 1
 92   ITER=5*MM
      GOTO 1
C  THOSE PERMUTATION ARRAY SLOTS THAT REPRESENT WORKBENCH POSITIONS
C  MUST BE FILLED BY ELEMENTS THAT HAVE NOT BEEN SELECTED.
 94   DO 39 KL=4*[illegible]1,MM
      DO 35 JJ=1,MM
      IF (T(JJ).EQ.-1) GOTO 35
      RERR(KL)=JJ
      T(JJ)=-1
      GOTO 39
 35   CONTINUE
 39   CONTINUE
      DO 36 JJ=1,MM
      IF(RERR(JJ).EQ.0) THEN
        DO 37 KK=1,MM
        IF (T(KK).EQ.-1) GOTO 37
        RERR(JJ)=KK
        T(KK)=-1
        GOTO 36
 37     CONTINUE
      ENDIF
 36   CONTINUE
C  PLACE EACH ARRAY IN THE SAME RANDOM ORDER.
      DO 30 KK=1,MM
      IMRT(KK)=IMR(NN(KK))
      RERT(KK)=RER(NN(KK))
      RERRT(KK)=RERR(NN(KK))
 30   CONTINUE
      DO 40 KK=1,MM
      RER(KK)=RERT(KK)
      IMR(KK)=IMRT(KK)
      RERR(KK)=RERRT(KK)
```

```
 40   CONTINUE
      RETURN
      END

      SUBROUTINE SETK
C  ROUTINE TO STORE THE MEAN VALUES AND VARIANCES FOR THE FIRST FOUR
C  ELEMENTS SELECTED FOR USE AS A REFERENCE POINT.
      INTEGER T(4)
      REAL K(8),A(2),V(2),AMP(200),PHASE(200)
      COMMON/COMM03/ AMP,PHASE
      COMMON/COMM05/ K,E
      COMMON/COMM07/ A,V
      COMMON/COMM09/ T
C  COMPUTE THE MEANS AND VARIANCES FOR THE FOUR SELECTED ELEMENTS.
      A(1)=0.0
      A(2)=0.0
      V(1)=0.0
      V(2)=0.0
      DO 10 JJ=1,4
      A(1)=A(1)+AMP(T(JJ))/4.0
      A(2)=A(2)+PHASE(T(JJ))/4.0
 10   CONTINUE
      DO 20 JJ=1,4
      V(1)=V(1)+(AMP(T(JJ))-A(1))**2.0
      V(2)=V(2)+(PHASE(T(JJ))-A(2))**2.0
 20   CONTINUE
      V(1)=SQRT(V(1)/3.0)
      V(2)=SQRT(V(2)/3.0)
      WRITE(4,500)
 500  FORMAT(5X,'THE MEANS AND VARIANCES OF THE FIRST FOUR CHOSEN ARE')
      WRITE(4,501) 100.*A(1),A(2),100.*V(1),V(2)
 501  FORMAT(5X,'AMPLITUDE MEAN: ',F12.8,'  PHASE MEAN: ',F12.8/7X,'VARI
     1ANCE: ',F12.8,4X,'VARIANCE: ',F12.8)
C  INITIALIZE THE REFERENCE MEANS AND VARIANCES.
      K(1)=V(1)
      K(2)=V(2)
      K(3)=A(1)
      K(4)=A(2)
      E=SQRT(K(1)**2.0+K(2)**2.0)
      RETURN
      END

      SUBROUTINE SETQ
C  ROUTINE TO KEEP TRACK OF HOW MANY ELEMENTS ARE AVAILABLE IN THE
C  SELECTION POOL.
      INTEGER U(200,2),Q
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM04/ Q
      Q=0
      DO 10 JJ=1,MM
      IF (U(JJ,2).EQ.0.AND.U(JJ,1).NE.-1) Q=1+Q
```

```
 10   CONTINUE
      RETURN
      END

      SUBROUTINE SORT
C  ROUTINE TO RANK THE REMAINING ELEMENTS, BEFORE THE OPTIMO
C  SEARCH, ACC^RDING TO THEIR DISTANCES FROM THE REFERENCE
C  MEAN VALUES.
      INTEGER U(200,2),BR(200),D,BRT(200),RER(200),RERR(200)
      COMPLEX*16 ELEMNT(200)
      CHARACTER*1 NAME(40)
      REAL AMP(200),PHASE(200),A(2),V(2),C(20),K(8),AMPT(200)
      REAL PHASET(200),RA(40),XA(40)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM03/ AMP,PHASE
      COMMON/COMM3A/ BR
      COMMON/COMM05/ K,E
      COMMON/COMM6A/ NAME
      COMMON/COMM08/ RA,XA,F,SIZE
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      COMMON/COMM16/ ELEMNT
      COMMON/COMM17/ C
      DATA KL/0/
      KL=KL+1
      D=0
      DO 10 JJ=1,MM
      IF (U(JJ,2).NE.0.OR.U(JJ,1).EQ.0) GOTO 10
      D=1+D
      C(D)=SQRT((DREAL(ELEMNT(JJ))-K(3))**2+(DIMAG(ELEMNT(JJ))-K(4))**2)
      AMP(D)=DREAL(ELEMNT(JJ))
      PHASE(D)=DIMAG(ELEMNT(JJ))
      BR(D)=U(JJ,1)
 10   CONTINUE
      DO 11 JJ=1,D
      CALL CMAX(C1,D)
      DO 12 KK=1,D
      IF(C(KK).EQ.C1) THEN
        I=KK
        GOTO 16
      ENDIF
      IF(C1.EQ.0.0) GOTO 17
 12   CONTINUE
 16   AMPT(D+1-JJ)=AMP(I)
      PHASET(D+1-JJ)=PHASE(I)
      BRT(D+1-JJ)=BR(I)
      C(I)=0.0
 11   CONTINUE
 17   DO 18 JJ=1,D
      AMP(JJ)=AMPT(JJ)
      PHASE(JJ)=PHASET(JJ)
```

```
      BR(JJ)=BRT(JJ)
 18   CONTINUE
C  DETERMINE THE MEANS AND PHASES OF THE ELEMENTS REMAINING IN THE POOL.
      A(1)=0.0
      A(2)=0.0
      V(1)=0.0
      V(2)=0.0
      DO 20 JJ=1,D
      A(1)=AMP(JJ)+A(1)
      A(2)=PHASE(JJ)+A(2)
 20   CONTINUE
      A(1)=A(1)/FLOAT(D)
      A(2)=A(2)/FLOAT(D)
      DO 30 JJ=1,D
      V(1)=(AMP(JJ)-A(1))**2+V(1)
      V(2)=(PHASE(JJ)-A(2))**2+V(2)
 30   CONTINUE
      V(1)=SQRT(V(1)/FLOAT(D-1))
      V(2)=SQRT(V(2)/FLOAT(D-1))
      WRITE(4,102) 100.*A(1),A(2)
 102  FORMAT(5X,'AVERAGES OF CHOSEN ELEMENTS:  AMPLITUDE = ',F12.8,3X,'
     1PHASE = ',F12.8)
      WRITE(4,103) 100.*V(1),V(2)
 103  FORMAT(5X,'VARIANCES OF CHOSEN ELEMENTS: AMPLITUDE = ',F12.8,3X,'
     1PHASE = ',F12.8)
C  IF AN ODD ARRAY IS BEING SELECTED FOR, THE CENTER ELEMENT IS THE
C  ONE WHICH IS CLOSEST TO THE MEAN AND VARIANCE OF THE REMAINING
C  ELEMENTS.
      IF(KL.EQ.1.AND.NEVEN.EQ.1) THEN
        RER(1)=BR(1)
        DO 40 JJ=1,MM
        IF(U(JJ,1).EQ.RER(1)) THEN
          U(JJ,2)=-1
          RERR(1)=JJ
        ELSE
          U(JJ,2)=0
        ENDIF
 40     CONTINUE
C   THE CENTER ELEMENTS MEAN AND THE GROUPS VARIANCE ARE USED AS THE
C   REFERENCE POINT.
        K(3)=AMP(1)
        K(4)=PHASE(1)
        K(1)=V(1)
        K(2)=V(2)
        E=SQRT(K(1)**2+K(2)**2)
        WRITE(4,104) 100.*AMP(1),PHASE(1),NAME(1),BR(1)
 104    FORMAT(5X,'CENTER ELEMENT : AMPLITUDE = ',F12.8,3X,' PHASE = ',F
     112.8/'+',91X,A1,19X,I3)
        WRITE(4,105) RA(1),XA(1)
 105    FORMAT(5X,'RADIATION LOADING:  REAL = ',E12.4,5X,' IMAGINARY =
     1',E12.4//)
      ENDIF
```

```
      RETURN
      END

      SUBROUTINE STAT(II)
C   ROUTINE TO DETERMINE THE MEAN VALUES AND VARIANCES FOR THE ELEMENTS
C   REMAINING IN THE SELECTION POOL.
C   ADDITIONALLY, IT SPECIFIES THE REFERENCE VALUES IN THE SEARCH FOR
C   THE ELEMENTS TO OCCUPY THE FIRST ARRAY POSITION.
      INTEGER U(200,2),Q
      COMPLEX*16 ELEMNT(200)
      REAL K(8),A(2),V(2)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM04/ Q
      COMMON/COMM05/ K,E
      COMMON/COMM07/ A,V
      COMMON/COMM16/ ELEMNT
      A(1)=0.0
      A(2)=0.0
      V(1)=0.0
      V(2)=0.0
      CALL SETQ
      DO 10 JJ=1,MM
      IF (U(JJ,2).NE.0.OR.U(JJ,1).EQ.0) GOTO 10
      A(1)=DREAL(ELEMNT(JJ))/FLOAT(Q)+A(1)
      A(2)=DIMAG(ELEMNT(JJ))/FLOAT(Q)+A(2)
 10   CONTINUE
      DO 20 JJ=1,MM
      IF (U(JJ,2).NE.0.OR.U(JJ,1).EQ.0) GOTO 20
      V(1)=(DREAL(ELEMNT(JJ))-A(1))**2+V(1)
      V(2)=(DIMAG(ELEMNT(JJ))-A(2))**2+V(2)
 20   CONTINUE
      V(1)=SQRT(V(1)/FLOAT(Q-1))
      V(2)=SQRT(V(2)/FLOAT(Q-1))
      IF (II.GT.1) RETURN
      K(1)=V(1)
      K(2)=V(2)
      K(3)=A(1)
      K(4)=A(2)
      E=SQRT(K(1)**2+K(2)**2)
      RETURN
      END

      SUBROUTINE SETU(S,U1,K2,L2,M2,N2)
C   ROUTINE TO DETERMINE THE VALUE OF THE KENDIG TERM SPECIFIED
C   FOR EACH PERMUTATION FOUND IN OPTIMO.
C   K CORRESPONDS TO POSITION 1, L TO POSITION 3, M TO POSITION 2
C   AND N TO POSITION 4.
      INTEGER TERM(3),S,OSS(40)
      REAL AMP(200),PHASE(200)
      COMMON/COMM03/ AMP,PHASE
      COMMON/COMM06/ OSS,TERM
```

```
C  KENDIG TERM 3.
      IF (TERM(S).EQ.3) THEN
      U1=ABS(AMP(L2)*PHASE(L2)-AMP(K2)*PHASE(K2)+AMP(N2)*PHASE(N2)-AMP(M
     12)*PHASE(M2))
C  KENDIG TERM 4.
      ELSEIF (TERM(S).EQ.4) THEN
      U1=ABS(AMP(L2)*PHASE(L2)-AMP(K2)*PHASE(K2)-AMP(N2)*PHASE(N2)+AMP(M
     12)*PHASE(M2))
C  KENDIG TERM 8.
      ELSE
      U1=ABS(AMP(L2)*PHASE(L2)+AMP(K2)*PHASE(K2)-AMP(N2)*PHASE(N2)-AMP(M
     12)*PHASE(M2))
      ENDIF
      RETURN
      END

      SUBROUTINE WRITEIT(L)
C  ROUTINE TO WRITE OUT THE RESULTS FOR EACH POSITION.
C  ADDITIONALLY, A MAP OF THE NUMBERS ASSIGNED TO EACH ELEMENT
C  IS PRINTED SHOWING WHERE IN THE ARRAY EACH ELEMENT IS LOCATED.
      INTEGER U(200,2),BR(200),RER(200),IMR(200),NN(200),OSS(40),T(4)
      INTEGER RERR(200)
      REAL RA(40),XA(40)
      CHARACTER*1 NAME(40)
      COMPLEX*16 ELEMNT(200)
      CHARACTER*3 RRER(200),BLANK
      DATA BLANK/'   '/
      REAL A(2),V(2)
      COMMON/COMM01/ M,MM,IM,IM1
      COMMON/COMM02/ U
      COMMON/COMM3A/ BR
      COMMON/COMM06/ OSS,TERM
      COMMON/COMM6A/ NAME
      COMMON/COMM08/ RA,XA,F,SIZE
      COMMON/COMM09/ T
      COMMON/COMM12/ RER,RERR,IMR
      COMMON/COMM13/ NEVEN,N1L,N2L,LSEED
      COMMON/COMM16/ ELEMNT
      IF (L.EQ.0) GOTO 200
      WRITE(4,501) NAME(OSS(L)),BR(T(1)),BR(T(2)),BR(T(3)),BR(T(4))
 501  FORMAT('+',91X,A1,19X,I3,3X,I3,3X,I3,3X,I3)
      WRITE(4,510) RA(OSS(L)),XA(OSS(L))
 510  FORMAT(5X,'RADIATION LOADING:  REAL = ',E12.4,5X,'IMAGINARY = ',E1
     12.4//)
      RETURN
C  DETERMINE THE MEANS AND VARIANCES OF ALL THE ELEMENTS IN THE ARRAY.
 200  A(1)=0.0
      A(2)=0.0
      V(1)=0.0
      V(2)=0.0
      N=0
      DO 10 JJ=1,MM
```

```
      IF (U(JJ,2).NE.-1) GOTO 10
      A(1)=DREAL(ELEMNT(JJ))+A(1)
      A(2)=DIMAG(ELEMNT(JJ))+A(2)
      N=N+1
 10   CONTINUE
      A(1)=A(1)/FLOAT(N)
      A(2)=A(2)/FLOAT(N)
      DO 20 JJ=1,MM
      IF (U(JJ,2).NE.-1) GOTO 20
      V(1)=(DREAL(ELEMNT(JJ))-A(1))**2+V(1)
      V(2)=(DIMAG(ELEMNT(JJ))-A(2))**2+V(2)
 20   CONTINUE
      V(1)=SQRT(V(1)/FLOAT(N-1))
      V(2)=SQRT(V(2)/FLOAT(N-1))
      WRITE(4,502) 100.*A(1),A(2)
 502  FORMAT(5X,'ARRAY AMPLITUDE MEAN = ',F12.8,':   ARRAY PHASE MEAN =
     1',F12.8)
      WRITE(4,503) 100.*V(1),V(2)
 503  FORMAT(5X,'AMPLITUDE  VARIANCE  = ',F12.8,':   PHASE  VARIANCE  =
     1',F12.8)
C  THE ARRAY RRER IS USED TO PRINT A MAP OF THE ASSIGNED NUMBERS
C  OF EACH SELECTED ELEMENT IN THE POSITION THAT IT OCCUPIES IN
C  THE ARRAY.
      DO 36 JJ=1,MM
      WRITE(RRER(JJ),505) BLANK
  36  IMR(JJ)=JJ
      N=0
      IF(NEVEN.EQ.1) THEN
        N=1
        WRITE(RRER(1),504) RER(1)
      ENDIF
      N1=1+N
      DO 40 LL=1+N,IM
      DO 50 KK=1+N,IM1
      IF (OSS(KK).EQ.IMR(LL)) THEN
        WRITE(RRER(N1),504) RER(LL)
        WRITE(RRER(N1+IM),504) RER(LL+IM)
        WRITE(RRER(N1+2*IM),504) RER(LL+2*IM)
        WRITE(RRER(N1+3*IM),504) RER(LL+3*IM)
 504    FORMAT(I3)
        N1=N1+1
        GOTO 40
      ENDIF
 50   CONTINUE
      WRITE(RRER(N1),505) BLANK
      WRITE(RRER(N1+IM),505) BLANK
      WRITE(RRER(N1+2*IM),505) BLANK
      WRITE(RRER(N1+3*IM),505) BLANK
 505  FORMAT(A3)
      N1=N1+1
 40   CONTINUE
C  THIS ROUTINE INITIALIZES AN ARRAY, NN, TO BE USED TO PRINT A MAP OF
```

```
C  THE ELEMENT NUMBERS REPRESENTING THEIR POSITION IN THE ARRAY.
C  THE SAME ROUTINE IS FOUND IN THE KENDANAL SUBROUTINE OF THE NEARPRES
C  PROGRAM.
      LL=1
      IF(NEVEN.NE.1) THEN
C  THIS BRANCH INITIALIZES NN FOR AN EVEN ARRAY.
        NL1=2*IM
        NL2=IM
        DO 110 KK=1,N2L
        DO 120 JJ=1,N1L
        I1=JJ-1
        NN(LL)=IMR(NL1-I1)
        LL=LL+1
 120    CONTINUE
        NL1=NL1-N1L
        DO 130 JJ=N1L,1,-1
        I1=JJ-1
        NN(LL)=IMR(NL2-I1)
        LL=LL+1
 130    CONTINUE
        NL2=NL2-N1L
 110    CONTINUE
        NL1=2*IM+1
        NL2=3*IM+1
        DO 140 KK=1,N2L
        DO 150 JJ=N1L,1,-1
        I1=JJ-1
        NN(LL)=IMR(NL1+I1)
        LL=LL+1
 150    CONTINUE
        NL1=NL1+N1L
        DO 160 JJ=1,N2L
        I1=JJ-1
        NN(LL)=IMR(NL2+I1)
        LL=LL+1
 160    CONTINUE
        NL2=NL2+N1L
 140    CONTINUE
      ELSE
C  THIS BRANCH INITIALIZES NN FOR AN ODD ARRAY, ( AN ARRAY WITH
C  AN ELEMENT AT THE CENTER.)
        LL=1
        NL1=2*IM+1
        NL2=IM+2-N1L
        DO 117 KK=1,N1L
        DO 127 JJ=1,N1L*N2L,N1L
        I1=JJ-1
        NN(LL)=IMR(NL1-I1)
 127    LL=LL+1
        NL1=NL1-1
        DO 137 JJ=1,N1L
        I1=JJ-1
```

```
      NN(LL)=IMR(NL2+I1)
137   LL=LL+1
      NL2=NL2-N1L
117   CONTINUE
      NL1=3*IM-(N2L-1)*N1L+1
      DO 147 JJ=1,N1L
      I1=JJ-1
      NN(LL)=IMR(NL1-I1)
147   LL=LL+1
      NL1=NL1+N1L
      NN(LL)=IMR(1)
      LL=LL+1
      DO 157 JJ=1,N1L
      I1=JJ-1
      NN(LL)=IMR(NL2+I1)
157   LL=LL+1
      NL2=4*IM+2-N1L*N2L
      DO 167 KK=1,N1L
      DO 177 JJ=1,N1L
      I1=JJ-1
      NN(LL)=IMR(NL1-I1)
177   LL=LL+1
      NL1=NL1+N1L
      DO 187 JJ=1,N1L*N2L,N1L
      I1=JJ-1
      NN(LL)=IMR(NL2+I1)
187   LL=LL+1
      NL2=NL2+1
167   CONTINUE
     ENDIF
C  THIS STATEMENT DETERMINES WHETHER THE MAP IS TO BE PRINTED
C  ON A NEW PAGE, BASED ON THE SIZE OF THE ARRAY.
     IF(M.LE.36) THEN
       WRITE(4,506)
506    FORMAT('0'/'0',35X,'PLACEMENT OF ELEMENTS IN THE ARRAY.'/'0')
     ELSE
       WRITE(4,509)
509    FORMAT('1',35X,'PLACEMENT OF ELEMENTS IN THE ARRAY.'/'0')
     ENDIF
     DO 197 KK=1,M,N1L+N2L
     WRITE(4,507)
     WRITE(4,508) (RRER(NN(LL)),LL=KK,KK+N2L+N1L-1)
     WRITE(4,507)
507  FORMAT(1X)
508  FORMAT(35X,12(A3,3X))
197  CONTINUE
     RETURN
     END
```

APPENDIX F

LISTING OF ARRAY RESPONSE PROGRAM

```
C*****************************************************************
C                      NEARPRES                                  *
C       DIRECTIONAL BEAM PATTERN MAPPING PROGRAM                 *
C             WRITTEN BY KING W. WIEMANN                         *
C*****************************************************************
      DIMENSION PHI(50),IDEG(50),THETA(200),AMPL(50),PHASE(50)
      DIMENSION SAMP(50),AAMP(50),NAMP(50)
      CHARACTER*1 JAMP(50),ICOLON,IBLANK
      COMMON/COMM01/ X(150),Y(150),ELEMNT(150),SHADE(150),AK
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
      COMMON/COMM04/ D1,D2,DELD
      COMMON/COMM05/ R
      COMMON/COMM06/ D
      COMMON/COMM07/ M
      COMMON/COMM16/ NSOURC
      COMPLEX ELEMNT
      DATA ICOLON/':'/,IBLANK/' '/
      L = 0
      NCOUNT = 0
C  READ TASK INDEX K, AND CALL THE APPROPRIATE SUBROUTINE.
      READ(3,10) NSOURC,K,NTYPE
 10   FORMAT(3I3)
      GOTO(1,2,3,4,5) K
 1    CALL ROUTE1(L,&20)
 2    CALL ROUTE2(L,&20)
 3    CALL ROUTE3(L,&20)
 4    CALL ROUTE4(L,&20)
 5    CALL ROUTE5(L,&20)
C  IF THE TASK CHOSEN IS TO BE DONE ITERATIVELY FOR NEW PARAMETERS
C  DATAIN IS CALLED FOR EACH SUCCESSIVE ITERATION.
C  AFTER EACH MAP IS PRINTED, CONTROL RETURNS TO LINE 11.
 11   CALL DATAIN(L)
      IF(NPRINT.EQ.1) CALL PRINT
C  INITIALIZING THE THETA AND PHI VALUES.
C  FOR K = 1,2,3 THETA IS HELD CONSTANT WHILE PHI CYCLES.
C  FOR K = 4,5, PHI IS HELD CONSTANT WHILE THETA CYCLES.
C
C  DETERMINE THE RESPONSE ON THE MAIN BEAM. (THETA = 0.0;PHI = 0.0)
C
 20   IF(NTYPE.NE.1) GOTO 12
      CALL PRESS (0.0,0.0,AMPL1,PHASE1,DNORM,D)
      SAMP1 = AMPL1/DNORM
      IF (ABS(SAMP1).GT.1.0592E-5) THEN
        AAMP1 = 20.0*ALOG10(SAMP1)
      ELSE
        AAMP1 = 99.0
      ENDIF
      Z=AAMP1
```

```
C  SET UP THE PHI AND THETA VALUES FOR TASKS 1, 2, OR 3.
 12   IF(K.LE.3) THEN
        II = (PHI2 - PHI1)/DELPHI +1.0
        IF (II.GT.37) II = 37
        JJ = (THETA2 - THETA1)/DELTHE +1.0
        IF (JJ.GT.181) JJ = 181
        DO 25 I = 1,II
        PHI(I) = PHI1 + (I-1)*DELPHI
 25     IDEG(I) = PHI(I)
        DO 30 J = 1,JJ
 30     THETA(J) = THETA1 + (J-1)*DELTHE
C  PRINT THE HORIZONTAL SCALE FOR THE MAP.
        WRITE(4,35)
 35     FORMAT('1',40X,'MAP OF ARRAY OUTPUT'/)
        WRITE (4,40) (IDEG(I),I=1,II,2)
 40     FORMAT(5X,'THETA',55X,'PHI (DEGREES)'/11X,18(I4,2X),I4)
      ELSE
C  SET UP THE THETA AND PHI VALUES FOR TASKS 4 AND 5.
 50     II = (THETA2 - THETA1)/DELTHE + 1.0
        IF(II.GT.37) II = 37
        JJ = (PHI2 - PHI1)/DELPHI + 1.0
        IF(JJ.GT.181) JJ = 181
        DO 55 I = 1,II
        THETA(I) = THETA1 + (I-1)*DELTHE
 55     IDEG(I) = THETA(I)
C   PRINT THE HORIZONTAL SCALE FOR THE MAP.
        WRITE(4,35)
        WRITE (4,45) (IDEG(I),I=1,II,2)
 45     FORMAT(5X,'PHI',55X,'THETA (DEGREES)'/11X,18(I4,2X),I4)
        DO 60 J = 1,JJ
 60     PHI(J) = PHI1 + (J-1)*DELPHI
      ENDIF
C  IF A HEIGHT SPECTRUM IS THE TASK, THETA AND PHI WILL CYCLE WHILE
C  THE VALUE OF D IS HELD CONSTANT.
 65   IF(K.NE.5) GOTO 67
 66   D=D1
 67   DO 80 J = 1,JJ
      DO 70 I = 1,II
      IF(K.GT.3) THEN
C  DEPENDING ON K, DIFFERENT ANGLE PARAMETERS ARE SENT TO PRESS.
        CALL PRESS (THETA(I),PHI(J),AMPL(I),PHASE(I),DNORM,D)
      ELSE
        CALL PRESS (THETA(J),PHI(I),AMPL(I),PHASE(I),DNORM,D)
      ENDIF
 74   SAMP(I) = AMPL(I)/DNORM
      IF (ABS(SAMP(I)).GT.1.0592E-5) THEN
        AAMP(I) = -20.0*ALOG10(SAMP(I))
      ELSE
        AAMP(I) = 99.0
      ENDIF
C  SUBRACT THE SIDE LOBE RESPONSE FROM THE MAIN BEAM RESPONSE.
      IF(NTYPE.NE.1) GOTO 78
      AAMP(I)=Z+AAMP(I)
      IF(AAMP(I).GT.99.5) AAMP(I)=99.0
```

```
      PHASE(I)=PHASE1-PHASE(I)
78    NAMP(I) = INT(AAMP(I) + SIGN (0.5,AAMP(I)))
      JAMP(I) = IBLANK
      IF(PHASE(I).GE.90.0.AND.PHASE(I).LE.270.0) JAMP(I) = ICOLON
      IF(PHASE(I).LE.-90.0) JAMP(I) = ICOLON
70    CONTINUE
      IF(K.GT.3) THEN
        WRITE(4,84) PHI(J),(NAMP(I),JAMP(I),I=1,II)
      ELSE
        WRITE (4,84) THETA(J),(NAMP(I),JAMP(I),I=1,II)
      END IF
84    FORMAT (5X,F6.2,2X,37(I2,A1))
80    CONTINUE
      IF(K.NE.5) GOTO 99
      D = D1 + DELD
      IF(D.GT.D2) GOTO 99
      GOTO 67
99    WRITE (4,110)
110   FORMAT ('0',5X,'THETA IS THE ANGLE MEASURED FROM THE Z-AXIS (ANGLE
     1 OF  INCIDENCE)'/5X,'PHI IS THE ANGLE MEASURED FROM THE X-AXIS (RO
     2LL PLANE)'/5X,'ALL NUMBERS ARE NEGATIVE DECIBELS'/1X,'**NOTE** LAB
     3ELLING OF THE HORIZONTAL SCALE IS FOR ALTERNATE COLUMNS AND IS TRU
     4NCATED TO INTEGER MODE'/)
      GOTO(101,102,103,104,105) K
101   WRITE(4,130) K
130   FORMAT(5X,'K='1X,I2,2X,'THIS IS A MAP OF THE RESPONSE OF THE ARRA
     1Y TO A SIGNAL INCIDENT AT ANGLES PHI AND THETA'//)
      GOTO 136
102   WRITE(4,131) K,R
131   FORMAT(5X,'K='1X,I2,2X,'THIS IS A MAP OF THE PRESSURE FIELD ON A H
     1EMISHPERICAL SURFACE'/1X,'CENTERED AT THE CENTER OF THE ARRAY AT A
     2 DISTANCE OF R=',1X,F8.4//)
      GOTO 136
103   WRITE(4,132) K,D
132   FORMAT(5X,'K=',1X,I2,2X,'THIS IS THE PRESSURE FIELD IN A PLANE PAR
     1ALLEL TO THE PLANE OF THE ARRAY AT HEIGHT D=',1X,F8.4//)
      GOTO 136
104   WRITE(4,133) K,D,PHI1
133   FORMAT(5X,'K=',1X,I2,2X,'THIS IS THE PRESSURE FIELD AT HEIGHT D=',
     11X,F8.4,2X,'AND ALONG THE ROLL PLANE PHI=',1X,F8.4//)
      GOTO 136
105   WRITE(4,134) K,PHI,DI,D2,DELD
134   FORMAT(5X,'K='1X,I2,2X,'THIS IS THE PRESSURE FIELD AT ROLL PLANE P
     1HI=',1X,F8.4/5X,'STARTING AT HEIGHT D1=',1X,F8.4,2X,'ENDING AT HEI
     2GHT D2=',1X,F8.4,2X,'AT AN INCREMENT OF DELTA D=',1X,F8.4//)
C
C  IF THE TASK IS TO BE REPEATED, THE COUNTER IS INCREMENTED AND THE NEW
C  VALUES READ IN.
136   NCOUNT=NCOUNT+1
      IF(NCOUNT.GE.ITER) GOTO 112
      L=NREPET
      GOTO 11
C  IF THE PROGRAM IS FINISHED, A NEW SET OF ARRAY PARAMETERS MAY BE
```

```
C  CHANGED AND THE PROGRAM RUN AGAIN.
C  HOWEVER, ONLY ONE PARAMETER MAY BE CHANGED AT THIS TIME.
 112  READ(3,137) NAGAIN
 137  FORMAT(I1)
      IF(NAGAIN.NE.1) GOTO 113
      READ(3,138) ITER,NREPET,NPRINT
 138  FORMAT(3I3)
      NCOUNT=0
      L=NREPET
      GOTO 11
 113  STOP
      END

      SUBROUTINE PRESS(THETAP,PHIP,AMPLP,PHASEP,DNORM,D)
C*********************************************************************
C     WRITTEN BY MARK SCHAFER                                        *
C     EXPANDED BY KING W. WIEMANN                                    *
C *******************************************************************
      COMMON/COMM01/ X(150),Y(150),ELEMNT(150),SHADE(150),AK
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM05/ R
      COMMON/COMM07/ M
      COMMON/COMM08/ RK
      DIMENSION ANGLE(150),A(150)
      REAL*8 U,SUMR,SUMI,DCOS,DSIN,DATAN2,DAMPL
      COMPLEX ELEMNT
C  CONSTANTS TO CONVERT DEGREES TO RADIANS AND VICE VERSA.
      DATA D2RAD/1.74532925 E-02/
C  INITIALIZE THE SUMS.
      SUMI =0.0
      SUMR = SUMI
      UCOS = SIN(THETAP*D2RAD)*COS(PHIP*D2RAD)
      USIN = SIN(THETAP*D2RAD)*SIN(PHIP*D2RAD)
C  SUM THE RESPONSE OVER ALL THE ELEMENTS.
C  A(J) IS THE ACTUAL SHADING OF EACH ELEMENT. IT IS THE PRODUCT OF
C  THE IDEAL SHADING FOR THE DESIRED ELEMENT (A FUNCTION OF POSITION),
C  AND THE ELEMENTS AMPLITUDE RESPONSE.
      DNORM = 0
      DO 10 J = 1,M
      A(J) = REAL(ELEMNT(J))*SHADE(J)
      DNORM = DNORM + SHADE(J)
      ANGLE(J) = AIMAG(ELEMNT(J))
      S=X(J)*UCOS+Y(J)*USIN
      U=DBLE(AK*S+ANGLE(J)*D2RAD)
C  DEPENDING ON THE TASK CHOSEN, THE RESPONSE IS COMPUTED DIFFERENTLY.
C  FOR THE PRESSURE FIELD, THERE IS A PHASE FACTOR INVOLVING THE DISTANCE
C  R FROM THE CENTER OF THE ARRAY. FOR K=1, R1=1 (i.e. DIVISION BY
C  UNITY).
      GOTO(5,4) K
      R = D/COS(THETAP)
```

```
 4      R1 = R - S
        GOTO 6
 5      R1 = 1.0
 6      DAMPL = DBLE(A(J))
        SUMR = SUMR + DCOS(U)*DAMPL/R1
        SUMI = SUMI + DSIN(U)*DAMPL/R1
 10     CONTINUE
C  TAKE THE MAGNITUDE OF THE AMPLITUDE AND CONVERT THE PHASE TO DEGREES.
        AMPLP = SNGL(SUMR*SUMR + SUMI*SUMI)
        AMPLP = SQRT(AMPLP)
        PHASEP = SNGL(DATAN2(SUMI,SUMR))/D2RAD
        IF(K.NE.1) PHASEP = PHASEP-AK*R/D2RAD
        RETURN
        END

        SUBROUTINE ROUTE1(L,*)
        COMMON/COMM02/ ITER,NREPET,NPRINT,K
        COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
        COMMON/COMM14/ NPOOL,MM
        READ(3,10) THETA1,THETA2,DELTHE
        READ(3,10) PHI1,PHI2,DELPHI
        READ(3,15) NPOOL
 10     FORMAT(3F8.4)
 15     FORMAT(I5)
        IF(NPOOL.EQ.1) READ(3,15) MM
        CALL DATAIN(L)
        IF(NPRINT.EQ.1) CALL PRINT
        RETURN 1
        END

        SUBROUTINE ROUTE2(L,*)
        COMMON/COMM02/ ITER,NREPET,NPRINT,K
        COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
        COMMON/COMM05/ R
        READ(3,10) THETA1,THETA2,DELTHE
        READ(3,10) PHI1,PHI2,DELPHI
        READ(3,15) R
 10     FORMAT(3F8.4)
 15     FORMAT(F8.4)
        CALL DATAIN(L)
        IF(NPRINT.EQ.1) CALL PRINT
        RETURN 1
        END

        SUBROUTINE ROUTE3(L,*)
        COMMON/COMM02/ ITER,NREPET,NPRINT,K
        COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
        COMMON/COMM06/ D
        READ(3,10) THETA1,THETA2,DELTHE
        READ(3,10) PHI1,PHI2,DELPHI
        READ(3,15) D
 10     FORMAT(3F8.4)
```

```
15    FORMAT(F8.4)
      CALL DATAIN(L)
      IF(NPRINT.EQ.1) CALL PRINT
      RETURN 1
      END

      SUBROUTINE ROUTE4(L,*)
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
      COMMON/COMM06/ D
      READ(3,10) THETA1,THETA2,DELTHE
      READ(3,15) PHI
      READ(3,15) D
10    FORMAT(3F8.4)
15    FORMAT(F8.4)
      PHI1 = -PHI
      PHI2 = PHI
      DELPHI = 2*PHI
      CALL DATAIN(L)
      IF(NPRINT.EQ.1) CALL PRINT
      RETURN 1
      END

      SUBROUTINE ROUTE5(L,*)
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
      COMMON/COMM04/ D1,D2,DELD
      READ(3,10) THETA1,THETA2,DELTHE
      READ(3,15) PHI
      READ(3,10) D1,D2,DELD
10    FORMAT(3F8.4)
15    FORMAT(F8.4)
      PHI1 = -PHI
      PHI2 = PHI
      DELPHI = 2*PHI
      CALL DATAIN(L)
      IF(NPRINT.EQ.1) CALL PRINT
      RETURN 1
      END

      SUBROUTINE PRINT
C   ROUTINE TO PRINT OUT THE ARRAY PARAMETERS FOR THE CURRENT MAP.
      COMMON/COMM01/ X(150),Y(150),ELEMNT(150),SHADE(150),AK
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM03/ THETA1,THETA2,DELTHE,PHI1,PHI2,DELPHI
      COMMON/COMM04/ D1,D2,DELD
      COMMON/COMM05/ R
      COMMON/COMM06/ D
      COMMON/COMM07/ M
      COMMON/COMM09/ DOLAM,WAVE
      COMMON/COMM15/ NEVEN,NROUTE,IM,N1L,N2L
      COMPLEX ELEMNT
```

```
      DIMENSION NN(150),IMR(150),AMP(150),PHASE(150)
      DATA L/0/
      L=L+1
      DO 1 KK=1,M
      AMP(KK)=REAL(ELEMNT(KK))
 1    PHASE(KK)=AIMAG(ELEMNT(KK))
      IF(NROUTE.EQ.1) THEN
        IF(L.GT.1) GOTO 153
        DO 4 KK=1,M
 4      IMR(KK)=KK
        LL=1
C  THIS BLOCK INITIALIZES AN ARRAY, NN, WHICH ALLOWS THE PRINTING OF
C  ELEMENT TOLERANCES IN THE OUTPUT FILE SO THAT IT MATCHES THEIR
C  RESPECTIVE PLACEMENTS IN THE ARRAY.
        IF(NEVEN.NE.1) THEN
C  THIS BRANCH INITIALIZES NN FOR AN EVEN ARRAY.
          NL1=2*IM
          NL2=IM
          DO 10 KK=1,N2L
          DO 20 JJ=1,N1L
          II=JJ-1
          NN(LL)=IMR(NL1-II)
          LL=LL+1
 20       CONTINUE
          NL1=NL1-N1L
          DO 30 JJ=N1L,1,-1
          II=JJ-1
          NN(LL)=IMR(NL2-II)
          LL=LL+1
 30       CONTINUE
          NL2=NL2-N1L
 10       CONTINUE
          NL1=2*IM+1
          NL2=3*IM+1
          DO 40 KK=1,N2L
          DO 50 JJ=N1L,1,-1
          II=JJ-1
          NN(LL)=IMR(NL1+II)
          LL=LL+1
 50       CONTINUE
          NL1=NL1+N1L
          DO 60 JJ=1,N2L
          II=JJ-1
          NN(LL)=IMR(NL2+II)
          LL=LL+1
 60       CONTINUE
          NL2=NL2+N1L
 40       CONTINUE
        ELSE
C  THIS BRANCH INITIALIZES NN FOR AN ODD ARRAY, ( AN ARRAY WITH
C  AN ELEMENT AT THE CENTER.)
          LL=1
```

```
          NL1=2*IM+1
          NL2=IM+2-N1L
          DO 17 KK=1,N1L
          DO 27 JJ=1,N1L*N2L,N1L
          II=JJ-1
          NN(LL)=IMR(NL1-II)
27        LL=LL+1
          NL1=NL1-1
          DO 37 JJ=1,N1L
          II=JJ-1
          NN(LL)=IMR(NL2+II)
37        LL=LL+1
          NL2=NL2-N1L
17        CONTINUE
          NL1=3*IM-(N2L-1)*N1L+1
          DO 47 JJ=1,N1L
          II=JJ-1
          NN(LL)=IMR(NL1-II)
47        LL=LL+1
          NL1=NL1+N1L
          NN(LL)=IMR(1)
          LL=LL+1
          DO 57 JJ=1,N1L
          II=JJ-1
          NN(LL)=IMR(NL2+II)
57        LL=LL+1
          NL2=4*IM+2-N1L*N2L
          DO 67 KK=1,N1L
          DO 77 JJ=1,N1L
          II=JJ-1
          NN(LL)=IMR(NL1-II)
77        LL=LL+1
          NL1=NL1+N1L
          DO 87 JJ=1,N1L*N2L,N1L
          II=JJ-1
          NN(LL)=IMR(NL2+II)
87        LL=LL+1
          NL2=NL2+1
67        CONTINUE
        ENDIF
153     WRITE(4,106)
106     FORMAT('1',4X,'PLACEMENT OF ELEMENT TOLERANCES IN THE ARRAY.')
        II=1
        DO 97 KK=1,M,N1L+N2L
        WRITE(4,108)
        WRITE(4,109) (AMP(NN(LL)),LL=KK,KK+N2L+N1L-1)
        WRITE(4,108)
        WRITE(4,109) (PHASE(NN(LL)),LL=KK,KK+N2L+N1L-1)
        WRITE(4,108)
108     FORMAT(1X)
109     FORMAT(5X,12(F9.5,2X))
        II=II+1
```

```
 97     CONTINUE
        CALL KENDANAL
      ELSE
C  THIS WILL PRINT OUT THE ARRAY PARAMETERS FOR NON-SYMMETRIC ARRAYS.
 118    WRITE(4,245)
 245    FORMAT('1SHADING, POSITION(X,Y), AND TOLERANCES (AMPL. AND PHASE
     1)'/1X,'FOR EACH ELEMENT')
        DO 279 J=1,M
        WRITE(4,250) SHADE(J), X(J), Y(J), ELEMNT(J)
 250    FORMAT(1X,5(F9.5,2X))
 279    CONTINUE
 291  ENDIF
      WRITE(4,12) DOLAM,WAVE
 12   FORMAT(5X,'D OVER LAMDA =',F8.4,2X,'WAVELENGTH=',F8.4///)
C  EACH TASK HAS A UNIQUE PARAMETER ASSOCIATED WITH IT. FOR EACH
C  TASK, THAT PARAMETER IS PRINTED.
      IF(K.NE.2) GOTO 21
      WRITE(4,15) K,R
 15   FORMAT(1X,'K = ',I2,2X,'THE RADIUS IS: ',F8.4///)
 21   IF(K.NE.3) GOTO 31
      WRITE(4,16) K,D
 16   FORMAT(1X,'K = ',I2,2X,'THE DISTANCE IS: ',F8.4///)
 31   IF(K.NE.4) GOTO 41
      WRITE(4,25) K,PHI2,D
 25   FORMAT(1X,'K = ',I2,2X,'THE ROLL PLANE IS: ',F8.4,2X,'THE DISTANCE
     1 IS : ',F8.4)
 41   IF(K.NE.5) GOTO 51
      WRITE(4,35) K,PHI2,D1,D2,DELD
 35   FORMAT(1X,'K = ',I2,2X,'THE ROLL PLANE IS: ',F8.4/1X,'THE INITIAL
     1AND FINAL DISTANCES ARE: ',2(F8.4,2X),'AND THE INCREMENT IS: ',F8.
     24)
 51   RETURN
      END

      SUBROUTINE KENDANAL
C  ROUTINE WHICH DETERMINES THE KENDIG TERMS, AND PLACES THEM IN
C  THEIR RESECTIVE POSITION IN THE FIRST QUADRANT.
      COMMON/COMM01/ X(150),Y(150),ELEMNT(150),SHADE(150),AK
      COMMON/COMM07/ M
      COMMON/COMM10/ IMR
      COMMON/COMM14/ NPOOL,MM
      COMMON/COMM15/ NEVEN,NROUTE,IM,N1L,N2L
      COMMON/COMM16/ NSOURC
      COMPLEX ELEMNT
      DIMENSION V1(40),V2(40),V3(40),V4(40),V5(40),V6(40),V7(40)
      DIMENSION V8(40),AMP(150),PHASE(150),NN(40),IMR(150)
      DATA D2RAD/1.74532925 E-02/
      DO 10 II=1,M
      AMP(II)=REAL(ELEMNT(II))
      PHASE(II)=AIMAG(ELEMNT(II))*D2RAD
 10   CONTINUE
      KK1=IM
```

```
      KK2=2*IM
      KK3=3*IM
      N=0
      IF(NEVEN.EQ.1) N=1
      DO 20 KK=1,IM+N
      K1=KK+KK1
      K2=KK+KK2
      K3=KK+KK3
      V11=AMP(KK)*PHASE(KK)
      V22=AMP(K1)*PHASE(K1)
      V33=AMP(K2)*PHASE(K2)
      V44=AMP(K3)*PHASE(K3)
      V1(KK)=AMP(KK)+AMP(K1)+AMP(K2)+AMP(K3)
      V2(KK)=AMP(K1)+AMP(K3)-AMP(KK)-AMP(K2)
      V3(KK)=V33-V11+V44-V22
      V4(KK)=V33-V11-V44+V22
      V5(KK)=AMP(KK)-AMP(K2)+AMP(K1)-AMP(K3)
      V6(KK)=AMP(KK)-AMP(K2)+AMP(K3)-AMP(K1)
      V7(KK)=V11+V22+V33+V44
      V8(KK)=V11+V33-V22-V44
 20   CONTINUE
C  THE ARRAY NN CONTROLS THE PLACEMENT OF THE KENDIG TERMS SIMILAR TO
C  THE WAY NN WORKS IN THE PRINT SUBROUTINE.
      IF(NEVEN.NE.1) THEN
C   INTITIALIZE NN FOR AN EVEN ARRAY.
        LL=1
        NL1=N2L
        NL2=1
        DO 40 KK=1,N2L
        DO 30 II=NL1,NL2,-1
        JJ=II-1
        NN(LL)=IM-JJ
        LL=LL+1
 30     CONTINUE
        NL1=NL1+N2L
        NL2=NL2+N2L
 40     CONTINUE
      ELSE
C  INITIALIZE NN FOR AN ODD ARRAY.
        LL=1
        NT=1
        DO 60 JJ=1,N2L
        DO 70 II=NT+N2L-1,NT,-1
        NN(LL)=IM+2-II
        LL=LL+1
 70     CONTINUE
        NT=NT+N1L
 60     CONTINUE
        DO 80 II=1,N1L
        NN(1+(II-1)*N2L)=0
 80     CONTINUE
      ENDIF
```

```
      NL1=1
      NL2=N2L
      WRITE(4,109)
109   FORMAT('1',15X,'KENDIG TERMS'/)
      DO 50 II=1,N2L
      WRITE(4,104)
      IF(NEVEN.EQ.1.AND.NN(NL1).EQ.0) THEN
        WRITE(4,106) (V1(NN(JJ)),V2(NN(JJ)),JJ=NL1+1,NL2)
        WRITE(4,106) (V3(NN(JJ)),V4(NN(JJ)),JJ=NL1+1,NL2)
        WRITE(4,106) (V5(NN(JJ)),V6(NN(JJ)),JJ=NL1+1,NL2)
        WRITE(4,106) (V7(NN(JJ)),V8(NN(JJ)),JJ=NL1+1,NL2)
      ELSE
        WRITE(4,105) (V1(NN(JJ)),V2(NN(JJ)),JJ=NL1,NL2)
        WRITE(4,105) (V3(NN(JJ)),V4(NN(JJ)),JJ=NL1,NL2)
        WRITE(4,105) (V5(NN(JJ)),V6(NN(JJ)),JJ=NL1,NL2)
        WRITE(4,105) (V7(NN(JJ)),V8(NN(JJ)),JJ=NL1,NL2)
      ENDIF
      WRITE(4,104)
      NL1=NL1+N2L
      NL2=NL2+N2L
50    CONTINUE
104   FORMAT(1X)
105   FORMAT(15X,5(2X,F8.5,2X,F8.5,2X))
106   FORMAT(37X,4(2X,F8.5,2X,F8.5,2X))
      RETURN
      END

      SUBROUTINE DATAIN(L)
C  SUBROUTINE TO INPUT DATA FOR NEARPRES.
      COMMON/COMM01/ X(150),Y(150),ELEMNT(150),SHADE(150),AK
      COMMON/COMM02/ ITER,NREPET,NPRINT,K
      COMMON/COMM07/ M
      COMMON/COMM08/ RK
      COMMON/COMM09/ DOLAM,WAVE
      COMMON/COMM10/ IMR
      COMMON/COMM14/ NPOOL,MM
      COMMON/COMM15/ NEVEN,NROUTE,IM,N1L,N2L
      COMMON/COMM16/ NSOURC
      COMPLEX ELEMNT
      DIMENSION IMR(150)
      PI = 4.0*DATAN(1.0D00)
C  THIS PASSES CONTROL TO THE SYMMETRIC LOOP ONLY IF DATAIN IS CALLED
C  FROM THE MAIN PROGRAM RATHER THAN ONE OF THE ROUTE SUBROUTINES.
      IF(NROUTE.EQ.1) GOTO 500
      GOTO (1,2,3,4) L
      READ(3,95) ITER,NREPET,NROUTE,M,NPRINT
95    FORMAT(5I3)
      IJ=M
      IF(NSOURC.EQ.1) IJ=MM
C  NROUTE SPECIFIES LOOP 1 OR LOOP 2
      IF(NROUTE.EQ.1) GOTO 500
C
```

```
C  LOOP ONE; A NONSYMMETRIC ARRAY
C
C  READ IN THE SHADING COEFFICIENTS
 1     DO 10 J=1,M
       READ(3,105) SHADE(J)
 105   FORMAT(F9.6)
 10    CONTINUE
       IF (L.GT.0) GOTO 50
C  READ IN THE ELEMENT TOLERANCES
 2     IF(NSOURC.EQ.1) THEN
         DO 93 J=1,IJ
         READ(3,94) IMR(J)
 94      FORMAT(I4)
 93      CONTINUE
       ELSE
         DO 96 JJ=1,M
         IMR(JJ)=JJ
 96      CONTINUE
       ENDIF
       DO 15 JJ=1,IJ
       READ (3,110) ELEMNT(IMR(JJ))
 110   FORMAT(2F12.6)
 15    CONTINUE
       IF (L.GT.0) GOTO 50
C  READ IN THE ELEMENT POSITIONS
 3     DO 20 J=1,M
       READ (3,115) X(J), Y(J)
 115   FORMAT(2F7.3)
 20    CONTINUE
       IF (L.GT.0) GOTO 50
C  READ IN THE WAVELENGTH AND CONVERT IT TO THE WAVE NUMBER AK
 4     READ (3,120) WAVE
 120   FORMAT(F9.4)
       AK=2*PI/WAVE
       DOLAM=X(2)-X(1)/WAVE
 50    RETURN
C
C  LOOP TWO; A SYMMETRIC ARRAY.
C
 500   GOTO(401,402,403,404) L
       READ(3,405) IM,NEVEN,N1L,N2L
 405   FORMAT(4I5)
C  INPUT THE SHADING COEFFICIENTS.
C  IF THE ARRAY IS 'ODD', NEVEN=1. ('ODD' ARRAY MEANS THERE IS A
C  CENTER ELEMENT, AT X=0,Y=0).
 401   IF(NEVEN.EQ.1) THEN
         READ(3,415) SHADE(1)
         DO 411 J=2,IM+1
         READ(3,415) SHADE(J)
C  ASSIGN EACH POSITION IN THE ARRAY WITH ITS PROPER SHADING.
         SHADE(J+IM)=SHADE(J)
         SHADE(J+2*IM)=SHADE(J)
```

```
        SHADE(J+3*IM)=SHADE(J)
 411    CONTINUE
      ELSE
C  INPUT THE VALUES FOR AN EVEN ARRAY.
 406    DO 410 J=1,IM
        READ(3,415) SHADE(J)
C  ASSIGN EACH POSITION IN THE ARRAY WITH ITS PROPER SHADING.
        SHADE(J+IM)=SHADE(J)
        SHADE(J+(2*IM))=SHADE(J)
        SHADE(J+(3*IM))=SHADE(J)
 415    FORMAT(F9.6)
 410    CONTINUE
      ENDIF
        IF (L.GT.0) GOTO 600
C  READ IN THE ELEMENT TOLERANCES.
 402    IF(NSOURC.EQ.1) THEN
          DO 493 JJ=1,IJ
          READ(3,494) IMR(JJ)
 494      FORMAT(I4)
 493      CONTINUE
        ELSE
 492      DO 496 JJ=1,IJ
          IMR(JJ)=JJ
 496      CONTINUE
        ENDIF
        DO 420 JJ=1,IJ
        READ (3,110) ELEMNT(IMR(JJ))
 420    CONTINUE
        IF (L.GT.0) GOTO 600
C  READ IN THE ELEMENT POSITIONS.
C  IF THE ARRAY IS ODD, NEVEN=1. (ODD ARRAY MEANS THERE IS A CENTER
C  ELEMENT).
 403    IF(NEVEN.EQ.1) THEN
          READ(3,433) X(1),Y(1)
          DO 438 J=2,IM+1
          READ(3,433) X(J),Y(J)
C  ASSIGN EACH QUADRANT WITH THE PROPER SIGN VALUES FOR X AND Y.
          Y(J+IM)=X(J)
          X(J+IM)=-Y(J)
          X(J+2*IM)=-X(J)
          Y(J+2*IM)=-Y(J)
          X(J+3*IM)=Y(J)
          Y(J+3*IM)=-X(J)
 438      CONTINUE
        ELSE
C  INPUT THE POSITIONS FOR AN EVEN ARRAY.
          DO 430 J=1,IM
          READ(3,433) X(J),Y(J)
C  ASSIGN EACH QUADRANT WITH THE PROPER SIGN VALUES FOR X AND Y.
          X(J+IM)=-X(J)
          X(J+(2*IM))=-X(J)
          X(J+(3*IM))=X(J)
```

```
          Y(J+IM)=Y(J)
          Y(J+(2*IM))=-Y(J)
          Y(J+(3*IM))=-Y(J)
 433      FORMAT(2F7.3)
 430      CONTINUE
        ENDIF
 308    IF (L.GT.0) GOTO 600
C  READ IN THE WAVELENGTH WAVE AND SPECIFY THE WAVENUMBER AK.
 404    READ(3,435) WAVE
        READ(3,435) DOLAM
 435    FORMAT(F10.6)
        AK=2*PI*DOLAM
        RK=2*PI/WAVE
 600  RETURN
      END
```

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