DYNAMIC TRACK-TERRAIN INTERACTION MODEL

FINAL REPORT ON CONTRACT DAAE07-86-C-R020

Leslie K. Karafiath
Corporate Research Center
Grumman Corporation
Bethpage, New York 11714

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RESEARCH, DEVELOPMENT & ENGINEERING CENTER
Warren, Michigan 48397-5000

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Dynamic Track - Terrain Interaction Model (U)

Leslie L. Karafiath

Final

FROM Jan 86 TO Oct 87
1988, April 28

Dynamic Soil Response Pressure-Sinkage relationship
Dynamic Plate Sinkage Tests Dynamic Track-Soil Interaction
Dynamic Ground Response Plate

Experimental information on dynamic soil response to plate penetration has been evaluated and approximate pressure-sinkage relationships, dependent on the rate of penetration, have been established for the penetration of track segments. The effect of tractive forces on track segment penetration, as well as the effect of adjacent track segments on the penetration are accounted for by factors applied in the pressure-sinkage relationship. The concept of a dynamic ground response plate has been developed as a unit in a ground response model.
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1.0 INTRODUCTION

This report, prepared by the Grumman Corporate Research Center under Contract No. DAAE07-86-C-R020, deals with the dynamic response of soil to loads generated by the tracks of military vehicles. The simulation of the dynamic behavior of vehicles traveling off-road is of great interest to the Army, since vibrations resulting from the unevenness of the terrain adversely affect the ability of the crew to operate the vehicle and its weaponry. Increased battlefield mobility, coupled with the demand that tanks be able to fire on the move, requires gun control systems which are stabilized irrespective of the motions of the vehicle. A capability to simulate the dynamic behavior of vehicles and its components is essential for the design of these vehicles and optimization of vehicle components to reduce harmful vibrations.

2.0 OBJECTIVES

The objectives of this work are to:

- Evaluate the experimental information on dynamic soil response to plate penetration with respect to its relevance to dynamic track-soil interaction
- Determine the applicability of state-of-the art methods of soil dynamics to the modeling of dynamic soil response to track segment loads
- Develop a concept of dynamic track-soil interaction, to be used in conjunction with tracked vehicle dynamic models for the simulation of this interaction by computer models
- Write a computer program as a subroutine to the dynamic track model under development at TACOM for the calculation of the dynamic soil response to track segment loads.

3.0 CONCLUSIONS

The dynamic response of soils to plate loads has been examined and analyzed in detail to establish the significant factors in the complex interaction between track segment and dynamic soil response. These were found as follows:

- The loading history of the soil
- The time rate of load application
- Tractive forces developing shear stresses at the track-soil interface
Interaction between adjacent track segments.

A concept of dynamic soil response model has been formulated which takes into account all significant factors enumerated above. A computer code for a subroutine suitable to use with dynamic track models was written. The limitations on CPU time allocated for the computation of dynamic soil response in the dynamic track model were met by performing all time-consuming computations in a preprocessor program and establishing in that program appropriate approximations for the interrelationships affecting the soil response.

4.0 RECOMMENDATIONS

4.1 Basic Research

It is recommended that a long-range research project be initiated and supported to advance the understanding of the dynamics of partially saturated surface soils and develop theories to describe soil response to dynamic loadings by off-road vehicles in terms of fundamental soil properties.

4.2 Dynamic Plate Sinkage Tests

It is recommended that laboratory and field testing apparatuses, capable of performing plate sinkage tests in soils at various penetration velocities, be designed and constructed and that dynamic plate sinkage tests be made a standard procedure in all field tests involving off-road vehicles. A data base, using the results of these tests, should be established for the estimation of soil dynamic parameters from other terrain descriptors.

5.0 DISCUSSION

5.1 Review of Theories and Experimental Information

Theoretical treatment of the behavior of soils under dynamic loading conditions requires the establishment of constitutive relations for nonlinear stress-strain properties which depend both on the loading history and the time rates of loading. Of the few constitutive relations proposed to simulate nonlinear stress-strain behavior experienced in triaxial tests, none meets the test of generality, although some, applied to specific type of soil and loading conditions, represent the behavior of that type of soil fairly well. Even if a general constitutive relation were available, its implementation into a nonlinear, three-dimensional finite element program would require a major research effort, clearly outside of the scope of the present work.

Theories of dynamic soil behavior with a limited scope have been developed in connection with the following engineering fields:
5.2. Principal Considerations in the Conceptual Development of Dynamic Track-Soil Interaction Model

In the conceptual development of the dynamic track-soil interaction model, it was necessary to reconcile the requirements of realistic simulation with the limitation imposed by the CPU time allocated in the dynamic track model to soil response computations. Finite element modeling (FEM) of the dynamic soil response would be desirable from the viewpoint of realistic simulation, yet it is prohibitive because of the considerable CPU time used by FEMs. In addition, FEMs generally solve for the inverse of the dynamic problem, namely, they calculate the displacements of soil for applied forces as input.

To resolve the problem of realistic simulation within the allocated CPU time, the following approach was adopted. The basic element in the soil response model is the pressure-sinkage relationship obtained when loading the soil by a rectangular plate of the size of a track segment. This relationship may be represented by a mathematical expression which lends itself readily to the solution of the problem as formulated in the dynamic track-model: calculation of soil response forces for given track segment displacements. Plate sinkage tests at low penetration rates have been regularly conducted, both in the laboratory and in the field, for off-road mobility research and analyses. Therefore, pressure-sinkage curves may easily be determined, and the parameters in the mathematical expressions may be easily
### Table 5-1. Dynamic Plate Penetration Tests

<table>
<thead>
<tr>
<th>Institutions:</th>
<th>WES</th>
<th>IIT</th>
<th>U. of DAYTON</th>
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<tr>
<td>Experiments in sand</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Properties:</td>
<td>Yuma Sand</td>
<td>Newport News sand</td>
<td>Riverwash sand</td>
</tr>
<tr>
<td>Median diameter (mm)</td>
<td>0.12</td>
<td>N.A.</td>
<td>0.8</td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td>Dry</td>
<td>8.1</td>
<td>4.75-5.9</td>
</tr>
<tr>
<td>Dry unit weight (pcf)</td>
<td>92-107</td>
<td>99.8**</td>
<td>113-125</td>
</tr>
<tr>
<td>Degree of saturation* (%)</td>
<td>0</td>
<td>32**</td>
<td>31-37</td>
</tr>
<tr>
<td>Cohesion* (psf)</td>
<td>0</td>
<td>10</td>
<td>17</td>
</tr>
<tr>
<td>Friction angle</td>
<td>39°-46°</td>
<td>?N.A</td>
<td>N.A</td>
</tr>
<tr>
<td>Experiments in clay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Alluvial Mississippi (Buckshot) clay</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Moisture content (%)</td>
<td>29.2-39.8</td>
<td>30.4**</td>
<td>29.4**</td>
</tr>
<tr>
<td>Wet density (pcf)</td>
<td>111-118</td>
<td>105.5**</td>
<td>107.8**</td>
</tr>
<tr>
<td>Dry density (pcf)</td>
<td>79.5-91.4</td>
<td>80.7**</td>
<td>83.4**</td>
</tr>
<tr>
<td>Degree of saturation (%)</td>
<td>95</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Cone index (psi)</td>
<td>30-116</td>
<td>N.A.</td>
<td>120**</td>
</tr>
</tbody>
</table>

* estimated average
evaluated. The dynamic effect at higher penetration rates, the effect of tractive forces on the pressure-sinkage relationship, and the effect of adjacent track segments may be considered by appropriate factors, or modifications of the parameters in the pressure-sinkage equation. An important consideration in the conceptual development of the dynamic soil response model has been to take into account the effect of the loading history of soil on its dynamic response. Soil is essentially a material with memory; it "remembers" the stresses and displacements imposed on it by previous actions when responding to a new excitation. Since the soil is loaded and unloaded again and again during the pass of a track, it is essential to consider the changes in soil response as the various track segments pass over it.

5.3 Pressure-Sinkage Relationship for Individual Track Segments

Pressure-sinkage relationships have been used in off-road mobility research to characterize soil behavior since the late fifties. The exponential relationship

\[ p = k z^n \]  \[1\]

where \( p = \) pressure
\( z = \) sinkage
\( k, n = \) parameters

and its modifications, advocated by Bekker for the determination of the pressure distribution beneath wheels, characterize the soil behavior by the two parameters \( k \) and \( n \). Since the dimension of the parameter "k" depends on the exponent "n," it is difficult to associate a "k" value with a physical property of the soil, or compare "k" values obtained in various soils.

A hyperbolic type of equation proposed by Kondner and Krizek\(^2\) reads as follows:

\[ p = q_t \frac{z / \sqrt{A}}{(B + R z / \sqrt{A})} \]  \[2\]

where \( q_t = \) triaxial (or reference) strength of soil
\( A = \) area of plate
\( B, R = \) parameters

In this equation, the dimensionless coefficients \( B \) and \( R \) characterize the behavior of the soil together with the value of \( q_t \), the triaxial strength of the soil, or a selected reference strength. The coefficient \( B \) is proportional to the tangent of the pressure-sinkage curve at \( z = 0 \), the coefficient of proportionality being \( 1/q_t \). The coefficient \( R \) is
defined as follows:

\[ R = \frac{q_t}{p} - \frac{B}{z/\sqrt{A}} \]  \hspace{1cm} [3]

Experience with plate penetration tests has shown that at penetrations in the range of \( z/\sqrt{A} > 0.5 \), the soil is in the plastic state. The second term in Equation [3] then becomes very small, and the value of \( R \) may be associated with the approximate value of the ratio of the triaxial strength to the bearing strength.

To determine the applicability of Equation [2] to the pressure-sinkage curves obtained in dynamic plate penetration tests, the experiments listed in Table 5-1 were analyzed in detail. The main findings of these analyses are summarized hereafter.

The graphically reported results of 74 pressure-controlled plate sinkage tests, done at IIT Research Institute, were digitized and analyzed by various computer programs to determine the degree of correlation with various parameters influencing the dynamic soil response. For example, correlations between the developed pressure and rate of penetration were found to be very poor, indicating that linear viscosity does not directly apply to the problem. In another analysis it was found that while correlation between the sum of pressure times the duration of loading was good within a certain group of tests, others negated this correlation.

This test series was also analyzed to determine how well the proposed pressure-sinkage equation fits the experimental data. Equation [3] may be rearranged and a new variable

\[ Y = \frac{q_t \cdot z}{p \cdot \sqrt{A}} \]  \hspace{1cm} [4]

may be introduced, resulting in the linear equation as follows:

\[ B + \frac{z}{\sqrt{A}} = Y \]  \hspace{1cm} [5]

Figures 5-1 and 5-2 show typical data points from this test series plotted against the new variable \( Y \). Clearly, the data points fit a straight line fairly well for the tests performed in both sand and clay, except for a few initial points in Figure 5-1. These are believed to be the result of seating inaccuracies rather than signifying a different behavior at very small sinkages.

Equation [2] also fits reasonably well the results of penetration-controlled tests. Figure 5-3 shows data points from the tests done at the University of Dayton in clay (see Table 5-1). The curve fits by
Figure 5-1 Pressure-controlled Plate Sinkage Test on Sand
DIAMETER OF PLATE: 4 IN.
RATE OF PRESSURE APPLICATION: 2500 PSI/SEC

Figure 5-2. Pressure-controlled Plate Sinkage Test on Clay
Figure 5-3. Penetration-controlled Plate Sinkage Tests on Clay
Figure 5-4. Results and Evaluation of Experiments Performed at WES on Yung Sand (Plate Size: 1 x 4 in.)
Equation [2] are shown by dashed lines. While the results of most of the plate sinkage tests may be represented by Equation [2] fairly well, some tests performed in sand show the phenomenon of brittle failure, caused by the strain softening stress-strain behavior of dense sands. Figure 5-4 shows the results of penetration-controlled experiments from the test series done at WES. Dotted lines show an approximation of the upper portion of the observed pressure-sinkage relations by Equation [2]. While the approximation in the range of brittle failure, where pressures drop with increasing sinkage, is poor, it would be necessarily so with any other smooth monotonic mathematical relationship which could be considered for representation of the soil response in the dynamic track model. Since surface sands encountered in off-road travel are highly unlikely to be in a dense state comparable to the 80% relative density to which the sand in the WES tests shown in Figure 5-4 was compacted, it was not deemed necessary to use a different type of pressure-sinkage relationship of great complexity for the better representation of this special case.

Shear stresses developed by tractive or braking forces at the track-soil interface interact with the soil response and modify the pressure sinkage relationship represented by Equation [2]. In the plastic state of soil this effect manifests itself in the reduction of bearing stresses. It was found expedient to express the effect of shear stresses in terms of the interface friction angle, $\theta$, which is the angle enclosed by the resultant of the shear and normal stresses and the normal to the interface. The reduction factor $F_\theta$, representing the ratio of bearing stresses at a given $\theta$ angle to that at $\theta = 0$, may then be closely approximated by a parabolic relationship. While this reduction factor strictly applies only when the soil is in the plastic state, it has been assumed that it approximately expresses the pressure reduction approximately in the elastic-plastic state of soil as well.

The effect of adjacent track segments on the pressure-sinkage relationship has been analyzed by various methods. It has been found that even if the interface friction angle is small, the pressure-sinkage relationship is significantly affected only by that track segment which abuts a segment in the direction of the applied shear stresses, and only if that track segment moves downward and is in contact with the ground surface at that point. Under these conditions the effect of an adjacent track segment may be approximated by a factor $F_a$, which expresses the ratio of bearing capacity of a plate twice the width of a track segment to that of a track segment standing alone. The dynamic plate sinkage tests listed in Table 5-1 were also analyzed to determine the effect of penetration velocity on the soil response and establish an approximation for the representation of the dynamic effect which correlates well with the experimental data. Of the several concepts which were evaluated, the following correlated best with the test results. The "B" and "R" parameters in Equation [2] were evaluated for the plate sinkage tests performed at various penetration rates. It was found that these parameters, when plotted against the logarithm of the rate of penetration, fit a straight line reasonably well. Thus, the parameters
"B" and "R" were replaced by the rate dependent parameters "BDYN" and "RDYN," and an approximate linear relationship was introduced for the quick calculation of each of these parameters in the dynamic track model. These relationships are as follows:

$$BDYN = B + F_{bd} \log(\dot{z})$$

$$RDYN = R + F_{rd} \log(\dot{z})$$

The coefficients $F_{bd}$ and $F_{rd}$ may be calculated by linear regression from test data. Figures 5-5 and 5-6 show BDYN and RDYN values, respectively, evaluated for the penetration controlled tests done at the University of Dayton on clay (see Table 5-1) and plotted against the logarithm of the rate of penetration. The slope of the best fitting straight line yields the coefficients $F_{bd}$ and $F_{rd}$.

### 5.4. Track-soil Interface for Dynamic Track Model

#### 5.4.1. Dynamic Ground Response Plate (DYGREP)

The basic element of the soil model is the Dynamic Ground Response Plate (DYGREP). For computational efficiency it is expedient to use plates of the same size as that of a track segment. The position of a plate is defined by the ground coordinates of its center. The DYGREP plate can move only in the direction of the $z'$ coordinate. ($z'$ is the downward coordinate in the local plate coordinate system in accordance with ISTVS nomenclature.) The plate remains horizontal irrespective of changes in its vertical position or the elevation of adjacent plates.

Plate response is activated by a forced downward displacement to a new position ($z'$) of the plate. This displacement is resisted by a uniform pressure over the area of the plate. As long as the $z'$ displacement is greater than any previous downward displacement ($z'_{\text{max}}$) of the plate (Figure 5-7), the magnitude of this pressure is controlled by the pressure-sinkage relationship expressed by Equation [2]. The first time the plate is forced downward from its initial position ($z_{\text{m}} = 0$), Equation [2] controls the resisting pressure, since $z' > z'_{\text{max}}$. Then, before a new $z'$ value is computed for subsequent time increments, the value of $z'_{\text{max}}$ is updated to $z'$. If there is no displacement constraint on the plate, it is assumed to rebound from its maximum displacement $z'_{\text{max}}$ as follows:

$$z'_{\text{o}} = z'_{\text{max}} - F_{r} \cdot z'_{\text{max}} = (1 - F_{r}) \cdot z'_{\text{max}}$$

where $z'_{\text{o}}$ = plate displacement after unloading  
$z'_{\text{max}}$ = max vertical displacement of the plate  
$F_{r}$ = coefficient of rebound

If a new displacement constraint $z'$ is such that

$$z'_{\text{o}} < z' < z'_{\text{max}}$$

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Figure 5-5. Coefficient "B" Evaluated from Penetration-controlled Plate Sinkage Tests on Clay
Figure 5-6. Coefficient "R" Evaluated from Penetration-controlled Plate Sinkage Tests on Clay

RDYN = 0.0141 - 0.002 LOG (\dot{z})
Figure 5-7. Displacement — Pressure Relationship for DYGREP Plates
then the resisting pressure is controlled by the relationship described by Equation [2], entered with an adjusted value of z' as follows:

$$z'^{\text{adj}} = \frac{(z' - z'_o)}{F_r}$$  \[8\]

A schematic illustration of the DYGREP plate is shown in Figure 5-8. The plate itself is supported by nonlinear springs and a center post connected to firm ground by a ratchet mechanism. The downward movement of the plate is resisted by the nonlinear spring forces, while the ratchet mechanism simulates the effect of loading history on the dynamic soils response.

5.4.2. Effect of Interface Shear Stresses on Soil Response. The effect of interface shear stresses on the soil response is taken into account by the factor $F_s$, which is approximated by the following equation:

$$F_s = FSC(1) + FSC(2) \times D_r + FSC(3) \times D_r^2$$

where $FSC(i) = \text{coefficients, } i = 1-3$

$D_r = \text{interface friction angle development ratio}$

The interface friction angle, $\delta$, is the angle enclosed by the resultant stress acting on the interface and the vertical. The ratio $D_r$ is defined as follows:

$$D_r = \frac{\delta}{\delta_{\text{max}}}$$

where $\delta_{\text{max}} = \text{the maximum value of the interface friction angle}$

The values of the $FSC(i)$ coefficients are determined in the preprocessor program (Section 5.0).

5.4.3. Effect of Adjacent Track Segments on Soil Response. The effect of adjacent track segments is taken into account by the factor $F_a$ which is approximated by the following equation:

$$F_a = A_{fa} + F_{fa} \times D_r$$

where $A_{fa}, F_{fa} = \text{parameters}$

The factor $F_a$ applies to the calculation of the dynamic response of the ground to track segment "n" when either of the adjacent track segments "n+1" or "n-1" meets the following conditions:

- The adjacent track segment is in the direction of the interface shear stresses acting on track segment "n"
Figure 5-8. Schematic Presentation of a DYGREP Plate
• The position of the adjacent track segment is lower than that of track segment "n"

• The velocity of the adjacent track segment is downward.

5.4.4. Ground Model for Use with TACOM Dynamic Track Model. The ground model consists of a series of DYGRIP plates, as shown in Figure 5-9. Realistic simulation requires that models of ground response be fixed to the ground in their horizontal position, as opposed to ground models which travel with the vehicle. In the ground model consisting of a series of DYGRIP plates, the problems arising from the motion of the vehicle relative to the ground model are resolved by adding a new DYGRIP plate in the front of the vehicle and omitting one at the rear whenever the distance traveled by the vehicle equals or exceeds the width of a DYGRIP plate. In computer methodology this is achieved by an overlay of the computer model of DYGRIP plates. The vertical position of omitted DYGRIP plates may be preserved in the program if it is desired to have a record of the rut depth made by the tracks of the vehicle.

In the upper part of Figure 5-9 a potential track geometry is shown which constitutes (together with the track segment velocities) the input to the ground model at a given time. This input geometry, when overlaid on the ground model of DYGRIP plates, shows the elevation differences between the momentary position of the plates and track segments. Whenever the elevation of a track segment is lower than that of the plate beneath it, the DYGRIP plate is forced to move downward. For computational efficiency, and in conformity with the horizontal rigidity of the DYGRIP plates (which does not allow rotation about its center), the pin displacements, instead of those of the track segments, have been chosen as agents which activate the DYGRIP plates. The effect of pin displacement is assumed to extend half of the width of the track segment each way and to exert a displacement constraint on the ground model over this range. Generally, a pin displacement constraint affects two adjacent DYGRIP plates, and, conversely, the forced displacement of a DYGRIP plate consists of a proportional allocation of two pin displacement constraints.

The sequence of calculations for the determination of the normal pressures resisting the forced displacement of a DYGRIP plate in the ground model is shown in Figure 5-10.

The tangential (shear) stresses generated at the track-soil interface are assumed to be related to the magnitude of slip between track segment and soil by the following equation:

\[ \tau = \tau_{\text{max}} (1 - e^{-s/K}) \]  

where  
\[ s \]  = slip

\[ \tau \]  = shear stress mobilized at the track-soil interface
Figure 5-9. Ground Model for Use with TACOM Dynamic Track Model
FLOW DIAGRAM FOR DYGREP PLATE RESPONSE COMPUTATION

\[ x_{pn} = 0 = f_1 \]

IDENTIFY DYGREP PLATE NUMBER \( j \) INTERACTING WITH PIN \( n \)

\[ x_{pn} < x_j \quad \text{and} \quad x_{pn} > x_{j+1} \]

Determine share of DYGREP plate in response to PIN action

\[ z_i = \partial_{zpn} x_{pn} + y_{pn} \]

FALSE

FALSE

\[ z_{pn} < z_l \]

TRUE

Determine \( F_a \)

\[ F_a = F_{a(p+1)} + F_{a(p-1)} \]

\[ 0 < F_{a(p+1)} < 0 \]

\[ z_{p(n+1)} < z_{p,n} \]

FALSE

TRUE

FALSE

\[ z_{p(n+1)} < z_{p,n} \]

TRUE

\[ z_{p(n-1)} < z_{p,n} \]

TRUE

Compute \( F_{a(n+1)} \) using coefficients obtained from the preprocessor

\[ F_{a(n+1)} = 0 \]

\[ F_{a(n-1)} = 0 \]

\[ z_l = z_i \]

\[ z_i = z_{p,n} - z_{p,n} \]

\[ z_{p,n} < 0 \]

\[ z_{p,n} > 0 < z_{p,n} \]

\[ z_{p,n} > 0 > z_{p,n} \]

\[ z_{p,n} = z_{p,n} + 0 < z_{p,n} \]

\[ z_{p,n} = z_{p,n} + 0 > z_{p,n} \]

Compute PINFRC \( i(n) \) from \( z_i \)

\[ PINFRC(i,n) = 0 \]

Legend

- \( x_{pn} \), \( y_{pn} \), \( z_{pn} \): Coordinates of PIN \( n \)
- \( x_{pn} \), \( y_{pn} \), \( z_{pn} \): Ground coordinates of DYGREP plate \( j \) (initial)
- \( x_{pn} \), \( y_{pn} \), \( z_{pn} \): Coordinates of DYGREP plate \( j \) (current)
- \( z_l \): Min. max. Z coordinate of DYREP plate
- \( z_{p,n} \): Min. max. Z coordinate of DYREP plate
- \( w \): Width of DYREP plate
- \( F_a \): Factor for effect of adjacent track segment
- \( F_r \): Factor for shear stress interaction
- \( B_R \): Pressure-singage parameters
- \( \text{DYNSDYN} \): Dynamic pressure-singage parameters

FALSE

FALSE
\[ \tau_{\text{max}} = \text{maximum value of interface shear stress} \]

\[ K = \text{slip parameter} \]

Slip is defined as follows:

\[ s = \frac{\dot{X}_{\text{cg}} - \dot{X}_{\text{tr}}}{\dot{X}_{\text{max}}} \]  \hspace{1cm} [13]

where \( \dot{X}_{\text{max}} = \max (\dot{X}_{\text{cg}}, \dot{X}_{\text{tr}}) \)

Because of the assumed inextensibility of the track in the dynamic track model, the velocity of the individual track segments, \( \dot{X}_{\text{cg}} \), differs but little from the track velocity. Therefore, for simplicity and to minimize computer time, the track velocity is used for the determination of slip.

5.5. Preprocessor Program

The preprocessor program serves the purpose of computing input values for the dynamic track-soil interface subroutine from experimental data or soil properties not directly usable in that program. The preprocessor program addresses the following three tasks:

- Determination of the factors \( F_s \) and \( F_a \)
- Estimation of the triaxial strength \( q_t \)
- Determination of the parameters \( B \) and \( R \) from test data.

For the determination of the factors \( F_s \) and \( F_a \) and the triaxial strength \( q_t \), the track segment geometry and the strength properties of the soil (cohesion, friction angle and unit weight) are needed as input values in the preprocessor program.

5.5.1. Determination of the Factors \( F_s \) and \( F_a \). A rectangle of the size of a track segment is subdivided into 10 subrectangles each way and the bearing stresses at the nodal points (corners of subrectangles) are computed by numerical integration of the differential equations of the plasticity theory for soils. The three-dimensional case of a rectangular plate is approximated by computing bearing stresses for two-dimensional failure in the \( x, x', y, y' \) directions and assuming that the three-dimensional bearing stress is the lowest one of the four computed for a nodal point. To determine the relationship between the \( F_s \) factor and the interface friction angle \( \delta \), the three-dimensional bearing stresses are determined for various ratios of \( \delta \) to its maximum value \( \delta_{\text{max}} \). It was found that a parabolic relationship between these ratios and \( F_s \) approximates the computed values of \( F_s \) reasonably well. The coefficients of the best fitting parabolic relationship are determined in the program for use in the dynamic track-soil interface model.
The factor $F_a$ is determined by computing the three-dimensional bearing stresses for the same ratios of $\delta/\delta_{\text{max}}$ as for the computation of $F_s$ but for a rectangle twice the width of the track segment. It was found that the computed $F_a$ values for the various $\delta/\delta_{\text{max}}$ ratios can be approximated reasonably well by a linear relationship. The coefficients of the best fitting linear relationship are determined in the program for use in the dynamic track-soil interface model.

5.5.2. Estimation of Triaxial Strength $q_t$. The triaxial strength of the soil, if not available, may be estimated from the soil strength parameters $c$ and $\phi$, and the unit weight of soil $\gamma$. The chamber (lateral) pressure is assumed to be equal to the lateral pressure in the soil at a depth equal to half the width of a track segment. The soil is assumed to be loaded at the surface by a uniform pressure exerted by the weight of a track segment. The triaxial pressure, $q_t$, is then computed from the following formula:

$$q_t = (\sigma_3 + c \cotan(\phi)) \cdot \frac{1 + \sin(\phi)}{1 - \sin(\phi)} - c \cotan(\phi) \quad [14]$$

where

- $c$ = cohesion
- $\phi$ = friction angle
- $\sigma_3$ = lateral pressure

5.5.3. Determination of the Parameters $B$ and $R$ from Test Data. A set of test data pairs, consisting of sinkage values in inches and pressure values in psi, is the input for this program. The values of $B$ and $R$ for this data set are determined in the program by calculating the new variables defined in Equation [5] and determining the best fitting values by linear regression.
LIST OF REFERENCES


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U.S. Army Tank-Automotive Command
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