The intent of this research is to study the dynamic behavior of a solid body resting on a moving surface. Results of the study are then used to propose methods for controlling the orientation of parts in preparation for automatic assembly. Two dynamic models are discussed in detail. The first examines the impacts required to cause reorientation of a part. The second investigates the use of oscillatory motion to selectively reorient parts. This study demonstrates that the dynamic behaviors of solid bodies, under the conditions mentioned above, vary considerably with small changes in geometry.
Block 20 cont.
or orientation.
Utilizing Dynamic Stability
To Orient Parts

Neil C. Singer
Warren P. Seering

Abstract: The intent of this research is to study the dynamic behavior of a solid body resting on a moving surface. Results of the study are then used to propose methods for controlling the orientation of parts in preparation for automatic assembly. Two dynamic models are discussed in detail. The first examines the impacts required to cause reorientation of a part. The second investigates the use of oscillatory motion to selectively reorient parts. This study demonstrates that the dynamic behaviors of solid bodies, under the conditions mentioned above, vary considerably with small changes in geometry or orientation.

Acknowledgements: The first author was sponsored by the Office of Naval Research Graduate Fellowship Program. The research described in this paper was performed at the Massachusetts Institute of Technology Artificial Intelligence Laboratory. The laboratory's research is funded in part by the Office of Naval Research under ONR contract N00014-81-K-0494 and in part by the Defense Advanced Research Projects Agency of the United States Department of Defense under ONR contract N00014-85-K-0124.

c 1988 Massachusetts Institute of Technology.

1 Introduction and Problem Definition

Conventionally, parts are oriented by bowl feeders. These machines vibrate and thereby convey parts through a series of filters which reject all but a particular orientation. Rejected parts are returned to storage. Many researchers have examined the implementation of programmable or adjustable filter stages, (Boothroyd, 1975) (Boothroyd and Ho, 1977) (Boothroyd and Murch, 1970) (Boothroyd et al., 1982) (Boothroyd et al. 1977) (Lozano-Pérez, 1986) (Murch, 1977) (Murch and Boothroyd, 1975) (Murch and Poli, 1977) (Redford et al., 1983a) (Redford et al., 1983b) (Singer, 1985) (Suzuki and Kohno, 1981). These techniques are often successful for limited classes of parts. The scope of this paper is to present theoretical results which may be useful in feeder designs. It is hoped that a more detailed understanding of the dynamic behavior of bodies resting on a moving surface will facilitate the design of new types of programmable parts feeders.

2 Impact Reorientation

In this section, we determine the conditions necessary to cause part reorientation. A part starts in a stable orientation (a natural resting aspect (Boothroyd and Ho, 1977)) on a flat surface. It is then given an initial horizontal velocity after which it impacts a wall near the surface upon which it is resting (figure 1). The equations describing the impact are based on conservation of angular momentum about the impact point. After impact, the principle of conservation of energy is applied in order to determine whether reorientation occurs.

The part shown in figure 1 is a cylinder, but the analysis easily generalizes to any object. The important parameters are the mass, \( m \) of the part; the moment of inertia, \( I \); and the constants, \( l \) and \( h \), which locate the center of mass, \( cm \), with respect to the impact point. The expressions for angular momentum, \( L_i \), before and, \( L_f \), after impact are (Klepner and Kolenkow, 1973):

\[
L_i = mV_i h \\
L_f = \frac{I_i V_f}{\sqrt{h^2 - l^2}}
\]  

(2.1)

where \( V_i \) is the speed of the center of mass prior to impact, \( V_f \) is its speed just after impact, and \( I_i \), the inertia about the impact point, is defined as

\[
I_i = I + (h^2 - l^2) m
\]

with \( I \), the inertia about the center of mass. Equating kinetic energy just after impact to the change in potential energy which will cause the part to reorient yields

\[
\frac{1}{2} I_i \left( \frac{V_f^2}{h^2 - l^2} \right) = mg \left[ (h^2 - l^2)^{\frac{1}{2}} - h \right]
\]  

(2.2)
Solving equations 2.1 for \( V_f \), substituting into equation 2.2, and then solving the result for the critical value of \( V_i \) yields

\[
V_{crit} = \frac{\sqrt{2gy \left( \frac{h^2 + l^2}{2} \right)^{\frac{1}{2}}}}{h} \left( \frac{h^2}{l^2} \right)^{\frac{1}{2}} \left( \frac{L}{h} \right)^{\frac{1}{2}}
\]  \hspace{1cm} (2.3)

which is an equation for the minimum initial horizontal velocity that will cause reorientation. For cylinders, \( L/m \) is \( \frac{1}{2}h^2 + \frac{1}{4}l^2 \); for rectangular parts, \( L/m \) is \( \frac{1}{2}(h^2 + l^2) \).

In order to test the ability of impact reorientation to discriminate among different orientations, it is useful to consider a part which has two very similar orientations. Figure 2a shows a rectangular solid part in two possible orientations. One edge is \( bb \) longer than the other where \( bb \) is expressed as a fraction less than one. As the part exchanges orientations, the center of mass location changes in both height and horizontal distance from the impact point by \( \delta \). The ratio of \( V_{crit} \) of orientations one and two (denoted by \( V_1 \) and \( V_2 \)) expanded in a power series in \( \delta \) is expressed as:

\[
\frac{V_2}{V_1} = 1 + 2.2\delta + 1.33\delta^2 \ldots
\]  \hspace{1cm} (2.4)

This equation illustrates that small differences in location of the center of mass result in significant variations in impact reorientation velocity.

To verify the equations and to get a measure of experimental error, tests were performed. Cylinders of different size were placed on a moving conveyor belt. The cylinders impacted a steel wall .889 mm (.035 inch) high. Fifteen trials were made for each velocity set point and the number of pegs which reoriented was counted. The velocity of the belt was monitored by a tachometer which was friction driven directly by the belt.

Several tests were performed. First, the lower threshold velocity, \( V_{fall} \), was determined by finding the maximum velocity at which no pegs fell over in fifteen trials. The upper threshold velocity, \( V_{fall} \), was determined similarly by finding the lowest velocity at which all pegs fell over. The median velocity \( V_{med} \) is simply the center of the velocity band defined by \( V_{fall} \) and \( V_{fall} \). Finally, two percentages have been determined from this data. The percentage of error is calculated between the theoretical velocity, \( V_{Theory} \), and the median experimental velocity, \( V_{med} \). The uncertainty band of the experiment is calculated from \( V_{fall} \) and \( V_{fall} \); this band is represented as a percentage of \( V_{med} \).

Results confirm that the system model adequately represents the experimental situation. All tests performed had narrow uncertainty bands. This indicates that there is a sharp cutoff between not having enough energy to cause reorientation and having enough energy to guarantee reorientation. In addition, the error percentages between \( V_{fall} \) and \( V_{fall} \) are small. Furthermore, the scaling of the velocity agrees with the scaling predicted in equation 2.3. Two pegs which vary greatly in size and length to diameter ratio can have the same reorientation velocity. Table 1 gives typical results for two different cylinders.

The ability to discriminate between orientations, or to reorient one object while not reorienting another, depends heavily on the quality of the velocity source. Fortunately,
excellent velocity sources are inexpensive and easy to build. Figure 2b presents a velocity line with the reorientation velocities for the two parts shown in figure 2a. The dark section of the line identifies the desired operating velocity range. Within this range part 1 will always be reoriented, while part 2 will never be reoriented. By cascading several of these impacts at different velocities, a set of like parts, initially oriented randomly can all be driven to the same orientation.

3 Vibratory Reorientation

3.1 Modeling

The model for this section is an object in a gravity field. The table on which the object rests oscillates vertically (Figure 3). The object is first given some initial angle, \( \phi_0 \), possibly by a sudden motion of the table. For a proper choice of \( X(t) \), the table driving function, the given object can be maintained in a rocking mode of fixed amplitude. The driving function of the table can then be changed, causing an increase in the amplitude at which the object will continue rocking. Finally, the amplitude can be increased until the object reorients.

Because of the discontinuity of motion as the part impacts the table, the solution to the equations governing the system must be broken into regions. First, the region in which there is no impact will be examined. The equations of this motion may be generated using Lagrange's method. The potential energy function can be expressed as

\[
V = mg(r \sin \phi + x),
\]

where \( V \) is the potential energy, \( m \) is the part mass, \( r \) is the distance from the contact point to the center of mass, \( \phi \) is the angle between the horizontal and the vector \( r \), and \( x \) is the vertical position of the table. The kinetic coenergy can be expressed as:

\[
T' = \frac{1}{2} I_c \left( \frac{d\phi}{dt} \right)^2 + \frac{1}{2} m \left( \frac{dx}{dt} - r \frac{d\phi}{dt} \cos \phi \right)^2 + \frac{1}{2} m \left[ r \frac{d\phi}{dt} \sin \phi \right]^2,
\]

where \( I_c \) is the moment of inertia of the part about the center of mass. These two terms can be used to form the Lagrangian which is then substituted into Lagrange’s equation, yielding the two equations of motion for the system,

\[
\left( I_c + mr^2 \right) \frac{d^2\phi}{dt^2} - m r \cos \phi \left( g + \frac{d^2x}{dt^2} \right) = 0
\]

\[
m \frac{d^2x}{dt^2} - mr^2 \frac{d^2\phi}{dt^2} \cos \phi - mr \frac{d\phi}{dt} \sin \phi + mg = F,
\]

The first of the equations gives the basic motion of the object, while the second equation is an expression for \( F \), the contact force. These equations are valid in the conservative regions without impact.
The next step in formulating the model is to represent the impact of a part with the table. At impact, the center of mass of the object has a vertical velocity which is reflected with a coefficient of restitution, \( \epsilon \). In a perfectly elastic impact, \( \epsilon \) is equal to one. Physical experiments were performed to determine the coefficient of restitution for aluminum parts rocking on an aluminum plate. The coefficient of restitution was determined to be between .7 and .75 for these tests. For all of the simulations in this paper, an \( \epsilon \) of .5 was used to be conservative.

As a part hits the table, the impact will also affect the horizontal velocity. If the table is frictionless, the part will slide. In this derivation the table was assumed to be rough enough so that the contact surface will not slide on the table and after impact the part will continue to rock by rotating about the new contact point.

The next step in creating steady rocking motion was to determine table acceleration profiles which would cause a part to oscillate at fixed amplitude. The amplitude of vibration is coupled with the frequency of oscillation. As the part gains energy, both the amplitude and the period of oscillation become greater. This response makes analysis difficult; few tools are available for predicting the behavior of this type of system (Meirovitch, 1975).

An analytical solution to the system equations for the time required for passage from an initial to a final angle can be derived for the case in which the driving function, \( \frac{d^2 \xi}{dt^2} \), remains constant. The derivation of this solution is based on conservation of energy in the region without impact. For a constant table acceleration, the part can be considered to be in a conservative field as shown in equation 3.3a. In this particular case the energy balance equation becomes

\[
\frac{1}{2} \left( I_c + mr^2 \right) \frac{d\phi}{dt}^2 + m\left( g + \frac{d^2 \xi}{dt^2} \right) r \sin \phi = \frac{1}{2} \left( I_c + mr^2 \right) \frac{d\phi}{dt}^2 + m\left( g + \frac{d^2 \xi}{dt^2} \right) r \sin \phi, \quad (3.1)
\]

where \( \phi_0 \) is the starting angle of the object, and \( \phi \) is some angle of interest. Note that the derivative of this expression is the result obtained in equation 3.3a. Equation 3.4 can be solved for \( \frac{d\phi}{dt} \):

\[
\frac{d\phi}{dt} = \sqrt{\frac{2m\left( g + \frac{d^2 \xi}{dt^2} \right) r}{I_c + mr^2}} \sqrt{\frac{(I_c + mr^2) \frac{d\phi}{dt}^2}{2m\left( g + \frac{d^2 \xi}{dt^2} \right) r} + \sin \phi_0 - \sin \phi}, \quad (3.5)
\]

This equation can in turn be solved by separation of variables and then once integrating with respect to time. For convenience the constant \( A \) is defined as

\[
A = \frac{(I_c + mr^2) \frac{d\phi}{dt}^2}{2m\left( g + \frac{d^2 \xi}{dt^2} \right) r} + \sin \phi_0, \quad (3.6)
\]

and the result is

\[
\int_{\phi_0}^{\phi} \frac{d\phi}{\sqrt{A - \sin \phi}} = \int_0^T \sqrt{\frac{2m\left( g + \frac{d^2 \xi}{dt^2} \right) r}{I_c + mr^2}} dt. \quad (3.7)
\]
where \( \phi \) is the ending angle of interest. The right side is a simple integral; the left side is an elliptic integral. The solution to this equation will be presented as a combination of elliptic functions. Though elliptic functions can not be expressed in terms of elementary functions, they are well understood and are easily computed. In order to show that equation 3.7 becomes an elliptic integral, a change of variables must be made. Let

\[
w = \phi - \frac{\pi}{2} ,
\]

so that

\[
\sin \phi = \cos w .
\]

and

\[
\cos \phi \, d\phi = \sin w \, dw .
\]

From these we get

\[
d\phi = \frac{\sin w \, dw}{\sqrt{1 - \cos^2 w}} + \, dw .
\]

Substituting equations 3.11 and 3.8 into the left hand side of equation 3.7 produces the following expression

\[
\int_{\phi_1}^{\phi_2} \frac{d\phi}{\sqrt{A \sin \phi}} = \int_{\phi_1 - \frac{\pi}{2}}^{\phi_2 - \frac{\pi}{2}} \frac{dw}{\sqrt{A + \cos w}} .
\]

In the case of impact

\[
\phi = \psi ,
\]

where \( \psi \) is defined as the angle between the bottom of the part and the vector \( r \) from the contact point to the center of mass. If the integral is then rewritten as two integrals starting from zero, equation 3.12 can be expressed in terms of the elliptic integral of the first kind (Gradshteyn and Ryzhik, 1980):

\[
\int_{0}^{\psi} \frac{dw}{\sqrt{A \cos w}} = \int_{0}^{\psi - \frac{\pi}{2}} \frac{dw}{\sqrt{A + \cos w}} = \begin{cases} \sqrt{2} \left( F(x_1, k_1) - F(x_2, k_1) \right) & A < 1 \\ \frac{2}{1 - A} \left( F(x_3, k_2) - F(x_4, k_2) \right) & A > 1 \end{cases} .
\]

The elliptic integral of the first kind expressed in normal trigonometric form is

\[
F(x, k) = \int_{\phi = 0}^{\phi = \sin \alpha} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} .
\]
and the limits are

\[
\begin{align*}
  x_1 &= \sqrt{\frac{1 - \sin \phi_1}{1 - A}} \\
  x_2 &= \sqrt{\frac{1 - \sin \phi_2}{1 - A}} \\
  k_1 &= \sqrt{\frac{1 - A}{2}} \\
  x_3 &= \frac{\phi_1}{2} - \frac{\pi}{4} \\
  x_4 &= \frac{\phi_2}{2} - \frac{\pi}{4} \\
  k_2 &= \sqrt{\frac{2}{1 - A}}
\end{align*}
\] (3.15)

in terms of the system parameters.

### 3.2 Choice of Driving Function

As mentioned previously, this analysis is correct only for constant values of \( \frac{d^2 \phi}{dt^2} \). The first driving function to be considered was a periodic rectangular wave in acceleration (figure 4). Four parameters must be selected to fully specify the waveform – the acceleration upward, \( AX_1 \); the acceleration downward, \( AX_2 \); and the switch times, \( T_1 \) and \( T_2 \). The selection of these parameters is constrained by several relations. First, the areas \( A_1 \) and \( A_2 \) must be equal in order for the table to return to its initial position and velocity after each cycle. Second, the part must start and finish the cycle at the same angle (the stability requirement). Lastly, the acceleration downward must never exceed gravity. This constraint is set so that the part does not lose contact with the table.

Equations 3.13 and 3.15 were substituted into a nonlinear solution program which calculated accelerations and switch times that brought a chosen part back to its initial angle in exactly one cycle. Figure 5 shows a time history of the acceleration of the table, \( \frac{d^2 \phi}{dt^2} \), and of the angle \( \phi \) as determined by numerical simulation of equation 3.3 including the model for impact. For this case, the rocking motion is stable. However, small parameter variations cause an instability. Figure 6 shows the rocking motion for an object given a slightly larger initial amplitude than that of figure 5. After the first impact, the peak amplitude is too low; after the second, it is too high. Eventually, the phasing between input and output is lost and the part ceases to rock.

Because our goal is to bring parts to a stable rocking motion from a range of initial conditions, the rectangular acceleration waveform will not do the job. A variety of waveforms were tested. One which exhibited good performance is shown in figure 7. The waveform is simply the rectangular waveform with the edges sloped. Figure 8 shows results from a numerical simulation using this forcing function as input. A part is started with an initial angle which is twice the steady state angle yet it still reaches a stable equilibrium motion. This suggests that by using the proper waveform, a great range of initial conditions of the part can be tolerated.
If this method is to be used to selectively reorient parts, different parts should exhibit significantly different responses. Figure 9 shows variation in response for two similar parts given the same initial angle and subjected to the same acceleration profile. For these simulations, upward acceleration was set to occur around the time of impact. Several simulations were executed with various length “windows” of upward acceleration. The ability to discriminate degraded as the window became very large in comparison with the difference in impact times.

An attempt was made to design an experimental apparatus for study of the behavior described above. No satisfactory configuration could be created at a reasonable cost. To drive reasonably large parts with periods of oscillation greater than .5 seconds would require a table with a range of motion substantially greater than that of available electromagnetic shakers. Hydraulic shakers are designed to deliver large forces and have servo valves which are too small to supply the flow rates necessary to achieve the required velocities. The ideal driving system would be a small diameter hydraulic actuator with a relatively large servo valve.

4 Conclusion

This study of part motion has demonstrated that small variations in inertial properties from one part to another cause significant differences in the dynamic behavior of the two parts. Two example techniques which capitalize on this property were presented. Practical implementation of the first technique, impact reorientation, is feasible given current design technology. The second technique, vibratory reorientation, will require construction of oscillating tables with performance characteristics available only through custom design.

5 References


Fig. 1. Impact reorientation model.

Fig. 2a. Two similar parts.

Fig. 2b. Distinguishing the parts in Fig. 2a.
Fig. 3. Vibratory reorientation model

Fig. 4. Square wave acceleration input.
Fig. 5a. Response of a part to input in fig 5b.

Fig. 5b. Square wave input.
Fig. 6. Unstable response for a part starting 6% above the desired level.

Fig. 7. Ramped square wave input.
Fig. 8a. Stable response to input in fig. 8b.

Fig. 8b. Ramped square wave input.
Fig. 9. Ability To Separate Orientations
END
DATE
FILMED
8-88
DTIC