A crack in the interior of a solid body can be characterized through its measurable effect on an externally applied ultrasonic field. This final report describes analytical and numerical solutions to the problem of scattering of ultrasonic (elastic) waves by cracks. Such solutions help in the design of ultrasonic tests for crack detection and characterization, and they provide invaluable aid in the interpretation of scattering data for the inverse problem. The configurations that have been considered included near surface cracks and surface breaking cracks, as well as configurations of macro and micro cracks. Solutions for the scattered field have generally been obtained by the boundary...
DIRECT AND INVERSE DIFFRACTION PROBLEMS
FOR QNDE APPLICATIONS

FINAL REPORT

by

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1. Introduction and Statement of Research Objectives

Non-destructive evaluation (NDE) procedures are needed for materials processing, as well as for post-process materials testing. They play important roles in product design, analysis of service-life expectancy, manufacturing, and quality control of manufactured products. They are also essential to on-line monitoring of the integrity of structural elements and complex systems. Rational accept and reject criteria should be based on NDE tests. Critical safety, efficiency and operational features of large-scale structures depend on adequate NDE capabilities.

For materials processing, sensors and related measurement techniques must be developed to monitor process variables which control the microstructure and the physical properties. The information obtained in this manner must be combined with predictive process modeling to produce desirable microstructures and physical properties of the material. "Intelligent" feedback control systems are needed, based on signal processing, artificial intelligence, expert systems and control theory.

For post-process material evaluation, and for testing of components and structural systems, procedures to decide on the suitability for further service must combine knowledge of the material state with considerations of the failure modes and the service conditions, to make a performance prediction. The role of quantitative nondestructive measurements is to assess the material state. Primary interest lies in identifying and sizing microscopic or macroscopic flaws which would ultimately lead to failure. Other measurements of direct interest relate to residual stresses and material properties such as fracture toughness, preferably by direct methods.

Most methods of non-destructive evaluation provide only limited information. It is, however, frequently not good enough just to detect a flaw or the presence of inferior material properties. Quantitative information is required. This need has given rise to a more rigorous and fundamental approach to non-destructive evaluation which is called quantitative NDE (QNDE). This project has been concerned with research in the area of QNDE.
Nondestructive evaluation (testing) methods include radiography, eddy current methods, dye penetrants, ultrasonic methods, optical methods, thermal wave imaging, x-ray and neutron scattering methods and methods based on nuclear magnetic resonance. Each method has its advantages and disadvantages for particular applications.

The great advantage of ultrasonic techniques is that they are relatively simple. Mechanical waves are used to penetrate a material and so mechanical properties and defects, that are most closely related to useful life and eventual failure, are measured directly. A considerable fraction of the research efforts in QNDE is, therefore, concerned with ultrasonic measurement techniques.

One of the most useful methods of quantitative non-destructive evaluation (QNDE) is based on the scattering of elastic (ultrasonic) waves by flaws. Two general approaches to ultrasonic flaw detection and characterization have been taken. The imaging approach seeks to process the scattered field in such a manner that a visual outline of the object is produced on a display. The scattered-field approach attempts to infer geometrical characteristics of a flaw from either the angular dependence of its far-field scattering amplitude at fixed frequency, or from the frequency dependence of its far-field scattering amplitude at fixed angles.

In experimental work on quantitative flaw definition by the ultrasonic pulse method, either the pulse-echo method with one transducer or the pitch-catch method with two transducers is used. The transducer(s) may be either in direct contact with the specimen, or transducer(s) and specimen may be immersed in a water bath. Most experimental setups include instrumentation to gate out and spectrum analyze the signal diffracted by a flaw. The raw scattering data generally need to be corrected for transducer transfer functions and other characteristics of the system, which have been obtained on the basis of appropriate calibrations. After processing, amplitudes and phase functions, are available as functions of the frequency and the scattering angle. These experimental data can then be directly compared with theoretical results. Inversely, the experimental data can be interpreted with the aid of analytical methods, to characterize the scatterer.

The solution to the direct diffraction problem, that is, the computation of the field generated when an ultrasonic wave is diffracted by a known flaw, is a necessary preliminary to the solution of the inverse problem, which is the problem of inferring the geometrical characteristics of an unknown flaw from either the angular dependence of the amplitude of the diffracted far-field at fixed frequency, or from the frequency dependence of the far-field amplitude at fixed angle. In recent years several analytical methods have been developed to investigate scattering of elastic waves by interior cracks as well as by surface-breaking cracks, in both the high- and the low-frequency domains. The appeal of the high-frequency approach is that the probing wavelength is of the same order of magnitude as the length-dimensions of the crack. This gives rise to interference phenomena which can easily be detected. The advantage of the low frequency approach is that useful approximations can be based on static results.
The work that has been completed on this Project, has been concerned with analytical work for the scattering approach to crack-like flaws. Both direct and inverse scattering problems have been considered.

2. Summary of Results

Papers completed in the course of research under Grant DAAG29-84-K-0163 have been published in Technical Journals and in Proceedings of Conferences and Symposia. A list of titles is given in Section 5. In addition, oral presentations have been made at Technical Meetings, Workshops, Symposia and at the Tenth U.S. National Congress of Applied Mechanics. In this Section we present a brief summary. Further details can be found in the semi-annual reports.

In the completed work, analytical methods have been developed to investigate scattering of elastic waves by interior cracks and volume flaws, as well as by surface-breaking cracks. Scattering of time-harmonic signals by flaws has been analyzed on the basis of linearized elasticity theory for homogeneous, isotropic solids.

From the theoretical point of view a crack is a planar surface across which the displacement can be discontinuous. The exact mathematical formulation of the elastodynamic field generated by the presence of a crack is rather complicated. Scattering of an incident wave by a crack is a mixed boundary value problem, whose exact solution satisfies one or more generally singular integral equations for the displacement discontinuities. The solution of the system of integral equations for a crack of finite dimensions requires a substantial amount of numerical analysis, which has been carried out for a number of configurations.

In an early stage of the work, a ray theory was developed to describe diffraction by crack edges. This work has been discussed in detail in an invited review paper [1].

Surface-breaking and near-surface cracks are generally the most detrimental to the integrity of structural elements. Hence a great deal of attention has been given to their detection and characterization. In recent years numerous results have become available for fields generated by scattering of ultrasonic waves by cracks.

In research on this project we have improved the modeling efforts of earlier work by including a number of aspects typical of practical configurations. Thus, we have taken into account that the transmitting and receiving transducers are often coupled to the specimen by placement in a water bath. The transmission of the signal from the water to the solid has been investigated. It was also taken into account that only part of the crack may be illuminated by the incident signal. The relation between the scattered field and the electrical signal recorded by the receiving transducer was made part of the overall modeling effort by relating this signal to the crack-opening displacement which is generated by the incident

*Numbers in brackets refer to papers listed in Section 5.
wave. The crack-opening-displacement has been computed by the use of frequency-domain methods. The process of modeling from the initial electrical pulse to the measured electrical pulse was completed by converting the frequency domain results to the time-domain by the use of the fast Fourier transform method. The results have been presented in paper [2].

The equations governing three-dimensional elastodynamic scattering from (internal) cracks in a full-space, and (surface-breaking) cracks that emanate from the traction-free surface of a half-space, have been formulated and solved in the frequency domain by boundary integral equation (BIE) methods. It has been shown that the system of Cauchy-singular integral equations for scattering by voids degenerates in the limit that the void becomes a crack. An alternate approach has been developed, which yields a (discretized) system of hyper-singular integral equations whose regularization is novel in that it employs an initial discretization of the boundary surfaces.

The full space Green's functions have been employed for the solution of both full-space and half-space problems. Their use for the solution of the half-space problem requires the surface of the half-space to be one of the surfaces of the system of boundary integral equations. The geometrical decay of Rayleigh-waves propagating along this traction-free surface allows, however, for a discretization of the surface over a finite region. Convergence of the crack-opening displacements with successively larger regions of discretization of the free-surface has been demonstrated. Papers [5], [7]-[10] report results obtained by the boundary element method.

Real cracks, particularly fatigue and stress corrosion cracks have rough faces, which may contact each other. Sometimes there is not a single crack, but rather a configuration of a principal crack and an adjoining satellite crack, for example, a macrocrack and a neighboring microcrack. The question which has motivated another effort on this project is to what extent these complicating features of real cracks or systems of cracks can be accounted for in an analysis. Complicated crack configurations have been modeled in papers [3], [5] and [7].

In paper [6] elastic wave scattering data has been utilized to characterize a crack of arbitrary shape by an equivalent elliptical crack contained in an unbounded solid. The inverse method is based on an integral representation for the scattered field in the frequency domain. The proposed method is valid for both small and intermediate wavelengths as compared with the size of the crack. In particular for intermediate wavelengths and normal incidence a simple method is discussed. Its solution gives the location of the geometrical center of the crack, the crack size, and the crack-opening displacement.

Paper [4] is a review paper which summarizes recent results.

3. Participating Scientific Personnel

Work on the subject Grant was carried out under the direction of
Dr. J. D. Achenbach, Walter P. Murphy Professor of Civil Engineering, Mechanical Engineering and Applied Mathematics. Also contributing to the research efforts for periods of time have been: Dr. Ch. Zhang, Dr. L. M. Keer, Dr. D. A. Sotiropoulos, Dr. Z. L. Li, Dr. W. Lin and Dr. D. E. Budreck.

4. Advanced Degrees

Dr. D. E. Budreck was employed (part-time) as a Research Assistant on the subject Grant. His work resulted in a Ph.D. Dissertation entitled "Three-Dimensional Elastodynamic Scattering from Internal and Surface-Breaking Cracks by Boundary Integral Equation Methods". Dr. Budreck is now employed by Iowa State University. His degree work was completed in January 1988.

Dr. W. Lin was also employed (part-time) as a Research Assistant on the subject Grant. His work resulted in a Ph.D. Dissertation entitled "Scattering of Ultrasonic Waves by a Subsurface Crack and Dynamic Stress Intensity Factors".

5. Titles of Completed Papers

The following papers have been published in Technical Journals and Proceedings of Conferences:

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