SIM WORKSHOP ON MATHEMATICS OF SYSTEMS AND SIGNAL PROCESSING HELD IN ST. LOUIS, SOCIETY FOR INDUSTRIAL AND APPLIED MATHEMATICS, PHILADELPHIA, P. S. BOYD ET AL., UNCLASSIFIED
The workshop brought together researchers on a variety of topics in signal processing, control and system theory, that share a common mathematical background. The emphasis of the workshop was on trying to uncover connections between the different fields, rather than on elaborating the fields themselves.

About fifty leading engineers and mathematicians from various countries attended the workshop along with several local participants from U.C. Berkeley and from Stanford. There were twenty major presentations, prepared in consultation with the organizers to ensure continuity and connection to the overall topics of the workshop; there were no parallel sessions. Time was also allotted for over a dozen short presentations by participants who wished to offer extended comments, to share recent results, or present open problems.
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The organizers of the workshop were Thomas Kailath and Stephen Boyd, both of the Electrical Engineering Department at Stanford University. The workshop was supported by grants to SIAM from the Air Force Office of Scientific Research, the Army Research Office, and the Office of Naval Research.

The first day of the conference was devoted to results and applications in inverse scattering theory, as encountered in a wide variety of areas in mathematics, physics, and engineering. Following a brief overview by Kailath, A. Bruckstein of the Technion (Haifa, Israel) gave an enlightening presentation of two generic procedures, called layer-peeling and layer-adjoining, for solving one-dimensional scattering problems. Bruckstein showed how this point of view could unify algorithms developed in several different signal processing problems: for example the Levinson and Schur algorithms for linear prediction; the Berlekamp-Massey and the generalized Lanczos algorithm for minimal partial realization. Padé approximation, and decoding of Reed-Solomon and BCH error-correcting codes; and certain algorithms for the design of cascade digital filters. More details can be found in review papers in the SIAM Review (Sept. 1987) and in the IEEE ASSP magazine (Apr. 1987).

A. Frazho of Purdue University then described the application of the so-called commutant lifting theorems of operator theory to an even wider range of inverse scattering problems. He showed that the lifting theorem is an important tool not only applicable to the Carathéodory and Nevanlinna-Pick interpolation and displacement structure problems underlying the results in Bruckstein's talk, but also to Nehari or Hankel interpolation, and to more
general $H_\infty$ optimization problems. Frazho's talk, based on a book he is co-authoring with C. Foias, thus provided a nice counterpoint to Bruckstein and Kailath's physically oriented development.

A. Yagle of Ann Arbor gave a lucid and well organized overview of recent results in the difficult area of multidimensional inverse scattering problems. In particular, he showed how stochastic interpretations, as now well-known in the one-dimensional case, could also illuminate topics such as Newton's "miracle equation" for three-dimensional problems and also suggest feasible numerical methods for these computationally challenging problems.

The final talk by H. Dym of the Weizmann Institute, Israel, introduced certain Hilbert spaces of analytic functions equipped with a reproducing kernel (deBrange spaces) as a powerful tool for the solution of general lossless inverse scattering problem that, among other things, encompass several of the previous results as special cases. This was joint work with P. Dewilde (Delft) and D. Alpay (Weizmann) that, along with considerable other material, will appear in a CBMS monograph under preparation by Dym.

The topics of the first day were extended in various ways by the operator-theoretic presentations on Tuesday by four leading researchers in this area: W. Helton (San Diego), N. Young (Glasgow), I. Gohberg (Tel Aviv) and J. Ball (Blacksburg). After an overview of the issues to be covered during the day, Helton surveyed the many relations of operator to theory control and especially circuit theory, themes he has elaborated in his recent AMS monograph. [Related papers can be found in another recent AMS monograph, "Moments in Mathematics", H. Landau, ed.] Young presented a summary of methods for computing the solution to Nevanlinna-Pick problems, with special emphasis on the case of data given as polynomial matrices.

Gohberg gave a wide survey of 'extension problems', showing how the Carathéodory, Nehari and various maximum entropy extension problems, among others, could be formulated and solved as problems of determining whether a matrix with some unspecified entries can be made positive definite by appropriate choice of the unspecified entries. In particular, he presented some interesting state-space formulas, recently obtained with colleagues in Amsterdam, that appear to have close links to formulas arising in Bruckstein's lecture of the first day. [It was decided that further explorations of these connections would be carried out during a special program on signal processing to be held at the Institute of Mathematics and its Applications, Minneapolis, in the summer of 1988.] The final talk presented several connections between model reduction and operator theory. J. Ball demonstrated that a study of quite abstract operator theory can in fact yield perfectly explicit state-space solutions to many practical model reduction, interpolation, and $H_\infty$ problems. In the course of discussions, Ball and Helton were able to identify various links with the reproducing kernel Hilbert space structures introduced in Dym's talk of the first day.

A variety of links with control theory was discussed on the third day of the conference. The day started with an introduction, overview, and survey by S. Boyd of Stanford. The other speakers were K. Glover (Cambridge), R. Francis (Toronto), and E. Jonckheere (USC).
Boyd gave an elementary derivation and explanation of how interpolation problems arise in linear control theory, and why they are important. The central result is that for a given plant the set of closed-loop maps achievable with controllers that stabilize the plant is an affine set that is readily described. The modern form of this so-called parametrization of stabilizing controllers is due to Youla and Boniorno, and Desoer, Saeks, Murray, and Liu; simpler versions (SISO plant) have been used in control design as early as 1956 and even earlier in circuit theory, by Youla and Helton, among others. With this parametrization, the minimum sensitivity controller design problem, pioneered by Zames, Francis, Helton, Doyle, Safonov, Tannenbaum, Pearson and Chang, and many other researchers, becomes an $H_\infty$-minimum norm interpolation problem.

M. Vidyasagar (Waterloo) was ill and unable to attend the workshop, so with a one weekend's notice Bruce Francis (Toronto) gallantly stepped in to give an introductory tutorial on $H_\infty$ control. As usual, Francis' talk was well organized and very clear -- as is his recent monograph on the subject (published by Springer-Verlag).

Keith Glover (Cambridge) gave a survey of his extensive work on state space formulas for Hankel norm and $H_\infty$-approximation problems. Finally, E. Jonckheere (USC) described several interesting relations between LQG ($H_2$) optimization and $H_\infty$-optimization, which was followed by an energetic discussion of many of the results in this and the earlier papers.

It was clear that this research area has many deep connections to the topics of the workshop. For example, one of the most basic results in $H_\infty$-optimal control is that the optimal residual transfer function is a multiple of an all-pass (inner) function. Interpreted as a scattering operator, an all-pass transfer function is nothing more than a lossless operator. Thus it is not surprising that we find many connections to circuit theory, nor that solutions of many of the problems are elegantly expressed in state-space formulas. The same circuit theory ideas underlie many of the inverse scattering results of the first day, and there is need for further work on developing the connections, with a view to efficient computational algorithms, between the control and signal processing applications.

The computational aspects have evident relations to linear algebra and matrix theory. several aspects of which were the focus of the presentations on Thursday and Friday. P. Fuhrmann (Bersheva University, Israel) showed how abstract state-space models and intertwining operator relationships could provide one way of organizing numerous results on stability tests and root-distribution problems for polynomials and polynomial matrices. H. Levari of Stanford used results on fast triangularization of matrices with displacement structure to provide another unified way of approaching root-distribution problems with respect to various regions, not only the unit circle and the left-half-plane. While these results are still in process of publication, some of the background theory can be found in the recent book, "Schur methods in Signal Processing" (I. C. Gohberg, ed., Birkhäuser-Verlag, 1986). Various supplementary results on inertia theory and Hankel and Toeplitz operators were provided in talks by B. Datta (N. Illinois University), G. Heinig (GDR) and V. Ptak (Czechoslovakia). The final major
presentations were on Friday with talks by J. Bunch (UC San Diego) on
different stability concepts for numerical algorithms and a survey by P.
Lancaster (Calgary) of a series of papers on fast algorithms and error bounds
for structured matrices arising in one-dimensional scattering problems, thus
once again making a link to the first talk of the workshop.

The rest of the time on Thursday and Friday was devoted to the previously
mentioned short presentations by various participants: J. Geronimo (Georgia
Tech) on iterations of tridiagonal matrices, L. Rodman (Arizona) on Riccati
equations, T. Georgiou (Ames, Iowa) on a Schur algorithm for spectral
factorization; H. Kimura (Osaka, Japan) on directional interpolation in
state-space, immittance domain algorithms for quasi-Toeplitz matrices by Y.
Bistritz (Bell Labs), a matrix proof of an AAK theorem by J. Meinguet
(Louvain-la-Neuve, Belgium), more on extension problems and cascade circuit
synthesis by S. Rao (Bell Labs), array methods for triangular and QR factor-
ization by Kailath (Stanford), fast evaluation of polynomials and functions by
N. Bose (Penn State), sensitivity reduction over a frequency band by Y. Ohta
/MIT, and group theory and linear estimation by M. Hazewinkel (Rotterdam).

Other participants in the workshop were A. Chan (UCLA), J. Chandra (ARO), J.
Corones (Ames, Iowa), J. Doyle (Honeywell and Cal Tech ), A. Friedman
(Minneapolis), G. Golub (Stanford), P. Khargonekar (Minneapolis), H. Landau
(Bell Labs), L. Lome (Washington), M. Mörf (Zürich), S. Parker (Monterey), E.
Parzen (Texas A&M), P. Sherman (Purdue), M. Verma (Austin) as well as several
faculty and students from UC Berkeley and Stanford.

The meeting was concluded with a few overview remarks by Tom Kailath and
Israel Gohberg.

In summary, the workshop explored several related problems in a number of
fields, especially inverse scattering stochastic estimation, root distribution
of polynomials, robust control design, and model reduction. The underlying
mathematics turns out to be a blend of complex analysis, matrix theory and
operator, especially related to interpolation problems. Different aspects of
this body of mathematics have been pursued by different investigators, often
blended with insights and tools of the particular application. The conference
provided a forum for the interlacing of application areas such inverse
scattering and control theory with theoretical mathematics from operator
theory, matrix theory and numerical analysis. One of the main achievements of
this workshop is the common exposure it gave to a group of problems will
stimulate a greater appreciation of the potential links between the different
areas. Hopefully, this will lead to new collaborations and useful results, as
in fact some post-workshop feedback indicates is already taking place.

Bridging the communication gap between mathematicians and engineers has been a
focus of several major conferences in the recent past. The 1984 AMS Summer
Research Conference at Maine and the SIAM Conference on Linear Algebra in
Signals, Systems and Control assembled mainly linear and numerical linear
algebraists with systems and control theorists; on the other hand, the
Stanford workshop brought together not only linear algebraists, but also
functional analysts and operator theorists as well. Reaction from some of the
participants may be of interest. Hazewinkel remarked, "All in all (linear)
network and signal processing theory has by now involved into a nice example of an area where very practical and powerful modern mathematics (both theoretical and computational) lives side by side in harmony. This conference amply testified to that fact." Louis Lome noted that ....... "In fact, the impetus for this sort of workshop came from the synthesis of theoretical work in operator theory and the much more practical algorithmic work deriving from very current design problems in modern solid state engineering. Indeed, the utility of encouraging this sort of alliance of theory and practice lies in the hope of identifying and exploiting the structure inherent in engineering problems for very high performance devices and systems. That it also enriches mathematics by drawing out new insights and fresh perspectives is an added advantage for which the mathematics community may be grateful." Israel Gohberg stated in his concluding remarks that, "When Schur, Nehari, Nevanlinna, Pick and later Adamjan, Avov, Krein and others were studying the interpolation problems that now bear their names and were devising algorithms to solve them, they did not know that these problems and algorithms (and all kinds of extensions) would one day be of prime importance in signal processing and systems theory. They just wanted to do beautiful mathematics." As Hazewinkel noted then, "in this they were guided by the inner dynamics and aesthetics of the fields in question, which are, as has been often observed, as reliable a guide to future usefulness and applicability as any. It also works the other way. The applications areas not only suggest new problems but also good new ways of thinking about old problems in terms of semi-abstract models that are then powerful guides for further thought both for other applications and for advances in theory." Examples from this workshop include the discrete-transmission line models for inverse scattering, the state-space representation of many operators, and the displacement structure and array methods for Toeplitz, Hankel and related computations.

Finally, we again quote Lome's comment that, "this workshop reveals an interesting, and perhaps for some a surprising, circumstance: namely that there are still plenty of fresh and creative ideas yet to be explored in linear operator theories and some very old engineering problems. This despite the recent explosion of interest in chaos and other nonlinear phenomena. One looks forward not only to the monographs that one hopes may come out of this SIAM workshop, but from future workshops in the same area as well."

Prepared by S. Boyd and T. Kailath
(with contributions from B. Datta and L. Lome)
March 1, 1988
SIAM

WORKSHOP ON THE MATHEMATICS OF SYSTEMS AND SIGNAL PROCESSING

AUGUST 31 - SEPTEMBER 4, 1987
SIAM WORKSHOP ON
THE MATHEMATICS OF
SYSTEMS AND SIGNAL PROCESSING

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SEPTEMBER 4, 1987

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